8.4 Data Envelopment Analysis - Example with one output and two inputs

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Summary	• DEA Example - One output and two inputs
• Formulate DEA – LP optimization problem for each DMU	**DEA – Linear programming** • Let us revisit the sales example where we had one output and two inputs. • Each sales office has the same sales target: INR 10,00,000 (output). They have their budgets approved and the respective team sizes (inputs). • Let us formulate linear programs to calculate the efficiency of each sales office.
	DEA – Linear programming • Notice that we will have to formulate an optimization for each of the sales offices independently. Let us start with office # 1. For this office, • The only Output $O_{jk} = O_{11} = 10{,}00{,}000$. • The two inputs: (I_{ik}) $Budget,\ I_{11} = 3{,}00{,}000;\ Team\ size,\ I_{21} = 13$. • We need one output weight (y_{11}), and two input weights (x_{11}, x_{21}). • We expect the optimization problem to tell us the optimal values of the weights.
	DEA – LP: Sales office 1 $$Max\ y_{11} * 1000000$$ $$subject\ to\ \ x_{11} * 300000 + x_{21} * 13 = 1$$ $$y_{11} * 1000000 \le x_{11} * 300000 + x_{21} * 13$$ $$y_{11} * 1000000 \le x_{11} * 256000 + x_{21} * 9$$ $$y_{11} * 1000000 \le x_{11} * 500000 + x_{21} * 7$$ $$y_{11} * 1000000 \le x_{11} * 390000 + x_{21} * 10$$ $$y_{11} * 1000000 \le x_{11} * 185000 + x_{21} * 14$$ $$Decision\ variables: x_{11}, x_{21}, y_{11} \ge 0$$
	DEA – LP: Sales office 2 $$Max\ y_{12} * 1000000$$ $$subject\ to\ \ x_{12} * 256000 + x_{22} * 9 = 1$$ $$y_{12} * 1000000 \le x_{12} * 300000 + x_{22} * 13$$ $$y_{12} * 1000000 \le x_{12} * 256000 + x_{22} * 9$$ $$y_{12} * 1000000 \le x_{12} * 500000 + x_{22} * 7$$ $$y_{12} * 1000000 \le x_{12} * 390000 + x_{22} * 10$$ $$y_{12} * 1000000 \le x_{12} * 185000 + x_{22} * 14$$

Sales office	Budget (INR)	Team size
1	3,00,000	13
2	2,56,000	9
3	5,00,000	7
4	3,90,000	10
5	1,85,000	14

$$Decision\ variables: x_{12}, x_{22}, y_{12} \geq 0$$

DEA – LP: Sales office 3

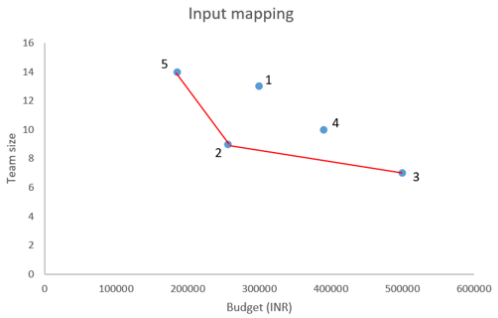
$$Max\ y_{13} * 1000000$$

$$subject\ to\ \ x_{13} * 500000 + x_{23} * 7 = 1$$
$$y_{13} * 1000000 \leq x_{13} * 300000 + x_{23} * 13$$
$$y_{13} * 1000000 \leq x_{13} * 256000 + x_{23} * 9$$
$$y_{13} * 1000000 \leq x_{13} * 500000 + x_{23} * 7$$
$$y_{13} * 1000000 \leq x_{13} * 390000 + x_{23} * 10$$
$$y_{13} * 1000000 \leq x_{13} * 185000 + x_{23} * 14$$

$$Decision\ variables: x_{13}, x_{23}, y_{13} \geq 0$$

DEA LP – Sales offices

- Similarly, the LP can be formulated for the other offices.
- See Excel sheet for the solution of the LP. Did we have the same solution in the graphical method?



Working has been done in the Excel sheet.

Results:

Sales office	inp1 wt	inp2 wt	out wt	
k	x1k	x2k	y1k	Efficier
1	2.06E-06	0.029302518	7.91993E-07	0.79
2	2.61E-06	0.036998437	0.000001	1
3	7.39E-07	0.090103397	0.000001	1
4	6.21E-07	0.075776398	8.40994E-07	0.84
5	2.61E-06	0.036998437	0.000001	1