i, $2x-3y+7z=5$

$3x+y-3z=13$

$2x+19y-47z=32$

$$A=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \quad B=\begin{bmatrix}5\\ 13\\ 32\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 0 & 22 & -54\end{bmatrix} \quad \& \quad B=\begin{bmatrix}5\\ 13\\ 27\end{bmatrix}$$

$$R_2 \to R_2 - \frac{3}{2}R_1$$

$$\begin{bmatrix}2 & -3 & 7\\ 0 & 11/2 & -27/2\\ 0 & 22 & -54\end{bmatrix} \quad B=\begin{bmatrix}5\\ 11/2\\ 5\end{bmatrix}$$

Rank of $A=2$ Rank of $A:B=3$

$\rho(A) \neq \rho(A:B)$ inconsistency

ii, $2x-y+3z=8$

$-x+2y+z=4$

$3x+y-4z=0$

$$A=\begin{bmatrix}2 & -1 & 3\\ -1 & 2 & 1\\ 3 & 1 & -4\end{bmatrix} \quad B=\begin{bmatrix}8\\ 4\\ 0\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$A=\begin{bmatrix}-1 & 2 & 1\\ 2 & -1 & 3\\ 3 & 1 & -4\end{bmatrix} \quad B=\begin{bmatrix}4\\ 8\\ 0\end{bmatrix}$$

Sameer

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix} \quad R_3 \to R_3 -$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -\frac{38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

Consistency

iii, $4x - y = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

Sameer

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

Consistency

b) For what values of λ and μ

$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$

i) No solution, ii) A unique solution

iii) infinite no. of solution.

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

i) No solution:

$\rho(A) \neq \rho(A:B)$

$\lambda = 3,\ \mu \neq 10$

ii) A unique solution $\rho(A) = \rho(A:B)$

$\lambda \neq 3$, μ any value

iii) Infinite solution $\rho(A) = \rho(A:B) < n$

$\lambda = 3,\ \mu = 10$

$n \rightarrow$ no. of unknowns

x, y, z

Samee

3) c) For what value of λ

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

Have a solution and solve for each val. of λ.

$$\begin{bmatrix}1&1&1\\1&2&4\\1&4&10\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\\lambda\\\lambda^2\end{bmatrix}$$

$$A=\begin{bmatrix}1&1&1\\0&1&3\\0&3&9\end{bmatrix}$$

$$A:B=\begin{bmatrix}1&1&1&1\\1&2&4&\lambda\\1&4&10&\lambda^2\end{bmatrix}$$

$$=\begin{bmatrix}1&1&1\\0&1&3\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}1&1&1&1\\0&1&3&\lambda-1\\0&3&9&\lambda^2-1\end{bmatrix}$$

$$=\begin{bmatrix}1&1&1&1\\0&1&3&\lambda-1\\0&0&0&\lambda^2-1-3\lambda\end{bmatrix}$$

$$\rho(A)=2$$
$$\rho(A:B)=2$$

$$=\begin{bmatrix}1&1&1&1\\0&1&3&\lambda-1\\0&0&0&\lambda^2-3\lambda+2\end{bmatrix}$$

$$\lambda^2-3\lambda+2=0$$
$$\lambda^2-2\lambda-\lambda+2=0$$
$$\lambda(\lambda-1)-2(\lambda-1)=0$$
$$(\lambda-1)(\lambda-2)=0$$
$$\lambda=1,2$$

$$\rho(A)=\rho(A:B)\neq n$$

Case-1 $\lambda=1$

$$\begin{bmatrix}1&1&1&1\\0&1&3&0\\0&0&0&0\end{bmatrix}$$

$$x+y+z=1$$
$$y+3z=0$$
$$z=k$$
$$y=-3k$$
$$x=1-y-z$$
$$=1+3k-k$$
$$=1+2k$$

Sameer

Case - 2 $\lambda = 2$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x+y+z=1$

$y+3z=1$

$y=1-3k, \quad z=k$

$x=1-1+3k-k=2k$

d) Find the sol of the system of eqns.

$x+3y-2z=0, \; 2x-y+4z=0, \; x-11y+14z=0$

$$A=\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad x=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A:B) = 2 \neq n$

Infinite solution

e) For what values of λ the given equation $3x+y-\lambda z=0, \; 4x-2y-3z=0, \; 2\lambda x+4y+\lambda z=0,$ may posses non-trivial solution and solve them completely in each case.

$$A=\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow{R_3 \to R_3+R_1} \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 4+1 & 0 \end{bmatrix} \begin{matrix} R_1 \to 4R_1 \\ R_2 \to 3R_2 \end{matrix}$$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2-R_1} \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1+R_2} \begin{bmatrix} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$$

Sameer

$$12x + 6y - 9z = 0$$
$$-10y + (-9 + 4\lambda)z = 0$$
$$(2\lambda + 3)x + 5y = 0$$
$$\boxed{\frac{-(2\lambda+3)x}{5} = y}$$

$$12x - 6y - 9z = 0$$
$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$
$$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{10}{(-9+4\lambda)}\right)\left(\frac{2\lambda+3}{5}\right)x$$
$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(4\lambda-9)} = 0$$
$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9)(12\lambda+8)$$
$$+18(5)(2\lambda+3) = 0$$
$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 16$$
$$48\lambda^2 + 384\lambda - 432 = 0$$
$$\lambda^2 + 8\lambda - 9 = 0$$
$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$
$$\lambda(\lambda - 1) + 9(\lambda - 1) = 0$$
$$(\lambda + 9)(\lambda - 1) = 0 \Rightarrow \lambda = 1,\ \lambda = -9$$

$\underline{\lambda = 1}$

$$-x = y,\quad z = -2y$$

$$12x - 6y - 9z = 0$$
$$12(-y) - 6y - 9(-2y) = 0$$
$$18y - 18y = 0$$
$$y = 0 \quad \text{Trivial solution.}$$

$d = -9$

$$y = \left(\frac{-(2d+3)\,x}{5} \right) \qquad z = \frac{10y}{(-9+4d)}$$

$$= \frac{-(-18+3)}{5}\,x = 3x \qquad = \frac{10y}{-45} = \frac{2y}{-9}$$

$$12x - 6y - 9z = 0$$

$$12\left(\frac{y}{3}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$$

$$4y - 6y + 2y = 0$$

$$y = 0 \quad \text{Trivial solution}$$

$\therefore$ It has no trivial solution

Samee

Assignment - 2

Q)

1) $[1, 0, 0]$, $[1, 1, 0]$, $[1, 1, 1]$

$[c_1+c_2+c_3 \quad c_2+c_3 \quad c_3] = [0 \quad 0 \quad 0]$

$c_3 = 0 \quad c_2 = 0 \quad c_1 = 0$

Unique solution. Linearly independent

2) $[7 \ -3 \ 11 \ -6]$, $[-56 \ 24 \ -88 \ 48]$

$[7c_1 - 56c_2 \quad -3c_1+24c_2 \quad 11c_1-88c_2 \quad -6c_1+4$

$= [0 \ 0 \ 0 \ 0]$

$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

$7c_1 = 56c_2$

$c_1 = 8c_2$

$c_1 = 8k$

Linearly dependent

3) $[-1 \ 5 \ 0]$, $[16 \ 8 \ -3]$ $[-64 \ 56 \ 9]$

$[-1c_1 + 16c_2 - 64c_3 \quad 5c_1+8c_2+56c_3 \quad -3c_2+9c_3]$

$= [0 \ 0 \ 0]$

$-3c_2 + 9c_3 = 0$

$9c_3 = 3c_2$

$c_2 = 3c_3$

$= 3k$

$-c_1 + 16c_2 - 64c_3 = 0$

$16c_2 = 64c_3 + c_1$

$16(3c_3) = 64c_3 + c_1$

$c_1 = 0$

$\text{Vector} = \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$

Linearly dependent

4) $[1 \quad -1 \quad 1]$ $[1 \quad 1 \quad -1]$ $[-1 \quad 1 \quad 1]$

$[0 \quad 1 \quad 0]$

$[c_1+c_2-c_3 \qquad -c_1+c_2+c_3+c_4 \qquad c_1-c_2+c_3]$

$c_1+c_2-c_3=0$

$c_1+c_2=c_3$

$c_1+c_1+c_3=c_3$

$2c_1=0$

$c_1=0$

$-c_1+c_2+c_3+c_4=0$

$c_1=c_2+c_3+c_4$

$c_4=-2k$

$c_1-c_2+c_3=0$

$c_2=c_1+c_3$

$c_2=c_3=k$

Vector $= k\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Linearly dependent

5) $[2 \quad -4]$ $[1 \quad 9]$ $[3 \quad 5]$

$[2c_1+c_2+3c_3 \qquad -4c_1+9c_2+5c_3] = [0 \quad 0]$

$2c_1+c_2+3c_3=0$

$4c_1+2c_2+6c_3=0$

$9c_2+5c_3+2c_2+6c_3=0$

$11c_2+11c_3=0$

$c_2=-c_3=-k$

$4c_1=9c_2+5c_3$

$4c_1=9(k)-5k$

$4c_1=4k$

$c_1=k$

Vector $= \begin{bmatrix} k \\ k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ Linearly dependent

Samee

c) $[3\ -2\ 0\ 4]$ $[5\ 0\ 0\ 1]$ $[6\ -1\ 0$ $[2\ 0\ 0\ 3]$

$[3c_1 + 5c_2 - 6c_3 + 2c_4 \quad -2c_1 + c_3 \quad 0 \quad 4c_1 + c_2 + c_3 + 3c_4] = [0\ 0\ 0$

$-2c_1 + c_3$
$2c_1 = c_3$
$c_1 = k$
$c_3 = 2c_1$
$= 2k$

$4c_1 + c_2 + c_3 + 3c_4 = 0$
$20c_1 + 5c_2 + 5c_3 + 15c_4 = 0$
$3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$
$- \quad - \quad + \quad -$

$17c_1 + 11c_3 - 13c_4 = 0$
$17k + 11(2k) = 13c_4$
$(17+22)k = 13c_4$
$\frac{\cancel{39}k}{\cancel{13}} = c_4 = 3k$

$4c_1 + c_2 + c_3 + 3c_4$
$4(k) + c_2 + 2k + 3(3k) = 0$
$c_2 + 15k = 0$
$c_2 = -15k$

Vector $= \begin{bmatrix} k \\ -15k \\ 2k \\ 3k \end{bmatrix} = k \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$

Linearly dependent

7) $[3\ 4\ 7]$ $[2\ 0\ 3]$ $[8\ 2\ 3]$ $[5\ 5\ 6]$

$[3c_1 + 2c_2 + 8c_3 + 5c_4 \quad 4c_1 + 2c_3 + 5c_4 \quad 7c_1 + 3c_2 + 3c_3 + 6c_4$
$= [0\ 0\ 0]$

$3c_1 + 2c_2 + 8c_3 + 5c_4 = 0$ $\quad 7c_1 + 3c.$
$4c_1 + 2c_3 + 5c_4 = 0$
$- \quad - \quad -$

$-c_1 + 2c_2 + 6c_3 = 0$
$c_1 = 2c_2 + 6c_3$

$$7c_1 + 3c_2 + 3c_3 + 6c_4 = 0$$

$$14c_1 + 6c_2 + 6c_3 + 12c_4 = 0$$
$$9c_1 + 6c_2 + 24c_3 + 15c_4 = 0$$
$$- \quad - \quad - \quad -$$

$$5c_1 - 18c_3 - 3c_4 = 0$$

$$5c_1 = 18c_3 + 3c_4$$

$$10c_2 + 30c_3 = 18c_3 + 3c_4$$

$$12c_3 = 3c_4 - 10c_2$$

$$4c_1 + 2c_3 + 5c_4 = 0$$

$$36c_1 + 18c_3 + 45c_4 = 0$$
$$5c_1 - 18c_3 - 3c_4 = 0$$

$$41c_1 + 42c_4 = 0$$

$$41c_1 = -42c_4$$

$$c_1 = \frac{-42}{41} c_4$$

$$= -\frac{42}{41} k$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = k \begin{bmatrix} -42/41 \\ 10543/492 \\ 1 \\ 1506/216 \end{bmatrix}$$

Linearly dependent

$$18c_3 = 5c_1 - 3c_4$$

$$c_3 = 5\left(\frac{-42}{41}\right)k - 3(k)\left(\frac{1}{18}\right)$$

$$= \frac{(-210(6) - 6(41))k}{41(6)}$$

$$\frac{(1260 + 216)k}{216}$$

$$= \frac{1506k}{216}$$

$$-\frac{42}{41}k = 2c_2 + 6\left(\left(\frac{251}{36}\right)\right)k$$

$$c_2 = \frac{-42k}{41} - \frac{251}{6}k$$

$$c_2 = -\left(\frac{252 + 10291}{2 \times 246}\right)k = \frac{10543k}{492}$$

Sanner

8) $[6\ 0\ 3\ 1\ 4\ 2], [0\ -1\ 2\ 7\ 0\ 5]$

$[12\ 3\ 0\ -19\ 8\ -11]$

$[6c_1+12c_3 \quad -1c_2+3c_3 \quad 3c_1+2c_2 \quad c_1+7c_2-19c_3$

$4c_1+8c_3 \quad 5c_2-11c_3] = [0\ 0\ 0\ 0\ 0\ 0]$

$c_1+2c_3=0$
$-c_2+3c_3=0$
$c_2=3c_3$
$c_2=0$

$c_1+2c_3=0$
$c_1+7c_2-19c_3=0$
$c_1=0$

$3c_1+2c_2=0$
$5c_2-11c_3=0$
$15c_3-11c_3=0$
$4c_3=0$
$c_3=0$

Unique solution.
Linearly independent

1) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ $\quad A-\lambda I = 0$

$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = (-2-\lambda)(-\lambda(1-\lambda)+6(2))$
$= -2(-\lambda(2)+6(-1))$
$-3(-4+1-\lambda)$

$= (-2-\lambda)(-\lambda+\lambda^2-12)+2(\lambda+6)+3(3+\lambda)$

$= 2\lambda-2\lambda^2+24+\lambda^2+\lambda-\lambda^3+12\lambda+2\lambda+12+9+3\lambda$

$= 19\lambda-\lambda^2-\lambda^3+45$

$\lambda^3+\lambda^2-19\lambda-45=0$

Sameer

2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A - \lambda I = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$$

$$= (4-\lambda-4\lambda+\lambda^2+2)$$

$$= (\lambda^2-5\lambda+6)$$

$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$\lambda = -1, 2, 3$

Eigen values

$\lambda = -1$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$5x+z=0$

$-2x+2y=0$

$-2x+2z=0$

$-10x+2z=0$

$-12x=0$

$x=0$

$z=0$

$y=0$

Eigen vector $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$2x+z=0$

$-2x-y=0$

$-2x-z=0$

$x=k \quad y=-2k$

$z=2k$

Eigen vector $= k\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x+z=0$

$-2x-2y=0$

$-2x-2z=0$

$x=-z=-k$

$z=k$

$y=-x$

$y=k$

Vector $= \begin{bmatrix} -k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5$$

$\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$5x = 0 \Rightarrow x = 0$

$-x + 3z = 0$

$z = 0$

$$\text{vector} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$2x = 0$

$-3y = 0$

$-x = 0$

$$\text{vector} = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$-5y = 0$

$-x - 2z = 0$

$x = 2z = -2k$

$$\text{vector} = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Samue

4)
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

$$\lambda((3-\lambda)(2+\lambda)) = 0$$
$$\lambda = 0, 3, -2$$

$\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 3y + 4z = 0 \\ -z = 0 \\ z = 0,\ y = 0 \end{matrix}$$

$$\text{vector} = K\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad \begin{matrix} 4z = 0 \\ -5z = 0 \end{matrix}$$

$$\text{vector} = K\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda = -2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \qquad \begin{matrix} 5y + 4z = 0 \\ 5y = -4z = -4K \\ y = \frac{-4}{5}K \end{matrix}$$

$$= K\begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$