Let $f(x) = -2x^2 + 5$. At $x = -3$, is $f(x)$ increasing or decreasing?

- ○ increasing
- ○ decreasing

For a function $f(x) = -x + 2x^2$, the global minimum occurs at

1 point

- ○ $x = -0.25$
- ○ $x = -0.5$
- ○ $x = 0.5$
- ○ $x = 0.25$

Consider two convex functions $f(x) = x^2$ and $g(x) = e^{3x^2}$. Choose the correct convex function(s) that is a resultant of combination of $f(x)$ and $g(x)$.

1 point

- ☐ $h(x) = x^2 + e^{3x^2}$
- ☐ $h(x) = x^2 e^{-3x^2}$
- ☐ $h(x) = x^2 e^{3x^2}$
- ☐ $h(x) = x^2 - e^{3x^2}$

Consider two functions $g(x) = 2x - 3$ and $f(x) = x - 10ln(5x)$. Select the true statement.

1 point

- ○ $h = fog$ is a convex function.
- ○ $h = fog$ is a concave function.

(Common data for Q5-Q7)

Given below is a set of data points and their labels.

X	y
$[1, 0]$	1.5
$[2, 1]$	2.9
$[3, 2]$	3.4
$[4, 2]$	3.8
$[5, 3]$	5.3

To perform linear regression on this data set, the sum of squares error with respect to w is to be minimized.

Which of the following is the optimal w^* computed using the analytical method? **1 point**

- ○ $\begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$
- ○ $\begin{bmatrix} 1.255 \\ -0.317 \end{bmatrix}$
- ○ $\begin{bmatrix} 1.512 \\ 0.004 \end{bmatrix}$
- ○ $\begin{bmatrix} -1.5 \\ 0 \end{bmatrix}$

Let w^1 be initialized to $\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$. Gradient descent optimization is used to find the value of optimal w^*. For the first iteration $t = 1$, which of the following is the gradient computed with respect to w^1? **1 point**

- ○ $\begin{bmatrix} 50.6 \\ 28.3 \end{bmatrix}$

- $\begin{bmatrix} 8.6 \\ 4.9 \end{bmatrix}$
- $\begin{bmatrix} -50.6 \\ -28.3 \end{bmatrix}$
- $\begin{bmatrix} -8.6 \\ -4.9 \end{bmatrix}$

Using the gradient descent update equation with a learning rate $\eta_t = 0.1$, compute the value of w at $t = 2$.

1 point

- $\begin{bmatrix} 5.16 \\ 2.83 \end{bmatrix}$
- $\begin{bmatrix} 5.16 \\ 2.93 \end{bmatrix}$
- $\begin{bmatrix} 5.5 \\ 3.5 \end{bmatrix}$
- $\begin{bmatrix} 5.5 \\ -3.5 \end{bmatrix}$

(Common data for Q8-Q10)

A rectangle has a perimeter of 20 m. Using the Lagrange multiplier method, find the height and width of the rectangle which results in maximum area.

What is the optimal height?

1 point

What is the optimal width?

1 point

Enter the value of Lagrange multiplier.

1 point

Which of the following statements about primal and dual problems is (are) true?. **1 point**

- [ ] Dual of dual is primal.
- [ ] If either the primal or dual problem has an infeasible solution, then the value of the objective function of the other is unbounded.
- [ ] If either the primal or dual problem has a solution then the other also has a solution and their optimum values are equal.
- [ ] If one of the variables in the primal has unrestricted sign, the corresponding constraint in the dual is satisfied with equality.

(Common data for Q12, Q13)

Rahul is a consumer who wants to maximize his utility subject to some constraints. He consumes two goods x and y and the utility function is the product of x and y. His budget is Rs.1000. The per unit price of goods x and y are Rs.15 and Rs.20 respectively.

Choose the correct optimization problem. **1 point**

- ○ maximize $x + y$ subject to $(15x)(20y) = 1000$
- ○ maximize xy subject to $15x + 20y = 1000$
- ○ minimize xy subject to $15x + 20y = 1000$
- ○ minimize $x + y$ subject to $(15x)(20y) = 1000$

Choose the equivalent Lagrange function for the problem. **1 point**

- $L(x, y, z) = x + y - \lambda(15x + 20y - 1000)$
- $L(x, y, z) = x + y + \lambda(15x + 20y + 1000)$
- $L(x, y, z) = xy + \lambda(15x + 20y - 1000)$
- $L(x, y, z) = xy - \lambda(15x + 20y + 1000)$

Minimize the function $f = x_1^2 + 60x_1 + x_2^2$ subject to the constraints $g_1 = x_1 - 80 \geq 0$ and $g_2 = x_1 + x_2 - 120 \geq 0$ using KKT conditions. Which of the following is the optimal solution set? **1 point**

- $[x_1^*, x_2^*] = [80, 40]$
- $[x_1^*, x_2^*] = [-80, -40]$
- $[x_1^*, x_2^*] = [45, 75]$
- $[x_1^*, x_2^*] = [-45, -75]$