Chp.1) SIMPLE EQUATIONS

(i) No. of variables = No. of equations , determinate equations

(ii) If, No. of variables > No. of equations.

↳ Indeterminate Equations.

→ Infinite solution set.

(iii) System of eq^n in 2 variables,

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

(1) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the equations are inconsistent.

→ No solution set . $(\{\phi\})$

(2) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the equations are consistent.

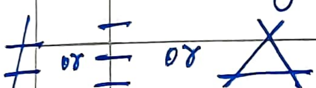
→ Infinite solution set.

(iv) System of eq^n in 2 variables,

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0$$
$$a_3 x + b_3 y + c_3 = 0$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(1) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then equations are inconsistent. $\boxed{\Delta \neq 0}$



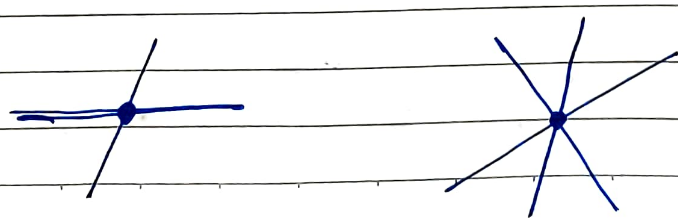
(All 3 lines aren't meeting at a unique point) → No solution set.

(2) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then equations are (All three can be expressed as one another) consistent.

↳ Infinite solution set.

(3) If $\Delta = 0$, then the equations are consistent but have only one solution (unique).

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ but $\frac{a_1}{a_3} \neq \frac{b_1}{b_3}$



(v) System of eq^n in 3 variables,

$$a_1 x + b_1 y + c_1 z + d_1 = 0$$
$$a_2 x + b_2 y + c_2 z + d_2 = 0$$
$$a_3 x + b_3 y + c_3 z + d_3 = 0$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- $\Delta_1 = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

- $x = \dfrac{\Delta_1}{\Delta}$; $y = \dfrac{\Delta_2}{\Delta}$; $z = \dfrac{\Delta_3}{\Delta}$

Solutions (Considering they are independent)

- $\Delta \neq 0$ (Consistent) One solution
  - Non-Trivial ↳ ∞ solutions
  - Trivial ↳ $(0,0,0)$ is the only solution.
- $\Delta = 0$
  - At least one of Δ_1, Δ_2 or $\Delta_3 \neq 0$ → No solution
  - $\Delta_1 = \Delta_2 = \Delta_3 = 0$
    - Infinite
    - No soln.

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Chpt. 2 : RPV

(i) $a : b$ or $\frac{a}{b}$ or a is to b

$a \rightarrow$ antecedent and $b \rightarrow$ consequent

(ii) $\frac{a}{b}$, if $a > b$, greater inequality

$a < b$, lesser inequality

$a = b$, equality

Useful in DI when comparing fractions.

If $\frac{a+x}{b+x}$, where x is a +ve number.

$\frac{a+x}{b+x} > \frac{a}{b}$, if $a < b$

$\frac{a+x}{b+x} < \frac{a}{b}$, if $a > b$

(iii) $\frac{a}{b} = \frac{c}{d} \Rightarrow \overbrace{a : \underbrace{b :: c}_{\text{Means}} : d}^{\text{Extremes}}$

Product of Means = Product of Extremes

If, $a : b = c : d$ then,

(1) $a : c = b : d \rightarrow$ ALTERNENDO

(2) $b : a = d : c \rightarrow$ INVERTENDO

(3) $(a+b) : b = (c+d) : d \rightarrow$ COMPONENDO

(4) $(a-b) : b = (c-d) : d \rightarrow$ DIVIDENDO

(5) $(a+b) : (a-b) = (c+d) : (c-d) \rightarrow$ COMPONENDO - DIVIDENDO

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$$\Rightarrow \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots\cdots = k$$

$$\frac{a+c+e+\cdots\cdots}{b+d+f+\cdots\cdots} = k$$ too.

(iv) If $a:b :: b:c$ then they are continued proportion,

$c \rightarrow$ Third proportional to a and b

$b \rightarrow$ Mean proportional of a and c

$\boxed{b^2 = ac}$; [a, b, c are in GP]

(v) Variation can be of 3 types,

$A \propto B \Rightarrow A = KB$

$A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$

$A \propto \frac{B}{C} \Rightarrow A = \frac{BK}{C}$ or $A \propto BC$
$\Rightarrow A = KBC$

** Always remember to take the <u>same units</u>, when forming a proportion.

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Chp 3 : P-PL-P

(i) Percentage increase = $\frac{\text{Actual Increase}}{\text{Original}} \times 100\%$

(ii) % decrease = $\frac{\text{Actual decrease}}{\text{Original}} \times 100\%$

(iii) If percentage increase is p% then,

New value = $\left(\frac{p}{100} + 1\right) \times$ Old Value

(iv) If the value of an item goes up/down by x%, the percentage reduction/increment to be now made to bring it back to the original level is,

$$\frac{100x}{(100 \pm x)}\%$$

(Applies to expenditure as well)

(v) If A is x% more or less than B, then B is $\frac{100x}{100 \pm x}\%$ less or more than A.

(vi) Percentage points : Difference between 2 percentages.

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(vii) C.P. = Cost Price ; S.P. = Selling Price

Profit = S.P. − C.P. ; Loss = C.P. − S.P.

* Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$; Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

* * When 2 articles are sold at the same price (S.P.) such as there is a Profit of p% and loss of p% then the net result of the transaction is <u>LOSS</u>.

$$\text{LOSS \%} = \frac{p^2}{100}$$

(viii) M.P. or L.P. ≠ C.P.

M.P. = S.P. + Discount

$$\text{Discount \%} = \frac{\text{M.P.} - \text{S.P. (Discount)}}{\text{M.P.}} \times 100$$

(ix) Discounted Price (Discount) = S.P. × $\frac{(100-p)}{100} \times \frac{(100-q)}{100} \times \frac{(100-r)}{100}$

where p, q and r are successive discounts

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(x) Working Partner → Involved in day to day work
Dormant Partner → Not involved in day to day work

(1) DIFF AMOUNT BUT SAME TIME
Profits in the ratio of amount.

(2) SAME AMOUNT BUT DIFF TIME
Profits in the ratio of time

(3) DIFF AMOUNT AND DIFF TIME
Profits in the ratio of amount x time

(xi) Share = Stock ; Face Value or Par Value can be 10 / 100.
Market value > Face value, Premium
" < " , Discount

$$\text{Dividend } \% = \frac{\text{Dividend}}{\text{Face value}} \times 100$$

* Dividend is always calculated on face Value.

(xii) Government bond / stock. Comes with a fixed rate of return, calculated on par value which is taken as 100.

5% stock at 95 → Bought Price
↓ Return% calculated on face value

Higher rate of return, better investment.

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Chptr 4 : SI & CI

(i) $SI = \frac{PRT}{100}$; $A = P\left(1 + \frac{RT}{100}\right)$

(ii) $A = P\left(1 + \frac{R}{100}\right)^T$; $CI = P\left[-1 + \left(1 + \frac{R}{100}\right)^T\right]$

(iii) When compounding is done more than once a year, the rate of interest is called NOMINAL RATE OF INTEREST.

If the same for SI, it is called Effective Rate of Interest.

$$A = P\left[1 + \frac{R}{K} \times \frac{1}{100}\right]^{T \times K}$$

If compounding is done for every (12/K) months.

(iv) If compounding is done for every moment i.e. infinitely

$$A = P \cdot e^{RT/100}$$

(v) $CI_{(K+1)^{th}} - CI_{K^{th}} =$ (Interest for one 1yr) on the $CI_{K^{th}}$

(vi) Present Value, in SI: $P = \frac{X}{(1 + RT/100)}$

in CI: $P = \frac{X}{(1 + R/100)^T}$

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• If Present value of r_1 which is ~~[illegible]~~ received at the end of year 1 is Z_1 and like wise, then Present value of $r_1, r_2, \ldots$ series is $Z_1 + Z_2 + \ldots$

(vii) Repayment in **n** equal installments,

Instalment X is paid after every year,

$$P_1 = \frac{X}{(1+R/100)}, \quad P_2 = \frac{X}{(1+R/100)^2}, \ldots\ldots$$

$$\frac{X}{\left(1+\frac{R}{100}\right)} + \frac{X}{\left(1+\frac{R}{100}\right)^2} + \ldots\ldots + \frac{X}{\left(1+\frac{R}{100}\right)^n} = P$$

Let us assume, $\left(1+\frac{R}{100}\right)^{-1} = k$

$$\frac{X}{k^{-1}} + \frac{X}{k^{-2}} + \ldots + \frac{X}{k^{-n}} = P$$

$$X[k + k^2 + \ldots + k^n] = P$$

$$X \cdot \frac{k[k^n - 1]}{[k-1]} = P \Rightarrow X = \frac{P(k-1)}{k(k^n-1)}$$

$$X \text{ (Each Instalment)} = \frac{P \cdot r}{100\left[1 - \left\{\frac{100}{100+r}\right\}^n\right]}$$

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Chptr 5: Time & Distance

(i) Distance = Speed × time.

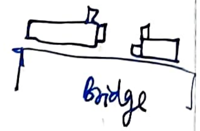
Km/hr → m/sec
Multiply by $\frac{5}{18}$.

Distance = Length of train

(a) 

(b) 

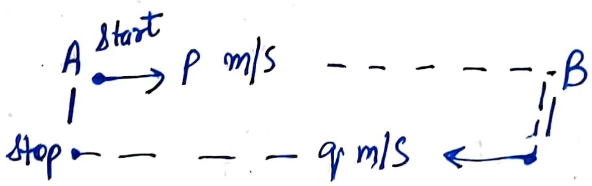
Distance = Length of train + Length of platform

(c) 

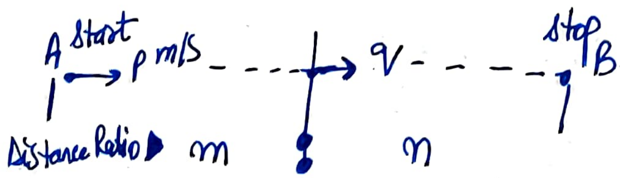
Distance = Length of train 1 + length of train 2

(ii) Avg. Speed = $\frac{\text{Total distance}}{\text{Total Time}}$

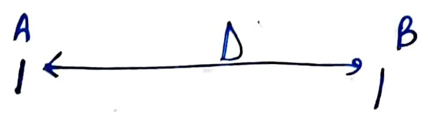
(iii)



Avg speed = $\frac{2pq}{(p+q)}$

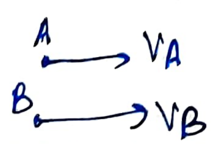


Avg speed = $\frac{(m+n)pq}{(np+mq)}$

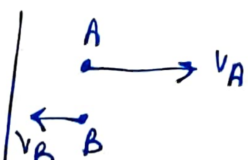
(iv) 

p hrs late at u km/hr
q hrs early at V km/hr

$D = \frac{Vu}{(V-u)}(p+q)$

(v) 

$V_{A,B} = V_A - V_B$



$V_{A,B} = V_A + V_B$

(vi) $V_{upstream} = V_{boat} - V_{stream}$
$V_{downstream} = V_{boat} + V_{stream}$ } Can be used in terms of elevator too.

(vii) Circular length $\to$ L

$\boxed{\text{faster complete 1 round more than slower}}$ for 2 people,

	Same direction	Opp. direction
first time ever	$\frac{L}{a-b}$	$\frac{L}{a+b}$
At the starting point	$LCM\left\{\frac{L}{a}, \frac{L}{b}\right\}$	$LCM\left\{\frac{L}{a}, \frac{L}{b}\right\}$

[dist. of L by two]

for 3 people,

At same time, same direction assumption.

first time ever $LCM\left\{\frac{L}{a-b}, \frac{L}{b-c}\right\}$

starting point $LCM\left\{\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right\}$

(viii) All measured in clockwise.

Hour hand, 12 hrs = 360°
[1 hr = 30°]

Minute hand, 1 hr = 360°
[1 min = 6°]

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(ix) Every angle is made twice in every hour by both the hands of clock.

(x) In every 12 hrs,
Hands make an angle of
- 0° – 11 times
  After every 12/11 hours or 65 5/11 min they coincide.
- 180° – 11 times
- 90° – 22 times

Chptr 6 : Time & Work

(i) If a man does three times the work than boy, then he takes $\frac{1}{3}$rd of the time.

(ii)
$$\frac{M_1 \times D_1 \times H_1}{W_1} = \frac{M_2 \times D_2 \times H_2}{W_2}$$

M_i → No. of people
D_i → No. of days
H_i → No. of hours per day
W_i → Piece of work done in the stipulated time

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(iii) A → p days alone ; B → q days alone

Together → $\left(\frac{pq}{p+q}\right)$ days.

(iv) Usually money is split in the ratio of the share of work.

(v) If people work for same no. of days then the money will be split in the ratio of the each day's work.

(vi) The pipe / tap emptying does negative work.

Chptr 7 : Averages & Alligations

(i) Weighted Averages ; $a : b$ → Number

A_1, A_2 → Averages

$$W.A. = \frac{a}{(a+b)} A_1 + \frac{b}{(a+b)} A_2$$

(ii) $A_1 < W.A. < A_2$

(iii) Weighted Average $= \dfrac{p_1 q_1 + p_2 q_2 + \ldots + p_n q_n}{q_1 + q_2 + \ldots + q_n}$

q_i → Quantity of p_i

p_i → Average.

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(iv) $$\frac{q_1}{q_2} = \frac{P_2 - P}{P - P_1}$$

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Rate of Dearer} - \text{Avg Rate}}{\text{Avg Rate} - \text{Rate of Cheaper}}$$

$$\begin{tikzcd}[column sep=small]
\text{Dearer Price} \arrow[dr] & : & \text{Cheaper Price} \arrow[dl] \\
& \text{Average Price} \arrow[dl] \arrow[dr] & \\
(\text{Avg Price} - \text{Cheaper Price}) & : & (\text{Dearer Price} - \text{Avg Price})
\end{tikzcd}$$

eg.

$$\begin{tikzcd}[column sep=small]
\underset{24}{(\text{Cheaper})} \arrow[dr] & & \underset{38}{(\text{Dearer})} \arrow[dl] \\
& 36\ (\text{Avg}) \arrow[dl] \arrow[dr] & \\
(38-36)\ 2 & & 12\ (36-24)
\end{tikzcd}$$

$$\frac{2}{12} = \frac{1}{6}$$

$$\frac{SP - CP}{CP} \times 100 = \frac{100}{9}$$

$$9SP = 10CP$$

$$\frac{9SP}{10} = CP$$

$$CP = \frac{9}{10} \times 40 = \underline{36}$$

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(v) P volume of pure liquid
Q volume is taken out and replaced by water.
This step carries for n steps.

$$K = \left\{\frac{P-Q}{P}\right\}^n$$ → Conc^n of liquid in the solution.

Volume of liquid = K P

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