i) $2x - 3y + 7z = 5$

$3x + y - 3z = 13$

$2x + 19y - 47z = 32$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{3}{2} R_1$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 11/2 & -27/2 \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 11/2 \\ 5 \end{bmatrix}$$

Rank of $A = 2$ Rank of $A:B = 3$

$\rho(A) \neq \rho(A:B)$ Inconsistency

ii) $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

Sameer

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix} \quad R_3 \to R_3 -$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & \frac{-38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

Consistency

iii, $4x - y = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

Sameer

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

Consistency

b) For what values of λ and μ

$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$

i) No solution, ii) A unique solution

iii) infinite no. of solution.

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

i) No solution:

$\rho(A) \neq \rho(A:B)$

$\lambda = 3,\ \mu \neq 10$

ii) A unique solution $\rho(A) = \rho(A:B)$

$\lambda \neq 3$, μ any value

iii) Infinite solution $\rho(A) = \rho(A:B) < n$

$\lambda = 3,\ \mu = 10$ $\quad$ $n \rightarrow$ no. of unknowns

x, y, z

Samee

8) c) For what value of λ

$x+y+z=1$
$x+2y+4z=\lambda$
$x+4y+10z=\lambda^2$

Have a solution and solve for each val. of λ.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \qquad A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda \end{bmatrix}$$

$\rho(A) = 2$
$\rho(A:B) = 2$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

$\lambda^2-3\lambda+2=0$
$\lambda^2-2\lambda-\lambda+2=0$
$\lambda(\lambda-1)-2(\lambda-1)=0$
$(\lambda-1)(\lambda-2)=0$
$\lambda = 1, 2$

$\rho(A) = \rho(A:B) \neq n$

Case-1 $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z=1$
$y+3z=0$
$z=k$
$y=-3k$
$x=1-y-z$
$=1+3k-k$
$=1+2k$

Samuel

Case - 2 $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z=1$

$y+3z=1$

$y=1-3k, \; z=k$

$x=1-1+3k-k=2k$

d) Find the sol of the system of eqns.

$x+3y-2z=0, \; 2x-y+4z=0, \; x-11y+14z=0$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A:B) = 2 \neq n$

Infinite solution

c) For what values of λ the given equation $3x+y-\lambda z=0, \; 4x-2y-3z=0, \; 2\lambda x+4y+\lambda z=0$, may posses non-trivial solution and solve them completely in each case.

$$A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow{R_3 \to R_3+R_1} \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 4+1 & 0 \end{bmatrix} \begin{matrix} R_1 \to 4R_1 \\ R_2 \to 3R_2 \end{matrix}$$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_2} \begin{bmatrix} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$$

Sameer

$$12x + 6y - 9z = 0$$
$$-10y + (-9 + 4\lambda)z = 0$$
$$(2\lambda + 3)x + 5y = 0$$
$$\boxed{\frac{-(2\lambda+3)x}{5} = y}$$

$$12x - 6y - 9z = 0$$
$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$
$$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{10}{(-9+4\lambda)}\right)\left(\frac{2\lambda+3}{5}\right)x$$
$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(4\lambda-9)} = 0$$
$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9)(12\lambda+8)$$
$$+18(5)(2\lambda+3) = 0$$
$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 1$$
$$48\lambda^2 + 384\lambda - 432 = 0$$
$$\lambda^2 + 8\lambda - 9 = 0$$
$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$
$$\lambda(\lambda - 1) + 9(\lambda - 1) = 0$$
$$(\lambda + 9)(\lambda - 1) = 0 \Rightarrow \lambda = 1,\ \lambda = -9$$

$\underline{\lambda = 1}$

$$-x = y, \quad z = -2y$$

$$12x - 6y - 9z = 0$$
$$12(-y) - 6y - 9(-2y) = 0$$
$$18y - 18y = 0$$
$$y = 0 \quad \text{Trivial solution.}$$

$d = -9$

$y = \left(-\frac{(2d+3)x}{5} \right)$

$= \frac{-(-18+3)}{5} x = 3x$

$z = \frac{10y}{(-9+4d)}$

$= \frac{10y}{-45} = \frac{2y}{-9}$

$12x - 6y - 9z = 0$

$12\left(\frac{y}{3}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$

$4y - 6y + 2y = 0$

$y = 0$ Trivial solution

$\therefore$ It has no trivial solution

Sameer

Assignment - 2

Q)

1) $[1, 0, 0]$, $[1, 1, 0]$, $[1, 1, 1]$

$[C_1 + C_2 + C_3 \quad C_2 + C_3 \quad C_3] = [0 \ 0 \ 0]$

$C_3 = 0 \quad C_2 = 0 \quad C_1 = 0$

Unique solution. Linearly independent

2) $[7 \ -3 \ 11 \ -6]$, $[-56 \ 24 \ -88 \ 48]$

$[7C_1 - 56C_2 \quad -3C_1 + 24C_2 \quad 11C_1 - 88C_2 \quad -6C_1 + 4\ldots$

$= [0 \ 0 \ 0 \ 0]$

$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 8K \\ K \end{bmatrix} = K\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

$7C_1 = 56C_2$

$C_1 = 8C_2$

$C_1 = 8K$

Linearly dependent

3) $[-1 \ 5 \ 0]$, $[16 \ 8 \ -3]$ $[-64 \ 56 \ 9]$

$[-1C_1 + 16C_2 - 64C_3 \quad 5C_1 + 8C_2 + 56C_3 \quad -3C_2 + 9C_3]$

$= [0 \ 0 \ 0]$

$-C_1 + 16C_2 - 64C_3 = 0$

$16C_2 = 64C_3 + C_1$

$16(3C_3) = 64C_3 + C_1$

$C_1 = 0$

$-3C_2 + 9C_3 = 0$

$9C_3 = 3C_2$

$C_2 = 3C_3$

$= 3K$

$\text{Vector} = \begin{bmatrix} 0 \\ 3K \\ K \end{bmatrix} = K\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$

Linearly dependent

4) $[1 \quad -1 \quad 1] \quad [1 \quad 1 \quad -1] \quad [-1 \quad 1 \quad 1]$

$[0 \quad 1 \quad 0]$

$[c_1+c_2-c_3 \qquad -c_1+c_2+c_3+c_4 \qquad c_1-c_2+c_3]$

$c_1+c_2-c_3=0$

$c_1+c_2=c_3$

$c_1+c_1+c_3=c_3$

$2c_1=0$

$c_1=0$

$-c_1+c_2+c_3+c_4=0$

$c_1=c_2+c_3+c_4$

$c_4=-2k$

$c_1-c_2+c_3=0$

$c_2=c_1+c_3$

$c_2=c_3=k$

Vector $= k\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Linearly dependent

5) $[2 \quad -4] \quad [1 \quad 9] \quad [3 \quad 5]$

$[2c_1+c_2+3c_3 \qquad -4c_1+9c_2+5c_3] = [0 \quad 0]$

$2c_1+c_2+3c_3=0$

$4c_1+2c_2+6c_3=0$

$9c_2+5c_3+2c_2+6c_3=0$

$11c_2+11c_3=0$

$c_2=-c_3=-k$

$4c_1=9c_2+5c_3$

$4c_1=9(k)-5k$

$4c_1=4k$

$c_1=k$

Vector $= \begin{bmatrix} k \\ k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ Linearly depend

Samee

c) $[3\ \ -2\ \ 0\ \ 4]$ $[5\ \ 0\ \ 0\ \ 1]$ $[6\ \ -1\ \ 0$ ... $[2\ \ 0\ \ 0\ \ 3]$

$[3c_1+5c_2-6c_3+2c_4 \quad -2c_1+c_3 \quad 0 \quad 4c_1+c_2+c_3+3c_4] = [0\ \ 0\ \ 0$ ...

$-2c_1+c_3$

$2c_1=c_3$

$c_1=k$

$c_3=2c_1$

$=2k$

$4c_1+c_2+c_3+3c_4=0$

$20c_1+5c_2+5c_3+15c_4=0$

$3c_1+5c_2-6c_3+2c_4=0$

$\underline{-\quad -\quad +\quad -}$

$17c_1+11c_3-13c_4=0$

$17k+11(2k)=13c_4$

$(17+22)k=13c_4$

$\frac{\cancel{39}k}{\cancel{13}}=c_4=3k$

$4c_1+c_2+c_3+3c_4$

$4(k)+c_2+2k+3(3k)=0$

$c_2+15k=0$

$c_2=-15k$

Vector $=\begin{bmatrix} k \\ -15k \\ 2k \\ 3k \end{bmatrix} = k\begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$

Linearly dependent

7) $[3\ \ 4\ \ 7]$ $[2\ \ 0\ \ 3]$ $[8\ \ 2\ \ 3]$ $[5\ \ 5\ \ 6]$

$[3c_1+2c_2+8c_3+5c_4 \quad 4c_1+2c_3+5c_4 \quad 7c_1+3c_2+3c_3+6c_4]$

$=[0\ \ 0\ \ 0]$

$3c_1+2c_2+8c_3+5c_4=0$ $\quad 7c_1+3c.$

$4c_1+2c_3+5c_4=0$

$\underline{-\quad -\quad -}$

$-c_1+2c_2+6c_3=0$

$c_1=2c_2+6c_3$

$$7c_1 + 3c_2 + 3c_3 + 6c_4 = 0$$

$$14c_1 + 6c_2 + 6c_3 + 12c_4 = 0$$
$$9c_1 + 6c_2 + 24c_3 + 15c_4 = 0$$
$$- \quad - \quad - \quad -$$

$$5c_1 - 18c_3 - 3c_4 = 0$$

$$5c_1 = 18c_3 + 3c_4$$

$$10c_2 + 30c_3 = 18c_3 + 3c_4$$

$$12c_3 = 3c_4 - 10c_2$$

$$4c_1 + 2c_3 + 5c_4 = 0$$

$$36c_1 + 18c_3 + 45c_4 = 0$$
$$5c_1 - 18c_3 - 3c_4 = 0$$

$$41c_1 + 42c_4 = 0$$
$$41c_1 = -42c_4$$
$$c_1 = -\frac{42}{41}c_4$$
$$= -\frac{42}{41}K$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = K \begin{bmatrix} -42/41 \\ 10543/492 \\ 1 \\ 1506/216 \end{bmatrix}$$

Linearly dependent

$$18c_3 = 5c_1 - 3c_4$$
$$c_3 = 5\left(\frac{-42}{41}\right)K - 3(K)\left(\frac{1}{18}\right)$$
$$= \frac{(-210(6) - 6(41))K}{41(6)}$$

$$\frac{(1260 + 216)K}{216}$$
$$= \frac{1506K}{216}$$

$$-\frac{42}{41}K = 2c_2 + 6\left(\left(\frac{251}{36}\right)\right)K$$

$$c_2 = \frac{-42K}{41} - \frac{251}{6}K$$
$$c_2 = -\left(\frac{252 + 10291}{2 \times 246}\right)K = \frac{10543K}{492}$$

Sanier

8) $[6\ 0\ 3\ 1\ 4\ 2], [0\ -1\ 2\ 7\ 0\ 5]$

$[12\ 3\ 0\ -19\ 8\ -11]$

$[6c_1+12c_3 \quad -1c_2+3c_3 \quad 3c_1+2c_2 \quad c_1+7c_2-19c_3$

$4c_1+8c_3 \quad 5c_2-11c_3] = [0\ 0\ 0\ 0\ 0\ 0]$

$c_1+2c_3=0$
$-c_2+3c_3=0$
$c_2=3c_3$
$c_2=0$

$c_1+2c_3=0$
$c_1+7c_2-19c_3=0$
$c_1=0$

$3c_1+2c_2=0$
$5c_2-11c_3=0$
$15c_3-11c_3=0$
$4c_3=0$
$c_3=0$

Unique solution.
Linearly independent

1) $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ $\quad A-\lambda I=0$

$=\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$ $= (-2-\lambda)(-\lambda(1-\lambda)+6(-2))$
$=-2(-\lambda(2)+6(-1))$
$-3(-4+1-\lambda)$

$=(-2-\lambda)(-\lambda+\lambda^2-12)+2(2\lambda+6)+3(3+\lambda)$

$=2\lambda-2\lambda^2+24+\lambda^2+\lambda-\lambda^3+12\lambda+2\lambda+12+9+3\lambda$

$=19\lambda-\lambda^2-\lambda^3+45$

$\lambda^3+\lambda^2-19\lambda-45=0$

Sameer

2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A - \lambda I = 0$

$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(2(1-\lambda))$

$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$

$= (4-\lambda-4\lambda+\lambda^2+2)$

$= (\lambda^2-5\lambda+6)$

$= (1-\lambda)(\lambda-2)(\lambda-3)$

$\lambda = -1, 2, 3$

Eigen values

$\lambda = -1$

$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$5x+z=0$
$-2x+2y=0$
$-2x+2z=0$
$-10x+2z=0$

$-12x=0$
$x=0$

$z=0$
$y=0$

Eigen vector $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$

$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$2x+z=0$
$-2x-y=0$
$-2x-z=0$

$x=k$ $\quad y=-2k$
$z=2k$

Eigen vector $= k\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$\lambda = 3$

$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$x+z=0$
$-2x-2y=0$
$-2x-2z=0$

$x=-z=-k$ $\quad y=-x$
$z=k$ $\quad y=k$

Vector $= \begin{bmatrix} -k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

Sam

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5$$

$\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$5x = 0 \Rightarrow x = 0$

$-x + 3z = 0$

$z = 0$

$$\text{vector} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$2x = 0$

$-3y = 0$

$-x = 0$

$$\text{vector} = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$-5y = 0$

$-x - 2z = 0$

$x = 2z = -2k$

$$\text{vector} = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Samee

4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$$\lambda((3-\lambda)(2+\lambda))=0$$
$$\lambda = 0, 3, -2$$

$\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 3y+4z=0 \\ -z=0 \\ z=0,\ y=0 \end{matrix}$$

$$\text{vector} = K\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad \begin{matrix} 4z=0 \\ -5z=0 \end{matrix}$$

$$\text{vector} = K\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda = -2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad \begin{matrix} 5y+4z=0 \\ 5y=-4z=-4K \\ y=-\frac{4}{5}K \end{matrix}$$

$$= K\begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$

papergrid

Date: / /

Assignments

1). $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ No. of nonzero rows

Rank = 3

2) $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$

Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$T(M) = (a-b) + (b-c)x + (c-a)x^2$

$= a - bx + c(x^2 - x + 1)$

∴ The set of all polynomials of degree at most 2 is denoted as P_2, where P_2 represents the image of T.

Rank of T:

The rank of T corresponds to the dimension of its image. Since P_2 deals with polynomials upto degree 2, which have 3 parts (coefficients for x^0, x^1, x^2).
The rank of T is 3.

The Null space of symmetric matrix.

$$T(M) = 0.$$

this leads to the system of equation.

$$a-b=0 \quad b-c=0 \quad c-a=0$$

$$\therefore a=b=c.$$

∴ T is the set of symmetric matrices of the form:

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scalar.

∴ Dimension = 1 (using only t)

∴ Rank T is 3

The nullity of T is 1

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \Rightarrow A^{-1} - \lambda I = \begin{bmatrix} 2/3-\lambda & 1/3 \\ 1/3 & 2/3-\lambda \end{bmatrix}$

$= (2/3-\lambda)^2 - (1/3)^2 = 0$

$(1/3-\lambda)(1-\lambda) = 0$

$\therefore$ Eigen values, $\lambda = 1, 1/3$

$\lambda = 1$

$$\begin{bmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$= -x(1/3) + y(1/3) = 0$

$\therefore$ Eigen vectors $= K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x = y = K$

$\lambda = 1/3$

$$\begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$x = -y$

Eigen vectors $= \begin{bmatrix} k \\ -k \end{bmatrix} = K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$Z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

$Z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$

$(6-\lambda)^2 = (6-\lambda-1)(6-\lambda+1)$
$= (5-\lambda)(7-\lambda) = 0$

$\therefore$ Eigen values $= 5, 7$

$\lambda = 5$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x - y = 0$
$x = y$

Eigen vectors $= k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 7$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x = -y$

Eigen vectors $= k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

4) $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

$x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$

$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$

$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$

1 => $z = y = 0$

$x_1 = \frac{7.85}{3} = 2.61$

$z = 0$

$y = \frac{1}{7}\left(-19.3 - 0.1\left(\frac{7.85}{3}\right)\right) = 2.79$

$y(1) = 2.79$

$z = \frac{1}{10}\left[71.4 - 0.3(2.61) + 0.2(2.79)\right]$

$z = \frac{1}{10}\left[71.4 - 0.783 + 0.558\right]$

$z_1 = 7.1175$

2 => $x_2 = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$

$= 2.9255$

$y_2 = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$

$= 3.0123$

$z_2 = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10}$

$= 7.0132$

3 => $x_3 = \frac{7.85 - 0.1(2.9255) - 0.2(7.0132)}{3} \approx 3.0032$

$y_3 = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} = 3.001$

$z_3 = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$

$x = 3.0032, y = 3.0001, z = 7.000$

5) $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$ $\quad R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ $\quad R_4 \to R_4 + R_3$, $R_3 \to R_3 - 2R_2$

$= \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This corresponds to the system.

$x + 3y + 2z = 0$

$-7y - z = 0$

Now, let's express the variables in terms of parameter. Let $y = t$,

$x = -3t, \; z = -7t$

So, the system has infinitely many solutions give by

$x = -3t, \; y = t, \; z = -7t$

$\therefore$ The system is consistent and dependent.

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6) $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$
is a linear transformation, so we need to check two properties.

1) Additivity : $T(u+v) = T(u)+T(v)$
2) Homogenity of degree 1:
$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

1) $T(u+v) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$
$= T((a_1+a_2)+(b_1+b_2)\cdot x+(c_1+c_2))$
$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$
$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x$
$+(c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$.
So, function is addictive.
∴ Homogenity of degre 1:

$T(ka) = T(k(a+bx+c))$
$= T(ka+kbx+kc) = (ka+1)+(kb+1)x+(kc+1)x^2$
$= k(a+1)+k(b+1)x+k(c+1)x^2$
$= kT(a+bx+c)$

So, the function is homogeneous of degree 1.
So, it is linear transformation.

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7) $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ can be arranged as matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \to R_3 + \frac{9R_2}{5}$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

The third row of zero suggests that the vectors in S are linearly dependent, forming the basis of for the subspace spanned by (S)

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$ The vectors $(1,3,2)$ and $(0,-5,5)$ constitute a basis for the subspace defined by (S).

∴ The dimension of the subspace spanned by (S) is 2.

∴ The set (S) cannot be a basis for R^3 because its row-reduced form includes a rows of zeros.

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∴ The vectors (1,3,-2) and (0,-5,5) form the basis for the subspace spanned by (S).

∴ The subspace has a dimension of 2.

8) $3x-6y+2z=23$
$-4x+y-z=-15$
$x-3y+7z=16$

Initial values ⇒ $x_0=1, y_0=1, z_0=1$

1 ⇒ $x_1 = \frac{23+6y_0-2z_0}{3} \approx 9.0$

$y_1 = \frac{-15+4(x)_1+z_0}{1} \approx -9.0$

$z_1 = \frac{16-x_1-3(y)_1}{7} \approx 2.0$

2 ⇒ $x_2 = \frac{23+6y_1-2z_1}{3} \approx 5.0$

$y_2 = -15+4x_1+z_1 \approx -5.0$

$z_2 = \frac{16-x_1+3y_1}{7} \approx 3.0$

3 ⇒ $x_3 = \frac{23+6y_2-2z_2}{3} \approx 6.0$

$y_3 = -15+4x_2+z_2 \approx -6.0$

$z_3 = \frac{16-x_2+3y_2}{7} \approx 2.0$