Seismic Wave Analysis and Core Mantle Boundary Calculations

February 3, 2025

1 Module 1: Understanding Seismic Wave Velocities

1.1 P and S Waves

P waves, known as primary compression waves, travel faster than S waves, which are secondary shear waves. Consequently, P waves are detected first by seismic sensors. These waves compress and expand materials in the direction of their propagation and can move through solids, liquids, and gases. In contrast, S waves arrive second and displace particles perpendicular to their direction of travel. Due to their reliance on shear stresses, S waves can only propagate through solids, rendering them unable to travel through liquids or gases. This distinction is instrumental in determining the composition of the planet's core.

P-Waves (Primary/Compressional Waves)

- **Nature:**
  - Longitudinal waves: The particle motion is parallel to the wave's direction of travel.
  - Compress and expand the material as they propagate (like a slinky).
- **Speed:**
  - Fastest seismic waves $V_p = \sqrt{\frac{\lambda+2\mu}{\rho}}$.
  - Arrive first at the seismic stations.
- **Propagation:**
  - Travel through solids, liquids, and gases.
  - Can pass through Earth's liquid outer core (though they slow down).

S-Waves (Secondary/Shear Waves)

- **Nature:**
  - Transverse waves: The particle motion is perpendicular to the wave's direction of travel.
  - Shear material side-to-side or up and down.
- **Speed:**
  - Slower than P-waves $V_s = \sqrt{\frac{\mu}{\rho}}$.
  - Arrive second at the seismic stations.
- **Propagation:**
  - Only travel through solids.
  - Cannot propagate through liquids or gases.

Note: It seemed more appropriate to us to derive the P and S wave equations and use the velocity formulae which we had obtained from them. In addition, we have sourced information on the relationship between Lame parameters and shear and bulk moduli from online references.

$$\lambda = K - \frac{2G}{3}$$
$$\mu = G$$

1.2 Deriving P and S waves in a homogeneous isotropic medium

Given:

$$
\begin{aligned}
\sigma_{ij} &= \lambda\delta_{ij}(\vec{\nabla}\cdot\vec{\mu}) + \mu\left(\frac{\partial\mu_i}{\partial x_j} + \frac{\partial\mu_j}{\partial x_i}\right) \\
\rho\frac{\partial^2 u_i}{\partial t^2} &= \sum_j \frac{\partial\sigma_{ij}}{\partial x_j} \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= \frac{\partial}{\partial x_i}\left(\lambda\delta_{ij}(\vec{\nabla}\cdot\vec{u}) + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right) \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= \lambda\delta_{ij}\frac{\partial}{\partial x_j}(\vec{\nabla}\cdot\vec{u}) + \mu\left(\frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j}\right)\right) \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= \sum_j \lambda\delta_{ij}\frac{\partial}{\partial x_j}\vec{\nabla}\cdot\vec{u} + \sum_j \mu\left(\frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j}\right)\right) \\
\text{since } \delta_{ij} &= \begin{cases} 1, & j = i \\ 0, & j \neq i \end{cases} \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= \lambda\frac{\partial}{\partial x_i}(\vec{\nabla}\cdot\vec{u}) + \mu\left(\sum_j \frac{\partial^2}{\partial x_j^2}\right)u_i + \mu\frac{\partial}{\partial x_i}\left(\sum_j \frac{\partial u_j}{\partial x_j}\right) \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= \lambda\frac{\partial}{\partial x_i}(\vec{\nabla}\cdot\vec{u}) + \mu\nabla^2 u_i + \mu\frac{\partial}{\partial x_i}(\vec{\nabla}\cdot\vec{u}) \\
\Rightarrow \rho\frac{\partial^2 u_i}{\partial t^2} &= (\lambda+\mu)\frac{\partial(\vec{\nabla}\cdot\vec{u})}{\partial x_i} + \mu\nabla^2 u_i
\end{aligned}
$$

$$\vec{u} = \vec{\nabla}V + \vec{\nabla}\times\vec{A}$$

Compressive Shear

$$
\begin{aligned}
\nabla^2\vec{u} &= \nabla^2(\vec{\nabla}V + \vec{\nabla}\times\vec{A}) \\
\vec{\nabla}\cdot\vec{u} &= \nabla^2 V \quad (\text{since } \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0)
\end{aligned}
$$

$$
\begin{aligned}
\sum_i \rho\frac{\partial^2 u_i\hat{i}}{\partial t^2} &= (\lambda+\mu)\sum_i \frac{\partial}{\partial x_i}(\vec{\nabla}\cdot\vec{u})\hat{i} + \mu\sum_i \nabla^2(u_i\hat{i}) \\
\rho\frac{\partial^2\vec{u}}{\partial t^2} &= (\lambda+\mu)\sum_i \vec{\nabla}\left(\nabla^2 V\right) + \mu\sum_i \nabla^2\vec{u} \\
\rho\frac{\partial^2}{\partial t^2}\left(\vec{\nabla}V + \vec{\nabla}\times\vec{A}\right) &= \underbrace{(\lambda+2\mu)\vec{\nabla}(\nabla^2 V)}_{V_p=\sqrt{\frac{\lambda+2\mu}{\rho}}} + \underbrace{\mu\left(\vec{\nabla}\times\nabla^2\vec{A}\right)}_{V_s=\sqrt{\frac{\mu}{\rho}}}
\end{aligned}
$$

1.3 Calculating P and S Wave Velocities

$$
\begin{aligned}
V_p &= \sqrt{\frac{K+\frac{4G}{3}}{\rho}} = \sqrt{\frac{2.5\times10^{10}+\frac{4}{3}(1\times10^{10})}{3000}} = 3.574\times10^3 \text{ m/s} \\
V_s &= \sqrt{\frac{G}{\rho}} = \sqrt{\frac{10^{10}}{3000}} = 1.826\times10^3 \text{ m/s}
\end{aligned}
$$

2 Module 2: Identifying Shadow Zones

2.1 S-Wave Shadow Zones

The shadow zone refers to a specific area on a planet's surface where certain seismic waves are absent following a seismic activity. For P-waves, this shadow zone typically spans approximately 105 to 140 degrees from the epicentre, a result of the waves being refracted by the planet's core, leaving a void where they cannot be detected. When we focus on S-waves, their shadow zone offers conclusive proof of the existence of a liquid core, as S-waves are unable to pass through liquids.

2.2 Core State

Their shadow zone extends across the entire region beyond 105 degrees from the epicentre, indicating that any S-waves departing in this direction would need to penetrate the core. Consequently, stations located beyond this angle would fail to detect S-waves, generating a clear shadow zone.

3 Module 3: Calculating the Core Mantle Boundary

3.1 Refraction at the Core Mantle Boundary

Since the velocity of P waves is lower inside the core than in the lower mantle, Snell's Law dictates that waves traveling from the mantle to the core refract towards the normal at the core-mantle boundary and refract away when exiting, forming a P-wave shadow zone.

3.2 Calculating the Angle of Refraction

Given:

$$
\begin{aligned}
&v_1 = 10 \text{ km/s}, \quad v_2 = 8 \text{ km/s}, \quad i = 30^\circ \\
&\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \\
&\frac{\sin(30^\circ)}{\sin(r)} = \frac{10}{8} \\
&\sin r = \frac{8}{10} \times \frac{1}{2} = 0.4 \\
&r = \sin^{-1}(0.4) = 23.58^\circ
\end{aligned}
$$

4 Module 4: Determining the Core Radius

Given:

Total radius of Mars, $(R) = 3390$ km

Depth to the core mantle boundary from the surface, $d = 560$ km

Radius od Core, $R_c = R - d = 2830$ km

Radius of the Core obtained from the given values is inconsistent with the estimated core radius using regression ().

5 Module 5: Verifying Core Status

From the InSight data, we have observed that S waves do not propagate through the Martian core, indicating that the core is liquid. Furthermore, the angles of refraction of the P waves obtained from the sizes of the shadow zone suggest that the velocities of the P waves reduce while they are inside the core. But the velocities do not change drastically since despite the density of core being higher than that of the mantle (due to its composition and pressure), the bulk modulus is higher in the core (as it is a liquid). Hence, we can conclude that the Martian core should be liquid.

6 Module 6: Seismic Signal Processing using ML

6.1 Introduction

This methodology outlines the process for extracting key seismic features such as *amplitude*, *frequency*, and *phase shift* from a waveform file in the MiniSEED (.mseed) format. These features are crucial for seismic data analysis, including the detection and characterization of seismic events.

6.2 Preprocessing

Before feature extraction, the waveform data undergoes several preprocessing steps:

- **Reading the Data**: The data is read using ObsPy, a Python library for seismology.
- **Denoising**: Filters (e.g., bandpass filters) are applied to remove noise.
- **Normalization**: The data is normalized to ensure consistency in amplitude measurements.
- **Detrending**: Linear trends are removed to focus on the seismic signal.

6.3 Feature Extraction

6.3.1 Amplitude Calculation

Amplitude represents the maximum extent of a seismic wave and is extracted as follows:

- **Instantaneous Amplitude**: Calculated using the Hilbert transform to obtain the envelope of the signal.
- **Root Mean Square (RMS) Amplitude**: Provides an average measure of the wave's power.

6.3.2 Frequency Calculation

Frequency analysis helps in understanding the energy distribution across different frequencies:

- **Instantaneous Frequency**: Derived from the analytic signal using the derivative of the phase.
- **RMS Frequency**: Represents the weighted average frequency of the seismic event.

Methods Used:

- Fast Fourier Transform (FFT) for spectral analysis.
- Hilbert transform for instantaneous frequency.

6.3.3 Phase Shift Calculation

Phase shift indicates the change in phase of a seismic wave over time:

- Obtained from the phase of the analytic signal derived via the Hilbert transform.
- Phase unwrapping techniques are applied to correct discontinuities.

6.4 Data Collection

The data required to predict whether a given station lies in a shadow zone or not was not directly available. Hence, we synthetically generate a dataset using Python library ObsPy, which has been trained on seismic data. ObsPy guarantees that the generated values will be valid for Mars. Training the model based on random data points affecting shadow zones will help improve model performance.
Features Generated:

- Relative time
- Distance of epicenter from station in degrees
- Instantaneous amplitude of received wave
- Instantaneous frequency of received wave
- RMS amplitude
- RMS frequency
- Phase shift
- Wave type
- P wave arrival time
- S wave arrival time
- Depth of source
- P wave velocity
- S wave velocity

6.5 Data Labelling

Based on known physics rules, we assigned feature weights to each feature. Using these feature weights and restricting feature values to those that classify a shadow zone, we label a given entry as either a shadow zone or non-shadow zone using a majority classifier. For example, if the distance angle lies in [103,143] degrees or amplitude values are lower, each feature is checked, and a decision is made based on the majority.

6.6 Data Preprocessing

We drop irrelevant or less correlated features and retain only the most relevant ones. Basic exploratory data analysis (EDA) is performed using heatmaps and statistical metrics.

6.7 Model Training

We split the data into training and test sets. A Robust Scaler is used so that scaling is not affected by outliers. A Random Forest Classifier is trained using Randomized Search CV to find the best parameters. Metrics like accuracy and precision are used to evaluate the model. Finally, user input is taken for feature values, and the model classifies the station as either a shadow zone or a non-shadow zone.

6.8 Challenges

- No specific dataset catered to the needs of the module.
- No specification on whether the station was fixed or variable.
- No mention of whether the shadow zone classification was for P-waves or S-waves.
- No input parameters specified for training the model.

6.9 Assumptions

- The station is assumed to be fixed on the surface of Mars, e.g., InSight SEIS (Seismic Experiment for Interior Structure), Landing Site: Elysium Planitia, Mars (Latitude: 4.5024°N, Longitude: 135.6234°E).
- These are P-wave shadow zones being classified.
- Final features used for prediction are based on astronomical analysis.

7 Module 7: Predicting Core Radius using Regression

7.1 Feature Selection Using mRMR (Minimum Redundancy Maximum Relevance)

Features and Target Selection:

- A list of seismic-related features is selected as independent variables (X), while *core_radius* is the target variable (y).
- The correlation matrix of features is computed to determine redundancy by finding the average correlation of each feature with others.

Feature Selection and Visualization:

- Features are sorted based on their mRMR scores.
- A horizontal bar plot is used to visualize the scores, highlighting the most important features.

7.2 Feature Importance Using Random Forest

- A `RandomForestRegressor` with 100 estimators is trained using the selected features to predict *core_radius*.
- The model calculates the importance of each feature based on how much they contribute to reducing the impurity in decision trees.
- The importance scores are visualized using a horizontal bar plot, emphasizing which features the Random Forest considers most influential.

7.3 Regression Modeling Using Multiple Algorithms

Data Preparation:

- Features (*vp, vs, density*) are standardized using `StandardScaler` to ensure uniform scaling.
- The dataset is split into training and testing sets (80% training, 20% testing).

Model Initialization:

- Five regression models are initialized:
  1. Linear Regression: A simple linear approach to model relationships.
  2. Random Forest Regressor: An ensemble learning method using multiple decision trees.
  3. Gradient Boosting Regressor: Sequentially builds trees to minimize prediction errors.
  4. XGBoost Regressor: An optimized boosting algorithm designed for performance.
  5. Support Vector Regressor (SVR): Uses an RBF kernel to capture non-linear relationships.
- The best model among these can be used to predict the *core_radius*.
- Performance metrics calculated: Mean Squared Error (MSE), R^2 Score.

8 Module 8: Anomaly Detection in Seismic Data

8.1 Dataset Features

- **Time Axis:** *relative_time* generated in 15-second intervals.
- **Geophysical Properties:**
  - Depth: Random values from 100 to 700 meters.
  - P-wave & S-wave Velocities: Simulated within typical Martian crustal properties, with Gaussian noise.
  - Density: Uniform distribution adjusted with noise to reflect Martian subsurface variations.
- **Derived Properties:**
  - Shear Modulus (μ)
  - Bulk Modulus (K)
  - Poisson's Ratio: Derived from wave velocities.
  - Wave Energy: Randomized to simulate different seismic event magnitudes.
  - Attenuation Factor: Reflects energy loss due to subsurface properties.
  - Phase Shift, RMS Amplitude, RMS Frequency: Simulated to mimic seismic signal characteristics.

8.2 Data Preprocessing

- **Standardization:** Applied `StandardScaler` to normalize all features for improved model performance.

8.3 Anomaly Detection

- **Technique Used:** Isolation Forest algorithm, optimized using Randomized Search CV.
- **Anomaly Detection Process:**
  1. Combined selected features into 2D arrays.
  2. Applied Isolation Forest with 100 estimators, a max sample size of 512, and a contamination rate of 2%.
  3. Identified anomalies based on model predictions (-1 for anomalies, 1 for normal data).

8.4 Visualization

- **Plotting Function:**
  - Visualizes anomalies against normal data points.
  - Anomalies are marked in red, while normal data appears in green.
  - Clear labeling and legends enhance interpretability.

8.5 Anomaly Detection Insights

- **P-wave Velocity vs S-wave Velocity Graph:**
  - Higher than expected V_p/V_s ratio $\rightarrow$ Fluid-filled zones (water, CO_2, brine pockets).
  - Lower than expected V_p/V_s ratio $\rightarrow$ Fractured, porous rocks, or ice inclusions.
  - Sudden drops in V_p and V_s together $\rightarrow$ Weak zones, unconsolidated sediments, or faulted regions.

- **Poisson's Ratio vs Depth of Source:**
  - Higher Poisson's ratio (0.3 - 0.5) → Fluid presence, porous rocks, or gas pockets.
  - Lower Poisson's ratio (∼0.2) → Denser rock, more rigid formations.
  - Abrupt shifts → Possible stratigraphic changes, fractures, or buried impact structures.
- **Attenuation Factor vs Depth of Source:**
  - High Attenuation (energy loss) → Loose sediments, gas-rich layers, or fractured zones.
  - Low Attenuation (energy retention) → Dense, compact formations like basaltic bedrock.
  - Abrupt changes → Could indicate underground reservoirs, faulted regions, or hidden impact craters.

8.6 Challenges

- The data required to train an anomaly detection model was not readily available.
- No specific anomaly was mentioned that should have been identified.

8.7 Assumptions

- Since no data was available, we synthetically generated values within known valid limits for Mars.
- We chose to detect the top 3 anomalies, from which we can infer a great deal about the Martian interior and geological features.
- The anomaly detection was performed mainly on 2 features to simplify visualization and anomaly identification.

9 Module 9: Simulation of Wave Propagation with Neural Networks

9.1 Data Generation

- **Mars Layer Constraints:** Defined bulk modulus, shear modulus, and density for core, mantle, and crust based on known physical constraints.
- **Synthetic Dataset:** Random values were generated within specified ranges for each layer. The P-wave (V_p) and S-wave (V_s) velocities were computed using the formulas:

$$V_p = \sqrt{\frac{(K + \frac{4}{3}\mu)}{\rho}}$$

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

- **Random Noise:** Generated random noise vectors as input for the trained Generator.

9.2 Data Preprocessing

- **Normalization:** Min-max scaling was applied to normalize the dataset for better GAN performance.
- **Tensor Conversion:** The normalized data was converted into PyTorch tensors and loaded into a DataLoader for batch processing.

9.3 GAN Architecture

- **Generator:** A neural network with 3 fully connected layers using ReLU activations, followed by a Sigmoid activation to output normalized values.
- **Discriminator:** A neural network with 3 fully connected layers using ReLU activations, followed by a Sigmoid activation to output the probability of data being real or synthetic.

9.4 Training the GAN

- **Loss Function:** Binary Cross-Entropy Loss (BCELoss) was used for both the Generator and Discriminator.
- **Optimizers:** Adam optimizer with a learning rate of 0.0002 was employed.
- **Penalties:**
  - **Velocity Penalty:** Ensured that generated V_p and V_s values are non-negative.
  - **Layer Consistency Check:** Verified if the generated data corresponds to consistent Martian layers.
- **Training Loop:** The Discriminator was trained to distinguish real and fake data, while the Generator was trained to produce realistic data.

9.5 Model Architecture

- **Network Structure:** Consists of multiple fully connected layers with 256 and 128 neurons.
- **Layer Normalization:** Applied after each linear layer to stabilize learning.
- **Leaky ReLU Activation:** Enhances gradient flow, preventing vanishing gradients.
- **Dropout:** Introduced (10%) for regularization to prevent overfitting.
- **Final Output Layer:** Predicts the P-wave velocity.

9.6 Loss Functions

- **Physics-Informed Loss:** Derived from the known P-wave/S-wave velocity equations. The loss is normalized using the mean of actual P-wave/S-wave velocities for stable training.
- **Supervised Loss:** Mean Squared Error (MSE) between predicted and true P-wave/S-wave velocities.
- **Total Loss:** Combined as a sum of the physics-informed loss and supervised loss to balance the influence of supervised learning.

9.7 Training Setup

- **Optimizer:** AdamW for better generalization with weight decay.
- **Learning Rate Scheduler:** Reduces learning rate on plateau to improve convergence.
- **Device Support:** Utilizes GPU if available for faster computation.

9.8 Data Preparation

- **Standardization:** Features (K, μ, ρ) are standardized to have zero mean and unit variance, ensuring efficient model training.
- **Physical P-wave Calculation:** Used for physics-based loss computation.

9.9 Training Loop

- **Epochs:** Model trained for 10,000 epochs.
- **Loss Monitoring:** Logs total, physics, and supervised losses every 500 epochs for performance tracking.
- **Best Model Selection:** Monitors the best loss for optimal model checkpointing.

9.10 Evaluation

- Metrics like Mean Squared Error (MSE), Mean Absolute Error (MAE), and R^2 score were used to evaluate the model.

9.11 Challenges

- No dataset was available which could have been used to train a PINN to predict wave velocities.
- Uncertainty regarding which features should be chosen for prediction.
- Defining the appropriate loss function for a PINN.

9.12 Assumptions

- We decided to train a GAN model that generates realistic data for each layer of the Martian interior, providing shear modulus, bulk modulus, and density at any given point.
- Based on physical principles, V_p depends on shear modulus, bulk modulus, and density, while V_s depends on shear modulus and density. Hence, these parameters were chosen as input features.
- The PINN loss function was defined as the sum of the physical loss (difference between physics-predicted and model-predicted values) and supervised loss (difference between model-predicted values and dataset values).