Question 1:

a. Solution

Let:

- L(x): Lorenz curve, representing cumulative income share by the bottom x-fraction of households.

- y=x: Line of perfect income equality

- The area between y=x and the Lorenz curve is:

$$A = \int_0^1 [x - L(x)]dx$$

The area under the line y=x is:

$$\int_0^1 xdx = \left[\frac{x^2}{2}\right]_0^1 = 1/2$$

So, the coefficient of inequality is defined as:

Coefficient of inequality=

$$\frac{Area\ between\ y{=}x\ and\ L(x)}{Area\ under\ y{=}x} = \frac{\int_0^1 [x-L(x)]dx}{1/2} = 2\int_0^1 [x - L(x)]dx$$

Hence proved.

b. Solution:

i) What is the percentage of total income received by the bottom 50% of the households?

We evaluate L(0.5):

$$L(0.5) = \frac{5(0.5)^2}{12} + \frac{7(0.5)}{12} = \frac{5\cdot 0.25}{12} + \frac{3.5}{12} = \frac{1.25+3.5}{12} = \frac{4.75}{12} = 0.3558\ or\ 39.58\%$$

The bottom 50% of households receive **39.58%** of total income.

(ii) Find the coefficient of inequality

We have proved:

Coefficient of inequality$= 2\int_0^1 [x - L(x)]dx$

Substituting $L(x) = \frac{5x^2}{12} + \frac{7x}{12}$:

$$x - L(x) = x - \left(\frac{5x^2}{12} + \frac{7x}{12}\right) = \frac{12x-7x-5x^2}{12} = \frac{5x-5x^2}{12} = \frac{5x(1-x)}{12}$$

$$Coefficient = 2\int_0^1 \frac{5x(1-x)}{12}dx = \frac{10}{12}\int_0^1 x(1 - x)dx = \frac{5}{6}\int_0^1 (x - x^2)dx$$

Evaluating the integral:

$$\int_0^1 (x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1/2 - 1/3 = 1/6$$

$$Coefficient = 5/6 \cdot 1/6 = 5/36 \approx 0.1386 \; or \; 13.89\%$$

Question 3:

```
#ASSIGNMENT 7 Q3
import numpy as np
from scipy.integrate import quad

# Define the integrand function
def integrand(x):
    return (1 + np.log(x)) * np.sqrt(1 + (x * np.log(x))**2)

# Define the limits of integration
lower_limit = 0.2
upper_limit = 1

# Perform the numerical integration
result, error = quad(integrand, lower_limit, upper_limit)

# Print the result
print("Result of the integral:", result)
print("Estimated error:", error)
```

```
Result of the integral: 0.3273627864342789
Estimated error: 1.8249311445990213e-11
```