Positive definiteness

* fn vanishes at (0,0) & strictly positive at all other points.

$$x = Ax^2 + 2Bxy + Cy^2 = [x \ y]\begin{bmatrix} A & B \\ B & C \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$[1\times2]\,[2\times2]\,[2\times1]$

$[1\times1]$

(i) $A>0$ & $AC>B^2 \rightarrow$ P.D

(ii.) $A>0$ & $AC=B^2 \rightarrow$ P.S.D

(iii.) $A<0$ & $AC=B^2 \rightarrow$ N.S.D

(iv.) $AC<B^2 \rightarrow$ SADDLE POINT

For Stationary point

$f_x=0,\ f_y=0 \quad \because$ [calculate x & y]

To know regarding maxima & minima

(1.) If (0,0) is stationary point & $A>0$ & $x^TAx>0$ for all other values then (0,0) is minima.

(ii) If (0,0) is stationary point & $A<0$ & $x^TAx>0$ for all other values then (0,0) is maxima.

To know regarding P.D

(1.) All $\lambda_i>0$

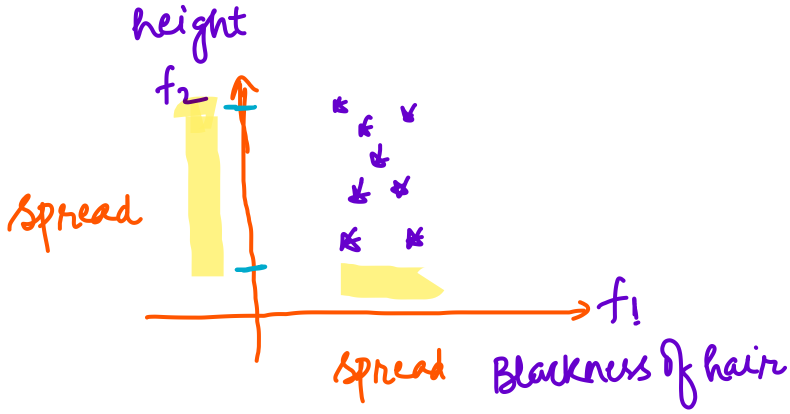
(2.) Pivot values >0

(3.) $x^TAx>0$ for all x.

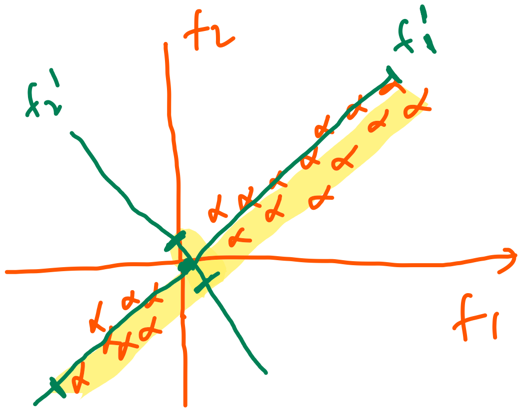
PCA

1) Why? → For visualization.
→ $d' < d$ for model training.

2) Geometric intuition



→ To reduce dimentionality from 2D → 1D, we can skip f_1.
→ By reducing f_1 we are loosing less information.
→ Preserving direction with maximum spread.
→



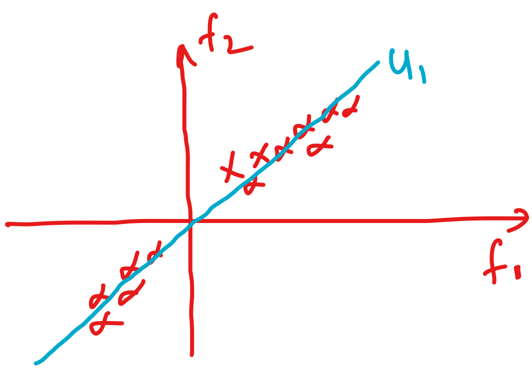
⇒ ① Find $f_1' \perp f_2'$
Spread on $f_2' << f_1'$

② Drop f_2'

③ Project x_i' onto f_1'
2D → 1D

So what we want :-

⇒ To find a direction f_1' such that the variance of x_i's projected on f_1' is maximum.



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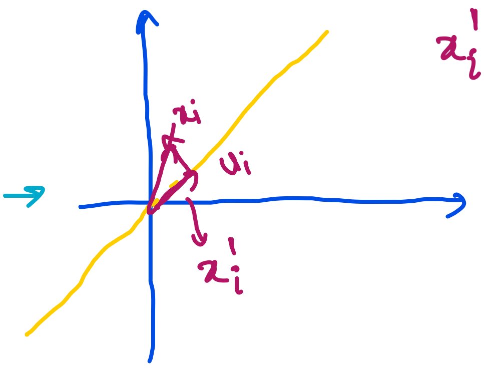


$x_i' = \mathrm{Proj}_{u_1} x_i$
$= \alpha x_i$
$= (x_i^T u) u$

Task:- find u_1 such that $Var\{Proj_{u_1} x_i\}$ is maximum

$$\Rightarrow Var\{Proj_{u_1} x_i\} = \frac{1}{n}\sum (u_1^T x_i - u_1^T \bar{x})^2$$

constrained $u_1^T u_1 = 1 = ||u_1||^2$

$$= \frac{1}{n}\sum u^T (x_i - \bar{x})(x_i - \bar{x})^T u$$

$$= u^T C u$$

problem:-

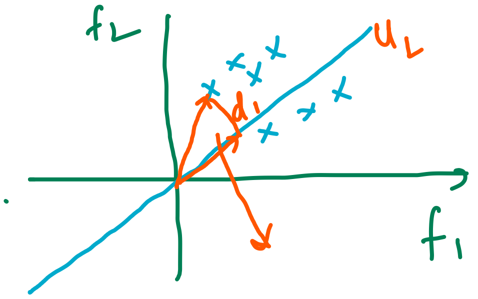
$$\max(u^T C u)$$

constrained $u^T u = 1$

After solving using Lagrangian.

u_1 denotes the eigenvector corresponding to max λ_1

2.) Minimising reconstruction error.



$$d_i^2 = ||x_i||^2 - (u_1^T x_i)^2$$

Problem

minimise $\sum (||x_i||^2 - (u_1^T x_i)^2)$

Constrained $u_1^T u_1 = 1$

Solution

$u_1 \longrightarrow$ corresponding to maximum λ_1

Things to remember:-

1.) $C = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(x_i-\bar{x})^T$

2.) $u_1, u_2, u_3 \longrightarrow$ Corresponding to $\lambda_1 > \lambda_2 > \lambda_3 \ldots\ldots$

3.) Projected data's $x_i' = \sum(x_i^T u_i)u_i$

4.) Reconstruction error $J^* = \frac{1}{n}\sum_{i=1}^{n}||x_i - x_i'||^2$