Chapter – 3

NUMBER SYSTEMS

The numbers that are commonly used are the decimal numbers which involve ten digits, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. If we consider the number 526 in the decimal system, it means $5 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$. Likewise, 85.67 means $8 \times 10^1 + 5 \times 10^0 + 6 \times 10^{-1} + 7 \times 10^{-2}$. The role played by "10" in the decimal system is termed as the "base" of the system. In this chapter we see the numbers expressed in various other bases.

Base: It is a number which decides the place value of a symbol or a digit in a number. Alternatively, it is the number of distinct digits/symbols that are used in that number system.

Note:
(1) The base of a number system can be any integer greater than 1.
(2) Base is also termed as radix or scale of notation.

The following table lists some number systems along with their respective base and symbols.

Number System	Base	Digits/Symbols
Binary	2	0,1
Septenuary	7	0,1,2,3,4,5,6
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Duo-decimal	12	0,1,2,3,4,5,6,7,8,9,A,B
Hexa decimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15. Some books denote ten as "E" and eleven as "e".

Representation:

Let N be any integer, r be the base of the system and let a_0, a_1, a_2, ……., a_n be the required digits by which N is expressed. Then
$N = a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \ldots\ldots\ldots + a_1 r + a_0$, where $0 \le a_i < r$.
We now look into some representations and their meaning in decimal system.

Examples

(i) $(100011)_2$
$= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 32 + 0 + 0 + 2 + 1 = (35)_{10}$

(ii) $(1741)_8$
$= 1 \times 8^3 + 7 \times 8^2 + 4 \times 8^1 + 1 \times 8^0$
$= 512 + 448 + 32 + 1$
$= 993_{10}$

(iii) $(A3D)_{16}$
$= A \times 16^2 + 3 \times 16^1 + D \times 16^0$
$= 10 \times 256 + 48 + 13 = 2621_{10}$

Conversions

1. Decimal to binary:

(a) $(253)_{10} = (11111101)_2$
Working:

$$\begin{array}{r|l} 2 & 253 \\ \hline 2 & 126 - 1 \\ \hline 2 & 63 \quad - 0 \\ \hline 2 & 31 \quad - 1 \\ \hline 2 & 15 \quad - 1 \\ \hline 2 & 7 \quad\; - 1 \\ \hline 2 & 3 \quad\; - 1 \\ \hline & 1 \quad\; - 1 \end{array}$$

Note: The remainders are written from bottom to top.

(b) $(37.3125)_{10} = (100101.0101)_2$
Working:
The given decimal number has 2 parts:
(i) Integral part 37,
(ii) Fractional part 0.3125.

(i) Conversion of integral part:

$$\begin{array}{r|l} 2 & 37 \\ \hline 2 & 18 \quad - 1 \\ \hline 2 & 9 \quad\; - 0 \\ \hline 2 & 4 \quad\; - 1 \\ \hline 2 & 2 \quad\; - 0 \\ \hline & 1 \quad\; - 0 \end{array} \qquad \therefore (37)_{10} = (100101)_2$$

(ii) Conversion of the fractional part:
Multiply the decimal part with 2 successively and take the integral part of all the products starting from the first.

	Binary digits
$0.3125 \times 2 = 0.6250$	0
$0.6250 \times 2 = 1.2500$	1
$0.2500 \times 2 = 0.500$	0
$0.5000 \times 2 = 1.0$	1

$\therefore (0.3125)_{10} = (0.0101)_2$

Note: We should stop multiplying the fractional part by 2, once we get 0 as a fraction or the fractional part is non-terminating. It can be decided depending on the number of digits in the fractional part required.

2. Binary to decimal:

(i) $(101011011)_2 = (347)_{10}$
Working :
$(101011001)_2 =$
$1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 256 + 0 + 64 + 0 + 16 + 8 + 0 + 2 + 1$
$= (347)_{10}$

(ii) $(0.11001)_2 = (0.78125)_{10}$
Working :
$(0.11001)_2$
$= 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$
$= 1/2 + 1/4 + 1/32 = 25/32 = (0.78125)_{10}$

3. Decimal to octal:

(i) $(2595)_{10} = (5043)_8$

Working :

$$\begin{array}{r|l} 8 & 2595 \\ \hline 8 & 324 - 3 \\ \hline 8 & 40 \;\; - 4 \\ \hline & 5 \;\;\; - 0 \end{array}$$

$\therefore (2595)_{10} = (5043)_8$

4. Octal to decimal:

(i) $(4721)_8 = (2513)_{10}$

Working:

$(4721)_8 = 4 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$

$= 2048 + 448 + 16 + 1 = (2513)_{10}$

(ii) $(365.74)_8 = (245.9375)_{10}$

Working:

(a) Integral part:

$(365)_8 = 3 \times 8^2 + 6 \times 8^1 + 5 \times 8^0$

$= 192 + 48 + 5 = 245$

$\therefore (365)_8 = (245)_{10}$

(b) Fractional part:

$(0.74)_8 = 7 \times 8^{-1} + 4 \times 8^{-2}$

$= \frac{56+4}{64} = \frac{60}{64} = 0.9375$

$\therefore (365.74)_8 = (245.9375)_{10}$

5. Decimal to hexa-decimal:

(i) $(47239)_{10} = (B887)_{16}$

Working:

$$\begin{array}{r|l} 16 & 47239 \\ \hline 16 & 2952 \;\; - 7 \\ \hline 16 & 184 \;\;\; - 8 \\ \hline & 11 \;\;\;\; - 8 \end{array}$$

Recall: 11 is B, in hexa-decimal system.

$\therefore (47239)_{10} = (B887)_{16}$

(ii) $(30014)_{10} = (753E)_{16}$

Working:

$$\begin{array}{r|l} 16 & 30014 \\ \hline 16 & 1875 - 14 \;\; = E \\ \hline 16 & 117 \;\;\; - 3 \\ \hline & 7 \;\;\;\;\; - 5 \end{array}$$

$\therefore (30014)_{10} = (753E)_{16}$

6. Hexa-decimal to decimal:

$(52B)_{16} = (1323)_{10}$

Working:

$(52B)_{16} = 5 \times 16^2 + 2 \times 16^1 + B \times 16^0$

$= 1280 + 32 + 11$

$= (1323)_{10}$

$\therefore (52B)_{16} = (1323)_{10}$

7. Decimal to duo-decimal or duodenary (base 12):

$(948)_{10} = (66C)_{12}$

Working:

$$\begin{array}{r|l} 12 & 948 \\ \hline 12 & 78 - 12 \text{ or C} \\ \hline & 6 \;\;\; - 6 \end{array}$$

$\therefore (948)_{10} = (66C)_{12}$

8. Duo-decimal to decimal:

$(5BC)_{12} = (864)_{10}$

Working:

$(5BC)_{12} = 5 \times 12^2 + B \times 12^1 + C \times 12^0$

$= 720 + 132 + 12 = (864)_{10}$

9. Binary to octal:

8 being the base of octal system and 2 being the base of binary system, there is a close relationship between both the systems. One can just club three digits of a binary number into a single block and write the decimal equivalent of each group (left to right).

Example:

(i) $(100101111)_2 = (100)_2\,(101)_2\,(111)_2 = (457)_8$

$\therefore (100101111)_2 = (457)_8$

(ii) $(11111110)_2 = (011)_2\,(111)_2\,(110)_2 = (376)_8$

$\therefore (11111110)_2 = (376)_8$

Note: Introduce leading zeros to form a block of 3 without changing the magnitude of the number.

10. Binary to hexa-decimal:

This is similar to the method disscused for octal; instead of clubbing 3, we club 4 digits.

Example:

$(10111110)_2 = (1011)_2\,(1110)_2 = (11)\,(14) = (BE)_{16}$

$\therefore (10111110)_2 = (BE)_{16}$

Note: If the number of digits is not a multiple of 4, introduce leading zeros as done earlier for octal conversion.

Binary Arithmetic:

Addition:

Elementary Rules

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$ (1 will be regarded as carry as we

$1 + 1 + 1 = 11$ do in decimal system)

Examples of binary addition:

1. $(110101)_2 + (110)_2$

$$\begin{array}{r l} 1 \quad\;\; & \rightarrow \text{ carry} \\ 1\;1\;0\;1\;0\;1 & \\ 0\;0\;0\;1\;1\;0 & \text{(Introduce leading zeros)} \\ \hline 1\;1\;1\;0\;1\;1 & \\ \hline \end{array}$$

2. $(101111)_2 + (111011)_2$

$$\begin{array}{r l} 1\;1\;\;1\;1\;1 & \rightarrow \text{carry} \\ 1\;0\;1\;1\;1\;1 & \\ 1\;1\;1\;0\;1\;1 & \\ \hline 1\;1\;0\;1\;0\;1\;0 & \\ \hline \end{array}$$

3. $(110)_2 + (100)_2 + (010)_2$

$$\begin{array}{r l} 1 \;\;\;\; & \rightarrow \text{carry} \\ 1\;1\;0 & \\ 1\;0\;0 & \\ 0\;1\;0 & \\ \hline 11\;0\;0 & \\ \hline \end{array}$$

Subtraction:

Subtract 1101 from 11010.

1.

```
             2
          0 0 2 0 2
          1 1 0 1 0
         – 1 1 0 1
result→    1 1 0 1
```

Explanation: Say N = 11010,
As 1 cannot be subtracted from 0, we borrow 2 from the next place. This gives 2 – 1 = 1, as the right most digit of the result. The penultimate digit of N would become 0. A similar calculation gives the 3rd digit of the result from the right as 1 and the 4th digit of N from the right becomes 0.
We now borrow a 2 from the 5th digit of N, this makes the 4th digit of N as 2, thereby resulting in 2 – 1 = 1 as the 4th digit of the result.

2. Subtract 11011 from 111001

```
  2 2 1
  0 0 2 2 →  Borrow
1 1 1 0 0 1
–1 1 0 1 1
  1 1 1 1 0
```

Examples

3.01. Show that the binary number 101011011001101 is equal to $(53315)_8$ and $(56CD)_{16}$.

Sol: (i) To represent any binary number in base 8 we divide it into blocks of 3 starting from the left.
If the number of digits in it is not a multiple of 3, leading zeros are introduced in it since inclusion of 0 to the left does not affect its value.
$(\underline{101011011001101})_2$
$= [(101)_2\ (011)_2\ (011)_2\ (001)_2\ (101)_2]$
$= (53315)_8$

(ii) To represent any binary number in base 16 we divide it into blocks of 4 starting from the left.
If the number of digits in it is not a multiple of 4, leading zeros are introduced in it since inclusion of 0 to the left does not affect its value.
$(\underline{0101011011001101})_2$
$= [(0101)_2\ (0110)_2\ (1100)_2\ (1101)_2]_{16}$
$= (56CD)_{16}$ ($\because 1100_2 = 12 = C$
and $1101_2 = 13 = D$)

3.02. Show that 144 is a perfect square in any base more than 4.

Sol: Let n be the base of a number system (n > 4).
$(144)_n = n^2 + 4n + 4 = (n + 2)^2$
which is a perfect square for all values of n, and hence the given number is perfect square for all values of n > 4

3.03. If a (p, q, r) = (p + q) (q + r) (p + r), find a $[(32)_4, (22)_8, (16)_{10}]$.

Sol: $(32)_4 = (4)(3) + 2 = (14)_{10}$
$(22)_8 = (8)(2) + 2 = (18)_{10}$
$(16)_{10} = (16)_{10}$
a $((32)_4, (22)_8, (16)_{10})$ = a(14, 18, 16) = (14 + 18) (18 + 16) (14 + 16) = (32) (34) (30) = 32640

3.04. If $(624)_7 = 312_k$ then find k.

Sol: $(624)_7 = (6)(7^2) + (2)(7) + (4)(7)^0 = 312.$
$(312)_k = 3k^2 + k + 2$
Given $(312)_k = (624)_7$
$3k^2 + k + 2 = 312$
$3k^2 + k – 310 = 0$
$(k – 10)(3k + 31) = 0$
$k > 0$
$\therefore k = 10$

3.05. Find the hexadecimal equivalent of the number $(234567)_8$.

Sol: $(234567)_8$
$= (10\quad 011\quad 100\quad 101\quad 110\quad 111)_2$
$= (0001\quad 0011\quad 1001\quad 0111\quad 0111)_2$
$= (1\quad 3\quad 9\quad 7\quad 7)_{16}$
$= (13977)_{16}$

3.06. Subtract $(124368)_{11}$ from $(987654)_{11}$.

Sol: We put the minuend at the top and the subtrahend at the bottom and align the digits from the right. If the lower digit is smaller or equal to the upper digit, we carry out at the usual subtraction. If the lower digit is greater, we borrow from the immediate neighbour on the left. But we have to remember that a 'loan' of 1 from the left neighbour represents not ten but the base of the system, in this case – eleven.

```
9    8    7    6    5    4
1    2    4    3    6    8
------------------------------
8    6    3    2    9    7
------------------------------
```

3.07. Which of these weights (all in kg) among 1, 2, 4, 8, ….. etc., are used in weighing 500 kg if not more than one weight of each denomination can be used for the weighing?

Sol: 500 = 256 + 128 + 64 + 32 + 16 + 4
Thus expressing 500 in binary scale, we get 111110100.
The place values of 1's are the weights required for weighing.

3.08. Multiply $(123)_4$ and $(330)_4$.

Sol: We convert each number to base 10 and then multiply the results.
$(123)_4 = (1)(4^2) + (2)(4) + (3)(1) = (27)_{10}$
$(330)_4 = (3)(4^2) + (3)(4) + (0)(1) = (60)_{10}$
$(27)(60) = (1620)_{10}$

4	1620	
4	405	0
4	101	1
4	25	1
4	6	1
	1	2

$(1620)_{10} = (121110)_4$

3.09. Find the binary equivalent of the fraction 0.625.

Sol: Consider a fraction x, i.e $0 \le x < 1$.
If $0 \le x < 1/2$, the first digit after the point (in base 2 representation) is 0.
If $1/2 \le x < 1$, the first digit after the point is 1.
i.e. If $0 \le 2x < 1$, the first digit is 0 and if $1 \le 2x < 2$, it is 1.
After this part (the integral part of 2x) of the fraction is represented (either as 0 or 1), we can proceed along the same lines and represent smaller and smaller parts of the residue.
If the denominator of the fraction is a power of 2, we get a terminating binary fraction. If there are other factors in the denominator, we get a non-terminating binary fraction. For the given fraction successive digits are calculated in the table below.

n	Residue	2 (Residue)	Digit
1	0.625	1.25	1
2	0.25	0.5	0
3	0.5	1	1

$\therefore 0.625 = (0.101)_2$

3.10. A non-zero number in base 8 is such that twice the number is the number formed by reversing its digits. Find it.

Sol: Let the number be $(xy)_8$,
where $0 \le x, y < 8$.
The number formed by reversing its digits is $(yx)_8$.
$2(xy)_8 = (yx)_8$
$2(8x + y) = 8y + x$
$$\frac{x}{y} = \frac{2}{5}$$
x = 2 and y = 5 is the only possibility.
$\therefore (xy)_8 = (25)_8$.

3.11. If $(23)_4 + (100)_2 = (x)_{10}$, then find the value of x.

Sol: $(23)_4 = 2\ (4) + 3 = 11$
$(100)_2 = 4$
$\therefore x = (23)_4 + (100)_2 = 11 + 4 = 15$.

3.12. If $(46)_7 - (13)_5 = (y)_{10}$, then find y.

Sol: $(46)_7 = ((7)\ (4) + 6)_{10} = (34)_{10}$
$(13)_5 = ((5)\ (1) + 3)_{10} = (8)_{10}$
$\therefore y = 26$.

Concept Review Questions

Directions for questions 1 to 20: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. To express a number in the binary system the digits we use are _______.
(A) 0, 1, 2 (B) 0, 1, 2, 3
(C) 0,1 (D) any digit from 0 to 9

2. The number of digits we use, to express a number in the octal system is ☐ .

3. In the duodecimal system, the numerical value of B is ☐ .

4. The number of digits we use to express a number in the hexadecimal system is ☐ .

5. Express 12 in the binary system.
(A) $(1010)_2$ (B) $(1100)_2$
(C) $(1001)_2$ (D) $(1011)_2$

6. If $(1221)_{10} = (n)_{12}$, then n = ☐ .

7. The highest digit used in the septenuary system is _______.
(A) 6 (B) 7 (C) 9 (D) 8

8. If $(2346)_{10} = (x)_{16}$, then x = _______.
(A) 29A (B) 92A (C) 9A2 (D) A29

9. Express $(13)_{10}$ in the octal system.
(A) 15 (B) 11 (C) 17 (D) 51

10. If $(121)_8 = (x)_2$, then x = _______.
(A) 101001 (B) 1010011
(C) 1010001 (D) 1011001

11. If $(3AB)_{12} = (x)_{10}$, then x = ☐ .

12. If $(ACD)_{16} = (x)_{10}$, then x = _______.
(A) 2765 (B) 6725 (C) 5672 (D) 7625

13. Which of the following is/are a perfect square?
(A) $(121)_{10}$ (B) $(171)_8$
(C) $(A1)_{12}$ (D) All the above

14. The decimal equivalent of the binary number (1.001) is ☐ .

15. $(101)_2 + (1101)_2 + (111111)_2$ = _______.
(A) $(1101011)_2$ (B) $(1010001)_2$
(C) $(1100101)_2$ (D) $(1010011)_2$

16. $(34)_7 + (25)_7$ = _______.
(A) $(65)_7$ (B) $(63)_7$ (C) $(62)_7$ (D) $(26)_7$

17. $(34)_6 - (25)_6$ = _______.
(A) $(4)_7$ (B) $(3)_7$ (C) $(6)_7$ (D) $(5)_7$

18. What is the largest three-digit number in the hexadecimal system?
(A) $(999)_{16}$ (B) $(AAA)_{16}$
(C) $(FFF)_{16}$ (D) $(100)_{16}$

19. The binary representation of an even number ends with _______.
(A) 1 (B) 0
(C) Either (A) or (B) (D) None of these

20. $(247)_{10}$ = _______.
(A) $(11110111)_2$ (B) $(367)_8$
(C) $(187)_{12}$ (D) All the above

Exercise – 3(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The binary equivalent of the decimal number 176 is _______.
(A) $(11010101)_2$ (B) $(101010101)_2$
(C) $(10110000)_2$ (D) $(11011011)_2$

2. The octal equivalent of the decimal number 472 is ☐.

3. The hexa-decimal equivalent of the decimal number 523 is _______.
(A) 19C (B) 20B (C) 1DF (D) 20D

4. The duo-decimal equivalent of the decimal number 2776 is _______.
(A) 951 (B) 1734 (C) 146A (D) 1728

5. The hexa-decimal equivalent of the octal number 7464 is _______.
(A) F34 (B) FC8 (C) 8CE (D) BD9

6. The octal equivalent of the number $(110001110)_2$ is _______.
(A) $(426)_8$ (B) $(507)_8$ (C) $(716)_8$ (D) $(616)_8$

7. The number $(1100111011011)_2$ in hexa-decimal scale is _______.
(A) $(1E9A)_{16}$ (B) $(18CD)_{16}$
(C) $(59A4)_{16}$ (D) $(19DB)_{16}$

8. The decimal equivalent of the binary number $(1101.0101)_2$ is ☐.

9. The decimal equivalent of $(BAD)_{16}$ is _______.
(A) $(2468)_{10}$ (B) $(2989)_{10}$
(C) $(2788)_{10}$ (D) $(2941)_{10}$

10. $(1101)_2 + (46)_8 + (97)_{10}$ = _______.
(A) $(148)_{10}$ (B) $(147)_{10}$
(C) $(146)_{10}$ (D) $(145)_{10}$

11. The minimum number of bits required to represent the decimal number 256 in binary system is ☐.

12. $(256)_{16} – (256)_8$ = _______.
(A) 0
(B) $(256)_8$
(C) $(424)_{10}$
(D) $(472)_{10}$

13. For $n \ge 3$;
$(n)_{n+2} + (n-1)_{n+1} + (n-2)_n + \ldots\ldots + (1)_3$ = _______.
(A) $(n+1)_{10}$
(B) $\left(\frac{n(n-1)}{2}\right)_{10}$
(C) $\left(\frac{n(n+1)}{2}\right)_{10}$
(D) $(n)_{10}$

Directions for questions 14 and 15: These questions are based on the data given below.

Kiran wanted to weigh 378 kg of wheat. The weights are available in denominations of 1 kg, 2 kg, 4 kg, 8 kg, 12 kg, 32 kg, ….. etc. He decides not to use more than one weight of each denomination.

14. How many weights does he use in all if only one side of the balance is used for weighing?
☐

15. In how many ways can he weigh 378 kg, if only one side of the balance is used for weighing?
☐

16. If f(x, y, z) = xy + yz + zx, then
$f((25)_8, (25)_{10}, (25)_{16})$ = _______.
(A) $(2178)_{10}$ (B) $(2227)_{10}$
(C) $(2463)_{10}$ (D) $(2841)_{10}$

17. If ‘1’ is concatenated to the right most digit of a positive binary integer, the number thus formed is _______.
(A) the same as the original number.
(B) half that of the original number.
(C) double the original number.
(D) 1 more than double the original number.

18. The square root of the octal number 1161 is _______.
(A) $(26)_8$ (B) $(25)_8$ (C) $(211)_8$ (D) $(31)_8$

19. The square of $(234)_6$ is _______.
(A) $(23425)_6$ (B) $(101423)_6$
(C) $(104524)_6$ (D) $(15235)_6$

20. The remainder obtained when $(1000111)_2$ is divided by $(101)_2$ is _______.
(A) $(0)_2$ (B) $(1)_2$ (C) $(10)_2$ (D) $(11)_2$

21. The L.C.M. of $(120)_8$ and $(24)_8$ is _______.
(A) $(20)_{10}$ (B) $(80)_8$ (C) $(160)_8$ (D) $(120)_8$

Directions for questions 22 and 23: These questions are based on the data given below.

Yogesh found a decimal number, which, when represented in bases 2, 3, 4 and 5 ends in 1, 2, 3 and 4 respectively.

22. What is the least positive integer satisfying this property?
☐

23. How many such three-digit decimal numbers are possible?
☐

24. In which of the following scales is 5016 a perfect cube?
(A) 8 (B) 9 (C) 11 (D) 7

25. If the arithmetic mean of $(12)_6$ and $(33)_7$ is $(10)_n$, then the value of n is ☐.

Exercise – 3(b)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The binary equivalent of the decimal number 108 is _______.
(A) $(10011001)_2$ (B) $(110110)_2$
(C) $(1101100)_2$ (D) $(1110100)_2$

2. The octal equivalent of the decimal number 567 is _______.
(A) $(1760)_8$ (B) $(1670)_8$ (C) $(1706)_8$ (D) $(1067)_8$

3. The duo-decimal equivalent of the decimal number 1896 is _______.
(A) $(1896)_{12}$ (B) $(1120)_{12}$ (C) $(1210)_{12}$ (D) $(AB0)_{12}$

4. The hexa-decimal equivalent of the decimal number 894 is _______.
(A) $(37E)_{16}$ (B) $(3D7)_{16}$ (C) $(D37)_{16}$ (D) $(73D)_{16}$

5. The hexa-decimal equivalent of octal number 7640 is _______.
(A) $(AB0)_{16}$ (B) $(CA0)_{16}$ (C) $(BA0)_{16}$ (D) $(EA0)_{16}$

6. The octal equivalent of the number $(10101101011)_2$ is _______.
(A) $(2355)_8$ (B) $(2553)_8$ (C) $(2535)_8$ (D) $(3253)_8$

7. The decimal equivalent of $(ABC)_{16}$ is ☐ .

8. If the difference of the numbers 6555 and 777 in a certain number system is 5556, then what is the sum of the numbers 5666 and 457 in the same system?
(A) $(3564)_9$ (B) $(5653)_{12}$ (C) $(4644)_{12}$ (D) $(6345)_8$

9. $(423)_9 - (423)_6$ = _______.
(A) $(256)_{12}$ (B) $(6B)_{12}$ (C) $(136)_{12}$ (D) $(163)_{12}$

10. The number of three-digit numbers in base 12 system is ☐ .

11. The number of three-digit numbers in base 5 system which are actually two-digit numbers in base 10 system is ☐ .

12. $(111)_2 + (222)_3 + (333)_4 + (444)_5 + (555)_6 + (666)_7$ = _______.
(A) $(999)_{10}$ (B) $(777)_8$ (C) $(777)_{10}$ (D) $(888)_9$

13. If $f(x, y, z) = x^2 + y^2 + z^2$, then $f((13)_5, (13)_8, (13)_{12})$ = _______.
(A) $(240)_{10}$ (B) $(140)_{10}$ (C) $(401)_{10}$ (D) $(410)_{10}$

14. The square root of the hexa-decimal number 310 is _______.
(A) $(3C)_{16}$ (B) $(2C)_{16}$ (C) $(1C)_{16}$ (D) $(C1)_{16}$

15. The square of $(43)_8$ is _______.
(A) $(2311)_8$ (B) $(2131)_8$ (C) $(1321)_8$ (D) $(1234)_8$

16. The remainder obtained when $(23232)_4$ is divided by $(232)_4$ is _______.
(A) $(32)_5$ (B) $(32)_4$ (C) $(24)_6$ (D) $(32)_{10}$

17. The L.C.M of $(210)_6$ and $(30)_6$ is _______.
(A) $(1030)_{10}$ (B) $(234)_6$ (C) $(1030)_6$ (D) $(432)_5$

18. The numbers $(11)_7$, $(55)_7$ and $(404)_7$ are in _______.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) arithmetic geometric progression

19. If the geometric mean of the numbers $(24)_6$ and $(34)_7$ is $(24)_n$, then n = ☐ .

20. In which of the following bases is 2454 a perfect cube?
(A) 11 (B) 12 (C) 9 (D) 6

***Directions for questions 21 and 22*:** These questions are based on the data given below.

Ajay wanted to weigh 456 kg of rice. The weights are available in denominations of 1 kg, 2 kg, 4 kg, 8 kg, 16 kg . . . . . . He decides not to use more than one weight of each denomination.

21. How many weights does he use in all if only one side of the balance is used for weighing?
☐

22. The minimum weight used for weighing 456 kg weight is ☐ Kg.

23. The minimum number of bits required to represent the decimal number 512 in the binary system is _______.
(A) 10 (B) 7 (C) 8 (D) 9

***Directions for questions 24 and 25*:** These questions are based on the data given below.

Rajesh found a decimal number which when represented in bases 2, 3, 4, 5 and 6 ends in 1, 2, 3, 4 and 5 respectively.

24. What is the smallest positive integer satisfying this property?
☐

25. How many such three-digit decimal numbers are possible?
☐

26. The decimal fraction 0.375 in the binary system is _______.
(A) $(0.111)_2$ (B) $(0.11)_2$
(C) $(0.0111)_2$ (D) $(0.011)_2$

27. The decimal number 13.34375 in the binary system is _______.
(A) $(1101.01011)_2$ (B) $(110.0111)_2$
(C) $(1011.01011)_2$ (D) $(1101.101011)_2$

28. A two-digit number in base 7 is such that the number is equal to thrice the number formed by reversing its digits. The number is _______.
(A) $(51)_{10}$ (B) $(36)_{10}$ (C) $(36)_7$ (D) $(15)_7$

29. $(11)_2 + (11)_3 + (11)_4 + - - + (11)_n =$ _______.
(A) $n^2 + 3n$ (B) $n^2 + 3n - 4$
(C) $\frac{n^2+3n}{2}$ (D) $\frac{n^2+3n-4}{2}$

30. If '0' is concatenated to the right most digit of a positive integer of base 'n' the number thus formed is _______.
(A) equal to as the original number
(B) (n + 1) times the original number
(C) n times the original number
(D) 3 times the original number

Key

Concept Review Questions

1. C	5. B	9. A	13. D	17. D
2. 8	6. 859	10. C	14. 1.125	18. C
3. 11	7. A	11. 563	15. B	19. B
4. 16	8. B	12. A	16. C	20. D

Exercise – 3(a)

1. C	6. D	11. 9	16. B	21. D
2. 730	7. D	12. C	17. D	22. 59
3. B	8. 13.3125	13. C	18. D	23. 15
4. B	9. B	14. 6	19. C	24. D
5. A	10. A	15. 1	20. B	25. 16

Exercise – 3(b)

1. C	6. B	11. 75	16. B	21. 4	26. D
2. D	7. 2748	12. C	17. C	22. 8	27. A
3. B	8. D	13. D	18. B	23. A	28. B
4. A	9. C	14. C	19. 8	24. 59	29. D
5. D	10. 1584	15. A	20. B	25. 15	30. C