CHAPTER – 2

RATIO – PROPORTION – VARIATION

RATIO

If the values of two quantities A and B are 4 and 6 respectively, then we say that they are in the ratio 4 : 6 (read as "four is to six"). Ratio is the relation which one quantity bears to another of the same kind, the comparison being made by considering what multiple, part or parts, one quantity is of the other. The ratio of two quantities "a" and "b" is represented as a : b and read as "a is to b". Here, "a" is called antecedent, "b" is the consequent. Since the ratio expresses the number of times one quantity contains the other, it's <u>an abstract</u> quantity.

Ratio of any number of quantities is expressed after removing any common factors that ALL the terms of the ratio have. For example, if there are two quantities having values of 4 and 6, their ratio is 4 : 6, i.e., 2 : 3 after taking the common factor 2 between them out. Similarly, if there are three quantities 6, 8 and 18, there is a common factor among all three of them. So, dividing each of the three terms by 2, we get the ratio as 3 : 4 : 9.

If two quantities whose values are A and B respectively are in the ratio a : b, since we know that some common factor k(>0) would have been removed from A and B to get the ratio a : b, we can write the original values of the two quantities (i.e., A and B) as ak and bk respectively. For example, if the salaries of two persons are in the ratio 7 : 5, we can write their individual salaries as 7k and 5k respectively.

A ratio a : b can also be expressed as a/b. So if two items are in the ratio 2 : 3, we can say that their ratio is 2/3. If two terms are in the ratio 2, it means that they are in the ratio of 2/1, i.e., 2 : 1.

"A ratio is said to be a ratio of greater or less inequality or of equality according as antecedent is greater than, less than or equal to consequent". In other words,

- the ratio a : b where a > b is called a ratio of greater inequality (example 3 : 2)
- the ratio a : b where a < b is called a ratio of less inequality (example 3 : 5)
- the ratio a : b where a = b is called a ratio of equality (example 1 : 1)

From this we can find that a ratio of greater inequality is diminished and a ratio of less inequality is increased by adding same quantity to both terms, i.e., in the ratio a : b, when we add the same quantity x (positive) to both the terms of the ratio, we have the following results

if a < b then   (a + x) : (b + x) > a : b
if a > b then   (a + x) : (b + x) < a : b
if a = b then   (a + x) : (b + x) = a : b

This idea can also be helpful in questions on Data Interpretation when we need to compare fractions to find the larger of two given fractions.

If two quantities are in the ratio a : b, then the first quantity will be a/(a + b) times the total of the two quantities and the second quantity will be equal to b/(a + b) times the total of the two quantities.

Examples

2.01. The scores of Mohan and Sohan in a test are in the ratio 5 : 4. If their total score is 135, find Mohan's score.

Sol: As their scores are in the ratio of 5 + 4, let their scores be 5k and 4k.
Given that the sum of their scores = 5k = 4k = 135
$\Rightarrow$ 9k = 135
$\Rightarrow$ k = 15
$\therefore$ Mohan's score = 5k = 5 $\times$ 15 = 75

Alternative method:
Mohan's score = $\dfrac{5}{5+9}(135) = 75$

2.02. If a : b = 3 : 4, find 3a + 4b : 4a + 5b.

Sol: 3a + 4b : 4a + 5b

$= \dfrac{3a+4b}{4a+5b} = \dfrac{\dfrac{3a+4b}{b}}{\dfrac{4a+5b}{b}}$

$= \dfrac{3\left(\dfrac{a}{b}\right)+4}{4\left(\dfrac{a}{b}\right)+5} = \dfrac{3\left(\dfrac{3}{4}\right)+4}{4\left(\dfrac{3}{4}\right)+5} = \dfrac{25}{32}$

2.03. The ratio of the number of marbles with Ram and Shyam is 19 : 13. If Ram gives Shyam 30 marbles, both will have equal number of marbles. Find the number of marbles with Ram.

Sol: Let the number of marbles with Ram and Shyam be 19x and 13x respectively. Total number of marbles with them = 32x
If Ram gives Shyam 30 marbles each will have
$\dfrac{32x}{2} = 16x$ marbles.
$\therefore$ 19x – 16x = 30
x = 10
19x = 190

2.04. Two numbers are in the ratio 3 : 4. What part of the larger number must be added to each number so that their ratio becomes 5 : 6?

Sol: Let the two numbers be 3x and 4x.
$\dfrac{3x+k}{4x+k} = \dfrac{5}{6}$
18x + 6k = 20x + 5k
k = 2x
$\therefore$ Half of the larger number must be added to each number.

2.05. 1400 is divided into 4 parts such that twice the first part, thrice the second part, 4 times the third part and 12 times the last part are all equal. Find the 4 parts.

Sol: Let the first part, second part, third part and last part be a, b, c and d respectively.

$2a = 3b = 4c = 12d$

$a = 6d$, $b = 4d$, $c = 3d$

Given, $a + b + c + d = 1400$

$\therefore 6d + 4d + 3d + d = 1400$

$\Rightarrow d = 100$

$\therefore a = 600$, $b = 400$, $c = 300$

2.06. 1400 is divided into 4 parts such that half of the first part, one third of the second part, one fourth of the third part and $\dfrac{1}{12}$ th of the last part are all equal. Find the 4 parts.

Sol: Let the first part, second part, third part and fourth part be a, b, c and d respectively.

$$\frac{1}{2}a = \frac{1}{3}b = \frac{1}{4}c = \frac{1}{12}d$$

$d = 6a$, $b = \dfrac{3}{2}a$, $c = 2a$

Given, $a + b + c + d = 1400$

$\Rightarrow a + \dfrac{3}{2}a + 2a + 6a = 1400$

$\Rightarrow a = \dfrac{400}{3}$

$\therefore b = 200$, $c = \dfrac{800}{3}$, $d = 800$

2.07. If $a : b = b : c = 2 : 3$, find $a : b : c$

Sol: $\dfrac{a}{b} = \dfrac{b}{c} = \dfrac{2}{3} \Rightarrow a = \dfrac{2}{3}b$ and $b = \dfrac{2}{3}c$

$\therefore a = \dfrac{2}{3}\left(\dfrac{2}{3}c\right) = \dfrac{4}{9}c$

$\therefore a : b : c = \dfrac{4}{9}c : \dfrac{2}{3}c : c = 4 : 6 : 9$

Alternative method:
As b is common to both ratios and since it is divisible by 3 (from the first ratio) and it is divisible by 2 (from the second ratio), it is divisible by L.C.M (3, 2) i.e., 6. Hence if $b = 6$, $a = 4$ and $c = 9$

$\therefore a : b : c = 4 : 6 : 9$

2.08. There are 2 classes A and B. If 10 students leave class A and join class B, then the ratio of the number of students in class A and class B would reverse. Find the difference in the numbers of students in class A and class B.

Sol: Let the numbers of students in class A and class B be ax and bx respectively.

Given, $\dfrac{ax - 10}{bx + 10} = \dfrac{b}{a}$

$a^2x - 10a = b^2x + 10b \Rightarrow a^2x - b^2x - 10a - 10b = 0$

$\Rightarrow (ax - bx - 10)(a + b) = 0$

$\therefore ax - bx = 10$

2.09. A husband's age exceeds that of his wife by 6 years. 10 years ago, the ratio of their ages was 5 : 4. Find the present age of the husband.

Sol: Let the present age of the husband be x years.

$\Rightarrow$ Present age of the wife $= (x - 6)$ years.

10 years ago, the ages of the husband and the wife will be $(x - 10)$ years and $(x - 16)$ years respectively.

Given $x - 10 = \dfrac{5}{4}(x - 16)$

$\therefore x = 40$

Alternative method:
Let the age of the husband 10 years ago be 5x years. Age of his wife at that time = 4x years. The husband would then also be 6 years older than his wife.

$\therefore 5x = 4x + 6 \Rightarrow x = 6$

Hence the present age of the husband
$= 5x + 10$ i.e. 40 years

PROPORTION

When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional i.e., if $a/b = c/d$, then a, b, c and d are proportional.

This is represented as $a : b :: c : d$ and is read as "a is to b (is) as c is to d".

When a, b, c and d are in proportion, then a and d are called the EXTREMES and b and c are called the MEANS. We also have the relationship:
Product of the MEANS = Product of the EXTREMES i.e.,
$bc = adv$

If	a : b	=	c : d then	
	b : a	=	d : c	..... (A)
	a : c	=	b : d	..... (B)
(a + b) : b	=	(c + d) : d		..... (C)

(obtained by adding 1 to both sides of the given relationship)

$\quad (a - b) : b \quad = \quad (c - d) : d \quad\quad ..... (D)$

(obtained by subtracting 1 from both sides of the given relationship)

$\quad (a + b) : (a - b) = (c + d) : (c - d) \quad ...... (E)$

{obtained by dividing relationship (C) above by (D)}

Relationship (A) above is called INVERTENDO
Relationship (B) is called ALTERNENDO;
Relationship (C) is called COMPONENDO;
Relationship (D) is called DIVIDENDO;
Relationship (E) is called COMPONENDO – DIVIDENDO.

The last relationship, i.e., COMPONENDO-DIVIDENDO is very helpful in simplifying problems. By this rule, whenever we know $a/b = c/d$, then we can write
$(a + b) / (a - b) = (c + d) / (c - d)$.

The converse of this is also true - whenever we know that $(a + b) / (a - b) = (c + d)/(c - d)$, then we can conclude that $a/b = c/d$.

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$ ......, then each of these ratios is equal to $\dfrac{a + c + e + ...}{b + d + f + ...}$.

If three quantities a, b and c are such that a : b : : b : c, then we say that they are in CONTINUED PROPROTION. We also get $b^2 = ac$. In such a case, c is said to be the third proportional of a and b. Also, b is said to be the mean proportional of a and c.

VARIATION

Two quantities A and B may be such that as one quantities changes in value, the other quantity also changes in value **bearing certain relationship** to the change in the value of the first quantity.

DIRECT VARIATION

One quantity A is said to vary directly as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A also increases in the same ratio and if B is decreased in a certain ratio, A also decreases in the same ratio.
This is denoted as A α B (A varies directly as B).

If A α B then A = kB, where k is a constant. It is called the constant of proportionality.

For example, when the quantity of sugar purchased by a housewife doubles from the normal quantity, the total amount she spends on sugar also doubles, i.e., the quantity and the total amount increase (or decrease) in the same ratio.

From the above definition of direct variation, we can see that when two quantities A and B vary directly with each other, then A/B = k or **the ratio of the two quantities is a constant**. Conversely, **when the ratio of two quantities is a constant, we can conclude that they vary directly with each other.**

If X varies directly with Y and we have two sets of values of the variables X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then, since XαY, we can write down

$$\frac{X_1}{Y_1} = \frac{X_2}{Y_2} \text{ or } \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

INVERSE VARIATION

A quantity A is said to vary inversely as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A gets decreased in the same ratio and if B is decreased in a certain ratio, then A gets increased in the same ratio.
It is the same as saying that A varies directly with 1/B. It is denoted as A α 1/B i.e., A = k/B where k is k the constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.

From the definition of inverse variation, we can see that when two quantities A and B vary inversely with each other, then AB = a constant, i.e., **the product of the two quantities is a constant**. Conversely, **if the product of two quantities is a constant, we can conclude that they vary inversely with each other**.

If X varies inversely with Y and we have two sets of values of X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then since X and Y are inversely related to each other, we can write down

$$X_1Y_1 = X_2Y_2 \text{ or } \frac{X_1}{X_2} = \frac{Y_2}{Y_1}$$

JOINT VARIATION

If there are three quantities A, B and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying. i.e., A α B when C is constant and Aα C when B is a constant; $\Rightarrow$ A α BC

A α BC $\Rightarrow$ A = kBC where k is the constant of proportionality.

2.10. Find the value(s) of x if $\frac{2x+5}{x+1} = \frac{x+2}{x-1}$.

Sol: $(2x + 5)(x – 1) = (x + 2)(x + 1)$
$2x^2 + 5x – 2x – 5 = x^2 + 2x + x + 2$
$\Rightarrow x^2 = 7$
$\therefore x = \pm \sqrt{7}$

2.11. X varies directly with $Y^2 + 18$. When Y = 18, X = 18. Find Y when X = 1.

Sol: $\frac{X_1}{X_2} = \frac{Y_1^2 + 18}{Y_2^2 + 18}$

$\frac{18}{1} = \frac{18^2 + 18}{Y_2^2 + 18}$

$Y_2^2 + 18 = 19$

$Y_2 = \pm 1$

In these types of problems on variation, there are typically three parts:

- the relationship between different variables is defined to frame an equation involving the variables and the constant of proportionality
- one set of values of all the values of all the variables is given to enable us find the value of the constant of proportionality
- the values of all but one variable of a second set are given and we are asked to find the value of the one variable whose value is not given.

2.12. The volume of a cone varies jointly as the area of its base and its height. The volume of a cone whose base area is A sq. cm and height is 10 cm is 270 cubic cm. Find the volume of a cone whose base area is 2A sq. cm and height is 30 cm.

Sol: V α A h

$\frac{V_1}{V_2} = \frac{A_1}{A_2} \frac{h_1}{h_2}$

$\frac{270}{V_2} = \left(\frac{A}{2A}\right) \frac{10}{30}$

$V_2 = 1620$ cubic cm

Note that the there should be consistency of the units used for the variables, i.e., whatever be the units used to express the variables when the constant of proportionality is being calculated, the same units should be used for different variables later on also when finding the value of the variable which we are asked to find out.

2.13. The ratio of the monthly incomes of A and B is 4 : 3. The ratio of their monthly expenditures is 5 : 4. If A saves one fourth of his monthly income, find the ratio of their monthly savings.

Sol: Let the monthly incomes of A and B be ₹4x and ₹3x respectively.
Let the monthly expenditures of A and B be ₹5y and ₹4y respectively. Monthly savings of A = ₹(4x – 5y). Monthly savings of B = ₹(3x – 4y).

Given that $4x - 5y = \dfrac{1}{4}(4x)$

3x = 5y.
∴ Monthly saving of B = 3x – 4y = 5y – 4y i.e. ₹y.

Required ratio = $\dfrac{1}{4}(4x) : y = x : y = 5 : 3$

2.14. If x varies directly with y, check whether $x^3 + y^3$ varies directly with $x^3 - y^3$.

Sol: Let $\dfrac{x}{y} = K$, where K is a constant
x = K y
$x^3 + y^3 = y^3 (K^3 + 1)$
$x^3 - y^3 = y^3 (K^3 - 1)$
$\dfrac{x^3 + y^3}{x^3 - y^3} = \dfrac{K^3 + 1}{K^3 - 1}$ a constant
∴ $(x^3 + y^3)$ varies directly with $(x^3 - y^3)$

2.15. The monthly expenses of Raja on his car are partly constant and partly vary with the number of kilometres he travels in a month. If he travels 100 km in a month his total car expenses will be ₹3500. If he travels 200 km in a month, his total car expenses will be ₹4000. If he travels 250 km in a month, what will be his total car expenses.

Sol: Let his total car expenses be ₹T. Let the fixed expense be ₹F. Let the variable expense be ₹V.
T = F + V

If he travels D km in a month, $\dfrac{V}{D} = K$, where K is a constant.
∴ T = F + K D
Given that
3500 = F + 100K --- (A)
4000 = F + 200K --- (B)
Solving (A) and (B),
F = 3000 and K = 5.
Total car expenses if he travels 250 km
= F + 250K = ₹4250.

The problems involving ratio and proportion are just different forms of the models of the basic problems we saw above. For example, the problem we just solved above might be reframed bringing in Mangoes, Bananas, Baskets, etc. Here, practice and perseverance pay you a lot. In entrance exams, there will be either direct problems on ratio, proportion and variation or indirect problems of application of these concepts just discussed to areas like Time and Work or Time and Distance.

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If a = $\frac{9}{8}$ b, then $\frac{3b}{4a}$ =

    (A) 2/3   (B) 1/3   (C) 4/3   (D) 2

2. If 2.4p = 0.08 q, then $\frac{q+p}{q-p}$ =

    (A) 31/28   (B) 31/27
    (C) 31/29   (D) 39/35

3. If a : b = 2 : 3, find $\frac{3a+4b}{4a+5b}$.

    (A) 6 : 7   (B) 18 : 23   (C) 19 : 23   (D) 20 : 23

4. If a : b = b : c = 2 : 3, find a : b : c.
    (A) 4 : 6 : 9   (B) 2 : 3 : 3
    (C) 3 : 3 : 4   (D) 9 : 6 : 4

5. If p : q = 5 : 6 and q : r = 4 : 7, then find p : q : r
    (A) 10 : 12 : 21   (B) 20 : 24 : 35
    (C) 15 : 18 : 28   (D) 25 : 30 : 35

6. Find a : e, if a : b = 2 : 3, b : c = 6 : 7, c : d = 14 : 25 and d : e = 1 : 2.
    (A) 14 : 25   (B) 6 : 25
    (C) 12 : 25   (D) 4 : 25

7. If p : q = 5 : 4 and p = a + b and q = a – b, find a : b.
    (A) 1 : 9   (B) 9 : 1   (C) 5 : 4   (D) 4 : 5

8. Thirty five sweets are distributed among Sita and Gita in the ratio 4 : 3. Find the number of sweets received by Sita.

9. Two numbers are in the ratio of 2 : 5. If 4 is added to each, they would be in the ratio 4 : 9. Find the numbers.
    (A) 16 and 40   (B) 18 and 45
    (C) 22 and 55   (D) 20 and 50

10. The ratio of the ages of A, B and C is 7 : 5 : 4. If C's age is 32 years, find the sum of the ages of A, B and C (in years).

11. The ratio of the ages of four members of a family is 9 : 8 : 3 : 2. The average age of the family is 22 years. What is the age of the eldest person in the family?
    (A) 36 years   (B) 32 years
    (C) 12 years   (D) 6 years

12. The number of marbles with A and B are in the ratio of 10 : 11. Which of the following cannot be a possible number of marbles with A and B together?
    (A) 189   (B) 210
    (C) 231   (D) 153

13. The strength of a class is 70. Which of the following cannot be the ratio of the number of boys and girls in the class?
    (A) 2 : 5   (B) 3 : 2
    (C) 11 : 3   (D) 9 : 2

14. The ratio of the number of ladies and gentlemen in a party is 3 : 4. If eight ladies join the party, the ratio of ladies and gentlemen would become 5 : 4. Find the number of gentlemen in the party.

15. The ratio of the present ages of a mother and a son is 4 : 1. Twelve years from now, the ratio of their ages will be 2 : 1. Find the present ages of mother and son respectively. (in years)
    (A) 36, 9   (B) 28, 6
    (C) 24, 6   (D) 32, 8

16. The ratio of the number of chocolates with A and B is 3 : 4. If A gives four chocolates to B, the ratio of the number of chocolates with them becomes 5 : 9. How many chocolates did A have initially?

17. The ratio of the number of students in three classes A, B and C is 3 : 7 : 8. If ten students, are transferred from C to B, B will have 80 students. Find the total number of students in the three classes.

18. The ratio of the incomes of A and B is 4 : 3. The ratio of the savings of A and B is 5 : 6. Find the ratio of the expenditures of A and B, if A spends three-fourth of his income.
    (A) 3 : 2   (B) 5 : 3
    (C) 3 : 5   (D) 2 : 3

19. The ratio of the monthly incomes of A and B is 3 : 4. The ratio of the monthly expenditures of A and B is 4 : 5. Which of the following represents a possible value of the ratio of their savings?
    (A) 9 : 10   (B) 3 : 4
    (C) 13 : 20   (D) 4 : 5

20. The ratio of the present ages of a husband and a wife is 5 : 4.

    (i) Which of the following can be a possible ratio of their ages 20 years ago?
    (A) 5 : 4   (B) 6 : 5
    (C) 23 : 20   (D) 13 : 10

    (ii) Which of the following can be a possible ratio of their ages 20 years hence?
    (A) 5 : 4   (B) 6 : 5
    (C) 7 : 5   (D) 13 : 10

21. Four positive numbers are in the ratio of 1 : 2 : 3 : 4. If the sum of their squares is equal to 480, find the numbers.
(A) 2, 4, 8 10
(B) 8, 12, 16, 20
(C) 4, 8, 12, 16
(D) 8, 10, 12, 16

22. Three positive numbers p, q and r satisfy $\dfrac{q+r}{p} = \dfrac{p+r}{q} = \dfrac{p+q}{r} = K$. K = _____.

(A) 3/2
(B) 5/2
(C) 3
(D) 2

23. x varies directly as y^2 and when y = 12, x = 4. Find x when y = 18.
(A) $9\frac{1}{16}$
(B) 9
(C) 1/9
(D) 16/9

24. The area of a square varies with the square of its diagonal. The area of a square whose diagonal measures eight units is 32 sq.units. Find the diagonal of a square of area 72 sq.units (in units).

25. The monthly salary of an employee in an office varies directly with the number of days he works in that month. An employee gets ₹10400 as salary in a month in which he works for 26 days. Find his salary (in ₹) in a month in which he works for 24 days.

26. For a given distance, speed is inversely proportional to the time taken. Raju would take six hours to cover it if his speed was 50 km/hr. Find the time taken (in hours) by him to cover it if his speed was 100 km/hr.
(A) 2
(B) 3
(C) 4
(D) 6

27. P, Q and R are three quantities, P varies directly with the sum of Q and R. If both Q and R decrease by 1, find the change in P.
(A) No change
(B) A decrease of 2
(C) An increase of 2
(D) Cannot be determined

28. A, B and C are three quantities. A varies directly with B when C is constant. A varies directly with C when B is constant. A = 6000 if B = 20 and C = 30. Find A if B = 40 and C = 60.

29. A varies directly with B when C is constant and inversely with C when B is constant. When A = 26, B = 65 and C = 30. Find A when B = 84 and C = 42.
(A) 24
(B) 36
(C) 14
(D) 32

30. X, Y and Z are three quantities. X varies inversely with Y when Z is constant. Y varies inversely with Z when X is constant. When Y = 8, and Z = 7, X = 30. Find X if Y = 16 and Z = 21.

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If a : b = 3 : 7, what is the value of $\dfrac{4a+5b}{2a+2b}$?

   (A) 47/20   (B) 3/2   (C) 7/4   (D) 5/2

2. If a : b = 2 : 3 b : c = 4 : 3 and c : d = 2 : 3, then find a : b : c : d.
   (A) 8 : 12 : 9 : 27
   (B) 16 : 24 : 18 : 27
   (C) 18 : 27 : 36 : 8
   (D) 12 : 18 : 15 : 20

3. The weights of Bimal and Basu are in the ratio 2 : 3 and the weights of Basu and Bali are in the ratio 4 : 3. What is Basu's weight (in kg) if the sum of the weights of Bimal, Basu and Bali is 203 kg?

4. The ratio of the number of boys to the number of girls in a school is 7:3. If an additional 15 girls were to join the class, the ratio of the number of boys to the number of girls would become 2:3. What is the initial number of girls in the class?
   (A) 4   (B) 10   (C) 12   (D) 6

5. If 3x – 4y + 2z = 0 and 4x – 2y – z = 0, find x : z : y.
   (A) 8 : 10 : 11   (B) 8 : 11 : 40
   (C) 11 : 40 : 8   (D) 8 : 40 : 11

6. What must be subtracted from p and added to q so that the ratio of the resultants becomes 1 : 3?

   (A) $\dfrac{p+q}{3}$ (B) $\dfrac{3p-q}{4}$

   (C) $\dfrac{p-q}{p+q}$ (D) $\dfrac{q-3p}{4}$

7. A certain number is added to each of a pair of numbers which are in the ratio 4 : 7. The sum of the resulting numbers is 75 and their ratio (taken in the same order as mentioned above) is 8 : 17. What is the number added?

8. Rohan and Sohan had certain amounts of money with them. The ratio of the amounts with them is 8 : 5. Each day Rohan spends a certain amount and Sohan earns one – sixth of the amount that Rohan spends. After 9 days, the ratio of the amounts with them is 10 : 11. After how many more days, will the ratio of the amounts with them be 18 : 35?

9. Vipin's present age is twice the age of Kishore one year ago. What is the sum of their present ages (in years), if the ratio of the sum of their present ages to the difference of their present ages is 19 : 5?

10. Three different types of balls priced at ₹5, ₹8 and ₹13 per piece are displayed in three different boxes by a trader. Mr. Paul bought from this shop all three types of balls spending a total sum of ₹768. The numbers of the balls he bought, taken in the order in which the prices are mentioned above, are in the ratio 5 : 4 : 3. How many balls of the costliest variety did he buy?
    (A) 104   (B) 64   (C) 48   (D) 24

11. A certain amount of money is divided among nine brothers. The second brother gets ₹2 more than twice the amount given to the first brother. The third brother gets ₹3 more than thrice the amount given to the first brother, and so on till the ninth brother. If the ratio of the amount with the ninth brother to the amount with the first brother is 10 : 1, how much did the fifth brother get (in ₹)?
    (A) 100   (B) 80   (C) 90   (D) 50

12. Some apples are divided among four people Karan, Kiran, Kumar and Khanna. The ratio of the number of apples given to Kiran to the total number of apples given to Karan and Khanna is 1 : 2. The ratio of the number of apples given to Kumar to that of the remaining apples is 2 : 5. Khanna gets 2 apples more than Kiran. Karan gets half the number of apples that Kumar gets. What is the total number of apples distributed?
    (A) 21   (B) 18
    (C) 19   (D) 24

13. Alok distributed a certain number of toffees among his brothers Bala, Chetan and David in the ratio 4 : 4 : 9. David distributed the toffees that he received among his sisters Amitha, Bama and Chandra in the ratio 1 : 7 : 8. If Amitha received 18 toffees, the number of toffees distributed by Alok is

Directions for questions 14 and 15: These questions are based on the data given below.

The amount used to purchase one litre of petrol can be used to purchase three litres of diesel or five litres of kerosene. Out of a certain amount, ₹510 is spent on diesel.

14. How much is spent on kerosene if equal volumes of the three liquids are purchased with the total amount?
    (A) ₹300   (B) ₹306
    (C) ₹382   (D) ₹354

15. What will be the amount spent on petrol if the total amount referred in the above question is instead spent to purchase equal volumes of petrol and kerosene only?
    (A) ₹1250   (B) ₹1275
    (C) ₹1955   (D) ₹1360

16. If $k = \dfrac{a + c}{b + d} = \dfrac{c + e}{d + f} = \dfrac{a + e}{b + f}$, when all quantities are positive, then which of the following must be true?

   (A) $k = \dfrac{e}{f}$ (B) $k = \dfrac{a}{b}$

   (C) $k = \dfrac{c}{d}$ (D) All of the above

17. A number of coins can be divided in the ratio 13 : 14 : 15. They can also be divided in the ratio 9 : 10 : 11 : 12. If the difference between the greatest parts in the two cases is 6, the number of coins is ____.

   [ ]

18. The mean proportional between two numbers is 9 and the third proportional of the two numbers is 243. Find the larger of the two numbers.
   (A) 27   (B) 81   (C) 9   (D) 54

19. P and Q are distinct two-digit numbers. Ps and Qs denote the sums of the digits in P and Q respectively. If $\dfrac{P}{P_s} = \dfrac{Q}{Q_s}$, then find the minimum possible value of $P_s + Q_s$.
   (A) 8   (B) 9   (C) 6   (D) 3

20. For a body starting from rest, the distance travelled (d) is directly proportional to the square of the time elapsed from the start (t). When t = 4 sec, d = 48 m. What is the value of d (in metres) at t = 7sec?
   (A) 128   (B) 150   (C) 115   (D) 147

21. A certain sum of money is sufficient to pay the wages of worker A for 55 days. It is also sufficient to pay the wages of worker B for 66 days. The number of days that the sum of money is sufficient to pay the wages of both workers is

   [ ]

22. The volume of a cone varies as the square of the radius of its base when its height is fixed and also varies as its height when radius of its base is fixed. If a radius of 3 cm and height of 7 cm give it a volume of 66 cubic centimetre, then what will be the radius, (in centimetres), if the cone has a height of 6 cm and volume of 308 cubic centimetres?
   (A) 9   (B) 15   (C) 7   (D) 11

23. The kinetic energy of a body is directly proportional to the square of its speed when the mass is kept constant and is directly proportional to mass when its speed is kept constant. A body with a mass of 2 kg and a speed of 10 m/s has a kinetic energy of 100 joules. What is the kinetic energy of a body whose mass is 20 kg and speed is 1 m/s?
   (A) 100 joules
   (B) 1000 joules
   (C) 10 joules
   (D) 20 joules

24. A writer gets a fixed amount for his book apart from opthe royalty he gets per book sold. He gets ₹46000 and ₹66000 for 2000 and 3000 books sold respectively. Find his income per book when 6000 books are sold (in ₹).

   [ ]

25. 'A' varies directly as the sum of two quantities 'B' and 'C'. 'B' in turn varies directly as 'x' and 'C' varies inversely as 'x'. When x = 1 or 2, A = 3. Find the value of A when x = 4.

   [ ]

26. A garrison of 900 soldiers had food-stock sufficient for 30 days when the rate of consumption is 2.5 kg/day/soldier. After some days of consumption at that rate, 300 soldiers were transferred to another garrison and the balance food lasted for 25 days for the remaining soldiers. If the rate of consumption of the remaining soldiers was 3.0 kg/day/ soldier, after how many days from the start, were the soldiers transferred?

   [ ]

27. The speed of a locomotive without any wagons attached to it is 40 kmph. It diminishes by a quantity which is proportional to the cube root of the number of wagons attached. If the speed of the locomotive is 34 kmph when 27 wagons are attached, what is the maximum number of wagons that can be attached if the condition is that the speed should not fall below 30 kmph?
   (A) 64   (B) 125   (C) 216   (D) 343

28. A precious stone worth ₹10872 fell and broke into three pieces, the weights of which are proportional to 1 : 2 : 3. The value of each stone is directly proportional to the square of its weight. Find the loss in the value caused by the breakage.
   (A) ₹3624   (B) ₹4228   (C) ₹6644   (D) ₹7510

29. The distance (in meters) to which a boy can throw a stone is inversely proportional to its weight (in kg). He breaks the stone into 3 pieces whose weights (in kg) are in the ratio 1 : 3 : 2. He then throws the stones one by one. The sum of the distances they cover is 22 meters. To what distance can he throw the unbroken stone? (in m)

   [ ]

30. The amount collected per month from a consumer of electrical power consists of two parts – a fixed charge for providing the service and a variable charge which is directly proportional to the number of units of power consumed. An amount of ₹700 is collected from the consumer when he consumed 50 units in a month. It is also noticed that when the consumption increases from 100 units per month to 200 units per month, the bill amount increases to 5/4 times that of the former. How much is the fixed charge per month?
   (A) ₹600   (B) ₹350   (C) ₹500   (D) ₹400

Exercise – 2(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If a + b – c : b + c – a : a + c – b = 5 : 6 : 7, then find a : b : c.
(A) 12 : 13 : 11
(B) 12 : 11 : 13
(C) 13 : 12 : 11
(D) 13 : 11 : 12

2. Which of the following represents a possible value of p : q satisfying $\dfrac{20p^2 - 40pq}{pq + 4q^2} = 20$?

(A) 3 : 1   (B) 1 : 4   (C) 4 : 1   (D) 5 : 1

3. If a : b = 3 : 4 and c : d = 2 : 3, then find $\dfrac{a^3c^2 + b^3d^2}{ab^2d^2 + a^2bcd}$.

(A) $\dfrac{19}{3}$

(B) $\dfrac{19}{4}$

(C) $\dfrac{19}{9}$

(D) $\dfrac{19}{18}$

4. Ajay and Vijay wrote a test. The sum of Ajay's score and twice Vijay's score is 310. The sum of Vijay's score and twice Ajay's score is 290. Find the ratio of the scores of Ajay and Vijay.
(A) 9 : 11
(B) 13 : 17
(C) 11 : 19
(D) 7 : 13

5. Ninety three is divided into two parts such that thrice the first part and twice the second part are in the ratio 25 : 4. Find the first part.

6. If three is subtracted from the numerator and five is added to the denominator of a fraction, the new fraction formed is 1/2. If two is added to the numerator of the initial fraction, the ratio of the new numerator to the denominator becomes 1 : 1. Find the original fraction.
(A) 11/13
(B) 18/23
(C) 13/15
(D) 13/11

7. The ratio of the number of students in classes A, B and C is 3 : 7 : 8. If 10 students leave C and join B, the ratio of the number of students in B and C would be reversed. Find the total number of students in the classes A, B and C.

8. Ajay distributed a total of 60 sweets among his sons Ram, Shyam and Tarun. For every five sweets received by Ram, Shyam received four sweets. For every two sweets received by Shyam, Tarun received three sweets. Find the number of sweets received by Tarun.

9. Divide 66 into three parts such that the sum of the first two parts equals the third part and the second part is three less than twice the first part. What is the ratio of the parts as arranged in the ascending order?
(A) 14 : 17 : 19   (B) 5 : 9 : 11
(C) 11 : 7 : 4   (D) 4 : 7 : 11

10. A person has with him a certain number of weighing stones of 100 gm, 500 gm and 1 kg in the ratio of 3 : 5 : 1. If a maximum of 5 kg can be measured using weighing stones of 500 gm alone, then what is the number of 100 gm stones he has?
(A) 6   (B) 3   (C) 9   (D) 5

11. In a class, the ratio of the number of boys and girls is 1 : 2. Among the boys, one-third are day scholars and one-fourth of these day scholars travel to school by bus. Among the girls, two-thirds are day scholars and half of these day scholars travel to school by bus. What part of the students in the class are day scholars travelling to school by bus?

(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{5}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{2}{5}$

12. In a bag, there are balls of three colours - white, black and green. The ratio of the number of white and black balls is 3 : 4. The ratio of the number of black and green balls is 3 : 5. Which of the following can be a possible value of the total number of white and green balls in the bag?
(A) 42   (B) 32   (C) 58   (D) 41

13. Amar, Bhavan and Chetan have some coins with them. The total number of coins with Bhavan and Chetan and those with Amar and Chetan are in the ratio 4 : 5. The total number of coins with Bhavan and Chetan and those with Amar and Bhavan are in the ratio 4 : 3. Which of the following is the ratio of the number of coins with Bhavan and Chetan?
(A) 7 : 4   (B) 3 : 2
(C) 7 : 5   (D) 1 : 3

14. The ratio of the prices of tea, last year and this year is 5 : 6. The ratio of the prices of coffee last year and this year is 7 : 8. The sum of prices of a kg of tea and a kg of coffee this year is ₹48. Find the price of tea (in ₹) last year if it was $\dfrac{20}{21}$ of the price of coffee last year.

15. ₹117 was supposed to be divided among Rohan, Sohan and Mohan in the ratio 2 : 3 : 4. By mistake, it was divided in the ratio $\dfrac{1}{2} : \dfrac{1}{3} : \dfrac{1}{4}$. Find the loss / gain of Rohan due to his mistake (in ₹)
(A) gain, 26   (B) loss, 26
(C) gain, 28   (D) loss, 28

16. Five vessels have equal capacities. Each vessel contains some milk. The ratio of the quantities of milk in these vessels is 3 : 4 : 5 : 6 : 7. The total quantity of milk is three-fifth of the combined capacity of the vessels. How many vessels contain at least half milk?

17. The monthly incomes of Ashok and Bala be 3 : 4. The ratio of the expenditures of Ashok and Bala is 4 : 5. Which of the followinjg cannot be the ratio of the savings of Ashok and Bala?
(A) 2 : 3   (B) 4 : 5   (C) 8 : 11   (D) 5 : 7

18. Manoj and Shiva, who are colleagues in an office, have their monthly savings in the ratio of 2 : 3. Manoj spends two-thirds of his income every month. If the ratio of their monthly incomes is 3 : 4, what is the ratio of their expenditures?
(A) 3 : 2   (B) 4 : 5   (C) 4 : 3   (D) 5 : 3

19. In a three digit number, the units digit is the sum of the other two digits and it exceeds the tens digit by as much as the latter exceeds the hundreds digit. How many such three digit numbers exist?
(A) 1   (B) 2   (C) 3   (D) 4

20. The sum of the present ages of a mother and her daughter is 60 years. When the mother attains her husband's present age, the ratio of her husband's age and her daughter's age would be 2 : 1. Find the present age of the daughter. (in years)

21. A person had two plots of land. In each plot, he cultivated maize and barley. The ratio of the area under maize cultivation and barley cultivation in the larger plot is 8 : 9. The ratio of the total areas under maize cultivation and barley cultivation in the two plots together is 29 : 33. The ratio of the areas under maize cultivation and barley cultivation in the smaller plot is 13 : 15. The ratio of the area under maize cultivation in the larger plot and that under barley cultivation in the smaller plot is _____.
(A) 8 : 15   (B) 16 : 15
(C) 15 : 16   (D) 15 : 8

Directions for questions 22 and 23: These questions are based on the data given below.

A test of 60 minutes contains questions on Mathematics and English only. The time taken to solve a Mathematics question is twice the time taken to answer an English question and the ratio of time taken to solve all Mathematics questions to time taken to answer all English questions is 8/7.

22. What is the ratio of the number of English questions to that of Mathematics?
(A) 11/7   (B) 7/4   (C) 9/4   (D) 7/5

23. If the total number of questions is 22, how many English questions can be answered in 18 minutes?
(A) 8   (B) 10   (C) 11   (D) 9

24. If $\dfrac{a^2+c^2}{a+c}=\dfrac{b^2+c^2}{b+c}=k$ and $a \neq b$, which of the following is equal to k?
(A) a + c   (B) b + c
(C) a + b   (D) a − c

25. If $\dfrac{p+q}{r}=\dfrac{q+r}{p}=\dfrac{p+r}{q}=k$, find the sum of all the possible values of k.
(A) 1   (B) 2   (C) 0   (D) 3

26. If $\dfrac{2x^2-4x+3}{4x-3}=\dfrac{2x^2-3x+5}{3x-5}$, find the value(s) of x.
(A) 0   (B) 2
(C) − 2   (D) Both (A) and (C)

27. There are 2 two-digit numbers. Their product equals the product of the numbers formed by reversing their digits. Which of the following holds true if the numbers are denoted by ab and cd?
(A) a, b, d and c are in proportion.
(B) a, b, c and d are in proportion.
(C) a, d, c and b are in proportion.
(D) a, c, b, d are in proportion.

28. A string is cut into two parts. The ratio of the lengths of the string and the smaller part is six times the ratio of the lengths of the smaller and the larger parts. Find the ratio of the lengths of the larger and smaller parts of the string.
(A) 2 : 1   (B) 3 : 1   (C) 4 : 1   (D) 5 : 1

29. The mean proportional between two numbers is 9. The third proportional of the same numbers is 6561. Find the greater of the 2 numbers.

30. The ratio of Ahmed's age to Mohammed's age is the same as the ratio of the ages of their respective elder brothers. The ratio of the difference of the ages of Ahmed and Mohammed to that of the difference of the ages of their respective brothers is 1 : 2. What is the ratio of the sum of the ages of their respective brothers to the sum of the ages of Ahmed and Mohammed?
(A) 2 : 1   (B) 4 : 1   (C) 3 : 1   (D) $\sqrt{2}$: 1

31. x is directly proportional to 4 more than the square of y . x is 39 when y is 3. What is the positive value of y when x is 60?

32. In a three digit number the tens digit is the average of the other two digits. The ratio of the number formed by its first two digits and their sum equals the ratio of the number formed by its last two digits and their sum. How many three digit numbers satisfy these conditions?
(A) 7   (B) 8   (C) 9   (D) 6

33. The distance that a body falls through when dropped from a certain height varies directly with the square of the time of fall. A body falls through a total of 500 m in 10 seconds. Find the distance it falls through, in the 9th second (in m).

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34. A quantity Q is obtained by adding three quantities. The first is a constant, the second varies directly with the square root of y and the third varies directly with the cube root of y. When y = 1, Q = 60. When y = 64, Q = 230 and when y = 729, Q = 660
Find the constant.

<div style="border:1px solid black; width:100px; height:40px"></div>

35. The cost of supply of a commodity equals the sum of three quantities. The first one is a constant, the second varies directly as the price per unit of the commodity and the third varies directly as the square of the price per unit. The costs of supply are ₹9, ₹24 and ₹47 when the prices per unit are ₹1, ₹2 and ₹3 respectively. Find the cost (in ₹) of supply when price per unit of commodity is ₹4.
(A) 80   (B) 72   (C) 76   (D) 78

36. The pressure of a gas varies directly with the temperature when the volume is constant and varies inversely with the volume when temperature is constant. If the present temperature is 100 K what will be the increase in temperature if the pressure triples and the volume doubles? (in K)

<div style="border:1px solid black; width:100px; height:40px"></div>

37. Two diaphragms partition a cylinder into three chambers, whose volumes are in the ratio of 1 : 2 : 3. Pressure of a gas, which is in joint variation with its mass and volume, is directly proportional to its mass and inversely proportional to its volume. If the masses of the gas in the three chambers are same, find the ratio of the pressures of the gases in the three chambers taking them in the same order as the volumes have been taken. (Assume no factors other than those mentioned play a role).
(A) 3 : 2 : 1   (B) 1 : 2 : 3
(C) 6 : 3 : 2   (D) 1 : 1 : 1

38. The heat radiated by a certain body per unit time varies directly with the square root of the excess of the temperature of the body over the ambient temperature which is 20ºC. The heat radiated by the body in one second is 16 joules when the temperature of the body is 36ºC. Find the temperature of the body when the heat radiated in one second is 20 joules.
(A) 56ºC   (B) 45ºC   (C) 50ºC   (D) 60ºC

39. In a colony, the expenses for yoga classes are partly constant and partly vary directly with the number of members. If there are 50 members, each member has to bear ₹110 per month. If there are 30 more members, each member has to bear ₹80 per month. How many members are there if the share of each member is ₹130 per month?

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40. The telephone bill consists of a fixed monthly rent and a variable part that varies with the number of units of calls made per month. The monthly rents charged for a telephone connection in Bangalore and Patna are in the ratio 5 : 3. The bill amounts corresponding to 100 calls made from a phone in Patna and 300 calls made from a phone in Bangalore are respectively ₹500 and ₹1100. What is the difference in the fixed monthly rent of a telephone connection in Bangalore and one in Patna, if call rate per unit is same in both cities?
(A) ₹200   (B) ₹100
(C) ₹500   (D) ₹300

Directions for questions 41 to 45: Each question is followed by two statements I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone
Mark (D) if the question cannot be answered even using I and II together.

41. A bowl contains ₹1, 50 paise and 25 paise coins. What is the total amount in the bowl?
I.   The total number of coins in the bowl is 20.
II. The total value of ₹1 coins is ₹6 and the number of 50 paise coins and 25 paise coins are in the ratio 6 : 1.

42. The ratio of technical staff and non-technical staff in a company is 3 : 10. What percentage of the staff are graduates?
I.   80% of the technical staff and 40% of the non-technical staff are graduates.
II. The ratio of the number of graduates to the total staff in the company is 32 : 65.

43. How many girls are there in the class?
I.   The ratio of the number of boys to girls is 5 : 4.
II. If four boys leave the class, the ratio of number of boys to girls in the class will be 1 : 1.

44. What is the percentage of students who are not intelligent?
I.   The ratio of number of boys and girls is 3 : 2.
II. 20% of the boys and 20% of the girls are intelligent.

45. If a cake is distributed among two boys A and B and three girls X, Y and Z, who received the largest part?
I.   X received one-eighth part of the cake, which is two-thirds of B's share.
II. A received twice B's share, which is equal to thrice the share of Y.

Key

Concept Review Questions

1. A	9. D	17. 180	24. 12
2. C	10. 128	18. B	25. 9600
3. B	11. A	19. C	26. B
4. A	12. D	20. (i) D	27. D
5. A	13. D	(ii) B	28. 24000
6. D	14. 16	21. C	29. A
7. B	15. C	22. D	30. 5
8. 20	16. 24	23. B	

Exercise – 2(a)

1. A	6. B	11. D	16. D	21. 30	26. 10
2. B	7. −12	12. A	17. 84	22. C	27. B
3. 84	8. 6	13. 544	18. A	23. C	28. C
4. D	9. 19	14. B	19. D	24. 21	29. 2
5. A	10. D	15. C	20. D	25. 4.5	30. A

Exercise – 2(b)

1. B	6. C	11. C	16. 3	21. B	26. D	31. 4	36. 500	41. C
2. C	7. 180	12. C	17. B	22. B	27. A	32. C	37. C	42. B
3. D	8. 24	13. D	18. B	23. D	28. A	33. 85	38. B	43. C
4. A	9. D	14. B	19. C	24. C	29. 81	34. 30	39. 40	44. A
5. 75	10. A	15. C	20. 20	25. A	30. A	35. D	40. A	45. C