

## CHAPTER – 8

### CUBES

A cube is a three dimensional solid having 6 faces, 12 edges and 8 corners. All the edges of a cube are equal in length and hence all the faces are square in shape.

In competitive exams a few questions may be asked based on cubes.

The questions on cubes may belong to anyone of the following categories.

- I. A cube is cut by making certain specified number of cuts. The directions in which the cuts are made may or may not be given. We are to find the number of identical pieces resulting out of the given cuts.

- II. The number of identical pieces, into which a cube is cut, is given and we need to find the number of cuts.

- III. A cube could be painted on all or some of its faces with the same colour or different colours and then cut into a certain specified number of identical pieces. Then questions of the form — "How many small cubes have 2 faces painted?". "How many smaller cubes have only one face painted?" etc. could then be framed.

### Exercise – 8(a)

**Directions for questions 1 to 5:** Select the correct alternative from the given choices.

1. 125 small but identical cubes have been put together to form a large cube. How many more such small cubes will be required to cover this large cube completely?  
(A) 208 (B) 212 (C) 254 (D) 218
2. What is the maximum possible number of identical pieces into which a cube can be cut by 3 cuts?  
(A) 9 (B) 8 (C) 7 (D) 6
3. What is the maximum possible number of identical pieces into which a cube can be cut by 4 cuts?  
(A) 10 (B) 12 (C) 15 (D) 5
4. A cube is cut parallel to one face by making 10 cuts [such that all the resulting pieces are identical]. What is the maximum number of identical pieces that can be obtained by now making two more cuts (in any direction)?  
(A) 33 (B) 40 (C) 55 (D) 44
5. What is the maximum number of identical pieces a cube can be cut into by 13 cuts?  
(A) 120 (B) 140 (C) 180 (D) 150

**Directions for questions 6 to 9:** These questions are based on the following data.

125 small but identical cubes are put together to form a large cube. This larger cube is now painted on all six faces.

6. How many of the smaller cubes have no face painted at all?  
(A) 27 (B) 64 (C) 8 (D) 36
7. How many of the smaller cubes have exactly one face painted?  
(A) 49 (B) 54 (C) 64 (D) 72
8. How many of the smaller cubes have exactly two faces painted?  
(A) 25 (B) 16 (C) 36 (D) 64
9. How many of the smaller cubes have exactly three faces painted?  
(A) 4 (B) 5 (C) 9 (D) 8

**Directions for questions 10 to 15:** Select the correct alternative from the given choices.

10. What is the least number of cuts required to cut a cube into 24 identical pieces?  
(A) 2 (B) 4 (C) 6 (D) 8
11. What is the maximum number of identical pieces a cube can be cut into by 7 cuts?  
(A) 36 (B) 49 (C) 25 (D) 56
12. What is the least number of identical cuboids, each of dimensions 2 cm x 4 cm x 5 cm, that are required to form a cube?  
(A) 160 (B) 240  
(C) 220 (D) 200

13. It was found that a cube can be cut into certain number of identical cuboids each measuring 1 cm x 2 cm x 5 cms. What is the side of the smallest such cube? How many such cuboids can be formed from such a cube?  
(A) 10 cm, 100 (B) 5 cm, 50  
(C) 20 cm, 800 (D) 15 cm, 100

14. A cube of side 6 cm, has been cut into 64 smaller but identical cubes. If it was estimated that it would take 4 ltrs. of paint to paint all the faces of the original cube, then how much paint is required to paint all the faces of all the smaller cubes?  
(A) 6 litres (B) 12 litres  
(C) 20 litres (D) 16 litres

15. Each face of a cube is painted either white or black. In how many different ways can the cube be painted?  
(A) 8 (B) 10 (C) 12 (D) 16

**Directions for questions 16 to 20:** These questions are based on the following data.

There is a cube in which one pair of opposite faces is painted red; the second pair of opposite faces is painted blue and the third pair of opposite faces is painted green. This cube is now cut into 216 smaller but identical cubes.

16. How many small cubes are there with no red paint at all?  
(A) 121 (B) 144 (C) 169 (D) 100
17. How many small cubes are there with at least two different colours on their faces?  
(A) 49 (B) 64 (C) 56 (D) 81
18. How many small cubes are there without any face painted?  
(A) 64 (B) 49 (C) 36 (D) 81
19. How many small cubes are there with only red and green on their faces?  
(A) 8 (B) 12 (C) 20 (D) 16
20. How many small cubes are there showing only green or only blue on their faces?  
(A) 64 (B) 81 (C) 125 (D) 100

**Directions for questions 21 to 23:** These questions are based on the following data.

A large cube is painted green on three of its faces, blue on two of its faces, red on one of its faces in such a way that there is exactly one pair of opposite faces having the same colour. The cube is now cut into 216 smaller and identical cubes.

21. How many smaller cubes will have the number of faces painted with different colours in the ratio of 2 : 1 or 1 : 2?

22. How many smaller cubes are painted with exactly two different colours?

23. How many smaller cubes are painted with only one colour?

**Directions for questions 24 to 27:** These questions are based on the following data.

A cube is formed by joining 216 smaller and identical cubes. The bigger cube is painted black colour on all its faces. From the bigger cube, 64 smaller cubes from a corner were taken out and the cube formed by joining these 64 smaller cubes is painted red colour in all its faces. Again, this cube is fitted back into its usual position in the large cube.

24. How many smaller cubes have no faces painted?

25. How many smaller cubes have three faces painted?

26. How many smaller cubes have only two faces painted?

27. How many smaller cubes have only one face painted?

**Directions for questions 28 to 30:** These questions are based on the following information.

A cube is formed by using 125 small and identical cubes in the form of five identical vertical layers from the front to the rear. On each small cube, a different number from 1 to 125 is painted in such way that in each layer of smaller cubes, the numbers are in ascending order from left to right i.e., 1 is painted on bottom left most of the front face, next to that is 2 and so on and 6 is painted on the cube just above the cube which is painted with 1 and above 2 is 7 and so on. Similar pattern is followed in the other layers also. The greatest number of each vertical layer from the front to the rear is in an ascending order.

28. What is the sum of numbers that form the diagonal connecting the right bottom cube of front face to the left top cube in the rear face?

29. What is the sum of the numbers on the smaller cubes that are not visible?

30. What is sum of the numbers on the middle cubes across all layers of the larger cube?

### Exercise – 8(b)

**Directions for questions 1 to 5:** These questions are based on the following data.

There is a cube in which one pair of adjacent faces is painted red; the second pair of adjacent faces is painted blue and a third pair of adjacent faces is painted green. This cube is now cut into 216 smaller but identical cubes.

- How many small cubes are there with no red paint at all?  
(A) 144 (B) 155 (C) 125 (D) 150
- How many small cubes are there with at least two different colours on their faces?  
(A) 64 (B) 54 (C) 33 (D) 44
- How many small cubes are there with one face painted red?  
(A) 64 (B) 81 (C) 60 (D) 100
- How many small cubes are with both red and green on their faces?  
(A) 8 (B) 12 (C) 16 (D) 32
- How many small cubes are there showing only green or only blue on their faces?  
(A) 64 (B) 72 (C) 81 (D) 90

**Directions for questions 6 to 13:** Select the correct alternative from the given choices.

- What is the maximum number of identical pieces a cube can be cut into by 6 cuts?  
(A) 12 (B) 36 (C) 18 (D) 27
- What is the maximum number of identical pieces a cube can be cut into by 5 cuts?  
(A) 25 (B) 20 (C) 18 (D) 16
- 64 smaller but identical cubes are placed on a table to form a large cube. How many more such smaller cubes are now required to enclose this large cube placed on the table completely?  
(A) 125 (B) 115 (C) 100 (D) 116
- If a cube is cut by three planes parallel to the faces to yield the maximum number of identical pieces, then what is the percentage increase in the total surface area?  
(A) 80% (B) 100% (C) 60% (D) 120%
- What is the least number of cuts required to divide a cube into 120 identical pieces?  
(A) 6 (B) 8 (C) 15 (D) 12

11. 125 small but identical cubes are put together on a table to form one large cube. A knife is passed through this cube starting along one edge of the top face to the diagonally opposite edge on the bottom face. How many of the small cubes are cut by this knife?  
 (A) 25 (B) 36  
 (C) 64 (D) 16
12. What is the maximum number of identical pieces into which a cube can be divided by 12 cuts?  
 (A) 100 (B) 144  
 (C) 150 (D) 125
13. 343 smaller but identical cubes are put together to form a large cube. A knife is passed through one edge AB of top face ABCD to the diagonally opposite edge of the bottom face. The knife is then again passed through the side CD of top face to the diagonally opposite edge of the bottom face? How many of the smaller cubes are not cut by the knife at all?  
 (A) 256 (B) 225  
 (C) 180 (D) 252

**Directions for questions 14 to 16:** These questions are based on the given data.

A large cube painted on all six faces is cut into 27 smaller but identical cubes.

14. How many of the smaller cubes have no faces painted at all?  
 (A) 0 (B) 1 (C) 3 (D) 4
15. How many of the smaller cubes have exactly one face painted?  
 (A) 3 (B) 6 (C) 12 (D) 15
16. How many of the smaller cubes have exactly two faces painted?  
 (A) 36 (B) 6 (C) 12 (D) 15

**Directions for questions 17 to 20:** These questions are based on the given data.

A large cube is painted on all six faces and then cut into a certain number of smaller but identical cubes. It was found that among the smaller cubes, there were eight cubes which had no face painted at all.

17. How many smaller cubes was the original large cube cut into?  
 (A) 27 (B) 48  
 (C) 64 (D) 125
18. How many small cubes have exactly one face painted?  
 (A) 12 (B) 24  
 (C) 16 (D) 32
19. How many small cubes have exactly two faces painted?  
 (A) 6 (B) 12  
 (C) 18 (D) 24
20. How many small cubes have three face painted?  
 (A) 0 (B) 8  
 (C) 27 (D) 64

**Directions for questions 21 to 23:** These questions are based on the following information.

A large cube is painted on only three of its faces with three different colours-red green and black. This cube is now cut into 125 smaller but identical faces.

21. What is the minimum number of cubes that have no face painted?

22. What is maximum number of cubes that have exactly two face painted?

23. What is the minimum number of cubes (respectively) that have exactly one face painted?

**Directions for questions 24 to 26:** These questions are based on the following data.

The faces of the cuboid are painted with three different colours-black, red, and yellow such that each colour is painted on at least one face. Now 4, 5 and 6 cuts are made in three different directions.

24. What is the maximum possible number of smaller pieces that have only black on their faces?

25. What is the maximum possible number of smaller pieces that have only black and yellow painted on their faces?

26. What is the maximum number of smaller pieces with three colours painted on them, respectively?

**Directions for questions 27 to 30:** These questions are based on the following information.

216 small but identical cubes are labelled 1 to 216 and a larger cube is built by placing the cubes in an ascending order. In each horizontal layer the front row is laid first from left to right and then the cubes are laid in a similar pattern in the second row and so on. The bottom most layer is laid first and then the layer just above that and so on.

27. What is the sum of numbers that form the diagonal connecting the left bottom corner cube of front face to the right bottom cube of the rear face?

28. What is the sum of the numbers of the cube in the vertical column which has its base as the fourth cube from the left in the row just behind the front row?

30. The sum of the numbers of the cubes forming diagonal from the bottom left of the front face of the larger cube to the top right corner of the front face of the larger cube is \_\_\_\_\_.

29. The sum of the numbers of the cubes forming diagonal from the bottom right of the front face of the larger cube to the top left corner of the rear face of the larger cube is \_\_\_\_\_.

### **Key**

#### **Exercise – 8(a)**

- |      |       |       |       |         |          |
|------|-------|-------|-------|---------|----------|
| 1. D | 6. A  | 11. A | 16. B | 21. 6   | 26. 63   |
| 2. B | 7. B  | 12. D | 17. C | 22. 42  | 27. 93   |
| 3. B | 8. C  | 13. A | 18. A | 23. 108 | 28. 315  |
| 4. D | 9. D  | 14. D | 19. D | 24. 45  | 29. 1701 |
| 5. D | 10. C | 15. B | 20. A | 25. 15  | 30. 315  |

#### **Exercise – 8(b)**

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|------|-------|-------|-------|--------|---------|
| 1. D | 6. D  | 11. A | 16. C | 21. 60 | 26. 8   |
| 2. D | 7. C  | 12. D | 17. C | 22. 12 | 27. 111 |
| 3. C | 8. D  | 13. D | 18. B | 23. 48 | 28. 600 |
| 4. C | 9. B  | 14. B | 19. D | 24. 90 | 29. 651 |
| 5. B | 10. D | 15. B | 20. B | 25. 32 | 30. 561 |