Chapter – 1

NUMBERS – I

NUMBERS is one of the most important topics required for competitive entrance exams. In this chapter, we have put together a number of models of problems - mainly based on various problems that have been appearing in different exams.

BASIC ARITHMETIC OPERATIONS

Addition is the most basic operation. We have an intuitive understanding of the operation. It is the process of finding out the single number or fraction equal to two or more quantities taken together. The two (or more) numbers that are added are called addends and the result of the addition is called the sum. For two numbers A and B, this is denoted as A + B.

Subtraction is the process of finding out the quantity left when a smaller quantity (number or fraction) is reduced from a larger one. This is called the difference of the two numbers. The word difference is taken to mean a positive quantity, i.e., the difference of 10 and 8 is 2. The difference of 8 and 10 is also 2. This is also referred to as the remainder.

Multiplication is repeated addition. The number that is added repeatedly is the multiplicand. The number of times it is added is the multiplier. The sum obtained is the product.

For example, in the multiplication 3 x 4 = 12, 3 is the multiplicand, 4 is the multiplier and 12 is the product.

Division is repeated subtraction. From a given number, we subtract another repeatedly until the remainder is less than the number that we are subtracting. The number from which we are subtracting the second one is the dividend. The number that is subtracted repeatedly (the second one) is the divisor. The number of times it is subtracted is the quotient. The number that remains after we are done subtracting is the remainder. Division can also be thought of as the inverse of multiplication. A/B is that number with which B has to be multiplied to get A.
For example, in the division 32/5, 32 is the dividend, 5 is the divisor, 6 is the quotient and 2 is the remainder.

Involution (or raising to the power n) is repeated multiplication. Thus, a^n is the the product of n a's. Here, a is the base, n is the index and a^n is the n^{th} power of a.
For example, a x a = a^2, which is the second power of a and a x a x a = a^3, which is the third power of a.

Evolution is the inverse of involution. The n^{th} root of a number is that number whose n^{th} power is the given number. The root of any number or expression is that quantity which when multiplied by itself the requisite number of times produces the given expression.

For example, the square root of a, $\sqrt{a}$ when multiplied by itself two times, gives a; similarly, the cube root of a, $\sqrt[3]{a}$ when multiplied by itself three times, gives a.

All the above operations are performed in Algebra also. Algebra treats quantities just as Arithmetic does, but with greater generality, for algebraic quantities are denoted by symbols which may take any value we choose to assign them as compared to definite values usually used in arithmetic operations.

Rule of Signs

The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.
Example : –1 x –1 = +1 ;
+1 x –1 = –1 ;
+1 x +1 = +1 ;
–1 x +1 = –1;

CLASSIFICATION OF REAL NUMBERS

Real Numbers are classified into rational and irrational numbers.

Rational Numbers

A number which can be expressed in the form p/q where p and q are integers and q $\neq$ 0 is called a rational number.

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator 1 $\neq$ 0. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly (For example, 0.333....., 0.111111...., 0.166666...., etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form p/q and hence it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

Irrational Numbers

Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt[4]{5}$, $\sqrt[3]{9}$, etc.
Numbers like π, e are also irrational numbers.

Between any two numbers, there are infinite number of irrational numbers.
Another way of looking at rational and irrational numbers is
Terminating decimals and recurring decimals are both rational numbers.
Any non-terminating, non-recurring decimal is an irrational number.

Integers

All integers are rational numbers. Integers are classified into negative integers, zero and positive integers. Positive integers can be classified as Prime Numbers and Composite Numbers. In problems on Numbers, we very often use the word "number" to mean an "integer."

Prime Numbers

A number other than 1 which does not have any factor apart from one and itself is called a prime number.

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. **Every prime number greater than 3 can be written in the form of (6k + 1) or (6k – 1) where k is an integer.** For the proof of this, refer to 4th point under "Some important points to note" given later on in this chapter.

Composite Numbers

Any number other than 1, which is not a prime number is called a composite number. In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

NOTE:

The number 1 is neither prime nor composite.
The only prime number that is even is 2.

There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

Even and odd numbers

Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

NOTE: Every even number ends in 0, 2, 4, 6 or 8.
The sum of any number of even numbers is always even.

The sum of odd number of odd numbers (i.e., the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.

The product of any number of odd numbers is always odd.

The product of any number of numbers where there is at least one even number is even.

PERFECT NUMBERS

A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

For example, 6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

HIERARCHY OF ARITHMETIC OPERATIONS

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc. a particular sequence of the operations has to be followed. For example, 2 + 3 x 4 has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 to 3 first to take the result 5 and multiply this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations are given by a rule called BODMAS rule. The operations have to be carried out in the order in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

B Brackets
O Of
D Division
M Multiplication
A Addition
S Subtraction

There are four types of brackets:

(i) Vinculum : This is represented by a bar on the top of the numbers. For example,

$2 + 3 - \overline{4+3}$; Here, the figures under the vinculum have to be calculated as 4 + 3 first and the "minus" sign before 4 is applicable to 7. Thus the given expression is equal to 2 + 3 – 7 which is equal to –2.

(ii) Simple Brackets: These are represented by ( )
(ii) Curly Brackets: These are represented by { }
(iv) Square Brackets: These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [{ (¯) }] to be opened fom inside outwards.

After brackets is O in the BODMAS rule standing for "of" which means multiplication. For example, 1/2 of 4 will be equal to 1/2 x 4 which is equal to 2.

After O, the next operation is D standing for division. This is followed by M standing for multiplication. After Multiplication, A standing for addition will be performed. Then, S standing for subtraction is performed.

Two operations that have not been mentioned in the BODMAS rule are taking powers and extracting roots, viz, involution and evolution respectively. When these operations are also involved in expressions, there is never any doubt about the order in which the steps of the simplification should be taken. The sign for root extraction is a variant of the vinculum and for powers, brackets are used to resolve ambiguities in the order.

Examples

1.01. Simplify:

$$\left[4+\frac{1}{9}\text{of}\left\{30-\left(19+\overline{8-6}\right)+\frac{1}{2}\text{of}\,54\right\}+3\right]$$

Sol. $\left[4+\frac{1}{9}\text{of}\left\{30-\left(19+\overline{8-6}\right)+\frac{1}{2}\text{of }54\right\}+3\right]$

$= \left[4+\frac{1}{9}\text{of}\left\{30-\left(19+2\right)+\frac{1}{2}\text{of }54\right\}+3\right]$

$= \left[4+\frac{1}{9}\text{of}\left\{30-21+27\right\}+3\right]$

$= \left[4+\frac{1}{9}\text{of}\left\{36\right\}+3\right] = \left[4+4+3\right] = 11$

RECURRING DECIMALS

A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, thus

$\frac{8}{3}$ = 2.666..... = $2.\dot{6}$ or $2.\overline{6}$;

$\frac{1}{7}$ = 0.142857142857142857... = $0.\overline{142857}$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that recur.

A recurring decimal like $0.\overline{3}$ is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\overline{6}$ (which is equal to 0.16666...) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a "circulator". The digit, or set of digits, which is repeated is called the "period" of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form p/q, where p and q are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

1.02. Express $0.\overline{6}$ in the form of a fraction.

Sol: $0.\overline{6}$ = 0.666………….
Let x = 0.66 -------- (1)
As the period is of one digit, we multiply the given number by 10^1 i.e., 10
Therefore,
10x = 6.666 -------- (2)
(2) – (1) gives, ⇒ 9x = 6
⇒ x = 6/9 = 2/3

1.03. Express $0.\overline{81}$ in the form of a fraction.

Sol: $0.\overline{81}$ = 0.818181 …………….
Let x = 0.8181 -------- (1)
As the period is containing 2 digits, we multiply by 10^2 i.e., 100
Therefore 100 x = 81.8181 -------- (2)
(2) – (1) gives, 99 x = 81
⇒ x = 81/99 = 9/11

1.04. Express the recurring decimal $0.\overline{024}$ in the form of a fraction.

Sol: $0.\overline{024}$ = 0.024024024
Let x = 0.024024 --------- (1)
As the period contains 3 digits, we multiply with 10^3 i.e., 1000, therefore
1000 x = 24.024024 -------- (2)
(2) – (1) gives, 999x = 24
⇒ x = 24/999 = 8/333

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits (called the period of the decimal) for its numerator, and for its denominator the number which has for its digits as many nines as there are digits in the period.

Thus $0.\overline{37}$ can be written as equal to $\frac{37}{99}$; $0.\overline{225}$ can be written as equal to $\frac{225}{999} = \frac{25}{111}$;

$0.\overline{63} = \frac{63}{99} = \frac{7}{11}$.

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

1.05. Express $0.2\overline{7}$ as a fraction.

Sol: Let x = $0.2\overline{7}$, then 10x = $2.\overline{7}$ = 2 + $0.\overline{7}$
= 2 + 7/9 (since $0.\overline{7}$ = 7/9)
⇒10x = 25/9
⇒ x = 25/90 = 5/18

1.06. Express $0.2\overline{79}$ in the form of a fraction.

Sol: Let x = $0.2\overline{79}$
10x = $2.\overline{79}$ = 2 + 79/99 = 277/99
x = 277/990

Now we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below:

In the numerator write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e., as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e. $0.1\overline{56} = \frac{156 - 1}{990} = \frac{155}{990} = \frac{31}{198}$

$0.7\overline{3} = \frac{73 - 7}{90} = \frac{66}{90} = \frac{11}{15}$

INTEGERS

A number of problems are based on the operation of division and the relation between the quantities involved in division.

Properties of Division

Before we take up the next area, the following simple points should be kept in mind.

1. A number when divided by d leaving a remainder of r is of the form dq + r where q is some integer from 0, 1, 2, ...

   For example, a number when divided by 4 leaving a remainder of 3 can be written in the form (4q + 3); a number when divided by 7 leaving a remainder of 4 can be written in the form (7q + 4)

2. When a number N is divided by divisor d if the remainder is r, then the number N – r is exactly divisible by d or in other words, when N – r is divided by d the remainder is 0.

   For example, when the number 37 is divided by 7, the remainder is 2; if this remainder 2 is subtracted from the number 37, the resulting number 35 is exactly divisible by 7.

3. When a number N is divided by a divisor d, if the remainder is r, then
   (a) the largest multiple of d which is less than or equal to N is obtained by subtracting r from N, i.e., N – r will be the largest multiple of d which is less than or equal to N.

   For example, when 27 is divided by 5, the remainder is 2; so 27 – 2, i.e., 25 is the largest multiple of 5 less than 27.

   (b) the smallest multiple of d which is greater than or equal to N is obtained by adding (d – r) to N, i.e., N + (d – r) will be the smallest multiple of d which is greater than N.

   For example, when 49 is divided by 8, the remainder is 1; hence the smallest multiple of 8 which is greater than 48 is 49 + (8 – 1) = 56

4. When a division is split into a sum of two divisions (with the same divisor as the original divisor), the original remainder will be equal to the sum of the remainders of the two individual divisions. Similarly, when a division is split into difference of two divisions, the original remainder will be equal to the difference of the remainders of the two divisions.

   For example, if we take the division 15/6 (where the remainder is 3), and write it as a SUM of two divisions 8/6 and 7/6 (where the remainders are respectively 2 and 1), the original remainder is equal to the SUM of the two remainders 2 and 1.

   $\frac{15}{6} = \frac{8}{6} + \frac{7}{6}$

   Remainder 3 = 2 + 1

   If we take the division 15/6 and write it as the DIFFERENCE of two divisions 29/6 and 14/6 (where the respective remainders are 5 and 2), the original remainder 3 is equal to the DIFFERENCE of the two remainders 5 and 2.

   $\frac{15}{6} = \frac{29}{6} - \frac{14}{6}$

   Remainder 3 = 5 – 2

5. If the remainder in a division is negative, then add the divisor repeatedly to the negative remainder till we get a positive remainder.

   For example, let us take the division 15/6 (where the remainder is 3) and split into difference of two divisions 25/6 and 10/6. The remainders of the two divisions are 1 and 4 respectively. The difference of these two remainders is 1 – 4 which is equal to –3 and this should be equal to the original remainder. Since this remainder is negative, add the divisor 6 to this negative remainder –3 to get the correct remainder 3.

   $\frac{15}{6} = \frac{25}{6} - \frac{10}{6}$

   Remainders are 3, 1, –4.
   Remainder 1 – 4 = –3 which is same as –3 + 6 = 3

6. In a division, if the dividend (the number which is being divided) is multiplied by a certain factor and then divided by the same divisor, then the new remainder will be obtained by multiplying the original remainder by the same factor with which the dividend has been multiplied.

   For example, when 11 is divided by 8, the remainder is 3. When the dividend 11 is multiplied by 2, we get 22 and when this number is divided by 8, the remainder is 6 which is same as the original remainder 3 multiplied by 2.

7. If the remainder is greater than the divisor, it means division is not complete. To get the correct remainder keep subtracting the divisor from the remainder till you obtain the positive remainder which is less than the divisor.

Factors, Multiples and Co-primes

Factors

If one number divides a second number exactly, then the first number is said to be a factor of the second number. For example, 5 is a factor of 15; 3 is a factor of 18. Factors are also called sub-multiples or divisors.

Multiples

If one number is divisible exactly by a second number, then the first number is said to be a multiple of the second number. For example, 15 is a multiple of 5; 24 is a multiple of 4.

Co-Primes

Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1. For example, the numbers 15 and 16 do not have any common factors and hence they are relative primes. Please note that none of the two numbers may individually be prime and still they can be relative primes. Unity is a relative prime to all numbers.

Rules for divisibility

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

Divisibility by 2

A number divisible by 2 will have an even number as its last digit (For example 128, 246, 2346, etc)

Divisibility by 3

A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is 9 + 1 + 2 + 3 = 15 which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789 etc,. are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence the number 74549 is not divisible by 3.

Divisibility by 4

A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 178564, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence the number 476854 is not divisible by 4.

Divisibility by 5

A number is divisible by 5 if its last digit is 5 or zero (eg. 15, 40, etc.)

Divisibility by 6

A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.)

Divisibility by 7

If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

Col(1) Number	Col (2) Twice the units digit	Col (3) Remaining part of the number	Col(3) - Col(2)
456745	10	45674	45664
45664	8	4566	4558
4558	16	455	439
439	18	43	25

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

Divisibility by 8

A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc. are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

In general, if the number formed by the last n digits of a number is divisible by 2^n, the number is divisible by 2^n.

Divisibility by 9

A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc. are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence the number 4763 is not divisible by 9.

Divisibility by 10

A number divisible by 10 should end in zero.

Divisibility by 11

A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11 - that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

If we take the number 89394811, the sum of the digits in odd places is 8 + 3 + 4 + 1, which is equal to 16. The sum of the digits in even places is 9 + 9 + 8 + 1, which is equal to 27. The difference between these two figures is 11 (27 – 16), which is a multiple of 11. Hence the given number 89394811 is divisible by 11.

The number 74537 is not divisible by 11 because the sum of the digits in odd places is 19 and the sum of the digits in even places is 7 and the difference of these two figures is 12 and this is not a multiple of 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

The next number that is of interest to us from divisibility point of view is 19.

Divisibility by 19

If the sum of the number of tens in the number and twice the units digit is divisible by 19, then the given number is divisible by 19. Otherwise it is not.

Take the units digit of the number, double it and add this figure to the remaining part of the number. If the result so obtained is divisible by 19, then the original number is divisible by 19. If that result is not divisible by 19, then the number is not divisible by 19.

For example let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the units digit doubled) is added to 66 we get 76. Since this result 76 is divisible by 19, the original number 665 is also divisible by 19.

Similarly, if we take 969, doubling the units digit gives 18 which when added to 96 gives a result of 114. Since 114 is divisible by 19, the number 969 is divisible by 19.

If we take 873, double the units digit (2 x 3 = 6) added to the remaining part of the number (87), we get 93 which is not divisible by 19. Hence the original number 873 is not divisible by 19.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 19. For example, take 456760. We will write down the figures in various steps as shown below.

Col(1) Number	Col (2) Twice the units digit	Col (3) Remaining part of the number	Col(3) + Col(2)
456760	0	45676	45676
45676	12	4567	4579
4579	18	457	475
475	10	47	57

Since 57 in the last step is divisible by 19, the original number 456760 is divisible by 19.

Let us take another example, the number 37895. Let us follow the above process step by step till we reach a manageable number.

37895 Double the units digit 5 and add the 10 so obtained to 3789. We get

3799 Double the units digit 9 and add the 18 so obtained to 379. We get 397 Double the units digit 7 and add the 14 so obtained to 39. We get 53.

Since 53 is not divisible by 19, 37895 is not divisible by 19.

FACTORS AND CO-PRIMES OF A NUMBER

Number of Factors of a Number

If N is a composite number such that N = $a^p . b^q . c^r$ .... where a, b, c are prime factors of N and p, q, r .... are positive integers, then the number of factors of N is given by the expression
(p + 1) (q + 1) (r + 1) ...
For example 140 = $2^2 \times 5^1 \times 7^1$.
Hence 140 has (2 + 1)(1 + 1)((1 + 1), i.e., 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So if you want to find the number of factors the given number has excluding 1 and the number itself, we find out (p + 1) (q + 1) (r + 1) and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and itself.

Number of ways of expressing a given number as a product of two factors

The given number N (which can be written as equal to $a^p. b^q. c^r$ ...... where a, b, c are prime factors of N and p, q, r..... are positive integers) can be expressed as the product of two factors in different ways.

The number of ways in which this can be done is given by the expression 1/2 {(p + 1)(q + 1)(r + 1).....}

So, 140 can be expressed as a product of two factors in 12/2 or 6 ways {because (p + 1) (q + 1) (r + 1) in the case of 140 is equal to 12}

If p, q, r, etc. are all even, then the product (p + 1) (q + 1) (r + 1) ……. becomes odd and the above rule will not be valid since we cannot take 1/2 of an odd number to get the number of ways. If p, q, r, ... are all even, it means that the number N is a perfect square. This situation arises in the specific cases of perfect squares because a perfect square can also be written as {square root x square root}. So, two different cases arise in case of perfect squares depending on whether we would like to consider writing the number as {square root x square root} also as one of the ways.

Thus, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following 2 rules

(1) as a product of two DIFFERENT factors:½ {(p + 1) (q + 1)(r + 1) ...... –1 } ways (excluding $\sqrt{N} \times \sqrt{N}$).

(2) as a product of two factors (including $\sqrt{N} \times \sqrt{N}$) in 1/2 {(p + 1)(q + 1)(r + 1) .... +1} ways.

1.07. Find the number of factors of 1225.

Sol: If a number can be expressed as a product of prime factors like
$a^p \times b^q \times c^r$ x - - - -
where a, b, c, …..are the prime numbers, then the number of factors of the number is (p + 1) (q + 1) (r + 1)…………..
First express 1225 as a product of its prime factors. (Note that to express a given number as a product of its prime factors, we first need to identify the prime factors of the given number by applying the rules of divisibility.)
$1225 = 5 \times 7 \times 5 \times 7 = 5^2 \times 7^2$
Hence, the number of factors 1225 has is (2 + 1) (2 + 1) = 9

1.08. How many divisors excluding 1 and itself does the number 4320 have?

Sol: Note that the two terms factors and divisors are used interchangeably. First express 4320 in terms of its prime factors.
4320 = 18 x 24 x 10
= 3 x 3 x 2 x 3 x 2 x 2 x 2 x 2 x 5
$= 3^3 \times 2^5 \times 5^1$
Hence 4320 has (3 + 1) (5 + 1) (1 + 1) = 48 factors. Excluding 1 and itself, the number has (48 – 2) = 46 factors.

1.09. In how many ways can 3420 be written as a product of two factors?

Sol: By prime factorisation 3420
$= 2 \times 5 \times 2 \times 19 \times 3^2 = 2^2 \times 3^2 \times 5^1 \times 19^1$
If a number is expressed as product of prime factors, like $a^p \times b^q \times c^r$ x…… where a, b, c,….. are prime numbers, then the number of ways in which the number can be expressed as a product of two factors = 1/2 [(p + 1) (q + 1) (r + 1)………]
Hence, 3420 can be written as product of two factors in ½[(2 + 1) (2 + 1) (1 + 1) (1 + 1)] = 18 ways

1.10. In how many ways can the number 52900 be written as a product of two different factors?

Sol: First expressing 52900 as a product of its prime factors, we get $52900 = 23^2 \times 2^2 \times 5^2$. Since all the powers are even, the given number is a perfect square. (Remember we can look at writing the number as a product of two factors either including or excluding the "square root x square root". Since we have to find the number of ways of writing the number as a product of two "different" factors, we cannot consider square root x square root)
So, required number of ways is
1/2 {(2 + 1) (2 + 1) (2 + 1) – 1}= 1/2{27 – 1} = 13

Sum of all the factors of a number

If a number $N = a^p \cdot b^q \cdot c^r$ ………. where a, b, c….. are prime numbers and p, q, r ….. are positive integers, then, the sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1}-1}{a-1}\right).\left(\frac{b^{q+1}-1}{b-1}\right).\left(\frac{c^{r+1}-1}{c-1}\right)............$$

The above can be verified by an example.
Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here a = 2, b = 3, p = 4, q = 1.
Hence, sum of all the factors

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1} \times \frac{8}{2} = 124$$

The list of factors of 48 is
1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

Product of all the factors of a number

The following examples explain the method of finding the product of all the factors of a number.

1.11. What is the product of all the factors of 180?

Sol: $180 = 4(45) = 2^2 3^2 5^1$. There are (2+1) (2+1) (1+1) or 18 factors.
If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).

∴ The required product is 180^9.

In general, if $N = p^a q^b r^c$ (where at least one of a, b, c is odd), the product of all the factors of N is $N^{\frac{d}{2}}$, where d is the number of factors of N and is given by (a+1)(b+1)(c+1).

1.12. Let us see what happens when N is a perfect square. Find the product of all the factors of 36.

Sol: $36 = 2^2 3^2$ (there are 9 factors)
1(36) = 2(18) = 3(12) = 4(9) = 6(6)
∴ The product of all the factors is 36^4 (6).

In general, let $N = p^a q^b r^c$ where each of a, b, c is even.
There are (a + 1)(b + 1)(c + 1) say d factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e., $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}} (\sqrt{N}) = N^{\frac{d}{2}}$
∴ Whether or not N is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where d is the number of factors of N.

1.13. What is the product of all the factors of 1728?

Sol: The product of the factors of a positive integer N is $N^{k/2}$, where k is the number of factors of N.
Now $1728 = 12^3 = 2^6 3^3$ and k = (6 + 1) (3 + 1) = 28
∴ The product of all the factors of 1728 $= 1728^{14}$

Number of ways of writing a number as product of two co-primes

Using the same notation and convention used earlier.
If $N = a^p \cdot b^q \cdot c^r$……………, then, the number of ways of writing N as a product of 2 co-primes is 2^{n-1}, where 'n' is the number of distinct prime factors of the given number N.

Taking the example of 48, which is 2^4 x 3^1, the value of 'n' is 2 because only two distinct prime factors (i.e. 2 and 3 only) are involved.
Hence, the number of ways = $2^{2-1} = 2^1 = 2$ i.e. 48 can be written as product of 2 co-primes, in two different ways. They are (1, 48) and (3, 16).

Number of co-primes to N, that are less than N

If N is a number that can be written as $a^p . b^q . c^r$ ……….., then, the number of co-primes of N, which are less than N, represented by ϕ (N) is,

$$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\text{...........}$$

For example if, 48 is considered,
N = $a^p . b^q . c^r$ ……….. i.e. 48 = $2^4 . 3^1$.
Hence, a = 2, b = 3, p = 4, q = 1.

$$\phi\,(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 48\times\frac{1}{2}\times\frac{2}{3} = 16.$$

Note : If numbers less than 48 are listed, and co-primes to 48 are spotted, the count of co-primes will be 16.

Sum of co-primes to N that are less than N

The sum of the co-primes of N, that are less than N is $\frac{N}{2} . \phi(N)$. If we consider the above example, already we have ϕ (48) = 16.
Hence, sum of co-primes of 48 that are less than 48

$$= \frac{N}{2} . \phi(N) = \frac{48}{2} \text{ x } 16 = 384$$

Note: After listing the co-primes of 48 that are less than 48, they can be added and the sum can be verified.

1.14. Find the largest three digit multiple of 32.

Sol. Largest three digit number = 999. When 999 is divided by 32, the remainder is 7
$\therefore$ 999 – 7 = 992 is the largest three digit multiple of 32.

1.15. Find the smallest four digit multiple of 32.

Sol. Smallest four digit multiple of 32
= Largest three digit multiple of 32 + 32
= 992 + 32 = 1024.

LEAST COMMON MULTIPLE (LCM) AND HIGHEST COMMON FACTOR (HCF)

Least Common Multiple (LCM) of two or more numbers is the least number which is divisible by each of these numbers (i.e. leaves no remainder; or remainder is zero). The same can be algebraically defined as "LCM of two or more expressions is the expression of the lowest dimension which is divisible by each of them i.e. leaves no remainder; or remainder is zero."

Highest Common Factor (HCF) is the largest factor of two or more given numbers. The same can be defined algebraically as " HCF of two or more algebraical expressions is the expression of highest dimension which divides each of them without remainder.
HCF is also called GCD (Greatest Common Divisor).

Product of two numbers = LCM x HCF

LCM is a multiple of HCF

For finding **LCM and HCF of fractions,** first reduce each fraction to its simplest form i.e., cancel out any common factors between the denominator and numerator and then apply appropriate formula from the following :

$$\textbf{HCF of fractions} = \frac{\textbf{HCF of numerators}}{\textbf{LCM of denominators}}$$

$$\textbf{LCM of fractions} = \frac{\textbf{LCM of numerators}}{\textbf{HCF of denominators}}$$

LCM and HCF can each be found by either one of two methods :
(1) Factorization (2) Long Division
We will look at both the methods.

LCM BY FACTORIZATION

Resolve the numbers into prime factors. Then multiply the product of all the prime factors of the first number by those prime factors of the second number, which are not common to the prime factors of the first number.

This product is then multiplied by those prime factors of the third number, which are not common to the prime factors of the first two numbers.

In this manner, all the given numbers have to be dealt with and the last product will be the required LCM.

In other words, take the product of ALL the prime factors of all the numbers except where a factor is occuring in more than one number, it is taken only ONCE in the product. This product is the LCM of all the numbers.

LCM BY DIVISION

Select any one prime factor common to at least two of the given numbers. Write the given numbers in a line and divide them by the above prime number. Write down the quotient for every number under the number itself. If any of the numbers is not divisible by the prime factor selected, write the number as it is in the line of quotients.

Repeat this process for the line of quotients until you get a line of quotients, which are prime to each other (i.e., no two "quotients" should have a common factor).
The product of all the divisors and the numbers in the last line will be the required LCM.

HCF BY FACTORIZATION

Resolve the given number into prime factors. The product of the prime factors common to all the numbers will be the required HCF.

1.16. Find the HCF and LCM of 288, 432 and 768 using factorisation method.

Sol. 288 = (24) (12)
432 = (24) (18) and 768 = (24) (32)
HCF 288, 432, 768 = 24 $\times$ HCF (12, 18, 32)
= 24 $\times$ 2 = 48
LCM (288, 432, 768) = 24 $\times$ LCM (12, 18, 32)
= 24 $\times$ 288 = 6912

HCF BY LONG DIVISION

Take two numbers. Divide the greater by the smaller; then divide the divisor by the remainder; divide the divisor of this division by the next remainder and so on until the remainder is zero. The last divisor is the HCF of the two numbers taken.

By the same method find the HCF of this HCF and the third number. This will be the HCF of the three numbers.

1.17. Find the HCF of 288 and 432 using long division method. Also find their LCM by division method.
Sol: HCF: Let us first find HCF of 288 and 432

```
              1
 288 ) 432
       288        2
      -----
       144 ) 288
             288
            -----
              0
```

$\therefore$ HCF of 288 and 432 is 144.
LCM:

	288,	432
2	144,	216
2	72,	108
2	36,	54
2	18,	27
3	6,	9
3	2,	3

LCM of 288 and 432
= (2) (2) (2) (2) (3) (3) (2) (3)
= $2^5 \times 3^3$ = (32) (27) = 864

LCM AND HCF MODELS

LCM - Model 1

In this model of problem, you will need to find out the smallest number (or number in a specified range like the largest five-digit number) which when divided by 2 or more other numbers (i.e., divisors) leaves the **same** remainder in all cases.

The basic distinguishing feature of this model of problems is that the remainder will be the **same** in all the cases (and that remainder will also be given).

The smallest such number will be the remainder itself. The next higher number that satisfies the given conditions is the LCM of the given numbers (i.e., divisors) plus the remainder given, i.e., add the remainder (which is the same in all cases) to the LCM of the given numbers (i.e., divisors).

To find any larger number that satisfies a given condition, we will first need to find out a multiple of the LCM in that range and add the remainder to this multiple of the LCM.

The general rule can be written as follows:

> Any number which when divided by p, q or r leaving the same remainder s in each case will be of the form **k (LCM of p, q and r) + s** where k = 0, 1, 2, ...
> If we take k = 0, then we get the smallest such number.

1.18. Find the HCF of 1363 and 1457.

Sol:

```
1363 ) 1457 ( 1
       1363
     94 ) 1362 ( 14
          1316
        47 ) 94 ( 2
             94
             0
```

$\therefore$ HCF (1367, 1457) = 47

1.19. Find the smallest number which when divided by 5 or 11 leaves a remainder of 4 and is greater than the remainder.

Sol: Set of such numbers are of the form K [LCM (5, 11)] + 4 where K is a whole number.
We get the required number when K = 1
$\therefore$ Smallest number = L.C.M (5, 11) + 4
= 55 + 4 = 59.

1.20. Find the largest three-digit number which when divided by 8 or 12 leaves a remainder of 2 in each case.

Sol: Required number must leave a remainder of 2 when divided by L.C.M (8, 12) = 24.
$\therefore$ It must be of the form 24K + 2, where K is the largest natural number satisfying 24K + 2 < 1000.
$\Rightarrow$ K < $41\frac{7}{12}$; $\therefore$ K = 41.
$\therefore$ Largest number = 986.

1.21. Find the smallest number which when divided by 4, 11 or 13 leaves a remainder of 1 and is greater than the remainder.

Sol: Required number = L.C.M (4, 11, 13) + 1 = 573.

LCM - Model 2

In this model, the remainders in the divisions given will not be the same but the difference between the divisor and the remainder (i.e. the complement of the remainder) will be the same in each case. For example, you may be asked to find out "the smallest number which when divided by 4 or 6 gives respective remainders of 3 and 5." Here, the remainders are not the same as in LCM - Model 1; but the difference between the divisor and the remainder is same in each case. In the first case the difference between the divisor and the remainder is 1(= 4 – 3). In the second case also the difference between the divisor and the remainder is 1(= 6 – 5).

The smallest such number is LCM minus constant difference (the constant difference being the difference between the divisor and the corresponding remainder in all cases).

Similarly, any multiple of the LCM minus the constant remainder also will satisfy the same condition.
In the example considered above, the LCM of 4 and 6 is 12 and hence the required number is 11 (which is equal to 12 – 1).
The general rule can be written as follows:

> Any number which when divided by p, q or r leaving respective remainders of s, t and u where (p – s) = (q – t) = (r – u) = v (say), will be of the form
> **k (LCM of p, q and r) -** v
> The smallest such number will be obtained by substituting k = 1.

1.22. Find the smallest number which when divided by 9 and 11 leaves remainders of 7 and 9 respectively.

Sol: Required number = L.C.M (9, 11) – 2 = 97.

1.23. Find the largest four-digit number which when divided by 9 and 11 leaves remainders of 7 and 9 respectively.

Sol: Required number must be in the form LCM (9, 11) K – 2 i.e. 99K – 2, where K is the largest natural number satisfying 99K – 2 < 10000.

$\therefore$ K < $101\frac{1}{33}$

$\therefore$ K = 101

$\therefore$ Largest number = 9997

1.24. Find the smallest six-digit number which leaves a remainder of 10 when divided by 13 and leaves a remainder of 4 when divided by 7.

Sol: Required number must be in the form
LCM (13, 7) K – 3 i.e. 91K – 3, where K is the smallest natural number satisfying 91K – 3 > 100000

K > $\frac{100003}{91} = 1098\frac{85}{91}$

$\therefore$ K = 1099

$\therefore$ Smallest number = 100006

LCM - Model 3

In this model the remainders will not be the same and even the differences between each of the given divisors and the corresponding remainders also will not remain the same.

Let us take an example and see how to solve this type of problem.

Find out the smallest number which when divided by 7 gives a remainder of 3 and when divided by 5 gives the remainder of 2.

Sol: Here, the remainders are not the same. The difference between the divisor and the remainder in the first case is 4 and in the second case, is 3.

Take the larger of the two given divisors –7 in this case. The required number, when divided by 7 gives a remainder of 3. We know that a number when divided by 7 giving a remainder of 3 is of the form 7k + 3, which means we are looking for a number of the form 7k + 3.
Since the same number, when divided by 5 gives a remainder of 2, this number (7k + 3) when divided by 5 gives a remainder of 2. We know that if there is a remainder in a division, by subtracting the remainder from the given number, the resulting number will then be exactly divisible by the divisor. This means, if 2 is subtracted from (7k + 3), the resulting number, i.e., 7k + 1 will be exactly divisible by 5. We should now give values of 0, 1, 2, ... to k and find out for what value of k, 7k + 1 will be divisible by 5.

The smallest value of k which satisfies the above condition, we notice, is 2 and hence k = 2 will give us a number that we are looking for. Since the number, we said, is 7k + 3 the number is 7 x 2 + 3 i.e. 17. So 17 is the smallest number which satisfies the two given conditions.

The next higher number which satisfies this condition is obtained by adding LCM of 7 and 5 to the smallest number 17 found above. In this manner by adding multiples of 35 (which is LCM of the two given numbers) to 17, we get a series of numbers that satisfy the given conditions. In other words any number of the form (35m + 17) will satisfy the given conditions.

From this, we can also find out the smallest 4 digit number, largest 5 digit number, etc. that will satisfy the given conditions.

For example, let us find out the largest five-digit number that satisfies the conditions that the remainders are 3 and 2 respectively when divided by 7 and 5.

Since we know that any number that satisfies the above condition will be of the form (35m + 17) and we want the largest 5-digit such number, we need to find a number close to 99999, i.e., 35m+17 $\leq$ 99999 $\Rightarrow$ 35m $\leq$ 99982 $\Rightarrow$ we need to find a multiple of 35 which less than or equal to 99982 (and we have already learnt how to find the multiple of a given number which is less than or equal to another given number). A multiple of 35 less than or equal to 99982 is 99960 (i.e., 35m = 99960). Hence the required number which is 35m + 17 will then be equal to 99960 + 17, i.e., 99977

1.25. Find the smallest number which leaves a remainder of 7 when divided by 11 and leaves a remainder of 12 when divided by 13.

Sol: Let the number be in the forms $11K_1 + 7$ and $13K_2 + 12$ where K_1 and K_2 have the least possible values.
$11K_1 + 7 = 13K_2 + 12$
$K_1 = K_2 + \dfrac{2K_2 + 5}{11}$
As K_1 is an integer, $2K_2 + 5$ must be divisible by 11.
Hence $K_2 = 3$.
$\therefore$ Smallest number = 51.

HCF - Model 1

In this model, we have to identify the largest number that exactly divides the given dividends (which are obtained by subtracting the respective remainders from the given numbers).

> The largest number with which the numbers p, q or r are divided giving remainders of s, t and u respectively will be the **HCF of the three numbers (p – s), (q – t) and (r – u).**

Let us understand this model with an example.

1.26. Find the largest number which leaves remainders of 2 and 3 when it divides 89 and 148 respectively.

Sol: Largest number = H.C.F (89 – 2, 148 – 3)
= 29

HCF - Model 2

In this model, the problem will be as follows:

"Find the largest number with which if we divide the numbers p, q and r, the remainders are the same."
Take the difference between any two pairs out of the three given numbers. Let us say we take the two differences (p – q) and (p – r). The HCF of these numbers will be the required number.

> Here, the required number = HCF of (p – q) and (p – r) = HCF of (p – q) and (q – r) = HCF of (q – r) and (p – r)

Let us take an example and look at this model.

1.27. Find the largest number which divides 444, 804 and 1344 leaving the same remainder in each case.

Sol: Largest number
= H.C.F (804 – 444, 1344 – 804)
= H.C.F (360, 540) = 180.

SUCCESSIVE DIVISION

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called "successive division." A successive division process can continue upto any number of steps – until the quotient in a division becomes zero for the first time. i.e., the quotient in the first division is taken as dividend and divided in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

If we say that 2479 is divided successively by 3, 5, 7 and 2, then the quotients and remainders are as follows in the successive division.

Dividend/Divisor		Quotient	Remainder
2479	3	826	1
826	5	165	1
165	7	23	4
23	2	11	1

Here we say that when 2479 is successively divided by 3, 5, 7 and 2 the respective remainders are 1, 1, 4 and 2.

1.28. A number when divided by 6 and 4 successively leaves remainders of 5 and 2 respectively. Find the remainder when the largest such two digit number is divided by 9.

Sol: Let the quotients obtained when the number is divided by 6 and 4 successively be q_1 and q_2 respectively.
Number = $6q_1 + 5$
In successive division, the quotient obtained for each division starting from the first, forms the dividend for the next division.
$\therefore q_1 = 4q_2 + 2$
$\therefore$ number = $6(4q_2 + 2) + 5 = 24q_2 + 17$
Largest two-digit number satisfying the given conditions is obtained when $24q_2 + 17 < 100$ and q_2 is maximum i.e. $q_2 < 3\frac{11}{24}$ and it is maximum i.e., $q_2 = 3$.
$\therefore$ number = 89. Required remainder = 8

Alternative Method:

Divisors : 6 $\times$ 4

Remainders : 5 + 2

The smallest number satisfying the given conditions is found by using the following method. Each divisor and the remainder it leaves are written as shown above. Starting with the last remainder, each remainder is multiplied with the previous divisor and added to that divisor's remainder. This procedure is carried out until the divisor's remainder is the first remainder.

Smallest possible value of the number
= (6) (2) + 5 = 17
General form of the number = $k(6 \times 4) + 17$
= $24k + 17$ where k is any whole number.
The number would be the largest two-digit number when $24k + 17 < 100$ and k is maximum i.e. $k < 3^{11}/_{24}$ and k is maximum i.e. k = 3.
$\therefore$ Largest two-digit number = 89
$\therefore$ required remainder = 8

1.29. A number when divided by 3, 5 and 6 successively leaves remainders of 1, 3 and 2 respectively. Find the number of possible values it can assume which are less than 1000.

Sol: Let the quotients obtained when the number is divided by 3, 5 and 6 successively be q_1, q_2 and q_3 respectively.

Number = $3\, q_1 + 1$
$q_1 = 5q_2 + 3$
$q_2 = 6q_3 + 2$
$\therefore$ number = $3\,(5q_2 + 3)$
$= 3\,(5(6q_3 + 2) + 3) + 1 = 90q_3 + 40$
$90q_3 + 40 < 1000$
$q_3 < 10\frac{2}{3}$
$\therefore$ q_3 has 11 possibilities i.e. 0 to 10.

Alternative method:

Divisors :	3	5	6
Remainders:	1	3	2

Smallest possible value of the number
$= ([(5 \times 2) + 3] \times 3) + 1 = 40$
General form of the number = $k \times (3 \times 5 \times 6) + 40 = 90\,k + 40$, where k is any whole number.

If $90\,k + 40 < 1000$, $k < 10\,\frac{2}{3}$

$\therefore$ k has 11 possibilities (i.e., 0 to 10)

1.30. A number when divided by 3, 5 and 6 successively leaves remainders of 1, 3 and 2 respectively. Find the remainders if its smallest possible value is divided successively by 6, 5 and 3.

Sol:

Divisors	3	5	6
Remainders	1	3	2

Smallest possible number
$= ((2)\,(5) + 3)\,3 + 1 = 40$
Required remainders are 4, 1 and 1.

Factorial

Factorial is first defined for positive integers. It is denoted by $\angle$ or !. Thus "Factorial n" is written as n! or $\underline{/n}$. n! is defined as the product of all the integers from 1 to n.

Thus $n! = 1.2.3. \ldots n(n - 1)\, n$.
0! is defined to be equal to 1.
0! = 1 and, 1! is also equal to 1.

IGP of a Divisor in a Number

Very often we would like to know how many times we can divide a given number by another and continue to get integral quotients. We first consider prime divisors and then other divisors.

If a single number is given we simply represent it in its canonical form (the simplest and must convenient form). For example, consider N = 258,048.

By trial, we express $N = 2^{12}3^{2}7^{1}$. We see immediately that N can be divided by 2 a total of 12 times, by 3 two times and by 7 just once. In other words the index of the greatest power (IGP) of 2 in N is 12, of 3 is 2 and of 7 is 1.

IGP of a number in N!

This model involves finding the index of the greatest power (**IGP**) of a divisor that divides the factorial of a given number (say N). (The statement 'a divides b' means the remainder of b divided by 'a' is 0. In this case, we also say 'b is divisible by a'.) Let us understand this type of problem with the help of an example.

1.31. Find the IGP of 7 that can divide 256!, without leaving any remainder. (This can be concisely stated as find the IGP of P in N!)

Sol: First we shall take a look at the detailed explanation and then look at a simple method for solving the problem. When we write N = 256! in its expanded form, we have 256 x 255 x 254 x ……. x 3 x 2 x 1
When we divide 256! by a power of 7, we have the first 256 natural numbers in the numerator. The denominator will have only 7's. The 256 numbers in the numerator have 36 multiples of 7 which are 7, 14, 21 ……. 252. Corresponding to each of these we can have a 7 in the denominator which will divide N completely without leaving any remainder i.e., 7^{36} can definitely divide 256! Further, every multiple of 49 after cancelling out 7 as above, will still have one more 7 left. Hence for every multiple of 49 N we can have an additional 7 in the denominator. There are 5 multiples of 49 in 256! Hence we can have a 7^5 in the denominator. As $7^{36+5} = 7^{41}$, 41 is the IGP.

The above calculation is summarised below. Successively dividing 256 by 7, we get:

$$\begin{array}{r|l} 7 & 256 \\ \hline 7 & 36 \\ \hline & 5 \end{array}$$

Add all the quotients to get 36 + 5 = 41.
So the IGP of 7 contained in 256! is 41.

Please note that this method is applicable only if the number whose greatest power is to be found out is a prime number.

1.32. Find the IGP of 3 in 599!

Sol: Divide 599 successively by 3

$$\begin{array}{r|l} 3 & 599 \\ \hline 3 & 199 \rightarrow \text{quotient} \\ \hline 3 & 66 \rightarrow \text{quotient} \\ \hline 3 & 22 \rightarrow \text{quotient} \\ \hline 3 & 7 \rightarrow \text{quotient} \\ \hline & 2 \end{array}$$

Add all the quotients,
199 + 66 + 22 + 7 + 2 = 296
Hence, 296 is the largest power of 3 that divides 599! without leaving any remainder.

1.33. Find the IGP of 10 that can divide 890!.

Sol: Here we cannot apply the successive division method as 10 is not a prime number. We know 10 can be written as 2 x 5 and these are prime numbers. So we find the largest powers of 2 and 5 respectively that can divide 890! and the smaller of the two indices is the index of the required power.

2 | 890
2 | 445
2 | 222
2 | 111
2 | 55
2 | 27
2 | 13
2 | 6
2 | 3
  1

Sum of the quotients = 883

5 | 890
5 | 178
5 | 35
5 | 7
  1

Sum of the quotients = 221
Since the largest power of 5 is the smaller, the largest power of 10 (i.e., 2 x 5) is 221.

If the divisor (say D) is not a prime number, we resolve it into its prime factors. Let D = $p^m q^n$ (where p, q are primes and m, n are positive integers). We first determine the IGP of p that divides N and the IGP of q that divides N. Let these be a and b respectively. Therefore, the IGP of p^m that divides N is $\left\lfloor \frac{a}{m} \right\rfloor$ and the IGP of q^n that divides N is $\left\lfloor \frac{b}{n} \right\rfloor$. Finally, the IGP of D that divides N is the smaller of $\left\lfloor \frac{a}{m} \right\rfloor$ and $\left\lfloor \frac{b}{n} \right\rfloor$. [$\lfloor x \rfloor$ is the greatest integer less than or equal to x.]

1.34. Find the IGP of 12 in 50!

Sol: $12 = 2^2 3$.
The IGP of 2 in 50! is obtained by successive division as shown below.

Number/Quotient	50	25	12	6	3	1
Divisor	2	2	2	2	2	

The IGP of 2 in 50! is 25 + 12 + 6 + 3 + 1 = 47
The IGP of 2^2 in 50! is $\left\lfloor \frac{47}{2} \right\rfloor = 23$
The IGP of 3 in 50! is 16 + 5 + 1 = 22
∴ The IGP of 12 in 50! is the smaller of 23 and 22, viz 22.

The following two results will prove to be extremely useful in problems on IGPs.

Let the IGP of p in A and B be m and n respectively.
(1) The IGP of p in AB is m + n.
(2) (a) If m ≠ n, the IGP of p in A + B is the smaller of m and n.
(b) If m = n, the IGP of p in A + B is at least m. It could be more. (For example the IGP of 2 in 58 is 1 and the IGP of 2 in 6 is also 1. But the IGP of 2 in 58 + 6 is 6.)

1.35. Find the IGP of 2 in 31! + 32! + 33! + ... + 40!.

Sol: The IGP of 2 in 31! is 15 + 7 + 3 + 1, viz 26.
The IGP of 2 in 32! is 16 + 8 + 4 + 2 + 1, viz 31.
The IGP of 2 in the other terms is 31 or more.
∴ The IGP of 2 in the given expression is 26.

ALGEBRAIC IDENTITIES

There are a number of identities that we have studied in lower classes. We consolidate them here. We can classify them on two criteria – the number of symbols that are used and the degree of each term in the identity.

Identities with two symbols (degree 2)
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 - b^2 = (a + b)(a - b)$

Identities with two symbols (degree 3)
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Identities with three symbols (degree 2)
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$
$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$
$(x + a)(x + b) = x^2 + x(a+b) + ab$

Identities with three symbols (degree 3)
$(a + b)(b + c)(c + a) = a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2) + 2abc$
$= a^2(b + c) + b^2(c + a) + c^2(a + b) + 2abc$
$= ab(a + b) + bc(b + c) + ca(c + a) + 2abc$
$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc)$

Examples:

$112^2 = (100 + 12)^2$
$= 100^2 + (2 \times 100 \times 12) + 12^2 = 12544$
$89^2 = (100 - 11)^2$
$= 100^2 - (2 \times 100 \times 11) + 11^2 = 7921$
$17 \times 23 = (20 - 3)(20 + 3) = 20^2 - 3^2 = 391$
$17^2 = (17 + 3)(17 - 3) + 3^2$
$= 20 \times 14 + 9 = 289$
$39^2 = (39 + 1)(39 - 1) + 1^2 = 40 \times 38 + 1^2$
$= 1520 + 1 = 1521$
$13^3 = (10 + 3)^3$
$= 10^3 + 3^3 + (3 \times 10 \times 3)(10 + 3) = 2197$

SOME IMPORTANT POINTS

Please note the following points which will be very useful in solving problems on Numbers.

1. When any two consecutive integers are taken, one of them is odd and the other is even. Hence the product of any two consecutive integers is always even i.e. divisible by 2.

   Two consecutive integers can be written in the form of n and n – 1 or n and n + 1. Hence, any number of the form n(n – 1) or n(n + 1) will always be even.

2. Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6.

   Three consecutive integers can be of the form (n – 1), n and (n + 1). The product of 3 consecutive integers will be of the form (n – 1)n(n + 1) or $n(n^2 - 1)$ or $(n^3 - n)$. Hence any number of the form (n – 1)n(n + 1) or $n(n^2 - 1)$ or $(n^3 - n)$ will always be divisible by 6.

3. Out of any n consecutive integers, exactly one number will be divided by n and the product of n consecutive integers will be divisible by n!

4. Any prime number greater than 3 can be written in the form of 6k + 1 or 6k – 1. The explanation is:

Let p be any prime number greater than 3. Consider the three consecutive integers (p – 1), p and (p + 1). Since p is a prime number greater than 3, p CANNOT be even. Since p is odd, both (p – 1) and (p + 1) will be even, i.e., both are divisible by 2. Also, since, out of any three consecutive integers, one number will be divisible by 3, one of the three numbers (p – 1), p or (p + 1) will be divisible by 3. But, since p is a prime number, that too greater than 3, p cannot be divisible by 3. Hence, either (p – 1) or (p + 1), one of them and only one of them, is definitely divisible by 3.

If (p – 1) is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form 6k. If (p – 1) is of the form 6k, then p will be of the form (6k + 1).

If (p + 1) is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form 6k. If (p + 1) is of the form 6k, then p will be of the form (6k – 1).

Hence any prime number greater than 3 will be of the form (6k + 1) or (6k – 1).

1.36. Find the HCF of $\frac{3}{5}, \frac{6}{10}$ and $\frac{9}{20}$.

Sol: To find the LCM or HCF of fractions, first express all the fractions in their simplest form.

HCF (fractions) =

$$\frac{\text{HCF}(\text{numerators})}{\text{LCM}(\text{denominators})} = \frac{\text{HCF}(3,3,9)}{\text{LCM}(5,5,20)} = \frac{3}{20}$$

1.37. Find the LCM of $\frac{3}{5}, \frac{6}{10}$ and $\frac{9}{20}$.

Sol: To find the LCM or HCF of fractions, first express all the fractions in their simplest form.

LCM (fractions)

$$= \frac{\text{LCM}(\text{numerators})}{\text{HCF}(\text{denominators})} = \frac{\text{LCM}(3,3,9)}{\text{HCF}(5,5,20)} = \frac{9}{5}$$

1.38. Arrange the following in ascending order $\frac{5}{7}, \frac{3}{4}$ and $\frac{7}{10}$.

Sol: LCM (10, 7, 4) = 140

$$\frac{7}{10} = \frac{98}{140}$$

$$\frac{5}{7} = \frac{100}{140}$$

$$\frac{3}{4} = \frac{105}{140}$$

$$\therefore \frac{7}{10} < \frac{5}{7} < \frac{3}{4}$$

1.39. Test whether the number 12320 is divisible by 2, 3, 4, 5, 6, 9, 10, 11 and 19.

Sol. The number has its last two digits divisible by 4 and ends with a 0.

$\therefore$ it is divisible by 4 and hence by 2 and also by 10 and 5.

The sum of the digits of the number is 8

$\therefore$ it is not divisible by 3. Hence it is neither divisible by 6 nor by 9.

The sum of the digits in the odd places = The sum of the digits in even places.

$\therefore$The number is divisible by 11.

Number of tens in the number + 2 (units digit of the number) = 1232 which is not divisible by 19.

$\therefore$ the number is not divisible by 19.

1.40. Simplify: $\frac{(3\cdot69 + 2\cdot16)^2 + (3\cdot69 - 2\cdot16)^2}{3\cdot69^2 + 2\cdot16^2}$.

Sol. The given expression is in the form

$$\left(\frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}\right)$$

where a = 3·69 and b = 2.16

$$\frac{(a+b)^2 + (a-b)^2}{(a^2 + b^2)} = \frac{2(a^2 + b^2)}{a^2 + b^2} = 2.$$

1.41. Simply:

$$\frac{[3.19\times3.19\times3.19 + 2.23\times2.23\times2.23 + 1.58\times1.58\times1.58 - 9.57\times2.23\times1.58]}{[3\cdot19^2 + 2\cdot23^2 + 1\cdot58^2 - (3\cdot19)(2\cdot23) - (3\cdot19)(1\cdot58) - (2\cdot23)(1\cdot58)]}$$

Sol. The given expression is in the form

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

where a = 3·19, b = 2·23 and c = 1·58

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = a + b + c$$

$\therefore$The given expression equals 7.

1.42. Simplify : $1\cdot42^2 + 2\cdot33^2 + 4\cdot25^2$ + (2·84) (2·33) + (4·66) (4·25) + (8·5) (1·42)

Sol: The given expression is in the form $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ where a = 1·42, b = 2·33 and c = 4·25

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

$\therefore$ The given expression equals 64.

1.43. Simplify : 4·56 × 4·56 × 4·56 + 3·44 × 3·44 + 3·44 + 13·68 × 4·56 × 3·44 + 10·32 × 4·56 × 3·44.

Sol: The given expression is in the form $a^3 + b^3 + 3a^2b + 3ab^2$ where a = 4·56 and b = 3·44

$a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3$

$\therefore$The given expression equals 512.

(ii) Simplify : 4·56 × 4·56 × 4·56 – 0·56 × 0·56 × 0·56 – 13·68 × 4·56 × 0·56 + 1·68 × 4·56 × 0·56

The given expression is in the form $a^3 - b^3 + 3ab^2 - 3a^2b$ where a = 4·56 and b = 0·56.

$a^3 - b^3 + 3ab^2 - 3a^2b = (a - b)^3$

$\therefore$The given expression equals 64.

Concept Review Questions

Directions for questions 1 to 80: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. $2^1 \times 2^2 \times 2^3 \times 2^4 \times 2^5 =$ _______.
(A) 37268 (B) 36728 (C) 32768 (D) 42768

2. **(a)** The sum of 36 odd numbers is _______.
(A) Even (B) Odd (C) Cannot say

(b) The product of 5 composite numbers is _______.
(A) Even (B) Odd (C) Cannot say

(c) The sum of 6 composite numbers is _______.
(A) Even (B) Odd (C) Cannot say

(d) The product of 5 prime numbers is _______.
(A) Even (B) Odd (C) Cannot say

(e) The sum of the first 10 prime numbers is _______.
(A) Even (B) Odd (C) Cannot say

3. Find the number of prime factors of 19019.

$\boxed{}$

4. **(a)** $0.2\overline{55} =$ _______.
(A) $\frac{23}{90}$ (B) $\frac{23}{99}$ (C) $\frac{253}{990}$ (D) $\frac{253}{900}$

(b) $0.3\overline{21} =$ _______.
(A) $\frac{53}{165}$ (B) $\frac{106}{333}$
(C) $\frac{10}{11}$ (D) None of these

(c) $0.32\overline{1} =$ _______.
(A) $\frac{289}{900}$ (B) $\frac{289}{990}$ (C) $\frac{32}{99}$ (D) $\frac{16}{45}$

(d) Express the recurring decimal $1.1\overline{16}$ in the form of a fraction.
(A) 367/330 (B) 221/198
(C) 62/55 (D) 223/198

5. Which of the following is a prime number?
(A) 851 (B) 589 (C) 429 (D) 307

6. Which of the following pairs of numbers are not twin primes?
(A) 131 and 133
(B) 191 and 193
(C) 157 and 159
(D) More than one of above

7. Which of the following is divisible by 11?
(A) 8787878
(B) 7777777
(C) 1234567
(D) More than one of the above

8. If the eight-digit number X7654321 is divisible by 9 where X is a single digit whole number, find X.

$\boxed{}$

9. Is the ten digit number PQRSTU8672 divisible by 32?
(A) Yes (B) No (C) Cannot say

10. Is the nine-digit number ABCDE9025 divisible by 625?
(A) Yes (B) No (C) Cannot say

11. The difference between a two digit number and the sum of its digits is always a multiple of _______.
(A) 6 (B) 9 (C) 11 (D) 1

12. If a is a single digit number and $a^n - a$ is divisible by 10, where n is any natural number, how many values can a take?
(A) 4 (B) 5 (C) 3 (D) 2

13. If n is a composite number and $(n - 1)!$ is not divisible by n, how many possibilities exist for n?

$\boxed{}$

14. The product of any 4 consecutive natural numbers is one less than _______.
(A) a perfect square (B) a perfect cube
(C) both (A) and (B) (D) Neither (A) nor (B)

15. How many odd natural numbers have the same parity as their factorials?

$\boxed{}$

16. The product of 7 consecutive natural numbers is always divisible by _______.
(A) 5040 (B) 10080 (C) 3430 (D) 6860

17. Find the number of factors of $2^{10} \times 10^2$.

$\boxed{}$

18. A number has an even number of factors. It must be _______.
(A) a perfect square
(B) a perfect cube
(C) both (A) and (B)
(D) None of these

19. In how many ways can $5^4\,7^6$ be written as a product of 2 coprimes?
(A) 1 (B) 2 (C) 3 (D) 4

20. In how many ways can $5^4\,7^6$ be written as a product of 2 distinct numbers?
(A) 35 (B) 36 (C) 18 (D) 17

21. What is the sum of the factors of $2^4 \times 3^3$?

$\boxed{}$

22. N is a perfect number. What is the ratio of the sum of the factors of N to N?
(A) 1 (B) 2 (C) 3 (D) 4

23. How many numbers less than 2^{24} are co prime to it?
(A) 2^{12} (B) 2^{23}
(C) 2^{22} (D) None of these

24. If $N = 2^a \times 3^b \times 5^c$, how many numbers are less than N and are co-prime to it?
(A) $\frac{2}{15}N$ (B) $\frac{4}{15}N$ (C) $\frac{8}{15}N$ (D) $\frac{2}{5}N$

25. The sum of all the co primes to 72 less than it is $\boxed{\quad}$.

26. (a) If $b + \frac{1}{b} = 4$, then $b^2 + \frac{1}{b^2} =$ _______.
(A) 10 (B) 8 (C) 12 (D) 14

(b) If $b - \frac{1}{b} = 4$, then $b^2 + \frac{1}{b^2} =$ _______.
(A) 22 (B) 18 (C) 20 (D) 24

27. If a + b = 30 and ab = 176, find $a^3 + b^3$.
(A) 10160 (B) 11060
(C) 11160 (D) None of these

28. If a + b + c = 0, then $a^3 + b^3 + c^3 =$ _______.
(A) 0 (B) abc (C) 3abc (D) 2abc

29. If $a^3 + b^3 + c^3 = 3abc$, then _______.
(A) a + b + c = 0
(B) a = b = c
(C) Both (A) and (B)
(D) At least one of (A) and (B)

30. $\dfrac{10.23^3 - 4.77^3}{10.23^2 + 4.77^2 + (10.23)(4.77)} = \boxed{\quad}$.

31. The LCM and HCF of 2 numbers are 120 and 6 respectively. If one of the numbers is 30, find the other.
(A) 27 (B) 33 (C) 24 (D) 18

32. If LCM (P, Q, R) = (P) (Q) (R) then H.C.F (P, R) = _______.
(A) 1 (B) 2
(C) Q (D) Cannot say

33. If P, Q and R are mutually co-prime, is LCM (P, Q, R) = (P) (Q) (R)?
(A) Yes (B) No (C) Cannot say

34. Find LCM (150, 180, 270).
$\boxed{\quad}$

35. Find HCF (63, 42, 105).
$\boxed{\quad}$

36. Find the respective values of LCM and HCF of the following numbers/fractions.
(a) 42, 72, 90
(A) 2520, 6 (B) 7560,12 (C) 1260, 3 (D) 3780,18

(b) 810,720
(A) 11664, 90 (B) 6480, 45
(C) 3880, 45 (D) 6480, 90

(c) 1830, 1098
(A) 2745, 122 (B) 10980, 45
(C) 5490, 366 (D) 1470, 183

(d) 1/6, 2/3, 5/8
(A) 5/12, 1 (B) 10, 1/24
(C) 1/10, 24 (D) 10, 24

(e) 2/5, 11/10, 16/25
(A) 176/5, 1/50 (B) 176, 10
(C) 5, 88/25 (D) 50, 1/176

37. Find LCM $\left(\frac{15}{4}, \frac{25}{6}, \frac{45}{8}\right)$.
(A) $\frac{135}{2}$ (B) $\frac{225}{2}$ (C) $\frac{405}{2}$ (D) $\frac{315}{2}$

38. Five bells toll at intervals of 5, 6, 10, 12 and 15 seconds respectively. If they toll together at the same time, after how many seconds will they toll together again, for the first time?
$\boxed{\quad}$

39. Find the area of the smallest square that can be formed with rectangles of dimensions 8×6.
$\boxed{\quad}$

40. The HCF of two numbers equals their LCM. The numbers must be _______.
(A) Prime (B) co prime
(C) equal (D) None of the above

41. What is the least natural number that should be added to 52341693 so that the sum is a multiple of 8?
$\boxed{\quad}$

42. Any prime number greater than 3 is of the form mk $\pm$ 1 where m and k are natural numbers. Which of the following represents a value that mk must always be divisible by?
(A) 9 (B) 12 (C) 18 (D) 6

43. The greatest number which always divides the product of any 10 even numbers is _______.
(A) $2^{10} \times 5!$ (B) 2^{10}
(C) $2^{10} \times 10!$ (D) None of these

44. (a) The least natural number by which $(2^8)(3^{17})(5^{13})$ must be multiplied so that the product is a perfect square is _______.
(A) 30 (B) 15 (C) 3 (D) 5

(b) The least natural number by which $(2^8)(3^{17})(5^{18})$ must be divided so that the product is a perfect square is _______.
(A) 30 (B) 15 (C) 3 (D) 5

45. The least natural number that must be added to 599 so that the sum is a perfect cube is _______.
(A) 120 (B) 125 (C) 130 (D) 135

46. If the number 1764 $\times$ k is a perfect cube, find the least value of k.
$\boxed{\quad}$

47. If N = $2^4 \times 3^2 \times 7^3 \times k$ is a perfect square as well as a perfect cube, find the number of factors of the least value of k, given k is a natural number.
(A) 40 (B) 120 (C) 80 (D) 60

48. A perfect square having its last 2 digits equal can have each of those digits equal to _______.
(A) 1 (B) 4 (C) 6 (D) 5

49. What is the least natural number which when divided by 7 and 8 leaves a remainder of 2 in each case?
(A) 54 (B) 56
(C) 58 (D) None of these

50. What is the least natural number which when divided by 18 and 24 leaves remainders of 11 and 17 respectively?

$\boxed{}$

51. What is the least natural number which when divided by 7 and 11 leaves remainders of 6 and 8 respectively?
(A) 34 (B) 30 (C) 41 (D) 52

52. What is the largest natural number, which divides 127 and 156 leaving remainders of 7 and 6 respectively?

$\boxed{}$

53. The number of positive integers which are co-prime to 349247 is ______.
(A) 4 (B) 5 (C) 3 (D) infinite

54. How many four digit numbers are divisible by 5, 12 and 18?

$\boxed{}$

55. **(a)** Find the least number which when successively divided by 8, 6 and 4 leaves respective remainders of 5, 2 and 3.
(A) 253 (B) 333 (C) 325 (D) 165

(b) In the above question if all the numbers satisfying the given conditions are written in ascending order, what is the tenth number in the sequence?
(A) 1893 (B) 1093 (C) 2085 (D) 2277

56. Find the number of zeros at the end of 150!.

$\boxed{}$

57. What is the index of the highest power of 2 that divides 256!?
(A) 256 (B) 255
(C) 254 (D) None of these

58. **(a)** Which of the following numbers is divisible by both 2 and 9?
(A) 7348 (B) 2436 (C) 4032 (D) 6201

(b) Which of the following numbers is divisible by 2, 3, 4, 6, 8 and 9?
(A) 6756 (B) 4608 (C) 2408 (D) 7270

(c) Which of the following numbers is divisible by 3, 8 and 12?
(A) 4248 (B) 2708 (C) 5824 (D) 8192

(d) Which of the following numbers is divisible by 2,4,8 and 11?
(A) 2304 (B) 5824 (C) 4752 (D) 3808

(e) Which of the following numbers is divisible by 2,3,9,5 and 10?
(A) 7140 (B) 3780 (C) 4290 (D) 1575

(f) Which of the following numbers is divisible by 24?
(A) 6666 (B) 3384 (C) 2732 (D) 8072

(g) Which of the following numbers is divisible by 22 and 33?
(A) 7752 (B) 4356 (C) 9856 (D) 1089

(h) Which of the following numbers is divisible by 36 and 24?
(A) 9856 (B) 9828 (C) 2268 (D) 6336

(i) Which of the following numbers is divisible by 40 and 72?
(A) 7560 (B) 3840 (C) 5670 (D) 3780

59. What is the least whole number that should be added to the following numbers to make them multiples of 9?
(a) 320526
(A) 2 (B) 3 (C) 0 (D) 5

(b) 24312
(A) 6 (B) 2 (C) 0 (D) 3

(c) 2347904
(A) 5 (B) 7 (C) 2 (D) 4

(d) 789457
(A) 2 (B) 3 (C) 4 (D) 5

(e) 123456789
(A) 3 (B) 6 (C) 5 (D) 0

60. What is the least whole number that should be added to the following numbers to make them multiples of 11?
(a) 243741
(A) 3 (B) 8 (C) 5 (D) 0

(b) 321423
(A) 6 (B) 5 (C) 3 (D) 8

(c) 243081
(A) 8 (B) 3 (C) 4 (D) 7

(d) 723111
(A) 4 (B) 8 (C) 7 (D) 3

(e) 123456789
(A) 3 (B) 6 (C) 5 (D) 8

61. **(a)** Prime factorise: 9000
(A) $2^2 \times 3^2 \times 5^2$ (B) $2^4 \times 3 \times 5^2$
(C) $2^3 \times 3^2 \times 5^3$ (D) $2^3 \times 3 \times 5^4$

(b) Prime factorise: 1936
(A) $2^2 \times 3 \times 11^3$ (B) $2^3 \times 11^3$
(C) $2^4 \times 11^2$ (D) $2^2 \times 3^2 \times 11^2$

(c) Write 3969 as a product of prime factors.
(A) $3^5 \times 7$ (B) $3^3 \times 7^3$ (C) $3^4 \times 7^2$ (D) $3^2 \times 7^4$

(d) Write 14553 as a product of prime numbers
(A) $3 \times 7^3 \times 11$ (B) $3^2 \times 7 \times 11^3$
(C) $3^3 \times 7^2 \times 11$ (D) $3 \times 7^2 \times 11^2$

62. (i) Simplify the following:
(a) $248 \times 555 + 148 \times 445$
(A) 203500 (B) 302500 (C) 205300 (D) 305200

(b) $4^1/_2 + 3^1/_5 - 2^1/_{10} - 4^1/_{20}$
(A) $1^1/_{10}$ (B) $1^{11}/_{20}$ (C) $1^1/_5$ (D) $1^{11}/_{40}$

(c) $$\frac{(3.37)^3 + 10.11(6.63)^2 + 19.89(3.37)^2 + (6.63)^3}{(3.37)^2 + 2 \times (6.63)(3.37) + (6.63)^2}$$
(A) 3.26 (B) 6.74 (C) 10 (D) 8

(ii) Simplify the following:
(a) $77 \times 335 + 37 \times 665 - 40 \times 335$
(A) 30000 (B) 77000 (C) 40000 (D) 37000

(b) $2^7/_{12} + 3^1/_4 - 1^1/_2 + 2^1/_6 - 2^1/_3$
(A) $4^1/_2$ (B) $4^1/_6$ (C) $4^1/_3$ (D) $4^1/_4$

(c) $(3.13)^2 + (4.25)^2 + (2.62)^2 + 6.26 \times 4.25 + 8.5 \times 2.62 + 5.24 \times 3.13$
(A) 10 (B) 100 (C) 9.125 (D) 81.75

63. Find the square root of the following numbers
(a) 17161
(A) 129 (B) 139 (C) 131 (D) 137

(b) 5929
(A) 77 (B) 73 (C) 87 (D) 83

(c) 24964
(A) 158 (B) 156 (C) 148 (D) 152

(d) 2809
(A) 57 (B) 51 (C) 59 (D) 53

(e) 231.04
(A) 15.04 (B) 15.8 (C) 15.08 (D) 15.2

(f) Find the square root of the following numbers 17689
(A) 143 (B) 137 (C) 133 (D) 147

64. Which of the following sets of numbers are relative primes?
(a) 57,61 (b) 396,455 (c) 693,132
(d) 6561,1024 (e) 384,352
(A) (c), (e) (B) (a), (b) and (d)
(C) (a), (c), (d) (D) (b), (e)

65. The LCM and HCF of two numbers are 432 and 18 respectively. If one of the numbers is 54, then find the other number.

66. Find the least number which when divided by 48 and 72 leaves a remainder of 9.

67. Find the smallest and the largest three-digit numbers which when divided by 22,33 and 55, leave a remainder of 5 in each case.
(A) 340, 980 (B) 335, 995
(C) 330, 990 (D) 325, 985

68. Find the smallest number which when divided by 8 and 12 leaves respective remainders of 3 and 7.
(A) 17 (B) 29 (C) 26 (D) 19

69. Find the greatest number which when divides 6850 and 2575 leaving respective remainders of 50 and 25.
(A) 425 (B) 850 (C) 1700 (D) 1275

70. Find the greatest number which divides 96,134 and 229 leaving the same remainder in each case.

71. Find the greatest number which divides 68, 140 and 248 leaving the same remainder in each case.
(A) 36 (B) 18 (C) 72 (D) 108

72. What is the largest number, which divides 218, 146 and 434 leaving the same remainder in each case?
(A) 36 (B) 72 (C) 18 (D) 144

73. Find the greatest number which divides 3300 and 3640 leaving respective remainders of 23 and 24.

74. Find the smallest number that must be added to 1994 such that a remainder of 28 is left when the number is divided by 38 or 57.

75. Find the GCD of the numbers p and q where
$p = 2^3 . 3^2 . 7^2 . 11^6$ and $q = 2^2 . 3^1 . 5^4 . 11^2 . 13^2$.
(A) 776 (B) 1452 (C) 1164 (D) 2028

76. If the least positive integer divisible by $2^2 \cdot 3 \cdot 5$, $3 \cdot 5^2 \cdot 7$ and $5 \cdot 7 \cdot 11^2$ has x distinct prime factors, then find x.

77. Is HCF [a, b, c, d] = HCF [HCF (a, c), HCF (b, d)]?
(A) Yes (B) No (C) Cannot say

78. Is LCM [a, b, c, d, e] = LCM [LCM (a, c), LCM (b, d), LCM (c, e), LCM (d, a), LCM (e, b)]?
(A) Yes (B) No (C) Cannot say

79. The sum of the first N natural numbers is equal to x^2 where x is an integer less than 100. What are the values that N can take?
(A) 1, 9, 27 (B) 1, 7, 26
(C) 1, 8, 48 (D) 1, 8, 49

80. If x and y are irrational numbers, then x + y – xy is _______.
(A) a real number (B) a complex number
(C) a rational number (D) an irrational number

Exercise – 1(a)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. What must be added to 315642 so that it becomes a multiple of 11?

2. If the eight-digit number 5668x25y is divisible by 48, find the least value of x + y.
(A) 10 (B) 9 (C) 8 (D) 7

3. N is a natural number obtained by adding 16 to the product of four consecutive even natural numbers. How many of the following statements are always true?
(1) N is divisible by 32
(2) N is divisible by 16
(3) N is divisible by 64
(4) N is a perfect square
(A) 0 (B) 1 (C) 2 (D) 3

4. There are four prime numbers written in ascending order. The product of the first three prime numbers is 2431 and that of the last three is 4199. Find the greatest of them.

5. Find the sum of all the possible distinct remainders which are obtained when cubes of prime numbers are divided by 6.
(A) 9 (B) 11 (C) 13 (D) 15

6. Two numbers 698 and 450 when divided by a certain divisor leave remainders of 9 and 8 respectively. Find the largest such divisor.
(A) 11 (B) 13 (C) 17 (D) 23

7. A three-digit number N leaves the same remainder upon dividing 68488 and 67516. How many possible values does N have?

8. Find the smallest three-digit number which when divided by 7 leaves a remainder of 3 and when divided by 5 leaves a remainder of 1 but when divided by 6 leaves a remainder of 5.
(A) 101 (B) 103 (C) 107 (D) 109

9. Find the smallest four-digit number which when divided by 9 leaves the remainder 5 and when divided by 11 leaves the remainder 7.
(A) 1058 (B) 1041 (C) 1089 (D) 1085

10. Find the largest four-digit number which when divided by 7, 9 and 11 leaves a remainder of 5 in each case.
(A) 9236 (B) 9467 (C) 9707 (D) 9763

11. There are a certain number of soldiers in a field. If the soldiers are arranged in rows of 8 or 15 or 20, one soldier is left out. If the soldiers are arranged in rows of 9 or 13, four soldiers only are left out. Find the number of soldiers in the field.
(A) 109 (B) 113 (C) 117 (D) 121

12. Rohan has a certain number of (less than 10000) sweets with him. He would be left with 1 sweet, if he distributes them equally among 12 or 16 or 18 children. If he distributes them equally among 17 children, he would be left with no sweets. Find the number of possibilities for the sweets he has.
(A) 2 (B) 3 (C) 4 (D) 5

13. What is the minimum number of identical square tiles required to completely cover a floor of dimensions 8 m 70 cm by 6 m 38 cm?

14. Four blocks of chocolates of weights $6\frac{1}{8}$ kg, $10\frac{1}{2}$ kg, $8\frac{3}{4}$ kg and $3\frac{15}{16}$ kg respectively were bought for a birthday party. The blocks were divided into pieces such that all the pieces are of the same weight. What is the least number of pieces that can be obtained?

15. The HCF of two numbers is 6 and the product of the two numbers is 4320. How many pairs of numbers exist, which satisfy the above conditions?
(A) 2 (B) 3 (C) 4 (D) 5

16. The LCM of two numbers is 196 and the HCF is 7. If the difference of the two numbers is 21, find the larger of the two numbers.
(A) 28 (B) 35 (C) 42 (D) 49

17. The sum of all the factors of 11111111 is

18. In how many ways can 152100 be expressed as a product of two different factors?
(A) 24 (B) 36 (C) 39 (D) 40

19. The total number of factors of a natural number N is 45. What is the maximum number of prime numbers by which N can be divided?

20. What is the product of the factors of 10!?
(A) $(10!)^{135}$ (B) $(10!)^{270}$
(C) $(5!)^{270}$ (D) None of these

21. What is the product of the factors of $6^8 \times 8^6$?
(A) $(3^8 \times 4^6)^{63}$ (B) $(6^4 \times 8^3)^{63}$
(C) $(6^4 \times 8^3)^{243}$ (D) $(3^8 \times 4^6)^{243}$

22. Y is an even natural number satisfying $Y \geq 4$. $X = Y^2 + 2Y$. The largest natural number that always divides $X^2 - 8X$ is
(A) 96 (B) 144 (C) 384 (D) 192

23. Find the number of three-digit numbers which are divisible neither by 2 nor by 3.

24. Find the sum of all the co-primes to 2016 which are less than 2016.
(A) 552108 (B) 5016
(C) 558432 (D) 580608

25. Find the index of the greatest power (IGP) of 24 contained in 360!.
(A) 108 (B) 118 (C) 128 (D) 178

26. If a, b, c and d are natural numbers such that $a^d + b^d = c^d$, which of the following is true?
(A) The minimum of a, b and c is at least d.
(B) The maximum of a, b and c is at most d.
(C) d lies between the minimum of a, b and c and the maximum of a, b and c.
(D) d = 1

27. A number when successively divided by 8, 6 and 5 leaves respective remainders of 1, 1 and 2. The number is a multiple of which of the following numbers?
(A) 3 (B) 5
(C) Either 3 or 5 (D) Both 3 and 5

28. Ramu was given a problem of adding a certain number of consecutive natural numbers starting from 1. By mistake, he missed a number during addition. He obtained the sum as 800. Find the number he missed.

29. N is a natural number greater than 6, find the remainder obtained when $N^7 – N$ is divided by 6.
(A) 0 (B) 1 (C) 2 (D) 3

30. A 1500 page dictionary was compiled on a computer by the Oxford University Press. Just before it went for print, it was spotted that none of the pages were numbered. Find the number of times the typist must press keys from 0 to 9 on the keyboard to number all the pages.

31. Let X be the set of integers {9, 15, 21, 27,…..375}. Y denotes a subset of X, such that the sum of no two elements of Y is 384. Find the maximum number of elements in Y.
(A) 29 (B) 30 (C) 31 (D) 32

32. Let w, x, y and z be four natural numbers such that their sum is 8m + 10 where m is a natural number. Given m, which of the following is necessarily true?
(A) The maximum possible value of $w^2 + x^2 + y^2 + z^2$ is $6m^2 + 40m + 26$.
(B) The maximum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 28$.
(C) The minimum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 28$.
(D) The minimum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 26$.

33. The sets S_x are defined to be (x, x +1, x + 2, x + 3, x + 4) where x = 1, 2, 3, ……80. How many of these sets contain 6 or its multiple?
(A) 65 (B) 66 (C) 59 (D) 60

34. What is the value of $0.7\overline{54} + 0.69\overline{2}$?
(A) $\frac{1813}{900}$ (B) $\frac{1783}{910}$
(C) $\frac{14323}{9900}$ (D) $\frac{13243}{9900}$

35. K(N) denotes the number of ways in which N can be expressed as a difference of two perfect squares. Which of the following is maximum?
(A) K(110) (B) K(105)
(C) K(216) (D) K(384)

36. From a book, in which the pages are numbered in the usual way with odd numbers appearing on the right pages and even numbers on the left pages, 31 consecutive leaves were torn off. Which of the following could be the sum of the 62 page numbers on these leaves?
(A) 1955 (B) 2201
(C) 2079 (D) None of these

37. The difference of any 40-digit number and its reverse is always divisible by _____.
(A) 9 but not always by 11
(B) 11 but not always by 9
(C) 99
(D) 198

38. The Index of the greatest power(IGP) of 2 in 32! + 33! + 34! + ….. + 90! is _______.
(A) 32 (B) 33 (C) 34 (D) 35

39. The remainders of 5x + 4y and 4x + 5y when divided by 9 are 4 and 5 respectively. Find the remainder of x – y divided by 9.

40. How many divisors of 21600 are divisible by 24 but not by 72?

Exercise – 1(b)

Directions for questions 1 to 60: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. **(a)** $2/3$ of $45 \div 5 \times (2^4 – 1 \div 90) =$
(A) 2 (B) 1 (C) 1/2 (D) 1/3

(b) $5 + 6 \times 1/3$ of $9 – \{4 – 5/8 + 2\frac{7}{8} + 3/4\} =$
(A) 12 (B) 14 (C) 16 (D) 18

2. Find the value of $75^3 – 50^3 – 25^3$.

3. Find the square root of 12345654321.
(A) 1111 (B) 11111
(C) 111111 (D) 1111111

4. Find the value of the expression below

$$\frac{(0.68)^3+(0.67)^3-(0.5)^3+(0.68)(0.67)(1.5)}{(0.68)^2+(0.67)^2+(0.5)^2-(0.68)(0.67)+(0.67)(0.5)+(0.68)(0.5)}$$

(A) 1.85 (B) 0.51 (C) 0.49 (D) 0.85

5. What should be subracted from 546789 so that it becomes a multiple of 7?
(A) 5 (B) 6 (C) 7 (D) 8

6. A, B and C are digits and 64A3B6C is divisible by 360. How many values can (A, B) take?
(A) 8 (B) 10 (C) 9 (D) 5

7. X is a natural number such that $X^4 + 2X^3 + 3X^2 + 4X + 36$ is divisible by X. How many values can X assume?

8. N is an even natural number. The greatest number that the expression $N^3 + 6N^2 + 8N$ is always divisible by is
(A) 96 (B) 24 (C) 16 (D) 48

9. If N = 1223334444……. and is a 100-digit number, find the remainder when N is divided by 16.

10. If n is a positive integer greater than 1, $n^5 – n$ is always divisible by ______.
(A) 5 (B) 7 (C) 11 (D) 13

11. If n is a positive integer greater than 1, then the highest number by which the function $n(2n + 1)(n^2 – 1)(4n^2 + 4n)$ is always divisible by is ______.
(A) 12 (B) 24 (C) 48 (D) 144

12. If abcde is a five-digit number the difference of abcde and acdbe would always be divisible by which of the following for all values of a, b, c, d and e?
(A) 9 (B) 18
(C) 99 (D) Both (A) and (B)

13. Find the sum of all possible distinct remainders which are obtained when squares of prime numbers are divided by 6.

14. When a natural number N is divided by D, the remainder is 35. When 50N is divided by D, the remainder is 11. Find D.
(A) 1739
(B) 43
(C) 47
(D) Cannot be determined

15. The LCM of the fractions 1/5, 4/15 and 8/25 is how many times their HCF?

16. Find the greatest number which divides 971 leaving a remainder 3 and divides 852 leaving a remainder 5.
(A) 11 (B) 33 (C) 121 (D) 88

17. When 5, 6 and 7 divide a number which is 6 less than a multiple of 47, they leave remainders of 2, 3 and 4 respectively. Find the least such number.
(A) 207 (B) 417 (C) 627 (D) 837

18. What is the largest three-digit number which when divided by 9 leaves a remainder 6 and when divided by 7 leaves a remainder 5?
(A) 988 (B) 989 (C) 992 (D) 978

19. Four runners started running simultaneously from a point on a circular track. They took 200 seconds, 300 seconds, 360 seconds and 450 seconds to complete one round. After how much time do they meet at the starting point (in seconds) for the first time?

20. A number when successively divided by 7 and 5 leaves remainders 4 and 2 respectively. What will be the remainder when the smallest such number is divided by 17?
(A) 1 (B) 2 (C) 3 (D) 5

21. Ravi distributed the chocolates with him equally among Rajesh and Suresh. He was left with a chocolate. Rajesh distributed his share equally among three of his friends and was also left with a chocolate. One of the three distributed his share equally among four of his friends and was left with no chocolate. Which of the following could be the number of chocolates that Rajesh received?
(A) 22 (B) 34 (C) 49 (D) 64

22. When a number is successively divided by 6, 5 and 4, it leaves remainders of 4, 3 and 2 respectively. Find the greatest such four-digit number.

23. Find the number of factors of 88400.
(A) 40 (B) 50 (C) 60 (D) 80

24. Find the number of ways in which 24700 can be expressed as a product of two coprime factors.
(A) 4 (B) 16 (C) 8 (D) 32

25. In how many ways can 3780 be expressed as a product of two numbers?
(A) 12 (B) 18 (C) 24 (D) 36

26. How many numbers are co-prime to 2304 and lie between 1000 and 2000?
(A) 500 (B) 334 (C) 333 (D) 332

27. What is the index of the greatest power (IGP) of 3 that exactly divides 100!?
(A) 97 (B) 68 (C) 48 (D) 32

28. If a, b and c are prime numbers satisfying a = b – 2 = c – 4. How many possible combinations exist for a, b and c?

29. If $1 \le R \le 50$, how many values of R are such that $(R - 1)!$ is not divisible by R?

30. Find the value of $2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{1}{2}}}}}$

(A) 103/52 (B) 113/52
(C) 123/52 (D) 133/52

31. Let p, q and r be distinct positive integers that are odd. Which of the following statements cannot always be true?
(A) $pq^2 r^3$ is odd.
(B) $(p + q)^2 r^3$ is even
(C) $(p - q + r)^2 (q + r)$ is even.
(D) If p, q and r are consecutive odd integers, the remainder of their product when divided by 4 is 3.

32. $(AB)^2$ = CCB where A, B and C are distinct single-digit natural numbers and 'AB' and 'CCB' are two-digit and three-digit natural numbers respectively. Find the number of possibilities for AB.
(A) 0 (B) 1 (C) 2 (D) 3

33. When the square of a number and the cube of a smaller number are added the result is 593. If the square of the smaller number exceeds the bigger number by 55, find the difference of the two numbers.

34. What is the product of the factors of 216^2?
(A) 216^{49} (B) 216^{98} (C) 216^{48} (D) 216^{50}

35. What is the product of the factors of 1296000?
(A) $(1296000)^{80}$ (B) $(3600)^{160}$
(C) $(360)^{1600}$ (D) None of these

36. Find the minimum number of coins required to pay three persons 69 paise, 105 paise and 85 paise, respectively using coins in the denominations of 2 paise, 5 paise, 10 paise, 25 paise and 50 paise.
(A) 9 (B) 10 (C) 14 (D) 11

37. A number when divided by a certain divisor leaves a remainder of 16. If twice the number is divided by the same divisor, the remainder is 9. Find the divisor.

38. Ramu was given a problem of adding a certain number of consecutive natural numbers starting from 1. By mistake, he added a number twice. He obtained the sum as 860. Find the number he added twice.
(A) 15 (B) 40 (C) 25 (D) 30

39. The difference between the LCM and HCF of two natural numbers a and b is 57. What is the minimum value of a + b?
(A) 22 (B) 27 (C) 31 (D) 58

40. Which of the following represents the sum of a certain number of consecutive natural numbers starting from 1?
(A) 206 (B) 1275 (C) 805 (D) 439

41. The index of the greast power (IGP) of 11 in $\frac{800!}{400!}$ is

42. How many integers when squared would exceed a perfect square by 113?
(A) 5 (B) 4 (C) 3 (D) 2

43. If x, y are positive integers and $x^2 - y^2 = 255$, how many ordered pairs $(x - y, x + y)$ are there?
(A) 4 (B) 8 (C) 6 (D) 12

44. An organization has 99 employees, who were all assigned numerical codes from 2 to 100. A certain number of rounds of an emergency exit drill took place. In the first round, all the employees whose codes were divisible by 2, made an exit. In the second round, all the remaining employees whose codes were divisible by 3 made an exit and so on until all the employees exit. In each round, at least one employee made an exit. How many rounds of the drill took place?
(A) 25 (B) 24 (C) 49 (D) 99

45. N is the least integer which leaves remainders of 5, 6, 7 when divided by the divisors 7, 8, 9 respectively. Find the remainder when N is divided by 17.
(A) 9 (B) 8 (C) 10 (D) 11

46. A four-digit number N exceeds the number formed by reversing its digits by K, where K is a positive multiple of 74. The least value of N lies between _____.
(A) 1000 and 1300
(B) 1900 and 2200
(C) 1300 and 1600
(D) 1600 and 1900

47. Let S be the set of positive integers such that the following conditions are satisfied.
I. The elements of S range from 2000 to 2400.
II. Each element of S has only even digits.
How many elements of S are divisible by 6?

48. There are three positive numbers. The sum of the squares of any two of these equals the sum of the product of these two numbers and the square of the third number. How many of the three numbers must be equal?
(A) 0
(B) 2
(C) 3
(D) Cannot be determined

49. The middle digit of a three-digit number is equal to the sum of the other two digits. Find the number of factors of the greatest such odd number.

50. How many positive integers up to 4624 are divisible by at least one of 2, 7 or 11?
(A) 2822 (B) 2766 (C) 2786 (D) 2854

51. The product of seven integers, which are not necessarily distinct, between 5 and 19, both exclusive, is 16081065. Find the sum of these integers.

52. The index of the greatest power (IGP) of 2 in 15! + 16! + 17! + … + 100! is
(A) 11 (B) 13 (C) 12 (D) 14

53. $E = (4a + 8b - 12c)^3 + (8a - 12b + 4c)^3 + (-12a + 4b + 8c)^3 - 3(-12a + 4b + 8c)(8a - 12b + 4c)(4a + 8b - 12c)$ where a, b and c are all more than 10000.
Consider the following statements.
I. E must be non-negative
II. E must be non-positive
Which of the following can be concluded?
(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Neither I nor II are true

54. $X(p, q, r, s\ t) = 32 - 16(\Sigma p) + 8(\Sigma pq) - 4(\Sigma pqr) + 2(\Sigma pqrs) - pqrst$.
$X\left(\frac{16}{15}, \frac{15}{14}, \frac{14}{13}, \frac{13}{12}, \frac{12}{11}\right)$ equals_____.
(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

55. $F(N) = \sum_{i=1}^{x} (-1)^{\frac{d_i - 1}{2}}$ where d_i is the i^{th} odd factor of N and x is the number of odd factors of N. Find the value of $F(3^4\ 5^2)$.
(A) 2 (B) 3 (C) 4 (D) 1

56. If $\lceil a \rceil$ is the least integer greater than or equal to a, $\left\lceil \frac{6^{2x}}{7} \right\rceil + \left\lceil \frac{6^{2x+1}}{7} \right\rceil$, where x is any positive integer more than 10, equals
(A) $6^{2x} - 1$ (B) $6^{2x} + 3$ (C) $6^{2x} + 1$ (D) $6^{2x} + 4$

57. $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\left(1 - \frac{1}{25}\right) \ldots \left(1 - \frac{1}{900}\right) =$
(A) $\frac{29}{60}$ (B) $\frac{31}{60}$ (C) $\frac{29}{30}$ (D) $\frac{31}{30}$

58. Find the sum of all the factors of 5400.

59. N_1 and N_2 are natural numbers not more than 100. $A \oplus B$ is defined as the remainder of A divided by B. A # B is defined as the product of A and B.
$(N_1 \oplus 8)\#(N_2 \oplus 7) = 21$. How many possible values does (N_1, N_2) have?

60. If three numbers are in the ratio 3 : 4 : 5, and their LCM is 480, then find the sum of the three numbers.
(A) 96 (B) 72 (C) 84 (D) 108

Directions for questions 61 to 75: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using I and II together.

61. What is the value of integer x, if 100 < x < 265?
I. When x is divided by 15, the remainder is 11.
II. When x is divided by 11, the remainder is 7.

62. If x is a negative number, is xyz < 0?
I. At least one out of y and z is negative.
II. Sum of y and z is positive.

63. If $abc \neq 0$, is $a^3 + b^3 + c^3 = 0$?
I. $a^2 + b^2 + c^2 = ab + bc + ca$.
II. $a + b + c = 0$.

64. Is x an even integer?
I. x is the square of an integer.
II. x is the cube of an integer.

65. The number of soldiers in a parade is less than 250. How many soldiers are there?
I. The soldiers can arrange themselves in rows of 3, 5 and 7.
II. The number of soldiers is an even number.

66. If x, y and d are positive integers and d is odd, are both x and y divisible by d?
I. x + y is divisible by d.
II. x – y is divisible by d.

67. For non-zero integers a, b and c, $\frac{a}{b - c} = 1$. What is the value of $\frac{b - c}{b}$?
I. a/b = 3/5.
II. a and b have no common factors greater than 1.

68. If n is an integer and $10 < 3^n < 300$, then what is the value of n?
I. n is the square of an integer.
II. 3^n is the square of an integer.

69. What is the smallest number among the five distinct natural numbers?
I. The sum of the five natural numbers is 16.
II. The product of the five natural numbers is 120.

70. What is the Greatest Common Divisor of the integers a and b?
I. The Greatest Common Divisor of 2a and 2b is 10.
II. Both a and b are odd.

71. Is N the HCF of two numbers x and y?
I. N divides x and y.
II. 2N divides x/2 and y/4.

72. Is x an odd integer?
- I. If x is divided by five, the remainder is an odd integer.
- II. If x is divided by four, the remainder is an odd integer.

73. When integer x is divided by 2, the remainder is 1. What is the remainder when x is divided by 4?
- I. When x is divided by 8, the remainder is 3.
- II. x is a multiple of 5.

74. If a, b and c are positive integers, is the sum of (2a + 4b) and (a – b + c) divisible by 3?
- I. (a + b) is divisible by 3.
- II. c is divisible by 3.

75. $p^q = r^q$ where q is a whole number. Is p = r?
- I. q is divisible by 3.
- II. q is odd.

Key

Concept Review Questions

1. C
2. a. A, b. C, c. C, d. C, e. B
3. 4
4. a. A, b. A, c. A, d. B
5. D
6. D
7. A
8. 8
9. C
10. B
11. B
12. A
13. 4
14. A
15. 1
16. A
17. 39
18. D
19. B
20. D
21. 1240
22. B
23. B
24. B
25. 864
26. a. D, b. B
27. C
28. C
29. D
30. 5.46
31. C
32. A
33. A
34. 2700
35. 21
36. a. A, b. D, c. C, d. B, e. A
37. B
38. 60
39. 576
40. C
41. 3
42. D
43. B
44. a. B, b. C
45. C
46. 42
47. D
48. B
49. C
50. 65
51. C
52. 30
53. D
54. 50
55. a. D, b. A
56. 37
57. B
58. a. C, b. B, c. A, d. C, e. B, f. B, g. B, h. D, i. A
59. a. C, b. A, c. B, d. D, e. D
60. a. B, b. D, c. A, d. C, e. B
61. a. C, b. C, c. C, d. C
62. (i) a. A, b. B, c. C; (ii) a. D, b. B, c. B
63. a. C, b. A, c. A, d. D, e. D, f. C
64. B
65. 144
66. 9
67. B
68. D
69. B
70. 19
71. A
72. B
73. 113
74. 86
75. B
76. 5
77. A
78. A
79. D
80. A

Exercise – 1(a)

1. 3	9. D	17. 12499488	25. B	33. B
2. A	10. C	18. D	26. A	34. C
3. C	11. D	19. 3	27. D	35. D
4. 19	12. C	20. A	28. 20	36. B
5. B	13. 165	21. C	29. A	37. A
6. B	14. 67	22. C	30. 4893	38. A
7. 6	15. C	23. 300	31. C	39. 8
8. A	16. D	24. D	32. D	40. 9

Exercise – 1(b)

1. (a) B	13. 8	26. B	39. B	52. A	65. C
(b) C	14. D	27. C	40. B	53. C	66. C
2. 281250	15. 120	28. 1	41. 39	54. A	67. A
3. C	16. C	29. 16	42. D	55. B	68. B
4. D	17. B	30. C	43. A	56. C	69. B
5. A	18. D	31. D	44. A	57. B	70. A
6. D	19. 1800	32. C	45. A	58. 18600	71. A
7. 9	20. A	33. 1	46. B	59. 168	72. A
8. D	21. C	34. A	47. 18	60. A	73. A
9. 9	22. 9922	35. A	48. C	61. C	74. A
10. A	23. C	36. D	49. 10	62. C	75. A
11. D	24. C	37. 23	50. A	63. B	
12. D	25. C	38. B	51. 79	64. D	