ISA 562 HW 4

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Q1:

a) $Pr[\tilde{T} \mid Trump]$ = probability of trueness + probability of trueness in noise.

$$= \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$$

b) $Pr[\tilde{T}] = Pr(\tilde{T} \mid Trump) * Pr[Trump] + Pr[\tilde{T} \mid \overline{Trump}] \cdot Pr[\overline{Trump}]$

$$= 0.75 * Pr[Trump] + (1 - 0.75) \cdot (1 - Pr[Trump])$$

$$= 0.75 \cdot Pr[Trump] + 0.25 - 0.25 \cdot Pr[Trump]$$

$$= 0.5\, Pr[Trump] + 0.25$$

c)

$$Pr[Trump \mid \tilde{T}] = \frac{Pr[\tilde{T} \mid Trump] \cdot Pr[Trump]}{Pr[\tilde{T}]}$$

When $Pr[Trump] = 1$,

$$Pr[\tilde{T}] = 0.25 + 0.5 \times 1 = 0.75$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.75 \times 1}{0.75} = \boxed{1}$$

When $Pr[Trump] = 0.62$,

$$Pr[\tilde{T}] = 0.25 + 0.5 \times 0.62 = 0.56$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.75 \times 0.62}{0.56} = \boxed{0.83}$$

When $Pr[Trump] = 0.5$,

$$Pr[\tilde{T}] = 0.25 + 0.5 \times 0.5 = 0.5$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.75 \times 0.5}{0.5} = \boxed{0.75}$$

When $Pr[Trump] = 0.25$,

$$Pr[\tilde{T}] = 0.25 + 0.5 \times 0.25 = 0.375$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.75 \times 0.25}{0.375} = \boxed{0.5}$$

d)

Here, $$Pr\left[\tilde{F} \mid Trump\right] = \frac{1}{4} + \frac{3}{4} \times \frac{1}{2} = \frac{5}{8} = 0.625$$

Then, $$\begin{aligned} Pr\left[\tilde{F}\right] &= Pr\left[\tilde{F} \mid Trump\right] \times Pr\left[Trump\right] + Pr\left[\tilde{F} \mid \overline{Trump}\right] \times Pr\left[\overline{Trump}\right] \\ &= 0.625 \times Pr\left[Trump\right] + (1 - 0.625) \times \left(1 - Pr\left[Trump\right]\right) \\ &= 0.625 \cdot Pr\left[Trump\right] + 0.375 - 0.375 \cdot Pr\left[Trump\right] \\ &= 0.25 \cdot Pr\left[Trump\right] + 0.375 \end{aligned}$$

When $Pr\left[Trump\right] = 1$,

$$Pr\left[\tilde{F}\right] = 0.25 \times 1 + 0.375 = 0.625$$

$$Pr\left[Trump \mid \tilde{F}\right] = \frac{0.625 \times 1}{0.625} = \boxed{1}$$

When $Pr\left[Trump\right] = 0.62$,

$$Pr\left[\tilde{F}\right] = 0.25 \times 0.62 + 0.375 = 0.53$$

$$Pr\left[Trump \mid \tilde{F}\right] = \frac{0.625 \times 0.62}{0.53} = \boxed{0.731}$$

When $Pr[Trump] = 0.5$,

$$Pr[\tilde{T}] = 0.375 + 0.25 \times 0.5 = 0.5$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.625 \times 0.5}{0.5} = \boxed{0.625}$$

When $Pr[Trump] = 0.25$,

$$Pr[\tilde{T}] = 0.375 + 0.25 \times 0.25 = 0.4375$$

$$Pr[Trump \mid \tilde{T}] = \frac{0.625 \times 0.25}{0.4375} = \boxed{0.357}$$

In this case the user privacy increases. The values of $Pr[Trump \mid \tilde{T}]$ get closer to the corresponding values of $Pr[Trump]$.