Question 1:

a. Solution

Let:

- $L(x)$: Lorenz curve, representing cumulative income share by the bottom x-fraction of households.

- $y=x$: Line of perfect income equality

- The area between y=x and the Lorenz curve is:

$$A = \int_0^1 [x - L(x)]dx$$

The area under the line y=x is:

$$\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = 1/2$$

So, the coefficient of inequality is defined as:

Coefficient of inequality=

$$\frac{Area\ between\ y=x\ and\ L(x)}{Area\ under\ y=x} = \frac{\int_0^1 [x-L(x)]dx}{1/2} = 2\int_0^1 [x - L(x)]dx$$

Hence proved.

b. Solution:

i) What is the percentage of total income received by the bottom 50% of the households?

We evaluate L(0.5):

$$L(0.5) = \frac{5(0.5)^2}{12} + \frac{7(0.5)}{12} = \frac{5\cdot 0.25}{12} + \frac{3.5}{12} = \frac{1.25+3.5}{12} = \frac{4.75}{12} = 0.3558\ or\ 39.58\%$$

The bottom 50% of households receive **39.58%** of total income.

(ii) Find the coefficient of inequality

We have proved:

Coefficient of inequality$= 2\int_0^1 [x - L(x)]dx$

Substituting $L(x) = \frac{5x^2}{12} + \frac{7x}{12}$:

$$x - L(x) = x - \left(\frac{5x^2}{12} + \frac{7x}{12}\right) = \frac{12x-7x-5x^2}{12} = \frac{5x-5x^2}{12} = \frac{5x(1-x)}{12}$$

$$Coefficient = 2\int_0^1 \frac{5x(1-x)}{12}dx = \frac{10}{12}\int_0^1 x(1 - x)dx = \frac{5}{6}\int_0^1 (x - x^2)dx$$

Evaluating the integral:

$$\int_0^1 (x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1/2 - 1/3 = 1/6$$

$$Coefficient = 5/6 \cdot 1/6 = 5/36 \approx 0.1386\ or\ 13.89\%$$

Question 2:

Solution:

1. Find the line equation

The line passes through:

- A=(0,0)
- B=$\left(\frac{7\pi}{6}, \frac{-1}{2}\right)$

Finding the slope m:

$$m = \frac{\frac{-1}{2}-0}{\frac{7\pi}{6}-0} = \frac{-1}{2} \cdot \frac{6}{7\pi} = \frac{3}{7\pi}$$

Now, using point-slope form:

$$y - 0 = \frac{3}{7\pi}x => y = \frac{3}{7\pi}x$$

Again, Area between two curves:

$$Area = \int_0^{7\pi/6} \left[sinx - \left(- \frac{3}{7\pi}x\right)\right]dx = \int_0^{7\pi/6} \left[sinx + \left(\frac{3}{7\pi}x\right)\right]dx$$

$$Area = \int_0^{7\pi/6} sinx\, dx + \int_0^{7\pi/6} \left(\frac{3}{7\pi}x\right)dx$$

First part::

$$\int_0^{7\pi/6} sinx\, dx = [- cosx]_0^{7\pi/6} =- cos(7\pi/6) + cos0 =- (- \sqrt{3}/2) + 1 = \sqrt{3}/2 + 1$$

Second part:

$$\int_{0}^{7\pi/6} \left(\frac{3}{7\pi}x\right)dx = \frac{3}{7\pi} \cdot \left[\frac{x^2}{2}\right]_0^{7\pi/6} = \frac{3}{7\pi} \cdot \frac{1}{2} \cdot \left(\frac{49\pi^2}{72}\right) = \frac{147\pi^2}{504\pi} = \frac{7\pi}{24}$$

Combining both parts:

$$Area = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{24} \approx 2.782$$

Monte carlo Simulation:

```python
#Q2

import random
import math

# Define functions
def f1(x):
    return math.sin(x)

def f2(x):
    return -3 * x / (7 * math.pi)

# Domain
a = 0
b = 7 * math.pi / 6

# Estimate bounding box (min and max y-values)
y_min = -0.5
y_max = 1

# Monte Carlo simulation
N = 100000
count = 0

for _ in range(N):
    x = random.uniform(a, b)
    y = random.uniform(y_min, y_max)
    top = f1(x)
    bottom = f2(x)
    if bottom < y < top:
        count += 1

# Area of bounding box
box_area = (b - a) * (y_max - y_min)

# Estimated area
area_estimate = (count / N) * box_area
print(f"Estimated area = {area_estimate}")
```

```
Estimated area = 2.779626202024811
```

Question 3:

```python
#ASSIGNMENT 7 Q3
import numpy as np
from scipy.integrate import quad

# Define the integrand function
def integrand(x):
    return (1 + np.log(x)) * np.sqrt(1 + (x * np.log(x))**2)

# Define the limits of integration
lower_limit = 0.2
upper_limit = 1

# Perform the numerical integration
result, error = quad(integrand, lower_limit, upper_limit)

# Print the result
print("Result of the integral:", result)
print("Estimated error:", error)
```

```
Result of the integral: 0.3273627864342789
Estimated error: 1.8249311445990213e-11
```