***1:*** Verify the following logic equivalence by the truth table in Excel.

1. ~ (𝑝 ⋃~𝑞) ⋃ (~𝑝⋂ ~𝑞) ≡~p

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **~(𝑝 ⋃~𝑞)⋃(~𝑝⋂ ~𝑞)≡~p** | | | | | | |
| p | q | ~p | ~q | p V ~q | ~p ∧ ~q | ~(p V ~q) V (~p ∧ ~q) |
| TRUE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |
| TRUE | FALSE | FALSE | TRUE | TRUE | FALSE | FALSE |
| FALSE | TRUE | TRUE | FALSE | FALSE | FALSE | TRUE |
| FALSE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |

1. ~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡𝑝** | | | | | | | | | | |
| p | q | ~p | ~q | ~p ∧ q | ~p ∧ ~q | p ∧ q | (~p∧q)V(~p∧~q) | ~((~p∧q)V(~p∧~q)) | ~((~p∧q)V(~p∧~q)) V (p∧q) |  |
| TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE |  |
| TRUE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | TRUE |  |
| FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |  |
| FALSE | FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE |  |

1. (𝑝 ⋂(~(~𝑝⋂𝑞))) ⋃ (𝑝⋂ 𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **(𝑝⋂(~(~𝑝⋂𝑞)))⋃(𝑝⋂𝑞)≡𝑝** | |  |  |
| p | q | ~p | p ∧ q | ~p ∧ q | ~(~p ∧ q) | p ∧ (~(~p ∧ q)) | (p ∧ (~(~p ∧q))) V (p ∧ q) |
| TRUE | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE |
| TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE |
| FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| FALSE | FALSE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |

***2.*** Let the symbol ⊕ denote exclusive or; so 𝑝 ⊕ 𝑞 ≡ (𝑝⋃𝑞) ⋂∼(𝑝⋃𝑞). Hence the   
truth table for p ⊕ q is as follows:  
 \*Note: p and q have the same value, the result of exclusive or will be 0 or   
false, otherwise it is 1 or true.  
# Find the following values or verify the logic equivalence in Excel.

a. (𝑝 ⊕𝑝) ⊕𝑝

|  |  |  |
| --- | --- | --- |
| **p** | **p ⊕p** | **(p ⊕ p) ⊕p** |
| FALSE | FALSE | FALSE |
| TRUE | TRUE | FALSE |

b. Is (𝑝 ⊕ 𝑞) ⊕ 𝑟≡𝑝 ⊕ (𝑞 ⊕ 𝑟)?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ⊕ q** | **q ⊕ r** | **(p⊕ q) ⊕ r** | **p ⊕ (q ⊕ r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | TRUE | TRUE |
| 0 | 1 | 0 | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | FALSE | TRUE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | TRUE | TRUE | FALSE | FALSE |
| 0 | 1 | 1 | TRUE | FALSE | FALSE | FALSE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE |

Therefore, (𝑝 ⊕ 𝑞) ⊕ 𝑟 ≡ 𝑝 ⊕ (𝑞 ⊕ 𝑟).

1. Is (𝑝 ⊕𝑞)⋂𝑟 ≡ (𝑝⋂𝑟) ⊕ (𝑞⋂𝑟) ?

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ⊕ q** | **(p ⊕ q) ∧ r** | **p ∧ r** | **q ∧ r** | **(p ∧ r) ⊕ (q ∧ r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 0 | 1 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 1 | 1 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 1 | TRUE | TRUE | TRUE | FALSE | TRUE |
| 0 | 1 | 1 | TRUE | TRUE | FALSE | TRUE | TRUE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE | FALSE |

Therefore, (𝑝 ⊕𝑞)⋂𝑟 ≡ (𝑝⋂𝑟) ⊕ (𝑞⋂𝑟).

***3:*** Suppose that p and q are statements so that p→q is false. Find the truth values of each of the following:

Answer: p→q is false meAnswer that q is False and p is true.

1. ~p→q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | ~p | p→q | ~p→q |
| TRUE | FALSE | FALSE | FALSE | TRUE |

1. p V q

|  |  |  |
| --- | --- | --- |
| p | q | p V q |
| TRUE | FALSE | TRUE |

1. q→p

|  |  |  |
| --- | --- | --- |
| p | q | q→p |
| TRUE | FALSE | TRUE |

***4***: If statement forms P and Q are logically equivalent, then 𝑃↔𝑄 is a tautology. Conversely, if 𝑃↔𝑄 is a tautology, then P and Q are logically equivalent. Use ↔ to convert each of the logical equivalences to a tautology. Then use a truth table to verify each tautology in Excel.

1. ***p→(𝑞V𝑟) ≡ (𝑝∧~𝑞) → 𝑟***

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ~p | ~q | qVr | p ∧ ~q | ~(p ∧ ~q) | p→(qVr) | (p∧~q)→r | p→(𝑞V𝑟) ≡ (𝑝∧~𝑞) → 𝑟 |
| 0 | 0 | 0 | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 0 | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 0 | 1 | 0 | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 0 | 0 | 1 | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |

1. ***p ∧ (𝑞V𝑟) ≡ (𝑝∧𝑞) V (𝑝∧𝑟)***

(𝑝∧𝑞) V (𝑝∧𝑟) ≡ p∧ (q V r)

Using Distributive law

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p ∧ q | q V r | p ∧ r | p∧(q V r) | (p ∧ q) V (p ∧ r) | p ∧ (𝑞V𝑟) ≡ (𝑝∧𝑞) V (𝑝∧𝑟) |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 1 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 0 | 1 | 0 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 1 | 0 | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE |
| 0 | 0 | 1 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 1 | 1 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |

1. ***p→(𝑞→𝑟)≡(𝑝∧𝑞)→𝑟***

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p ∧q | ~(p∧q) | ~p | ~q | q→ r | p→(q→ r) | (𝑝∧𝑞)→𝑟 | p→(𝑞→𝑟)≡(𝑝∧𝑞)→𝑟 |
| 0 | 0 | 0 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 0 | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 0 | FALSE | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 0 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | FALSE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 1 | TRUE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE | TRUE |

***5:*** The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

Answer: Fact C is incorrect since Lord Hazelton did not pass away from a fatal amount of strychnine. Thus, the cook was not present in the kitchen. Sara wasn't in the dining room if the cook wasn't in the kitchen. If Sara wasn't there, Lady Hazelton was probably in the dining area. Therefore, if Lady Hazelton was present, Lord Hazelton was killed by the chauffeur.