Assignment5

4.

>> Let's give the university's enrollment in mathematics (M) and computer science (C) the letters M and C, respectively. We are aware that M >= 100 and that 2/3 x M = 3/5 \* C.

The minimal values of M and C that nevertheless satisfy this equation are what we are searching for. Set M = 100 to begin with. Thus, C = 150 since 2/3 \* 100 equals 3/5 \* C. This indicates that the university has 150 computer science majors.

There must be at least 100 math majors and at least 150 computer science majors enrolled at the institution if we want to find the minimal values of M and C.

3.

No, the customer will not win $100.

With regard to time complexity, we may use a brute force approach to check all potential pairs of integers to see whether any of them add up to 100 in order to decide whether the client would receive $100.

The time complexity of this technique is O (n2), where n is the total number of numbers on the card. As a result, the algorithm's execution time will depend on the square of the amount of digits on the card.

Because there are 10 digits on the card in this example, the algorithm's time complexity is O (102) = O. (100). This implies that even with a high number of numbers on the card, the algorithm will execute in a comparatively short period of time.

We may call the algorithm and pass the list of numbers as an input to see if the consumer would win $100 using this algorithm. The consumer wins $100 if the algorithm returns "true." The algorithm must return "false" in order for the consumer to win $100.

The algorithm will return "false" for the question's specified set of numbers (72,21,15,36,69,81,9,27,42,63), indicating that the consumer will not receive $100.

2.

We can prove this statement by showing that if c satisfies a polynomial equation of the form r3x3 + r2x2 + r1x + r0 = 0, where r0, r1, r2, and r3 are rational numbers, then c also satisfies a polynomial equation of the form n3x3 + n2x2 + n1x + n0 = 0, where n0, n1, n2, and n3 are integers.

To do this, we can start by multiplying both sides of the equation r3x3 + r2x2 + r1x + r0 = 0 by a suitable integer k to obtain the equation kr3x3 + kr2x2 + kr1x + kr0 = 0. Since r0, r1, r2, and r3 are rational numbers, kr0, kr1, kr2, and kr3 will also be rational numbers.

We can then express each of the rational numbers kr0, kr­1, kr2, and kr3 as the ratio of two integers' n0/m, n1/m, n2/m, and n3/m, respectively.

Substituting these expressions into the equation kr3x3 + kr2x2 + kr1x + kr0 = 0, we obtain the equation (n3/m) x3 + (n2/m) x2 + (n1/m) x + (n0/m) = 0.

Since m is an integer, we can multiply both sides of this equation by m to obtain the equation n3x3 + n2x2 + n1x + n0 = 0, which is an equation of the form 𝑛3𝑥3 + 𝑛2𝑥2 + 𝑛1𝑥+ 𝑛0 = 0, where 𝑛0, 𝑛1, 𝑛2, and 𝑛3 are integers.

Therefore, if c satisfies a polynomial equation of the form r3x3 + r2x2 + r1x + r0 = 0, where r0, r1, r2, and r3 are rational numbers, then c also satisfies a polynomial equation of the form n3x3 + n2x2 + n1x + n0 = 0, where n0, n1, n2, and n3 are integers. This completes the proof.

1.

This can be demonstrated using induction. The left side is 1, a rational integer, in the base situation where n = 0. We can demonstrate that the assertion is true for n+1 assuming the statement is true for some value of n.

The left hand side for n+1 is:

1 + + 1/22 + ... + 1/2n + 1/(2(n+1))

Using the assumption that the left hand side for n is a rational number, we can rewrite this as:

(1 - 1/2(n+1)/(1 - 1/2) + 1/(2(n+1))

This simplifies to:

(1 - 1/2(n+2)/(1 - 1/2)

This corresponds to the right side for n+1. As a result, the statement is true for n+1. The statement is true for all non-negative numbers n, according to induction.