Merge Sort

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Programming, Data Structures and Algorithms using Python

Week 2

Beating the $O(n^2)$ barrier

- Both selection sort and insertion sort take time $O(n^2)$
- This is infeasible for $n > 10000$

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- Divide the list into two halves
- Separately sort the left and right half
- Combine the two sorted halves to get a fully sorted list

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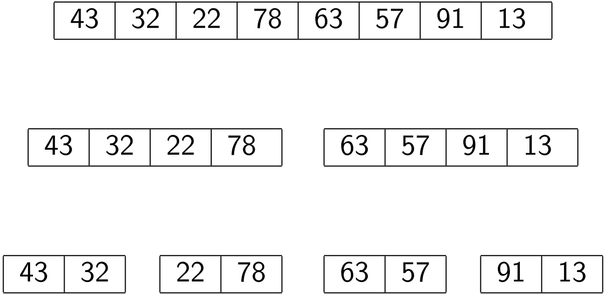

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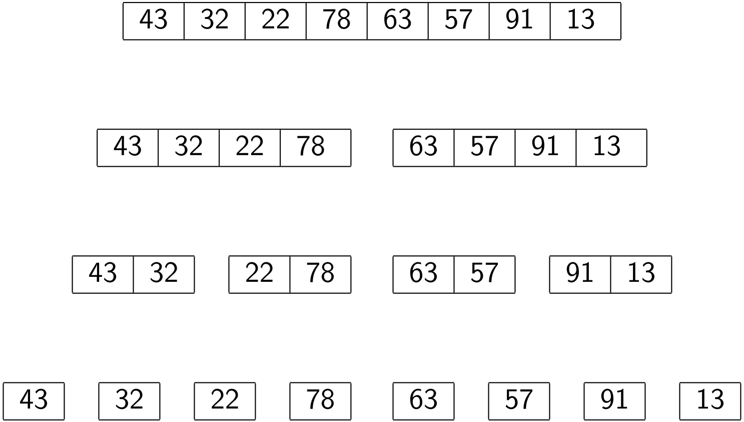

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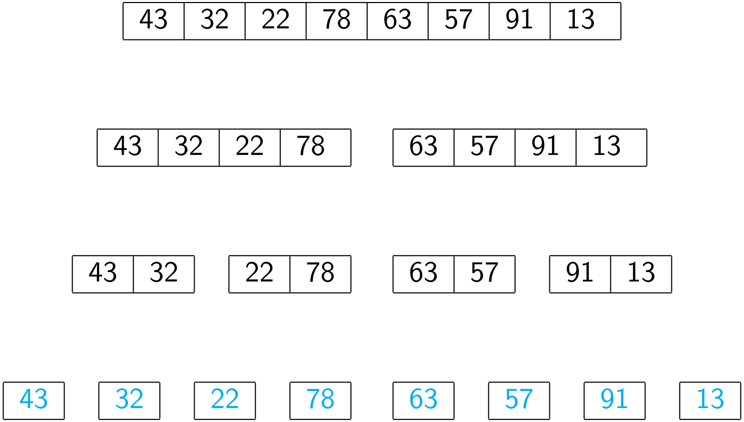

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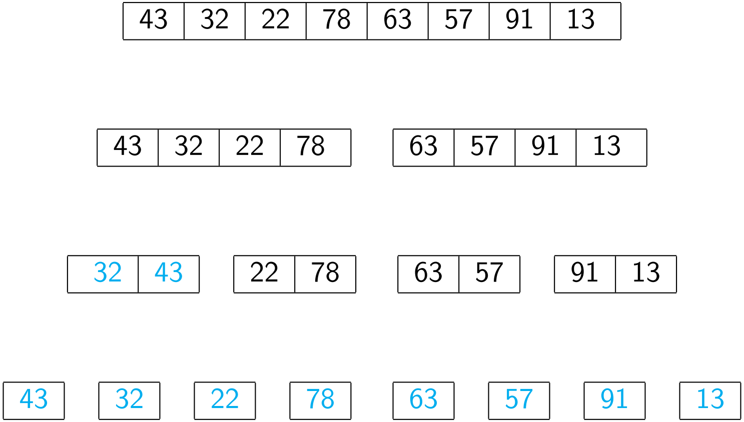

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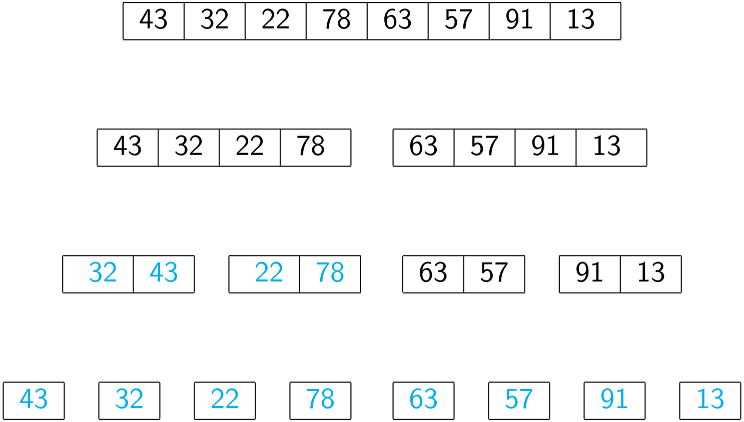

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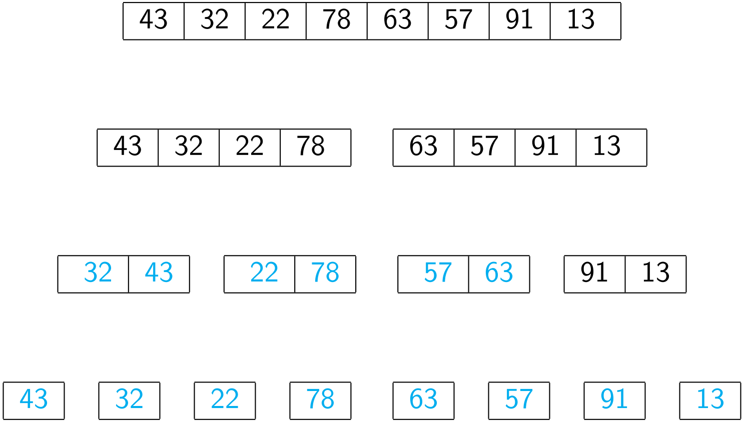

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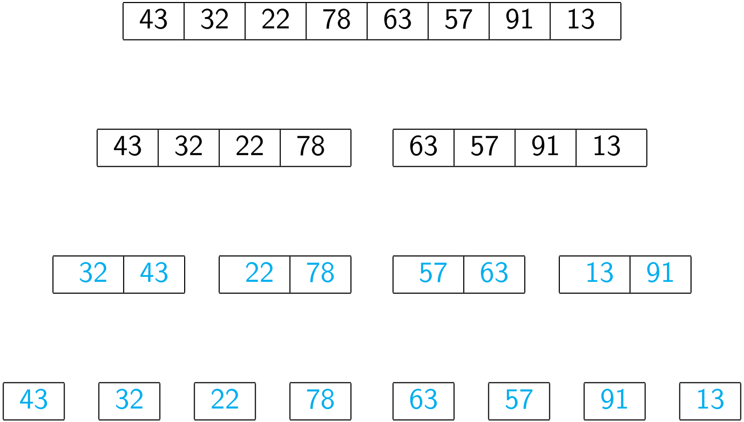

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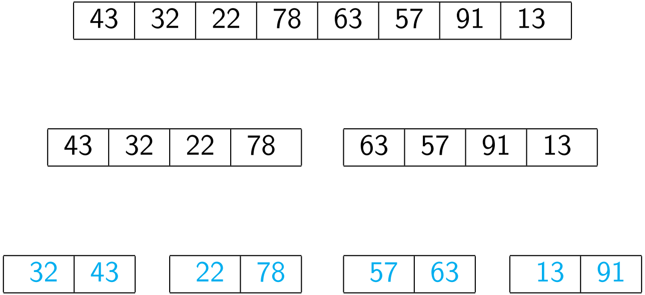

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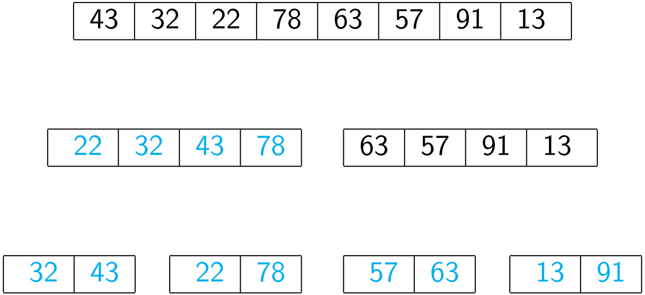

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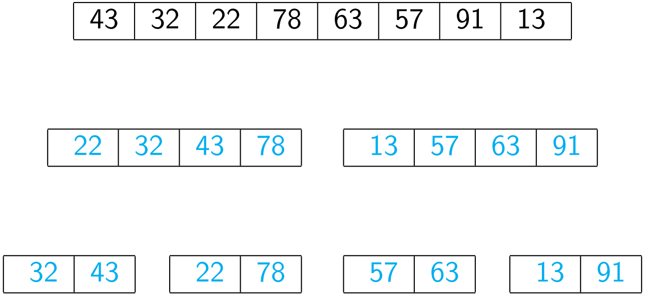

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Divide and Conquer

- Break up the problem into disjoint parts
- Solve each part separately
- Combine the solutions efficiently

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  - Repeat till all elements of `A` and `B` have been moved

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  - If B is empty, copy A into C
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```
def merge(A,B):
  (m,n) = (len(A),len(B))
  (C,i,j,k) = ([],0,0,0)
  while k < m+n:
    if i == m:
      C.extend(B[j:])
      k = k + (n-j)
    elif j == n:
      C.extend(A[i:])
      k = k + (n-i)
    elif A[i] < B[j]:
      C.append(A[i])
      (i,k) = (i+1,k+1)
    else:
      C.append(B[j])
      (j,k) = (j+1,k+1)
  return(C)
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- If $n \leq 1$, nothing to be done
- Otherwise
  - Sort `A[:n//2]` into `L`
  - Sort `A[n//2:]` into `R`
  - Merge `L` and `R` into `B`

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- To sort A into B, both of length n
- If $n \leq 1$, nothing to be done
- Otherwise
  - Sort A[:n//2] into L
  - Sort A[n//2:] into R
  - Merge L and R into B

```
def mergesort(A):
  n = len(A)

  if n <= 1:
     return(A)

  L = mergesort(A[:n//2])
  R = mergesort(A[n//2:])

  B = merge(L,R)

  return(B)
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- Merge the sorted halves
- Next, we have to check that the complexity is less than $O(n^2)$