Course: Machine Learning - Foundations
Week 11 Questions

PRACTICE QUESTIONS

1. (1 point) The continuous random variable X represents the amount of sunshine in hours between noon and 8 pm at a skiing resort in the high season. The probability density function, $f(x)$, of X is modelled by

$$f(x) = \begin{cases} kx^2, & \text{for } 0 \leq x \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

   Find the probability that on a particular day in the high season there is more than two hours of sunshine between noon and 8 pm.

   Answer: 0.98

   Solution:
   We know that $\int_{-\infty}^{\infty} f(x)dx = 1$
   $\Rightarrow \int_0^8 kx^2 dx = 1$

   value of k can be calculated while solving the above equation.
   and then calculate $P(2 \leq X \leq 8) \Longrightarrow \int_2^8 kx^2 dx$

2. (1 point) Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 6x + bx^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Calculate Value of b

   Answer: -6

   Solution:
   We know that $\int_{-\infty}^{\infty} f(x)dx = 1$
   $\Rightarrow \int_0^1 6x + bx^2 dx = 1$

   value of b can be calculated while solving the above equation.

3. (1 point) Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 4x^k & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

find $E(X)$.

Answer: 0.8

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$
$\Rightarrow \int_0^1 4x^k = 1$

value of k can be calculated while solving the above equation.i.e $k = 3$

$E(X) = \int_0^1 x \times 4x^3 dx$

4. (1 point) The time that passenger train will reach the station is uniformly distributed between 2:00 PM and 4:00 PM. What is the probability that the train reaches station exactly at 04:00 PM?

Answer: 0

The probability that train reaches exactly at 4:30 PM is $\int_{4:30}^{4:30} f(x)dx = 0$
Since the area under curve at a particular instant of x value is zero.

5. (1 point) If X is an exponential random variable with rate parameter λ then which of the following statement(s) is(are) correct.

A. $P(X > x + k|X > k) = P(X > x)$ for $k, x \leq 0$.
B. $P(X > x + k|X > k) = P(X > k)$ for $k, x \leq 0$.
C. $P(X > x + k|X > k) = P(X > x)$ for $k, x \geq 0$.
D. $P(X > x + k|X > k) = P(X > k)$ for $k, x \geq 0$.

Answer: C

Solution :

$$P(X > x + k|X > k) = \frac{P((X > x + k) \cap (X > k)}{P(X > k)}$$

$$\Rightarrow \frac{P(X > x + k)}{P(X > k)} = \frac{e^{-\lambda \times (x+k)}}{e^{-\lambda k}}$$

$$\Rightarrow e^{-\lambda x}$$

Hence, Option C is correct

6. (1 point) The lifetime of a electric bulb is exponentially distributed with a mean life of 18 months. If there is 60% chances that a electric bulb will last mostly for t months, then what is the value of t?

Solution:
Given mean of exponential random variable (life of light bulb)=18 months.
$\Rightarrow \frac{1}{\lambda} = 18, \Rightarrow \lambda = \frac{1}{18}$.
Given $P(X \leq t) = 0.6$
$\Rightarrow 1 - e^{-\lambda t} = 0.6$
$\Rightarrow e^{-\lambda t} = 0.4$
$\lambda t = \ln 2.5$
$\frac{1}{18} t = \ln 2.5$
$t = 18 \ln 2.5$

7. (1 point) **(Multiple Select)**Let X be uniformly distributed with parameters a and b, then which of the following is/are true:

 A. $E(X^2) = \dfrac{(b^2 + a^2 + ab)}{3}$

 B. $f(x) = \dfrac{-1}{(a-b)} \quad ; \quad a \leq x \leq b$

 C. $E(X) = \dfrac{(b+a)}{2}$

 D. $V(X) = \dfrac{(b-a)^2}{12}$

Answer: A, B, C, D

Solution:
From the properties of uniform distribution, options (a), (b), (c) and (d) are correct.

8. (1 point) **(Multiple Select)** Which of the following option is/are correct?

 A. The shape of the Normal Curve is bell curved.

 B. Normal Distribution is symmetric is about mean.

 C. For a standard normal variate, the value of mean is 0.

 D. The area under a standard normal curve is 1.

 E. The standard normal curve is symmetric about the value 1.

Answer: A, B, C, D, E

9. (1 point) Let X and Y be continuous random variables with joint density

$$f_{XY}(x,y) \begin{cases} cxy & \text{for } 0 < x < 1,\, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P(0 < X < \frac{1}{2}, 0 < Y < \frac{1}{2})$

Answer: 0.015

Solution:
We know that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)dydx = 1$

$\int_0^1\int_0^1 cxydydx = 1$
c can be calculated from above equation.

10. (points) Let X is a uniformly distributed random variable with $\mu_x = 15$ and $\sigma_x^2 = \dfrac{25}{3}$. Calculate $P(X > 17)$

Answer: 0.5
$E[X] = \dfrac{b+a}{2} = 15.$

$Var(X) = \dfrac{(b-a)^2}{12} = 12$

a and b can be calculated from above equation.

Then calculate $P(X \geq 17)$

11. (1 point) Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} ax & \text{for } 0 < x < 3 \\ a(6-x) & \text{for } 3 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P(0 \leq x \leq 4)$

Answer: 0.77

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

Solving above equation taking required $f(x)$ we will get, $a = \dfrac{1}{9}$

12. Let X be exponentially distributed with parameter λ, then which of the following is/are true about the variance of X:

    a. $V(X) = E[X^2] - (E[X])^2$
    b. $V(X) = E[X - E[X]]^2$
    c. $V(X) = (E[X])^2$
    d. $V(X) = E[X^2]$

    Answer: A, B, C

    Solution: From definition A and B are correct and for exponential distribution option C are correct.