

## 6.2 Multiple Linear Regression - Example

Thursday, 27 October 2022 20:54

Summary	<ul style="list-style-type: none"><li>Implementing MLR in Excel</li></ul>																																																																				
	<p>This is the data we're given</p> <table><thead><tr><th></th><th>Y</th><th>X1</th><th>X2</th></tr><tr><th>record</th><th>GPA at college</th><th>Entrance exam</th><th>interview</th></tr></thead><tbody><tr><td>1</td><td>9.5</td><td>9.8</td><td>9.1</td></tr><tr><td>2</td><td>6.3</td><td>7.5</td><td>7.1</td></tr><tr><td>3</td><td>8.2</td><td>7.9</td><td>7.7</td></tr><tr><td>4</td><td>9.1</td><td>9.5</td><td>9.6</td></tr><tr><td>5</td><td>8.2</td><td>9.1</td><td>7.5</td></tr><tr><td>6</td><td>8.32</td><td>8.5</td><td>8.4</td></tr><tr><td>7</td><td>9.6</td><td>7.54</td><td>9.5</td></tr><tr><td>8</td><td>7.6</td><td>8.4</td><td>7.8</td></tr><tr><td>9</td><td>6.5</td><td>5.6</td><td>7.8</td></tr><tr><td>10</td><td>8.64</td><td>8</td><td>8.5</td></tr><tr><td>11</td><td>9.5</td><td>9.8</td><td>9.9</td></tr><tr><td>12</td><td>8.1</td><td>8</td><td>8.9</td></tr><tr><td>13</td><td>7.95</td><td>7.5</td><td>6.9</td></tr><tr><td>14</td><td>9.99</td><td>10</td><td>8.9</td></tr><tr><td>15</td><td>6.87</td><td>7.6</td><td>7.9</td></tr></tbody></table> <p><math>\hat{Y}</math> GPA at college <math>X_1</math> Entrance exam <math>X_2</math> Interview</p> <p>Response variable: <math>Y</math> Explanatory variables: <math>X_1, X_2</math></p> <ul style="list-style-type: none"><li>We want to explain the variation in the response variable using these two explanatory variables.</li></ul>		Y	X1	X2	record	GPA at college	Entrance exam	interview	1	9.5	9.8	9.1	2	6.3	7.5	7.1	3	8.2	7.9	7.7	4	9.1	9.5	9.6	5	8.2	9.1	7.5	6	8.32	8.5	8.4	7	9.6	7.54	9.5	8	7.6	8.4	7.8	9	6.5	5.6	7.8	10	8.64	8	8.5	11	9.5	9.8	9.9	12	8.1	8	8.9	13	7.95	7.5	6.9	14	9.99	10	8.9	15	6.87	7.6	7.9
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## GPA at college interview

### SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.763282985
R Square	0.582600915
Adjusted R Square	0.550493293
Standard Error	0.76313828
Observations	15

ANOVA					
	df	SS	MS	F	Significance F
Regression	1	10.56743289	10.56743289	18.14525272	0.00093024
Residual	13	7.570940441	0.582380034		
Total	14	18.13837333			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.470298787	1.846585182	0.254685672	0.802950357	-3.519005962	4.459603536
interview	0.934785006	0.219447292	4.259724488	0.00093024	0.460697953	1.408872058

- $F$  values tells us that the regression is significant (  $F$  ).
- Coefficient of interview:  $\beta_1$  Estimate of  $\beta_1$  (marginal slope).
- This SLR tells us that the interview score is a good explanatory variable for  $Y$ .

## Multiple Linear Regression (MLR)

- Interpret:
  - Multiple R
  - R Square
  - Adjusted R Squared

- Regression  $df$ ?
- Residual  $df$ ?
- Total  $df$ ?

Y X1 X2  
GPA at college Entrance exam interview

### SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.860528625
R Square	0.740509514
Adjusted R Square	0.6972611
Standard Error	0.626281041
Observations	15

ANOVA					
	df	SS	MS	F	Significance F
Regression	2	13.43163802	6.715819012	17.12223502	0.000305301
Residual	12	4.70673531	0.392227942		
Total	14	18.13837333			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	-0.704401949	1.576544	-0.446801326	0.662975358	-4.139396243	2.730592345
Entrance exam	0.455442321	0.168539069	2.702295228	0.019227553	0.088227236	0.822657406
interview	0.62250322	0.213981085	2.909150685	0.013101725	0.156278487	1.088727953

- Multiple R:  $R$  correlation coefficient between observed value( $y$ ) and the fitted/predicted value( $\hat{y}$ ).
- R Square: explanatory power of this model.  
 $R^2$  these two explanatory variables are able to explain  $R^2$  in the variation of GPA.
- $R^2$  (two explanatory variables)
- $R^2$ .
- Adjusted R Square:  $R^2_{adj}$  more realistic view of  $R^2$ . The explanatory power of the model is actually  $R^2_{adj}$  (and not  $R^2$ ).

	df	
Regression	2	• We're estimating $\beta_0, \beta_1, \beta_2$ parameters. $R^2$ Regression $R^2$ .
Residual	12	• Residual $R^2$ .
Total	14	• Total $R^2$ .

- $F$  values tells us that the regression is significant (  $F$  ).

$R^2$  for Entrance Exam

