Samir Khadka (19701) Assignment 3

1. Based on the experience of customs clearance inspections at the airport, the probability of luggage containing prohibited items is 10^{-4}. Given that the X-ray inspection machine has a probability of 1/10 of falsely identifying a regular luggage as containing prohibited items, and the probability of falsely identifying prohibited items as regular luggage is 10^{-6}, find the probability that it actually does contain prohibited items for a luggage belonging to someone determined by the X-ray inspection machine to contain prohibited items

Answer:
Suppose:
A be the luggage containing prohibited items
& B be the X-ray machine indicating that the luggage contains prohibited items.
The probability of luggage containing prohibited items is given as
P(A)=10^{-4}
.
The probability of the X-ray machine mistaking identification of regular luggage for containing prohibited items is
P(B|A')=$\frac{1}{10}$
Let: A' be the complement of A .
The probability of the X-ray machine mistaking identification of prohibited items as regular luggage is
P(B'|A)=10^{-6}
Now, Required is:
P(A|B), the probability that, despite what the X-ray system says, the luggage does prohibited goods.
Using Bayes' Theorem:
P(A|B)=

$$\frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Then,
From the law of total probability:
We know,
P(B)=P(B|A)·P(A)+P(B|A')·P(A')

P(A|B)=$\frac{P(B \mid A) \cdot P(A)}{P(B \mid \sim A) \cdot P(\sim A) + \ P(B \mid A) \cdot P(A)}$

P(A|B)=$\frac{(1/10) \cdot (10^{-4})}{(1/10)(10^{-4}) \cdot + (10^{-6}) \cdot (1 - 10^{-4}} = \frac{10^{-5}}{1.1}$

Therefore, the required answer is $\frac{10^{-5}}{1.1}$.

2. There are two basketball teams A and B in the final for a 7-game series, where each team needs to win 4 out of 7 games. The probability of A winning in each game is 0.6 and B winning is 0.4 respectively. If defining the total game as N, N ∈ {4,5,6,7}, find the probability distribution of N and the expected value of N.

Supposer:
p be the probability of team A winning a single game (0.6)
&
q be the probability of team B winning a single game (0.4).
The probability distribution for N is given by the binomial distribution formula:
P(N=k)=$\binom{6}{k-4}p^4q^{k-4}$
Where k=4,5,6,7
For N=4:
P(N=4)=$\binom{6}{0}\times 0.6^4\times 0.4^0$
For N=5:
P(N=4)=$\binom{6}{1}\times 0.6^4\times 0.4^1$
For N=6:
P(N=4)=$\binom{6}{2}\times 0.6^4\times 0.4^2$
For N=7:
P(N=4)=$\binom{6}{3}\times 0.6^4\times 0.4^3$
Now, calculate each probability:
P(N=4)=0.1296
P(N=5)=0.3456
P(N=6)=0.3456
P(N=7)=0.1728
Now,
E(N)=$\Sigma_{k=4}^{7}$P(N=k)×k
E(N)=4×P(N=4)+5×P(N=5)+6×P(N=6)+7×P(N=7)
E(N)=4×0.1296+5×0.3456+6×0.3456+7×0.1728
E(N)=5.6
Therefore, the expected value of N is 5.6 games.

3. The values of the discrete random variable X are 0, 1, 2, 3 and the probability P(X) is

as follows. Find the expected value of X.

X	**0**	**1**	**2**	**3**
P(X)	0.2	0.1*(k+1)	0.3*(k+1)	0.2

E(X)=$\Sigma_x x \cdot P(X = x)$

Given,

Samir Khadka (19701) Assignment 3

Probability distribution for X:

P(X=0)=0.2

P(X=1)=0.1·(k+1)

P(X=2)=0.3·(k+1)

P(X=3)=0.2
Substituting the probabilities,
E(X)=0·0.2+1·(0.1·(k+1))+2·(0.3·(k+1))+3·0.2
E(X)=0+0.1·(k+1)+0.6·(k+1)+0.6
E(X)=0.7·(k+1)+0.6 [Combining like terms]

4. Assuming that there are 30 male students and 20 female students in a class, the five students will be randomly selected to attend the speech contests organized by the student association. If the random variable X is the number of female students in the selected group, find the probability distribution of X

Answer:
Using binomial probability distribution formula:
P(X=k)=$\binom{n}{k} \cdot p^n \cdot (1-p)^{n-k}$
where:
n is the number of trials (students selected),
k is the number of successes (female students),
p is the probability of success on each trial (probability of selecting a female student), and
$\binom{n}{k}$is the binomial coefficient, representing the number of ways to choose k successes out of n trials.
Then,
If 5 students were selected then, n=5
p is the probability of selecting a female student, which is $\frac{20}{50}$=$\frac{2}{5}$
k can range from 0 to 5.
Now, the probabilities for X=0,1,2,3,4,5:
For X=0:

P(X=0)=$\binom{5}{0} \cdot (\frac{2}{5})^0 \cdot (\frac{3}{5})^5$
For X=1:
P(X=1)=$\binom{5}{1} \cdot (\frac{2}{5})^1 \cdot (\frac{3}{5})^4$
For X=2:

P(X=2)=$\binom{5}{2} \cdot (\frac{2}{5})^2 \cdot (\frac{3}{5})^3$

For X=3:

P(X=3)=$\binom{5}{3} \cdot (\frac{2}{5})^3 \cdot (\frac{3}{5})^2$

For X=4:

P(X=4)=$\binom{5}{4} \cdot (\frac{2}{5})^4 \cdot (\frac{3}{5})^1$

For X=5:

P(X=5)=$\binom{5}{5} \cdot (\frac{2}{5})^5 \cdot (\frac{3}{5})^0$

5. In a batch of 12 TV sets, three of them are defective. Three TV sets will be randomly selected for inspection and the random variable X represents the number of good quality units in the inspection. If all three are good, the entire batch is accepted, otherwise, it is returned. Please answer the following questions

a. If the sampling is done without replacement, write the probability distribution of X, the mean, and the variance. Also, find the probability that the entire batch of TV sets can be accepted.
b. If the sampling is done with replacement, write the probability distribution of X, the mean, and the variance. find the probability that the entire batch of TV sets can be accepted.
c. If the sampling is done without replacement, calculate the probability that the third one is defective.

Answer:

Given,

12 TV sets with 3 defective ones.

Suppose, 3 TV sets are selected without replacement.

Probability Distribution of X:

X represents the number of good quality units. The possibilities are

X=0,1,2,3

For X=0:

P(X=0)=$\frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \frac{1}{220}$

For X=1:

P(X=1)=$\binom{3}{1} \cdot \frac{9}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = \frac{54}{220}$

For X=2:

P(X=2)=$\binom{3}{2} \cdot \frac{9}{12} \cdot \frac{8}{11} \cdot \frac{1}{10} = \frac{216}{220}$

For X=3:

P(X=3)=$\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} = \frac{168}{220}$

Mean:

E(X)=$\Sigma_{x=0}^{3}$x⋅P(X=x)=0 x $\frac{1}{220}$x 1 x$\frac{54}{220}$x 2 x$\frac{216}{220}$x 3 x $\frac{168}{220}$=$\frac{36}{55}$

Variance:

E(X^2)=$\Sigma_{x=0}^{3}$x⋅P(X=x)=0^2 x $\frac{1}{220}$x 1^2 x$\frac{54}{220}$x 2^2 x$\frac{216}{220}$x 3^2 x $\frac{168}{220}$=$\frac{1128}{55}$

Var(X)=E(X^2)-$[E(X)]^2$=$\frac{1128}{55}$-$(\frac{36}{55})^2 = 20.0806$

Probability that the Entire Batch Can be Accepted:

P(X=3)=$\frac{168}{220}$

b. For with Replacement:

Probability Distribution of X:

P(X=k)=$\binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$

Where, n is the number of trials,

k is the number of successes, and

p is the probability of success.

P(X=0)=$\binom{3}{0} \cdot (\frac{3}{12})^0 \cdot (\frac{9}{12})^3 = \frac{27}{64}$

P(X=1)=$\binom{3}{1} \cdot (\frac{3}{12})^1 \cdot (\frac{9}{12})^2 = \frac{27}{64}$

P(X=2)=$\binom{3}{2} \cdot (\frac{3}{12})^2 \cdot (\frac{9}{12})^1 = \frac{9}{64}$

P(X=3)=$\binom{3}{3} \cdot (\frac{3}{12})^3 \cdot (\frac{9}{12})^0 = \frac{1}{64}$

Mean:

E(X)=n⋅p=$3 \cdot \frac{3}{12} = \frac{3}{4}$

Variance:

Var(X)=n⋅p⋅(1−p)=$3 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{9}{16}$

C. For without Replacement:

Probability of the Third One Being Defective:

Since the sampling is without replacement, the probability that the third TV set is defective is $\frac{3}{10}$.