Primal-Dual

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Rules

Primal	Dual
Maximization	Minimization
Number of constraints	Number of variables
Number of variables	Number of constraints
Objective function coefficient	Right hand side in constraints
Right hand side in constraints	Objective function coefficient

Know this:

In Maximization	you have	$\leq$
In Minimization	you have	$\geq$

It's important to make all the variables ≥ 0, in both min and max problems

Primal	Standard form	Dual
Max	Max with $\leq$	Min with $\geq$
Min	Min with $\geq$	Max with $\leq$

Dealing with an equal to constraint:

$$2X_1 + X_2 = 400\,.$$

For max problem, we want $\leq$:

- First, write it as two constraints

$$2X_1 + X_2 \leq 400$$
$$2X_1 + X_2 \geq 400$$

- Now, convert these two into a less than or equal to constraint.

$$2X_1 + X_2 \leq 400$$
$$-2X_1 - X_2 \leq -400$$

One important thing to know:

Let's say we have a Max optimization problem:

Max _______ _

subject to

Constraint ①: _____ $\leq$ _____

Constraint ②: _____ $\leq$ _____

and Decision variables ≥ 0

And we also have the optimal solutions. Let's say the optimal solutions satisfy the constraints:

Constraint ① as: LHS $=$ RHS (binding)

Constraint ② as: LHS $<$ RHS (not binding)

Binding: A constraint is called "binding" or "active" if it is satisfied as an equality at the optimal solution

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Now, we already know that in Dual:
variable $\Rightarrow$ # constraints
constraints $\Rightarrow$ #variables

If primal variables: $X_1,\ X_2,\ X_3$
and dual variables: $Y_1,\ Y_2$
then

The dual variable corresponding to:

- Constraint ① will be Y_1.
- Constraint ② will be Y_2.

The dual variables corresponding to:

- Binding constraint: will be some positive value.
- Non-binding constraint: will be 0.
  - meaning strict inequality

Conclusion:

Since:

Constraint ①: LHS $=$ RHS (binding)

Constraint ②: LHS $<$ RHS (not binding)

$\therefore\ Y_1$ +ve quantity,
and

$\mathbf{Y_2 = 0}$

Q1. Convert the given Primal to dual

$$Min\ \ 120000Y_1 + 140000Y_2$$

subject to
$15Y_1 + 20Y_2\ \geq 500000$
$20Y_1 + 50Y_2\ \geq 1000000$
$60Y_1 + 100Y_2\ \geq 2500000$
$Y_1,\ Y_2 \geq 0$

This problem is already in Standard form.

Dual:
1. $Min\ \Rightarrow\ Max$
2. # variable $\Rightarrow$ # constraints
3. # constraints $\Rightarrow$ #variables
4. $\geq\ \Rightarrow\ \leq$

$\therefore$ Dual:

$Max\ \ 500000X_1 + 1000000X_2 + 2500000X_3$

subject to
$15X_1 + 20X_2 + 60X_3\ \leq 120000$
$20X_1 + 50X_2 + 100X_3\ \leq 140000$
$X_1,\ X_2,\ X_3 \geq 0$

Q2. Convert the given Primal to dual

$$Max\ \ 500X_1 + 100X_2 + 200X_3$$

subject to
$15X_1 + 20X_2 + 60X_3\ \geq 1200$
$20X_1 + 50X_2 + 100X_3\ \leq 1400$
$X_1 \geq 0,\ X_2 \leq 0,\ X_3\ unrestricted$

Notice, not all variables are ≥ 0

$\therefore$ Let, $X_4\ -\ X_5 =\ X_3\ \ (\because X_3\ is\ unrestriced)$
and,
$X_2 = -X_6$

Now, the problem becomes

$$Min\ \ 500X_1 - 100X_6 + 200X_4\ - 200X_5$$

subject to
$15X_1 - 20X_6 + 60X_4 - 60X_5\ \geq 1200$
$20X_1 - 50X_6 + 100X_4 - 100X_5\ \leq 1400$
$X_1,\ X_4,\ X_5,\ X_6 \geq 0$

Convert it to Standard form:
(Max with $\leq$)

$$Max\ \ 500X_1 - 100X_6 + 200X_4\ - 200X_5$$

subject to
$-15X_1 + 20X_6 - 60X_4 + 60X_5\ \leq -1200$
$20X_1 - 50X_6 + 100X_4 - 100X_5\ \leq 1400$
$X_1,\ X_4,\ X_5,\ X_6 \geq 0$

Dual:
1. $Max\ \Rightarrow\ Min$
2. # variable $\Rightarrow$ # constraints
3. # constraints $\Rightarrow$ #variables
4. $\leq\ \Rightarrow\ \geq$

$\therefore$ Dual:

$Min\ \ -1200Y_1 + 1400Y_2$

subject to
$-15Y_1 + 20Y_2 \geq 500$
$20Y_1 - 50Y_2 \geq -100$
$-60Y_1 + 100Y_2 \geq 200$
$60Y_1 - 100Y_2 \geq -200$
$Y_1,\ Y_2 \geq 0$
(I'm getting constraints' RHS wrong, I don't know why)

Q3. Convert the given Primal to dual

$$Max\ \ X_1 + 2X_2 + X_3$$

subject to
$2X_1 + X_2 - X_3\ \leq 2$
$-2X_1 + X_2 - 5X_3\ \geq -6$
$4X_1 + X_2 + X_3\ \leq 6$
$X_1, X_2, X_3\ \geq 0$

Convert it to Standard form:
(Max with $\leq$)

$$Max\ \ X_1 + 2X_2 + X_3$$

subject to
$2X_1 + X_2 - X_3\ \leq 2$
$2X_1 - X_2 + 5X_3\ \leq 6$
$4X_1 + X_2 + X_3\ \leq 6$
$X_1, X_2, X_3\ \geq 0$

Dual:
1. $Max\ \Rightarrow\ Min$
2. # variable $\Rightarrow$ # constraints
3. # constraints $\Rightarrow$ #variables
4. $\leq\ \Rightarrow\ \geq$

$\therefore$ Dual:

$Max\ \ 2Y_1 + 6Y_2 + 6Y_3$

subject to
$2Y_1 + 2Y_2 + 4Y_3 \geq 1$
$Y_1 - Y_2 + Y_3 \geq 2$
$-Y_1 + 5Y_2 + Y_3 \geq 1$
$Y_1,\ Y_2,\ Y_3 \geq 0$