

CHAPTER – 7

AVERAGES, MIXTURES AND ALLIGATIONS

AVERAGES

"Average is a very simple but effective way of representing an entire group by a single value."

"Average" of a group is defined as:

$$\text{Average} = \frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$$

"Sum of all the items in the group" means "sum of the values of all the items in the group".

A batsman's performance can be expressed as the average number of runs scored per innings rather than giving the scores in individual innings. For example, let us say a cricketer scored the following runs in 9 different innings in a year: 35, 56, 124, 29, 0, 87, 98, 45 and 75. Then his average score (per innings) for the year is

$$\frac{35 + 56 + 124 + 29 + 0 + 87 + 98 + 45 + 75}{9} = 61$$

Similarly, if there are 60 students in a class, instead of talking of the height of each individual student, we can talk of "average" height of the class. The average height of the class of students is equal to the sum of the heights of all the students of the class divided by the number of students in the class.

Average is also called the "mean" or mean value of all the values.

POINTS TO REMEMBER

- (i) If the value of each item is increased by the same value p , then the average of the group or items will also increase by p .
- (ii) If the value of each item is decreased by the same value p , then the average of the group or items will also decrease by p .
- (iii) If the value of each item is multiplied by the same value p , then the average of the group or items will also get multiplied by p .
- (iv) If the value of each item is divided by the same value p ($p \neq 0$), then the average of the group or items will also get divided by p .
- (v) The average of a group of items will always lie between the smallest value in the group and largest value in the group - i.e., the average will be greater than the smallest value and less than the largest value in the group.

AN EASY METHOD TO CALCULATE AVERAGES

As already discussed, the average of a group of items whose values are given can be found out by the rule given at the beginning of this section. However, in most of the cases, we do not need to perform such elaborate additions and divisions. The calculation of averages can be simplified greatly by taking some arbitrary number (P) as a starting point, take the deviations (differences) of the given items (Q_i) from this arbitrary number, find the average of all these deviations ($Q_i - P$) and algebraically add it to the arbitrary number (P) to give the correct average of the given items.

If there are n items and they are denoted by $Q_1, Q_2, Q_3, \dots, Q_n$, then the average of these n items is given by

$$\text{Average} = P + \frac{1}{n} \sum_{i=1}^n (Q_i - P)$$

The extent to which this method will simplify the calculation will depend on the selection of the arbitrary value P . It should be selected in such a way that the positive and negative deviations cancel out each other to the extent possible. Then the final figure left for division will be relatively small making the division easier.

For example, the cricketer that we considered above scored the following runs in seven innings: 35, 56, 45, 43, 67, 70 and 48. Now, to find his average, we take an arbitrary figure, say 50 and first find the deviations of each of the scores from this figure. The deviations of the scores from 50 are -15, +6, -5, -7, +17, +20 and -2. The sum of these deviations is +14.

Hence the average of the cricketer's scores is

$$50 + \frac{14}{7} = 52$$

Please note that the number P (= 50 above) can be any value. Let us work out the same example taking a different value for P . Let us take P equal to 45. The deviations of the scores from P are -10, +11, 0, -2, +22, +25 and +3. The sum of these deviations is 49. Hence the average is $45 + 49/7 = 45 + 7 = 52$.

WEIGHTED AVERAGE

When two groups of items are combined together, then we can talk of the average of the entire group. However, if we know only the average of the two groups individually, we cannot find out the average of the combined group of items.

For example, there are two sections A and B of a class where the average height of section A is 150 cm and that of section B is 160 cm. On the basis of this information alone, we cannot find the average of the entire class (of the two sections). As discussed earlier, the average height of the entire class is

$$\frac{\text{total height of the entire class}}{\text{total number of students in the entire class}}$$

Since we do not have any information regarding the number of students in the two sections, we cannot find the average of the entire class. Now, suppose that we are given that there are 60 students in the section A and 40 students in section B, then we can calculate the average height of the entire class which, in this case will be

$$\text{equal to } \frac{60 \times 150 + 40 \times 160}{60 + 40} = 154 \text{ cm.}$$

This average height 154 cm of the entire class is called "**weighted average**" of the class.

The above step in calculating the weighted average of the class can be rewritten as below:

$$\begin{aligned} & \frac{60 \times 150 + 40 \times 160}{60 + 40} \\ &= \frac{60}{100} 150 + \frac{40}{100} 160 = \frac{3}{5} 150 + \frac{2}{5} 160 \end{aligned}$$

It is clear from the above step that we would have been able to calculate the average height of the entire class even if we had not been given the number of students in the individual sections but only the **ratio** of the number of students in the two sections (which in this case is 3 : 2).

Even if there are more than two groups of items to be combined, then also the weighted average can be calculated by the same method. For example, if three sections in a class have their average marks as 75, 76 and 79 respectively and their respective strengths are 30, 35 and 35, then the average mark of the entire class is given by

$$\frac{30 \times 75 + 35 \times 76 + 35 \times 79}{30 + 35 + 35} = 76.75$$

The method of deviations we used for calculating averages can be applied to calculate weighted average also. Here, that method will involve finding out deviations from the arbitrarily chosen number and calculating the weighted average of these deviations. In the above example, if we take 70 as the arbitrary figure, then the deviations of the three observed values given from 70 are +5, +6 and +9. The weighted average of these deviations is

$$\frac{30 \times 5 + 35 \times 6 + 35 \times 9}{30 + 35 + 35} = \frac{675}{100} = 6.75.$$

Hence, the weighted average will be
70 + 6.75 = 76.75

The arbitrary figure chosen can be any figure and if it is selected, as in the previous case, between the smallest and largest observed figures, some of the deviations will be positive and some negative making the final division relatively simpler. For example, in the above case, if we take with 76 as the arbitrary figure, the deviations are -1, 0 and +3. Then the weighted average will be

$$\frac{30 \times (-1) + 35 \times 0 + 35 \times (+3)}{30 + 35 + 35} = \frac{75}{100} = 0.75$$

Hence, the weighted average will be
76 + 0.75 = 76.75

"Weighted Average" can be defined or calculated for any MIXTURE.

MIXTURES

Mixing of two or more qualities of things produces a mixture. When two items of different qualities are thus mixed, the quality of the resultant mixture lies in between the qualities of the original constituent items, i.e., it will be higher than the lowest quality and lower than the highest quality of the items being mixed.

In the above example that we took, the "quality" that we looked at was the height of the students. We could also have taken their weights or the marks scored by them or any other "quality" or "parameter" and calculated the "weighted average" value of that particular "quality" for the entire group.

Similarly, if two types of a product of different prices per unit are mixed, the unit price of the resultant mixture will lie between the prices of the two types that form the mixture.

Here, the average quality is essentially the **weighted average** of the two constituent items.

If q_1 is the quantity (or number of items) of one particular item of quality p_1 , and q_2 be the quantity (or number of items) of the second item of quality p_2 are mixed together to give a new mixture, then the **weighted average** value (p) of the quality of the mixture is given by

$$p = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2}$$

Even if there are more than two groups of items mixed, the weighted average rule can be applied. We will only have to take figures (as shown in the formula for the two groups) for all the groups in the numerator as well as the denominator and calculate the weighted average. For example, if there are four groups of quantities q_1, q_2, q_3 and q_4 whose respective qualities are p_1, p_2, p_3 and p_4 , then the weighted average quality of the group can be written as

$$p = \frac{p_1 q_1 + p_2 q_2 + p_3 q_3 + p_4 q_4}{q_1 + q_2 + q_3 + q_4}$$

A mixture can also be a solution - that is, a liquid mixed with another liquid which is normally water. The concentration of the solution is expressed as the proportion (or percentage) of the liquid in the total solution.

For example, if 10 litres of pure alcohol is mixed with 40 litres of water, then in a total solution of 50 litres, there is 10 litres of alcohol. Hence the concentration of this solution is 0.2 (= 10/50) or 20%.

Similarly, if 30 litres of pure milk is mixed with 10 litres of water, the concentration of this solution can be expressed as 75% (= 30/40) milk or 25% water.

We can also have two solutions mixed together to give a new solution. Such problems can also be handled in the same manner as other mixtures. In the weighted average rule, the quality of the constituents (p_1, p_2 , etc.) will then be the concentrations of various solutions mixed together.

Examples

7.01. Find the average of the scores of the tests taken by Ram given that his scores were 78, 82, 84, 86, 87, 89, 93 and 95.

Sol: Average score = $\frac{\text{Total score in all the tests}}{\text{Number of tests taken}}$
 $= \frac{78 + 82 + 84 + 86 + 87 + 89 + 93 + 95}{8} = 86.75$

7.02. The monthly incomes of Raja, his wife and their son are ₹6000, ₹4000 and ₹1880 respectively. Find their average monthly income.

Sol: Average income
 $= \frac{6000 + 4000 + 1880}{3} = ₹ 3960$

- 7.03.** Rajiv purchased three dozen mangoes at ₹10 per dozen, two dozen mangoes at ₹15 per dozen and five dozen mangoes at ₹16 per dozen. Find the average cost per dozen of the mangoes that he purchased.

Sol: The cost of first three dozen mangoes = (3) (10) = ₹30
The cost of next two dozen mangoes = (2) (15) = ₹30
The cost of next five dozen mangoes = (5) (16) = ₹80
Total cost of the mangoes purchased = ₹140.
Average cost per dozen

$$= \frac{\text{Total cost of Mangoes}}{\text{Number of dozens}} = \frac{140}{10} = ₹14.$$

- 7.04.** The average age of 5 men is 20 years. Their average age increased by 1 year when a new man joined them. Find the age of the new man.

Sol: Total age of 5 men = (5) (20) = 100 years
Total age of 6 men = (6) (21) = 126 years
The age of the new man = 126 – 100
i.e. 26 years

- 7.05.** Six kilograms of wheat costing ₹18 per kg is mixed with nine kilograms of wheat costing ₹12 per kg. Find the price per kg of the mixture.

Sol: Total cost of 6 kg = (6) (18) = ₹108.
Total cost of 9 kg = (9) (12) = ₹108.
Average cost of the mixture

$$= \frac{\text{Total cost}}{\text{Total quantity}} = \frac{108 + 108}{15} = ₹14.40 \text{ per kg.}$$

- 7.06.** A batsman had an average of 20 runs in the first 6 tests. In the second to the seventh tests his average score was 21. If he scored a total of 30 runs in the first and the seventh tests, find his score in the seventh test.

Sol: Total score in the first 6 tests = (6) (20) = 120.
Total score in the second to the seventh test = (6) (21) = 126.
Let his scores in the first and seventh tests be a and b respectively. Total score in the second to the seventh test – Total score in the first 6 tests = 6

$$\therefore b - a = 6 \quad \text{----- (1)}$$

Given, $b + a = 30$ ----- (2)
Solving (1) and (2), $b = 18$.

- 7.07.** The average marks of three sections in the tenth class were 90, 120 and 150. If the number of students in these sections are 30, 40 and 50 respectively, find the average mark of the tenth class.

Sol: Total mark of the first section = (90) (30) = 2700
Total mark of the second section = (120) (40) = 4800
Total mark of the third section = (150) (50) = 7500
Average mark of class X = $\frac{\text{Total mark}}{\text{Number of students}}$

$$= \frac{2700 + 4800 + 7500}{30 + 40 + 50} = \frac{15000}{120} = 125.$$

- 7.08.** The sales recorded by a salesman in 6 days of a week are as follows: ₹14,000 per day for the first 3 days, ₹13000 for the fourth day, ₹24000 for the fifth day and ₹26000 for the sixth day. Find his daily average sales over this period.

Sol: Total sales for the first 3 days = (3) (14000) = ₹42000
Sales on the fourth day = ₹13000
Sales on the fifth day = ₹24,000
Sales on the sixth day = ₹26000
Total sales on the 6 days = ₹105000
Average sales per day = $\frac{105000}{6} = ₹17500$

- 7.09.** Tarun earned an average of ₹1500 per month from January to April in a year. He earned an average of ₹1600 per month from May to October in that year. His earning in the month of December of that year was ₹300 more than his earning in the month of November of that year. His average monthly earnings for that year was ₹1675. Find his earnings in the month of November.

Sol: Total earnings of Tarun from January to April = (1500) (4) = ₹6000
Total earnings of Tarun from May to October = (1600) (6) = ₹9600.
Let his earnings in November be ₹x
His earnings in December = ₹(x + 300)

$$\frac{6000 + 9600 + x + x + 300}{12} = 1675$$

$$x = 2100.$$

- 7.10.** The average age of a group of friends is 37 years. If 6 new friends whose average age is 35 years join them, the average age of the entire group becomes 36 years. How many people were there in the group initially?

Sol: Let the initial number of people in the group be n.
The total age of the initial group of friends = 37n years
The total age of the six friends who joined the group = 35 × 6 i.e. 210 years.
Given that, $37n + 210 = 36(n+6)$

$$\Rightarrow 37n - 36n = 216 - 210$$

$$\therefore n = 6.$$

- 7.11.** The average age of the boys in a class is 18 years. The average age of the girls in that class is 12 years. If the ratio of the numbers of boys and girls in that class is 3 : 2, find the average age of the class.

Sol: Let the number of boys in the class be 3x.

$$\Rightarrow \text{Number of girls in that class} = 2x$$

Total age of the boys = (18) (3x) = 54x years.
Total age of the girls = (12) (2x) = 24x years.
Average age of the class

$$= \frac{54x + 24x}{3x + 2x} = 15.6 \text{ years}$$

- 7.12.** If 10 litres of 80% alcohol is mixed with 15 litres of 60% alcohol, find the concentration of the resulting solution.

Sol: Quantity of alcohol in 10 litres = (0.8) (10) = 8 litres. Quantity of alcohol in 15 litres = (0.6) (15) = 9 litres.
Concentration of the resulting solution

$$= \frac{8+9}{10+15} \times 100 = 68\%.$$

ALLIGATIONS

We will take the Weighted Average rule discussed in the previous section and rewrite the formula such that the quantity terms come on one side and the price terms come on the other side. If we do this we get the rule

$$\frac{q_1}{q_2} = \frac{p_2 - p}{p_1 - p}$$

This is called the RULE OF ALLIGATION. This rule connects quantities and prices in mixtures. This can also

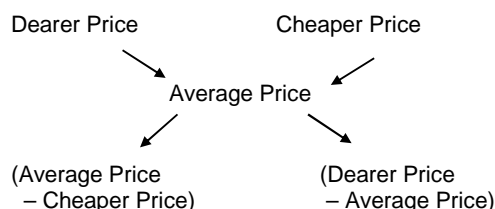
be written as
$$\frac{q_1}{q_2} = \frac{p_2 - p}{p_1 - p}$$

In a descriptive manner, the Rule of Alligation can be written as
$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

$$= \frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

This rule is a very powerful rule and is useful in problems on weighted averages and mixtures. This rule is also useful in a number of problems which can be treated as mixtures and applied to parameters other than price also. We will take examples where alligation rule can be applied.

In actual practice, to apply alligation rule, we do not need to remember the above formula at all. It can be made very simple by representing the rule pictorially. The above formula can be represented as follows:



{We write the dearer and cheaper prices in one line with some gap in between them. Then, we write the average price in between these two but in the line below the line in which dearer and cheaper prices are written. Then, take the differences of quantities as shown along the arrows and write along the same direction of the arrows continued, i.e., in a diagonally opposite corner. The difference between the top left hand quantity (Dearer Price) and Average Price has to be written at the bottom right hand corner. Similarly, the difference between the top right hand corner (Cheaper Price) and the Average Price has to be written at the bottom left hand corner. Now the ratio of the two quantities in the bottommost line will give us the ratio of the quantities of Dearer and Cheaper varieties. Please note that since we took Dearer Price on the top left corner, the ratio of the bottom left figure to that of the bottom right figure will give the ratio of Dearer Quantity to Cheaper Quantity}

7.13. How many kilograms of rice costing ₹6 per kg must be mixed with 6 kg of rice costing ₹8 per kg so that the resulting mixture costs ₹7.20 per kg?

Sol: Let x kg of rice costing ₹6 per kg be used for mixing.

Using alligation rule,

$$\frac{x}{6} = \frac{8 - 7.20}{7.20 - 6} = \frac{2}{3} \Rightarrow x = 4$$

7.14. A bag contains a total of 120 coins in the denominations of 50 p and ₹1. Find the number of 50 p coins in the bag if the total value of the coins is ₹100.

Sol: Let the number of 50 p coins be x.

$$\text{Total value of the coins} = ₹ \left[\frac{50x}{100} + 1(120 - x) \right]$$

$$\frac{50x}{100} + 120 - x = 100$$

$$\Rightarrow x = 40$$

Alternative method:

$$\text{Average value per coin} = \frac{(100)(100)}{120}$$

$$= \frac{500}{6} \text{ paise.}$$

$$\text{Using allegation rule, } \frac{x}{120 - x} = \frac{100 - \frac{500}{6}}{\frac{500}{6} - 50} = \frac{1}{2}$$

$$\Rightarrow 2x = 120 - x \Rightarrow x = 40.$$

7.15. A vessel has 60 litres of solution of milk and water having 80% milk. How much water should be added to it to make it a solution in which milk is 60%?

Sol: Let the quantity of water to be added be x litres.

$$\text{Quantity of milk in the vessel} = \left(\frac{80}{100} \right) (60)$$

$$= 48 \text{ litres} \Rightarrow 48 = 0.6 (60 + x)$$

$$\therefore x = 20$$

7.16. Two vessels contain mixtures of milk and water. The ratios of milk and water are 3 : 10 and 4 : 5 in the first and second vessels respectively. In what ratio should they be mixed so that the resulting mixture contains milk and water in the ratio 79 : 155?

Sol: Milk content in the first milk solution

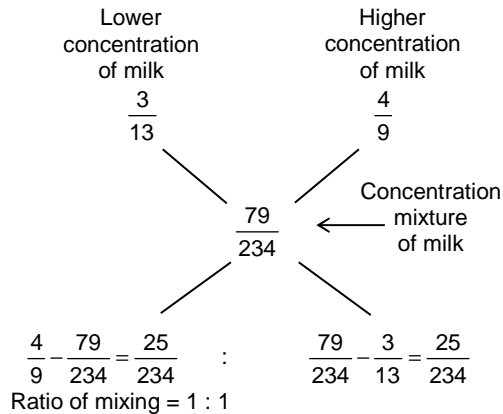
$$= \frac{3}{3+10} = \frac{3}{13}$$

Milk content in the second milk solution

$$= \frac{4}{4+5} = \frac{4}{9}$$

$$\text{Given that the milk content of mixture of the above two solutions} = \frac{79}{79+155}$$

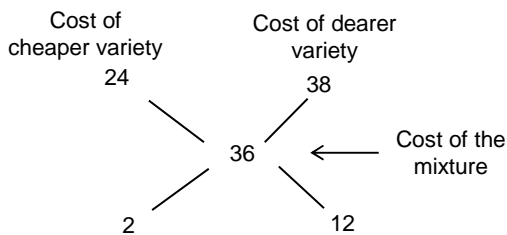
$$= \frac{79}{234}$$



- 7.17. In what ratio must Anand mix two varieties of barley costing ₹24 per kg and ₹38 per kg so that by selling the mixture at ₹40 per kg he would make $11\frac{1}{9}\%$ profit?

Sol: Cost price of the mixture = $\frac{40(100)}{100 + 11\frac{1}{9}}$

= ₹36 per kg



∴ The ratio of the costs of the cheaper to the dearer variety = 2 : 12 = 1 : 6.

If there is P volume of pure liquid initially and in each operation, Q volume is taken out and replaced by Q volume of water, then at the end of n such operations, the concentration (k) of the liquid in the solution is given by $\left\{\frac{P-Q}{P}\right\}^n = k$

This gives the concentration (k) of the liquid as a PROPORTION of the total volume of the solution.

If the concentration has to be expressed as a percentage, then it will be equal to 100k.

If the volume of the liquid is to be found out at the end of n operations, it is given by kP, i.e., the concentration k multiplied by the total volume P of the solution.

- 7.18. A vessel has 400 litres of pure milk. 40 litres of milk is removed from the vessel and replaced by water. 40 litres of the mixture thus formed is replaced by water. This procedure is repeated once again. Find the percentage of milk in the resultant solution.

Sol: Let v litre be volume of milk with a concentration of $c_1\%$.

If x litres of the solution is removed and replaced with water, the new concentration is $\left(\frac{v-x}{v} \times c_1\right)\%$

Given that initial concentration is 100%. (pure milk), $v = 400$, $x = 40$ and the replacement is done thrice.

∴ Concentration of milk in the resultant solution

$$= \left(\frac{400-40}{400}\right)^3 \times 100 = 72.9\%$$

Concept Review Questions

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The average of the numbers 41, 45, 49, 53, 57, 61, 65, 69 and 73 is
2. Find x , if the average of 17, 26 and x is 22.
(A) 18 (B) 22 (C) 1 (D) 23
3. The average of N numbers is x . If each number is doubled, find the new average.
(A) x (B) $2x$ (C) $x/2$ (D) $x + 2$
4. The average of a set of numbers is b . If each number is decreased by 50%, find the new average.
(A) b (B) $2b$
(C) $b/2$ (D) None of these
5. Find the average of the numbers from 21 to 30.
6. Find the average of all the two digit numbers divisible by 10.
(A) 40 (B) 50 (C) 45 (D) 60
7. Find the average of all odd numbers less than 50.
(A) 26.5 (B) 25.5 (C) 26 (D) 25
8. Find the average of all the multiples of 12 less than 100.
(A) 48 (B) 54 (C) 60 (D) 66
9. The average age of a class of 15 students is 15 years. A student whose age is 15 years joins the class. Find the new average age of the class (in years).
10. The average age of a class of 20 students is 20 years. A student whose age is 20 years leaves the class. Find the new average age of the class (in years).
(A) 20 (B) 21 (C) 22 (D) 19
11. There are five members in a family. Their present average age is 25 years. Find their average age after five years.
(A) 26 years (B) 27 years
(C) 30 years (D) 35 years
12. The present average age of a group of friends is 65 years. What is their average age thirty years ago, given that all the friends are at least 35-years old now?
(A) 35 years (B) 30 years
(C) 40 years (D) 25 years
13. The monthly income of Amar, Bhavan and Chetan in January 2005 were ₹4000, ₹4500 and ₹6500 respectively. Find their average monthly income in that month.
(A) ₹5000 (B) ₹6000
(C) ₹5500 (D) ₹6500
14. Sachin wrote a total of six tests. His average mark in those was 65. His average mark in the first five tests was 60. Find his mark in the sixth test.
15. The average age of a class of twenty students is 15 years. The average increases by three years, if the age of a teacher is included. What is the age of the teacher?
(A) 68 years (B) 86 years
(C) 78 years (D) 87 years
16. The average age of a group of students is 16 years. If a 66 year old teacher joins the group, the average increases by one year. What is the number of students in the group?
17. A total of 55 chocolates were distributed among 13 children such that each girl gets 3 chocolates and each boy gets 5 chocolates. Which of the following is the number of girls?
(A) 8 (B) 3 (C) 4 (D) 5
18. The average salary per month of a man for the first four months, next four months and the last four months of a year are ₹6000, ₹8000 and ₹13000 respectively. Find his average salary per month in that year (in ₹)
19. In an office there are 20 employees. The average heights of the male employees is 180 cm. The average height of the female employees is 170 cm. Find the average height of all the employees (in cm).
(A) 172 (B) 174
(C) 176 (D) Cannot be determined
20. In a class, there are 90 students. 30 of them are boys. Their average mark in a test is 75. The average mark of the girls in the test is 90. Find the average mark of the class in the test.
21. In a class, there are 50 students. The average weight of all the girls is 30 kg. The average weight of 30 of the boys is 40 kg. Which of the following can be a possible value of the average weight of the entire class (in kg)?
(A) 31 (B) 33 (C) 35 (D) 37
22. The average age of the boys in a class is ten years. The average age of the girls in the class is eight years. There are 50% more boys than girls in the class. Find the average age of the class (in years).

23. In a class there are 50 boys and 30 girls. The heights of boys and girls are in the ratio 4 : 3. Find the average height of the boys and girls in the class. (taken in cm)
 (A) 155
 (B) 160
 (C) 165
 (D) Cannot be determined
24. The average weight of all the students in a class equals the average of the average weight of the boys and that of the girls, which of the following holds true?
 (A) The numbers of boys and girls in the class are equal.
 (B) The average weights of the boys and the girls are equal.
 (C) At least one of (A) and (B)
 (D) Neither (A) nor (B)
25. A vessel has 20 litres of a mixture of milk and water having 60% milk. Five litres of pure milk is added to the vessel. Find the percentage of milk in the new solution. %
26. In what ratio must two kinds of coffee which cost ₹80 per kg and ₹108 per kg be mixed such that the resultant mixture costs ₹96 per kg?
 (A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 2 : 1
27. A trader mixes two varieties of rice one costing at ₹15 per kg and the other at ₹25 per kg so that by selling the mixture at ₹30 per kg he makes a profit. In what ratio he mixed them?
 (A) 1 : 2 (B) 1 : 1
 (C) 2 : 1 (D) Cannot be determined
28. Five kilograms of sugar costing ₹15 per kg is mixed with ten kilograms of sugar costing ₹12 per kg. The mixture is sold at 20% profit. Find the selling price of the mixture (in ₹/kg).
29. In what ratio should water and 84% milk solution be mixed to obtain 48% milk solution?
 (A) 4 : 3 (B) 9 : 13 (C) 2 : 3 (D) 3 : 4
30. Vessel A has 20 litres of a mixture of milk and water having 75% milk. Vessel B has x litres of a mixture of milk and water having 60% milk. The contents of the vessels are mixed to form a mixture having 66% milk. Find x.
31. A vessel has 30 litres of a mixture of milk and water containing 80% milk. How many litres of water must be added to the vessel to reverse the ratio of milk and water?
 (A) 60 (B) 75 (C) 90 (D) 120
32. Two solutions of sulphuric acid are mixed in the ratio 2 : 3. The concentrations of sulphuric acid in the first and second solutions are 10% and 20% respectively. Find the concentration of sulphuric acid in the final mixture.
 (A) 16% (B) 14% (C) 12% (D) 18%
33. A milkman has 15 litres of pure milk. How many litres of water have to be added to it so that he gets a 60% profit by selling at cost price?
 (A) 9 (B) 10 (C) 8 (D) 12
34. From 90 litres of pure milk, 9 litres is withdrawn and replaced by water. 9 litres of the mixture is then withdrawn and replaced by water. Find the ratio of milk and water in the present mixture.
 (A) 19 : 81 (B) 19 : 100
 (C) 81 : 19 (D) 81 : 100
35. In the above question, find the quantity of milk in the final mixture (in litres).

Exercise – 7(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

- Find the average of the first 20 multiples of 50.
(A) 500 (B) 512.5
(C) 525 (D) 537.5
- The average of eleven distinct positive integers is 21. If the average of the first 6 is 23 and the average of the last six is 22, find the sixth integer.
- The temperatures on Sunday, Monday and Tuesday were 28° C, 30° C and 32° C respectively. If the temperature of each day from Tuesday to Friday (both days inclusive) was 1° C less than that of the following day, what was the difference in the average temperatures of the first four days of the week and that of the last three days of the week?
(A) 4.5° C (B) 4.25° C
(C) 0.5° C (D) 4.75° C
- The average age of a board of 10 advisors of a company is the same as it was 3 years ago, on account of the replacement of one of the older advisors by a younger man. How much younger is the new man than the director whom he replaced?
(A) 24 years (B) 40 years
(C) 52 years (D) None of these
- The average weight of A, B, C, D is 60 kg. The average weight of A, B, C, D, E is 57 kg. The weight of F is 5 kg more than that of E. The average weight of B, C, D, E, F is 61 kg. Find the weight of A (in kg).
- The average of n numbers is a . If one of the numbers, y , is replaced by x , the average becomes b . Find the relation between n , a , x , y and b .
(A) $\frac{1}{n} = \frac{x-y}{a-b}$ (B) $\frac{1}{n} = \frac{a-b}{y-x}$
(C) $\frac{1}{n} = \frac{x+y}{a+b}$ (D) $\frac{1}{n} = \frac{a+b}{x+y}$
- The average weight of N boys in a group is 36 kg. If 20 other boys whose average weight is 30 kg join the group, the average weight of the group would be the same as what it would be if 5 boys whose average weight is 40 kg leave the group. Find N .
(A) 10 (B) 15 (C) 20 (D) 25
- The students of three classes X, Y and Z wrote a test. The average marks of the students of X, Y and Z are 70, 81 and 90 respectively. The average marks of the students of X and Y is 76. The average marks of the students of Y and Z is 86. Find the average marks of the students of all the three classes to the nearest integer.

- The students of three classes, A, B and C take a test. The average per student marks of the classes A and B put together is 71. The average marks per student of the classes B and C put together is 76. The average per student marks of the class A and C put together is 79. Find the range of the average marks (p) of all the three classes put together.
(A) $72 < p < 76$ (B) $73 < p < 75$
(C) $71 < p < 77$ (D) $73.5 < p < 77.5$
- Just before the last match in a season, the total number of runs scored by Sachin Tendulkar added up to 2100. In his last match he scored 101 runs. As a result his average score for the season went up by one run. Find the total number of matches he played in that season if he got out in every match.
(A) 31 (B) 5
(C) 71 (D) Either 31 or 71

11.

Class	Number of students	Average marks
A	32	83
B	58	76
C	82	85
D	48	90

Four classes A, B, C and D take an algebra test and the average score of each of the classes is as indicated above. Find the average score of all the four classes combined.

- (A) 82.3 (B) 84.5
(C) 85.6 (D) None of these
- The average of n numbers is 16. If $\frac{5}{8}$ th of the numbers are doubled and $\frac{3}{8}$ th of the numbers become $\frac{10}{3}$ times their original values, by what percentage does the current average exceed the original average?
(A) 20%
(B) 30%
(C) 25%
(D) Cannot be determined
- The average weight of all the students of classes I and II equals the average of the average weight of the students of the two classes. There are twice as many students in class II as in class I. The sum of twice the average weight of the students of class I and the average weight of the students of class II is 60 kg. Find the average weight of class I (in kg).
- Ajay attempted to add ten two-digit numbers. One of them, a , was the reverse of one of the others. If a was replaced by another two-digit number, b and the reverse of a was replaced by the reverse of b and the average was found, it would be 2.2 more. The sum of the digits in b exceeds the sum of the digits in a by
(A) 1 (B) 2 (C) 3 (D) 4

15. There are N students in a class. Their class teacher gave them a task of finding the average of the first N natural numbers and each of them left out a different number and found the average of the remaining numbers. The average of the averages obtained by all the students was 21. Find N.
(A) 39 (B) 40 (C) 41 (D) 38
16. Two alloys A and B contain copper and zinc in the ratio 4 : 9 and 5 : 6 respectively. If equal weights of the two are melted together to form a third alloy, find the ratio of the weights of copper and zinc in the third alloy named C.
(A) 109 : 167 (B) 113 : 164
(C) 109 : 177 (D) 107 : 158
17. Two varieties of wheat are mixed in the proportion of 3 : 4 and the mixture is sold at ₹28 per kg at a profit of 40%. If the second variety of wheat costs ₹3 more than the first variety of wheat, find the cost price of the first variety of wheat.
(A) ₹128/7 per kg (B) ₹120/7 per kg
(C) ₹141/7 per kg (D) ₹ $\frac{149}{7}$ per kg
18. A man buys milk at ₹4 per litre, mixes it with water and sells the mixture at the same price. If his profit is 25%, find the amount of water mixed with each litre of milk.
(A) 0.25 litres (B) 0.5 litres
(C) 0.75 litres (D) 0.6 litres
19. A vessel has 10 ml of a solution of milk and water containing 20% milk. x ml of milk was added to the vessel to reverse this ratio. y ml of water was then added to the vessel to reverse the ratio once again. Find x + y.
20. The average salary of the employees of two groups A and B in an organization is ₹4000. The average salary of the group A employees is ₹4800. The average salary of the group B employees is ₹3000. If there are 10 more employees in A than in B, find the number of group A employees.
(A) 35 (B) 50 (C) 45 (D) 40
21. Raju bought 40 kg of a certain variety of coffee powder costing ₹240/kg and 60 kg of another variety of coffee powder costing ₹360/kg. He mixed the two varieties of coffee powder and sold $\frac{4}{5}$ of the mixture at ₹360/kg and the rest at ₹270/kg. The overall profit percentage made by Raju is
(A) $9\frac{8}{13}\%$ (B) $11\frac{4}{7}\%$
(C) $5\frac{5}{11}\%$ (D) $17\frac{2}{19}\%$
22. In what proportion can three varieties of sugar priced at ₹10 per kg, ₹12 per kg and ₹18 per kg, be mixed so that the price of the mixture is ₹14 per kg?
(A) 2 : 2 : 5 (B) 2 : 3 : 4
(C) 1 : 3 : 4 (D) 3 : 4 : 5
23. There are two containers having mixtures of hydrochloric acid and water. In container 1, the ratio of hydrochloric acid and water is 1 : 2 and in container 2 the ratio of hydrochloric acid and water is 4 : 1. Find the amount of the mixture that should be taken from container 1 in order to make 28 litres of a mixture containing equal amount of water and hydrochloric acid. (in litres)
24. The ratio of alcohol and water in three mixtures of alcohol and water is 3 : 2, 4 : 1 and 7 : 3. If equal quantities of the mixtures are drawn and mixed, the concentration of alcohol in the resulting mixture will be _____.
(A) 65% (B) 70% (C) 75% (D) 80%
25. Fresh dates contain 90% water while dry dates contain 28% water. How many kilograms of dry dates can be obtained from 36 kg of fresh dates?
26. From 700 litres of milk, 70 litres are drawn and replaced with water. 70 litres of this mixture is then again drawn and replaced with water and the process is continued two more times. Find the quantity of milk present in the solution (in litres) now.
(A) 432.6 (B) 459.2
(C) 481.2 (D) 495.6
27. From a vessel containing only alcohol, six litres are drawn and replaced with water. six litres of the mixture is now taken out and replaced with water. The ratio of alcohol to water now is 9 : 16. How many litres of alcohol was there initially?
28. In what proportion should milk and water be mixed to reduce the cost of litre of milk from ₹18 per litre to ₹16?
(A) 8 : 1 (B) 6 : 1 (C) 10 : 1 (D) 7 : 1
29. A vessel contains a mixture of 100 litres of milk and water. The concentration of milk is 90%. 10% of the contents of the vessel are withdrawn and replaced with an equal amount of water. The minimum number of times that this procedure must be carried out such that the concentration of milk in the vessel is less than $66\frac{2}{3}\%$ is
30. Two vessels P and Q were partially filled with milk. 80% of the contents of P were transferred to Q. 60% of the contents of Q were then transferred back to P. At this stage, the ratio of the quantities of milk in P and Q is 23 : 12. Find the ratio of the initial volumes of milk is P and Q.
(A) 5 : 3 (B) 10 : 6
(C) 15 : 11 (D) 5 : 2

Exercise – 7(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. A's weight equals the average weight of B, C and D. B's weight equals the average weight of A, C and D. The average weight of C and D is 30 kg. Find the average weight of A and B.
(A) 15 kg (B) 30 kg
(C) 60 kg (D) 45 kg
2. The average weight of 18 students of a class is 15 kg. If a student leaves the class, the average weight of the class drops by 0.8 kg. Find the weight of the student (in kg) who left the class.
3. In a group of 11 people, x is 32 years old and y is 4 years younger than x. If x and y are replaced by two other people, the average age of the group drops by 1 year. Find the average age of the two people replacing x and y.
(A) 26 years
(B) 28 years
(C) 29 years
(D) 24.5 years
4. A basket ball player played nine matches. The average number of points he scored per match was 16. His points in the i^{th} match were two less than that in the $(i - 1)^{\text{th}}$ match. Find the average number of points scored in the second and the eighth matches.
(A) 8 (B) 12 (C) 16 (D) 18
5. There are nine two-digit numbers with distinct tens digits. The units digit of each number is one less than its tens digit. Find the average of the units digits.
6. The average score in Mathematics of a class of 36 students is 60. If the top two scores are excluded, the average goes down by 2. If the second highest score of the class is 90, find the highest score of the class.
(A) 98 (B) 96 (C) 97 (D) 95
7. The average score of a class of 32 students is 70. If the top score and the least score, which differ by 70, are excluded, the average score of the class drops by 1. Find the top score.
8. Of five numbers, the first number is thrice the third, the fourth number is two less than the first, the fifth number is one-seventh of the second and the second number is three less than thrice the first. Find the fifth number, if the average of the numbers is 16.2.
(A) 3 (B) 4 (C) 5 (D) 6
9. There are eleven people in a group. The average age of the oldest and the youngest is 11 years. If any one person leaves the group, the maximum and the minimum average of the remaining are 12 years and 11 years respectively. Find the average age of the entire group, in years.
(A) 9 (B) 10 (C) $11\frac{7}{11}$ (D) $11\frac{5}{11}$
10. A boy finds the average of ten positive integers. Each integer is a two-digit number. By mistake, the boy interchanges the digits of one of the numbers. Due to this, the average becomes 1.8 less than the previous one. Find the difference between the two digits of the number.
(A) 1 (B) 2 (C) 3 (D) 4
11. There are 60 persons. The age of each person (in years) is a two digit number. The average age of the persons is A years. One of the persons whose age is 'ab' is replaced by a new person whose age is 'ba'. If the average age of the new group is 0.8A years. Find the maximum value of A.
(A) 12 (B) 10.8
(C) 11.4 (D) Data inconsistent
12. Just before the last match of a season, the total runs of Rahul for the season added up to 1200. In his last match, he scored 20 runs and his average (Number of runs / Total matches) remained unchanged. Find the number of matches that he played in all.
13. The members of a club meet for lunch every Monday. Last week, just before the bill was presented, six of the members were called for an official meeting and hence they left. The remaining members were presented with a bill of ₹2160. It was customary to divide the bill equally. To cover the share of those who left, each member had to pay ₹81 more. How many members met for lunch?
(A) 16 (B) 12 (C) 18 (D) 24
14. Arjun ate a number of chocolates on each of the 5 week days of a certain week. On Tuesday, he ate 2 more than on Monday and 8 less than on Wednesday. On Friday, he ate 4 more than on Thursday and 6 less than on Wednesday. The average number of chocolates he ate on the first three days and the last two days are in the ratio 4 : 3. Find the number of chocolates he ate on Thursday.
(A) 3 (B) 4 (C) 5 (D) 6
15. There are nine three-digits numbers with distinct units digits. Each number is reversed and the reversed number is subtracted from the original number. The results were found to have an average of 0. If for each number, the hundreds digit is not less than its units digit, find the average of the hundreds digits of the greatest and the least numbers.
(A) 4 (B) 5 (C) 6 (D) 7

16. The average of the ages of a man and his only son exceeds the average of the ages of his wife and his son by 25%. The age of his son is 30 years less than the age of his wife whose age is 10 years less than his age. Find the age of his son (in years).

17. There are two classes A and B. The average weight of the students in class A is 40 kg. The average weight of the students in class B is 60 kg. A student, whose weight is x kg left A and joined B. As a result, the average weight of A, as well as that of B decreased. Which of the following must be true?
 (A) $35 < x \leq 40$ (B) $40 < x < 60$
 (C) $60 \leq x < 65$ (D) $30 < x \leq 35$

Directions for questions 18 and 19: These questions are based on the following data.

Each of four persons A, B, C and D, has a certain sum of money. The average amount with them is ₹200. If the money with A increases by 100% and that with B increases by 200%, then the average money with the four increases by ₹62.50. If the percentage increases of the amounts with A and B gets swapped, average money with A and B will be ₹275.

18. Find the money A originally has.
 (A) ₹150 (B) ₹200 (C) ₹175 (D) ₹180
19. If C gives ₹50 to D, he will have twice the money that D would then have. Find the amount that C originally has.
 (A) ₹500 (B) ₹400
 (C) ₹475 (D) None of these
20. In an office, the average salary of the employees is ₹8000. If the average salary of all the 50 clerks is ₹3000 and the average salary of all the managers is ₹10000, find the difference in the number of clerks and the number of managers.
-
21. The average cost per fruit of 5 apples and 4 mangoes is ₹48 and the average cost per fruit of 7 apples and 8 mangoes is ₹60. Find the total cost of 24 apples and 24 mangoes.
 (A) ₹1332 (B) ₹2424 (C) ₹2664 (D) ₹3996
22. The students of three classes P, Q and R take a test. The average mark per student of P and Q put together is 60. The average mark per student of Q and R put together is 64. The average mark per student of P and R put together is 72. If the average mark per student of all the classes put together is x , which of the following is a possible value of x ?
 (A) 61.5 (B) 61.9 (C) 62.5 (D) 68.3
23. Four classes P, Q, R and S take up a test. The ratio of the number of students in P, Q, R and S is 2 : 3 : 6 : 4. The ratio of the average marks of P, Q, R and S is 5 : 4 : 3 : 2. If the average marks of all combinations of three sections are considered, the greatest is 52. Find the least.
-

24. A sum of ₹7.75 is made up of 100 coins, which are in the denominations of 5 paise and 10 paise. Find the number of 5 paise coins.
 (A) 50 (B) 55 (C) 75 (D) 45
25. ₹8000 is lent in two parts. A part is lent at 10% p.a. and the other part is lent at 20% p.a. Both parts are lent at simple interest for a year. The total interest realized from them is ₹960. Find the difference of the parts (in ₹).
 (A) 4800 (B) 2400 (C) 1600 (D) 6400
26. A businessman lends ₹1800 in two parts, one at 10% and the other at 12% interest. At the end of the year, the average interest he obtained worked out to be 10.5%. Find the interest earned by the businessman from the part which was lent at 10%.
 (A) ₹135 (B) ₹150
 (C) ₹200 (D) ₹250
27. Raju covered a part of a total distance of 250 km at 30 kmph and the remaining part at 80 kmph. He covered the entire distance in 5 hours. Find the distance he covered at 80 kmph (in km).
-
28. A person travels 250 km in 5 hours in two parts. He travels the first part of the journey by scooter at an average speed of 45 kmph, and the second part by car at an average speed of 70 kmph. Find the distance he travelled by scooter.
 (A) 160 km (B) 180 km
 (C) 150 km (D) 200 km
29. A litre of pure milk is added to 6 litres of 30% milk. Find the percentage water in the solution.
 (A) 50% (B) 55% (C) 65% (D) 60%
30. A 350 ml solution contains 10% of milk. How much water should be added to it in order to bring down the percentage of milk to 7% in the solution?
 (A) 15 ml (B) 80 ml
 (C) 125 ml (D) 150 ml
31. A vessel contains pure milk costing ₹20 per litre. A certain quantity of water, which is free of cost, is added to it. As a result the cost of the mixture becomes ₹18 per litre. Find the ratio of milk and water in the mixture.
 (A) 8 : 1 (B) 1 : 9
 (C) 1 : 8 (D) 9 : 1
32. A milkman buys milk at ₹5 per litre. He mixes it with water and sells the mixture at the cost price and thereby makes a profit of 25%. Find the quantity of water he mixed with each litre of milk (in litres).
-
33. Vessels A and B contain mixtures of milk and water. The ratios of milk and water in A and B is 4 : 5 and 3 : 1 respectively. In what ratio should the contents of A and B be drawn and mixed, to obtain a mixture having milk and water in the ratio 3 : 2?
 (A) 27 : 28 (B) 23 : 24
 (C) 11 : 12 (D) 35 : 36

34. Two vessels contain alcohol and water in the ratio 3 : 4 and 65 : 79. In what ratio should the solution in the first vessel be mixed with the solution in the second, so as to get a solution with alcohol and water in the ratio 4 : 5?

(A) 7 : 16 (B) 16 : 7
(C) 36 : 85 (D) 85 : 36

35. A grocer had two varieties of rice, A and B. A and B were priced at ₹20 per kg and ₹30 per kg respectively. He mixed them in the same ratio as their prices and sold the mixture at ₹39 per kg.

(i) Find the cost of the mixture which he would have obtained if he had interchanged the quantities of A and B (in ₹ per kg).

(A) 20 (B) 22 (C) 24 (D) 26

(ii) Find the percentage of profit that he currently makes.

(A) 40% (B) 45%
(C) 50% (D) 62.5%

36. Three varieties of rice, A, B and C costing ₹6/kg, ₹9/kg and ₹12/kg are mixed together in a certain

ratio. The mixture is sold at $66\frac{2}{3}\%$ profit for ₹15/kg. Of

the total of 100 kg of the mixture, 50 kg is variety B. Find the quantity of variety A (in kg).

37. Two varieties of tea costing ₹280 per kg and ₹180 per kg are mixed in certain ratio to form varieties P and Q. P and Q are mixed in the ratio 1 : 2 to form variety R which is sold for ₹288 per kg at 20% profit. If P costs ₹200 per kg, in what ratio were the two varieties mixed to form Q?

(A) 2/1 (B) 5/2 (C) 4/1 (D) 8/3

38. A local grocer mixed three qualities of coffee T_1 , T_2 and T_3 at ₹56 per kg, ₹64 per kg and ₹80 per kg in the ratio 1 : 2 : 4. To 4 kg of this mixture, he added p kg of T_1 and 4p kg of T_3 . The final mixture so formed is sold for ₹87.60 per kg at 20% profit. Find p.

(A) 1/3 (B) 5/9 (C) 2/3 (D) 4/11

39. A vessel is full of a mixture of milk and water, with 9% milk. Nine litres are withdrawn and then replaced with pure water. If the milk is now 6%, how much does the vessel hold?

(A) 27 litres (B) 18 litres
(C) 36 litres (D) 40 litres

40. A vessel is filled to its capacity with pure milk. Nine litres are withdrawn from the vessel and replaced with an equal amount of water. Nine litres of the mixture is again withdrawn and then replaced with an equal amount of water. After these changes, the vessel contains 17.1 litres of milk less than it did initially.

(i) Find the capacity of the vessel. (in litres)

(ii) What is the least number of such additional replacements required, so that the vessel contains less than 75% milk?

Directions for questions 41 to 45: Each question is followed by two statements I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.

Mark (B) if the question can be answered using either statement alone.

Mark (C) if the question can be answered using I and II together but not using I or II alone

Mark (D) if the question cannot be answered even using I and II together.

41. Is the average age in a class of 20 students more than 15 years?

I. Eighteen students in the class are each 15 years old.

II. The average age of four of the students in the class is less than 15 years.

42. A certain alloy contains lead, copper and tin. How many kilograms of tin is contained in 60 kilograms of the alloy?

I. By weight, the alloy has $\frac{2}{5}$ th lead and $\frac{3}{16}$ th copper.

II. By volume, the alloy has $\frac{1}{3}$ rd lead and $\frac{1}{3}$ rd copper.

43. If the arithmetic mean of six numbers is 30, then how many numbers are greater than 30?

I. Three of the six numbers are equal to 40.

II. Three of the six numbers are equal to 20.

44. Between two mixtures x and y – each containing milk and water, the concentration of which mixture is more?

I. x has three parts of water to seven parts of milk.

II. y has seven parts of water to thirteen parts of milk.

45. In a certain class, x boys and y girls took a test. Was the average score of x boys greater than 83?

I. The sum of all the scores was $85(x + y)$.

II. The average score of y girls was 86.

Key

Concept Review Questions

1. 57	8. B	15. C	22. 9.2	29. D
2. D	9. 15	16. 49	23. D	30. 30
3. B	10. A	17. D	24. C	31. C
4. C	11. C	18. 9000	25. 68	32. A
5. 25.5	12. A	19. D	26. C	33. A
6. B	13. A	20. 85	27. D	34. C
7. D	14. 90	21. D	28. 15.60	35. 72.9

Exercise – 7(a)

1. C	6. B	11. D	16. C	21. A	26. B
2. 39	7. A	12. D	17. A	22. D	27. 15
3. B	8. 82	13. 20	18. A	23. 18	28. A
4. D	9. D	14. B	19. 150	24. B	29. 3
5. 30	10. D	15. C	20. B	25. 5	30. D

Exercise – 7(b)

1. B	9. D	17. B	25. A	33. A	40. (i) 90
2. 28.6	10. B	18. A	26. A	34. A	(ii) 1
3. D	11. D	19. D	27. 160	35. (i) C	41. C
4. C	12. 61	20. 75	28. B	(ii) C	42. A
5. 4	13. A	21. C	29. D	36. 25	43. C
6. A	14. B	22. C	30. D	37. C	44. C
7. 120	15. B	23. 41.8	31. D	38. D	45. D
8. D	16. 5	24. D	32. 0.25	39. A	