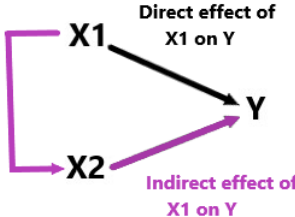
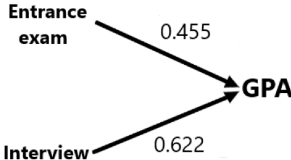


6.3 Multiple Linear Regression - Path Diagram

Monday, 07 November 2022 14:51

Summary	<ul style="list-style-type: none">Path Diagram																																																				
	<p>From the correlation matrix</p> <table><tr><td></td><td><i>GPA at college</i></td><td><i>Entrance exam</i></td><td><i>interview</i></td></tr><tr><td>GPA at college</td><td>1</td><td>0.74665952</td><td>0.763283</td></tr><tr><td>Entrance exam</td><td>0.74665952</td><td>1</td><td>0.540056</td></tr><tr><td>interview</td><td>0.763282985</td><td>0.5400556</td><td>1</td></tr></table> <ul style="list-style-type: none">We can conclude that the Entrance exam scores as well as the interview scores are highly correlated to the GPA.Also, notice that the correlation between the Entrance exam and interview variables is 0.54, which also significant.		<i>GPA at college</i>	<i>Entrance exam</i>	<i>interview</i>	GPA at college	1	0.74665952	0.763283	Entrance exam	0.74665952	1	0.540056	interview	0.763282985	0.5400556	1																																				
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<ul style="list-style-type: none">Direct and Indirect Effect of X on Y																																																					
<ul style="list-style-type: none">How do you calculate total effect of X_1 and total effect of X_2 on Y?Observe how we form these SLR and MLR equations	<table><tr><td></td><td><i>Coefficients</i></td></tr><tr><td>Intercept</td><td>-0.704401949</td></tr><tr><td>Entrance exam</td><td>0.455442321</td></tr><tr><td>interview</td><td>0.62250322</td></tr></table> <p>We saw from the MLR analysis that the regression coefficients, $\beta_1 = 0.455$, and $\beta_2 = 0.644$. These are the partial slopes, i.e., these are the direct effects on Y.</p> <p>$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ $Y = \beta_0 + 0.455X_1 + 0.622X_2$ $GPA = \beta_0 + 0.455 \text{ Entrance exam} + 0.622 \text{ interview}$</p> 		<i>Coefficients</i>	Intercept	-0.704401949	Entrance exam	0.455442321	interview	0.62250322																																												
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Analysing effect of X_1 (Entrance exam) on Y (GPA)																																																					
	<p>Indirect effect</p> <p>We will first analyse the indirect effect of Entrance Exam on GPA. For that we will analyse effect of the Entrance exam on the interview. So, we'll treat</p> <p>Response variable: Interview Explanatory variable: Entrance Exam</p> <table><tr><td>X</td><td>Y</td></tr><tr><td>Entrance exam</td><td>interview</td></tr></table> <p>SUMMARY OUTPUT</p> <table><tr><td colspan="2"><i>Regression Statistics</i></td></tr><tr><td>Multiple R</td><td>0.5400556</td></tr><tr><td>R Square</td><td>0.291660052</td></tr><tr><td>Adjusted R Square</td><td>0.237172363</td></tr><tr><td>Standard Error</td><td>0.811749825</td></tr><tr><td>Observations</td><td>15</td></tr></table> <table><tr><td colspan="6">ANOVA</td></tr><tr><td></td><td><i>df</i></td><td><i>SS</i></td><td><i>MS</i></td><td><i>F</i></td><td><i>Significance F</i></td></tr><tr><td>Regression</td><td>1</td><td>3.527142224</td><td>3.527142224</td><td>5.352769781</td><td>0.037694522</td></tr><tr><td>Residual</td><td>13</td><td>8.566191109</td><td>0.658937778</td><td></td><td></td></tr><tr><td>Total</td><td>14</td><td>12.09333333</td><td></td><td></td><td></td></tr></table> <table><tr><td><i>Coefficients</i></td><td><i>Standard Error</i></td><td><i>t Stat</i></td><td><i>P-value</i></td><td><i>Lower 95%</i></td><td><i>Upper 95%</i></td></tr></table>	X	Y	Entrance exam	interview	<i>Regression Statistics</i>		Multiple R	0.5400556	R Square	0.291660052	Adjusted R Square	0.237172363	Standard Error	0.811749825	Observations	15	ANOVA							<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	Regression	1	3.527142224	3.527142224	5.352769781	0.037694522	Residual	13	8.566191109	0.658937778			Total	14	12.09333333				<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
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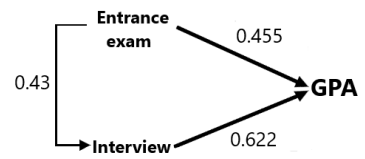
Intercept	4.829315637	1.543233569	3.129348489	0.007982787	1.495362205	8.16326907
Entrance exam	0.425366887	0.183854556	2.313605364	0.037694522	0.028173267	0.822560506

$$Y = \beta_0 + \beta_1 X$$

$$Y = \beta_0 + 0.42X$$

$$\text{interview} = \beta_0 + 0.43 \text{ Entrance exam}$$

i.e., one unit change in entrance exam score changes the interview score by 0.42 units.



Total effect

$$GPA = \beta_0 + 0.455 \text{ Entrance exam} + 0.622 \text{ interview}$$

$$\text{interview} = \beta_0 + 0.43 \text{ Entrance exam}$$

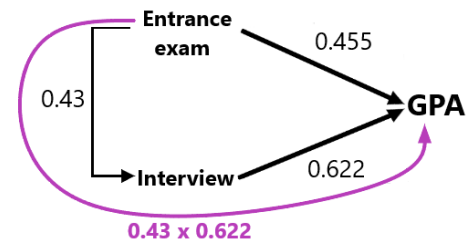
∴ The indirect effect of Entrance exam on GPA
 $= 0.43 \times 0.622 = 0.27$

Direct effect of Entrance exam on GPA = 0.455

Total effect = Direct effect + Indirect effect
 ∴ Total effect of Entrance exam on GPA
 $= 0.455 + 0.27 = 0.725$

This value is same as the marginal slope.

Y	X1
GPA at college	Entrance exam
Coefficients	
Intercept	2.301862586
Entrance exam	0.720234578



Analysing effect of x2 (interview) on y (GPA)

Indirect effect

We will first analyse the indirect effect of interview on GPA.
 For that we will analyse **effect of the interview on the Entrance exam**.
 So, we'll treat

Response variable: Entrance exam
Explanatory variable: Interview

Y	X
Entrance exam	interview

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.5400556
R Square	0.291660052
Adjusted R Square	0.237172363
Standard Error	1.030616281
Observations	15

ANOVA					
	df	SS	MS	F	Significance F
Regression	1	5.685551047	5.685551047	5.352769781	0.037694522
Residual	13	13.80820895	1.062169919		
Total	14	19.49376			

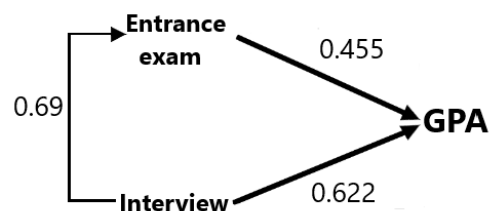
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2.579252481	2.493808533	1.034262433	0.319870981	-2.808293309	7.96679827
interview	0.685667034	0.296363003	2.313605364	0.037694522	0.045413691	1.325920377

$$Y = \beta_0 + \beta_1 X$$

$$Y = \beta_0 + 0.69X$$

$$\text{Entrance exam} = \beta_0 + 0.69 \text{ interview}$$

i.e., one unit change in entrance exam score changes the interview score by 0.69 units.



Total effect

$$GPA = \beta_0 + 0.455 \text{ Entrance exam} + 0.622 \text{ interview}$$

$$0.69 \times 0.455$$

$$\text{Entrance exam} = \beta_0 + 0.69 \text{ interview}$$

∴ The indirect effect of interview on GPA
 $= 0.69 \times 0.455 = 0.31$

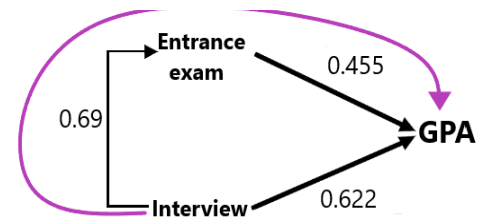
Direct effect of interview on GPA = 0.622

Total effect = Direct effect + Indirect effect
 ∴ Total effect of interview on GPA = 0.622 + 0.31
 = 0.932

This value is same as the marginal slope.

Y X2
 GPA at college interview

	Coefficients
Intercept	0.470298787
interview	0.934785006



- What if the indirect effect was zero?

If indirect effect were zero, then
 Partial slope = Marginal slope