# Chapter – 5
# MENSURATION

## Areas of Plane Figures

Mensuration is the branch of mathematics which deals with the study of geometric shapes, their area, volume and related parameters. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of "solids." Solids are objects, which have three dimensions (plane figures have only two dimensions).

Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

## TRIANGLES

The area of a triangle is represented by the symbol Δ. For any triangle, the three sides are represented by a, b and c and the angles opposite these sides represented by A, B and C respectively.

(i) For any triangle in general,

(a) When the measurements of three sides a, b, c are given,
**Area =** $\sqrt{\mathbf{s(s-a)(s-b)(s-c)}}$ where
s = $\dfrac{a+b+c}{2}$
This is called Hero's formula.

(b) When base (b) and altitude (height) to that base are given,
**Area = ½ x base x altitude = ½ b.h**

(c) **Area = ½ ab . sinC = ½ bc.sinA = ½ ca.sinB**

(d) **Area =** $\dfrac{\mathbf{abc}}{\mathbf{4R}}$ where R is the circumradius of the triangle.

(e) **Area = r.s** where r is the inradius of the triangle and s, the semi-perimeter.

Out of these five formulae, the first and the second are the most commonly used and are also more important from the examination point of view.

(ii) For a right angled triangle,

**Area = ½ X Product of the sides containing the right angle**

(iii) For an equilateral triangle

**Area =** $\dfrac{\sqrt{3}\cdot a^2}{4}$ where "a" is the side of the triangle

The height of an equilateral triangle $=\dfrac{\sqrt{3}\cdot a}{2}$

(iv) For an isosceles triangle

**Area =** $\dfrac{b}{4}\sqrt{4a^2-b^2}$ where "a" is length of each of the two equal sides and b is the third side

## QUADRILATERALS

(i) For any quadrilateral

**Area of the quadrilateral = ½ x One diagonal x Sum of the offsets drawn to that diagonal**
Hence, for the quadrilateral ABCD shown in Fig. 5.23, area of quadrilateral ABCD = 1/2 x AC × (BE + DF)

(ii) For a cyclic quadrilateral where the four sides measure a, b, c and d respectively,

**Area =** $\sqrt{\mathbf{(s-a)(s-b)(s-c)(s-d)}}$ where s is the semi-perimeter, i.e., s = (a + b + c + d)/2

(iii) For a trapezium
**Area of a trapezium = ½ x Sum of parallel sides x Distance between them**
**= ½ x (AD + BC) x AE** (refer to Fig. 5.25)

(iv) For a parallelogram
(a) **Area = Base x Height**

(b) **Area = Product of two sides x Sine of included angle**

(v) For a rhombus
**Area = ½ x Product of the diagonals**
**Perimeter** = 4 x Side of the rhombus

(vi) For a rectangle
**Area = Length x Breadth**
**Perimeter** = 2(l + b), where l and b are the length and the breadth of the rectangle respectively

(vii) For a square
(a) **Area = Side²**
(b) **Area = ½ x Diagonal²**
[where the diagonal = $\sqrt{2}$ × side]
Perimeter = 4 × Side

(viii) For a polygon
(a) **Area of a regular polygon = ½ x Perimeter x Perpendicular distance from the centre of the polygon to any side**
(Please note that the centre of a regular polygon is equidistant from all its sides)
Area of a regular hexagon = $\dfrac{3\sqrt{3}}{2}$ (side)$^2$

(b) For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon.

## CIRCLE

(i) **Area of the circle** = $\pi r^2$ where r is the radius of the circle
Circumference = $2\pi r$

(ii) Sector of a circle
Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$
**Area** = $\frac{\theta}{360^\circ} \times \pi r^2$ where θ is the angle of the sector in degrees and r is the radius of the circle.
Area = (1/2)lr; l is length of arc and r is radius.

(iii) Ring : Ring is the space enclosed by two concentric circles.

**Area = $\pi R^2 - \pi r^2 = \pi(R + r)(R - r)$** where R is the radius of the outer circle and r is the radius of the inner circle.

## ELLIPSE

**Area = $\pi$ ab** where "a" is semi-major axis and "b" is semi-minor axis.
**Perimeter** = $\pi$ (a + b)

## AREAS AND VOLUMES OF SOLIDS

Solids are three-dimensional objects which, in addition to area, have volume also. For solids, two different types of areas are defined

(a) Lateral surface area or curved surface area and

(b) Total surface area

As the name itself indicates, lateral surface area is the area of the LATERAL surfaces of the solid. Total surface area includes the areas of the top and the bottom surfaces also of the solid. Hence, Total surface area = Lateral surface area + Area of the top face + Area of the bottom face

In solids (like cylinder, cone, sphere) where the lateral surface is curved, the lateral surface area is usually referred to as the "curved surface area."

For any solid, whose faces are regular polygons, there is a definite relationship between the number of vertices, the number of sides and the number of edges of the solid. This relationship is given by "Euler's Rule".

**Number of faces + Number of vertices**
**= Number of edges + 2(Euler's Rule)**

## PRISM

A right prism is a solid whose top and bottom faces (bottom face is called base) are parallel to each other and are identical polygons (of any number of sides) that are parallel. The faces joining the top and bottom faces are rectangles and are called lateral faces. There are as many lateral faces as there are sides in the base. The distance between the base and the top is called height or length of the right prism.

In a right prism, if a perpendicular is drawn from the centre of the top face, it passes through the centre of the base.

For any prism,

**Lateral Surface Area = Perimeter of base × Height of the prism**
**Total Surface Area = Lateral Surface Area + 2 × Area of base**

**Volume = Area of base x Height of the prism**

## CUBOID OR RECTANGULAR SOLID

A right prism whose base is a rectangle is called a rectangular solid or cuboid. If l and b are respectively the length and breadth of the base and h, the height, then

**Volume = lbh**
**Lateral Surface Area = 2(l + b) . h**
**Total Surface Area = 2(l + b)h + 2lb**
**= 2(lb + lh + bh)**
**Longest diagonal of the cuboid =** $\sqrt{l^2 + b^2 + h^2}$

## CUBE

A right prism whose base is a square and height is equal to the side of the base is called a cube.

**Volume = $a^3$ where a is the edge of the cube**
**Lateral Surface Area = $4a^2$**
**Total Surface Area = $6a^2$**
The longest diagonal of the cube (i.e., the line joining one vertex on the top face to the diagonally opposite vertex on the bottom face) is called the diagonal of the cube. The length of the diagonal of the cube is $a\sqrt{3}$ .

## CYLINDER

A cylinder <u>is equivalent to</u> a right prism whose base is a circle. A cylinder has a single curved surface as its lateral faces. If r is the radius of the base and h is the height of the cylinder,

**Volume = $\pi r^2 h$**
**Curved Surface Area = $2\pi rh$**
**Total Surface Area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$**

A hollow cylinder has a cross-section of a ring.
Volume of the material contained in a hollow cylindrical ring
= $\pi (R^2 - r^2)h$ where R is the outer radius, r is the inner radius and h, the height.

## PYRAMID

A solid whose base is a polygon and whose faces are triangles is called a pyramid. The triangular faces meet at a common point called vertex. The perpendicular from the vertex to the base is called the height of the pyramid.

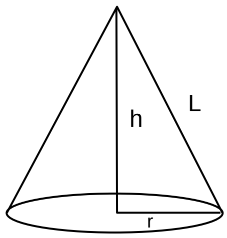
A pyramid whose base is a regular polygon and the foot of the perpendicular from the vertex to the base coincides with the centre of the base, is called a right pyramid.

The length of the perpendicular from the vertex to any side of the base (please note that this side will be the base of one of the triangular lateral faces of the prism) along the slant lateral surface is called the slant height of the prism.

**Volume of a pyramid = $^{1}/_{3}$ x Area of base x Height**
**Lateral Surface area = $^{1}/_{2}$ x Perimeter of the base x Slant height**
**Total Surface Area = Lateral Surface Area + Area of the base.**

## CONE

**Fig. 5.01**




A cone is equivalent to a right pyramid whose base is a circle. The lateral surface of a cone does not consist of triangles like in a right pyramid but is a single curved surface.
If r is the radius of the base of the cone, h is height of the cone and' l is the slant height of the cone, then we have the relationship (Fig. 5.41)

**$l^2 = r^2 + h^2$**

**Volume = $^{1}/_{3}\,\pi\, r^2h$**
**Curved Surface Area = $\pi$ r.l**
**Total Surface Area = $\pi$ rl + $\pi$ r$^2$ = $\pi$ r( l + r)**

A cone can be formed by taking the sector of a circle and joining together its straight edges. If the radius of the sector is R and the angle of the sector is θ°, then we have the following relationships between the length of the arc and area of the sector on the one hand and base perimeter of the cone and curved surface area of the cone on the other hand.

Radius of the sector = Slant height of the cone
i.e., R = l

Length of the arc of the sector = Circumference of the base of the cone

i.e., $\frac{\theta}{360}\times 2\times \pi R = 2\pi r$

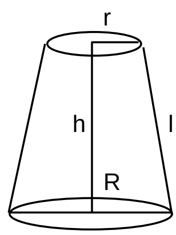
$\Rightarrow r = \frac{\theta}{360}\times R$

and Area of the sector = Curved surface area

(Actually, from this last equation, substituting the values from the first two equations, we can get the curved surface area of the cone, which is what is given previously as equal to $\pi$rl)

## CONE FRUSTUM

**Fig. 5.02**




If a cone is cut into two parts by a plane parallel to the base, the portion that contains the base is called the frustum of a cone.

If r is the top radius ; R, the radius of the base; h the height and l the slant height of a frustum of a cone (Fig. 5.42), then,

**Lateral Surface Area of the cone = $\pi$ l(R + r)**
**Total Surface Area = $\pi$ (R$^2$ + r$^2$ + R.l + r.l)**
**Volume = 1/3 $\pi$ h (R$^2$ + Rr + r$^2$)**
**$l^2 = (R - r)^2 + h^2$**

If H is the height of the complete cone from which the frustum is cut, then from similar triangles, we can write the following relationship.

$$\frac{r}{R} = \frac{H-h}{H}$$

A bucket that is normally used in a house is a good example of the frustum of a cone. The bucket is actually the inverted form of the frustum that is shown in the figure above.

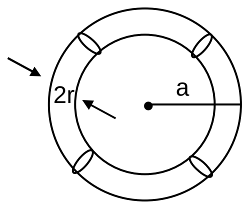
## FRUSTUM OF A PYRAMID

A pyramid left after cutting of a portion at the top by a plane parallel to the base is called a frustum of a pyramid.
If $A_1$ is the area of the base; $A_2$ the area of the top and h, the height of the frustum,

**Volume of frustum = $^{1}/_{3}\times h\times\left(A_1 + A_2 + \sqrt{A_1A_2}\right)$**
**Lateral Surface Area = ½ x (Sum of perimeters of base and top) x Slant height**
**Total Surface Area = Lateral Surface Area + $A_1$ + $A_2$**

## TORUS

**Fig. 5.03**




A torus is a three-dimensional figure produced by the revolution of a circle about an axis lying in its plane but not intersecting the circle. The shape of the rubber tube in a bicycle (when it is inflated fully) is an example of a torus. If r is the radius of the circle that rotates and a is the distance between the centre of the circle and the axis of revolution,

Surface Area of the torus = $4\pi^2ra$
Volume of the torus = $2\pi^2r^2a$

A torus is also referred to as a solid ring. (Fig. 5.43)

## SPHERE

Any point on the surface of a sphere is equidistant from the centre of the sphere. This distance is the radius of the sphere.

**Surface Area of a sphere = $4\pi r^2$**
**Volume of a sphere = $(4/3)\pi r^3$**

The curved surface area of a hemisphere is equal to half the surface area of a sphere, i.e., $2\pi r^2$

{Note: Among the solids discussed above, Pyramid, Frustum of a Pyramid and Torus are not important from the point of view of the entrance exams and hence can be ignored if you so wish. Similarly, among the plane figures, Ellipse may be ignored if you so wish.}

The following examples cover various properties / theorems discussed in Geometry as well as areas and volumes discussed in Mensuration. You should learn all the properties of triangles, quadrilaterals and circles as well as areas/volumes of plane figures and solids thoroughly before starting with the worked out examples and the exercise that follows the worked out examples.

## Examples

**5.01.** The sides of a triangle are 12 cm, 18 cm and 24 cm. Find its area.

**Sol:** The semi – perimeter (s)

$= \dfrac{12+18+24}{2} = 27$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(27)(27-12)(27-18)(27-24)}$

$= 27\sqrt{15}$ sq.cm

**5.02.** The wheel of a motorcar makes 800 revolutions in covering 880 m. Find its diameter $\left(\text{Take } \pi = \dfrac{22}{7}\right)$.

**Sol:** Let the diameter be d m.
Distance covered = (Number of revolutions) (circumference of the wheel).

$\therefore 880 = (800)\left(\dfrac{22}{7}\right)(d)$

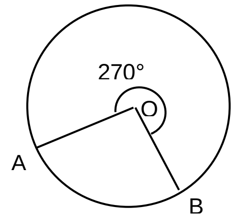
d = 0.35

**5.03.** A copper wire would enclose an area of 484 sq.cm if it was bent in the shape of a square. If it was bent in the form of a circle, find the radius of the circle. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$

**Sol:** Area of the square = 484. Hence perimeter of the square = 88. If the wire was bent in the form of a circle, it would have its circumference as 88 cm.

Hence radius = $\dfrac{88}{(2)\left(\dfrac{22}{7}\right)} = 14$ cm

**5.04.**




In the figure above, O is the centre of the circle. Find the length of the minor arc AB if OA = 7 cm. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$

**Sol:** $\angle AOB = 360° - 270° = 90°$

Length of the minor arc AB = $\dfrac{\theta}{360°} \times 2\pi r$

$= \left(\dfrac{90°}{360°}\right)\left((2)\left(\dfrac{22}{7}\right)(7)\right) = 11$ cm

**5.05.** In the previous example, find the area of the minor sector AOB $\left(\text{Take } \pi = \dfrac{22}{7}\right)$.

**Sol:** Area of the sector = $\dfrac{\theta}{360°} \times \pi r^2$

$= \left(\dfrac{90°}{360°}\right)\left(\dfrac{22}{7}\right)(7)^2 = 38{\cdot}5$ sq.cm

**5.06.** The circumference of a circle is $2\dfrac{1}{2}$ times that of another circle. How many times the area of the smaller circle is the area of the larger circle?

**Sol:** Let the radius of the smaller circle be r cm.
Radius $\alpha$ circumference.

Hence radius of the larger circle = $\dfrac{5}{2}$ (Radius of the smaller circle) = $\dfrac{5}{2}r$ cm .

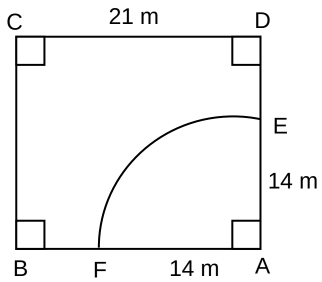
Area = $\pi$ (radius)$^2$

$\therefore$ Area of the larger circle = $\pi\left(\dfrac{5}{2}r\right)^2$

$= \dfrac{25}{4}\left[\pi r^2\right] = \dfrac{25}{4}$ (Area of the smaller circle)

**5.07.** A goat is tied to a corner of a square field of side 21 m with a rope of length 14 m. Find the area of the square field that the goat cannot graze $\left(\text{Take } \pi = \dfrac{22}{7}\right)$.

**Sol:**




Area that the goat cannot graze = (Area of ABCD) – (Area of sector AEF)

$= (212) - \left(\frac{90°}{360°}\pi(14)^2\right) = 287$ sq.m

**5.08.** A circular garden has a diameter of 56 m. It has a circular path running all around and outside it. The difference between the circumferences of the larger and the smaller gardens is 44 m. Find the width of the path $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Difference between the circumferences
= $2\pi$ (radius of the larger garden) – $2\pi$ (radius of the smaller garden) = $2\pi$ (width of the path)

$(2)\left(\frac{22}{7}\right)$ (width of the path) = 44

Width of the path = 7 m

**5.09.** A circular garden has a radius of 15 m. It is surrounded by a circular path of width 7 m. If the path is to be polished at a rate of ₹15 per sq.m, find the total cost of polishing the path. $\left(\text{Take } \pi = \frac{22}{7}\right)$

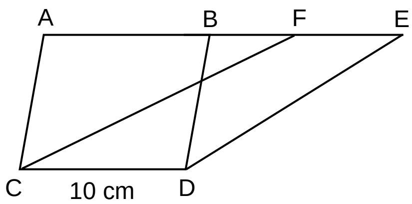
**Sol:** Total cost of polishing = (15) (Area to be covered)

$= 15\left(\frac{22}{7}\left(22^2 - 15^2\right)\right)$ = ₹12210

**5.10.** The area of parallelogram ABCD, whose height on side AB is 9 cm, is 108 sq.cm. Find length of side AB.

**Sol:** Area = (AB) (height) = 108 = (AB) (9)
∴ AB = 12 cm

**5.11.**




In the figure above, ACDE is a trapezium. If the area of parallelogram ABCD is 100 sq.cm, find the area of parallelogram CDEF.

**Sol:** Area of ABCD = Area of CDEF (the parallelograms are between the same parallel lines and hence their areas will be equal).
∴ Area of CDEF = 100 sq.cm

**5.12.** The areas of a rectangle and a square are in the ratio 3 : 4. The length of the rectangle is 8 cm more than that of the square. The breadth of the rectangle is 8 cm less than that of the square. Find the perimeter of the square.

**Sol:** Let the side of the square the a cm. Length of the rectangle and its breadth are (a + 8) cm and (a – 8) cm respectively. Given that

$(a + 8)(a - 8) = \frac{3}{4}a^2$

$\Rightarrow a^2 - 64 = \frac{3}{4}a^2$

$\Rightarrow a = 16$

Hence perimeter of the square = 4a = 64 cm

**5.13.** If the square on the diagonal of a rectangle is four the area of the rectangle, find the ratio of its length and breadth.

**Sol:** Let the length and breadth of the rectangle be $\ell$ and b respectively ($\ell$ > b). Its diagonal $= \sqrt{\ell^2 + b^2}$

Given that $\left(\sqrt{\ell^2 + b^2}\right)^2 = 4(\ell b)$

$\Rightarrow \ell^2 + b^2 - 4\ell b = 0$

By dividing throughout with $\ell$b and taking $\frac{\ell}{b}$ As K, we get $K^2 - 4K + 1 = 0$

$\Rightarrow K = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 + \sqrt{5}$

As K > 1, K = $2 \pm \sqrt{5}$

**5.14.** Find the total surface area and the volume of a cuboid whose length, breadth and height are 20 cm, 15 cm and 12 cm respectively.

**Sol:** Total surface area = $2(\ell b + bh + \ell h)$
= 2 ((20) (15) + (15) (12) + (20) (12))
= 1440 sq.cm
Volume = $\ell bh$ = (20) (15) (12) = 3600 cubic cm

**5.15.** A wall of measurements 30 m × 12 m × 4 m was constructed with bricks of dimensions
8 cm × 6 cm × 6 cm. If 80% of the wall consists of bricks, find the number of bricks used for the construction.

**Sol:** Volume of the wall
= (30) (100) (12) (100) (D) (100) cubic cm.
Total volume of the bricks = 0.8 [volume of the wall]
Volume of each brick = (8) (6) (6) cubic cm
Number of bricks required

$= \frac{(0.8)(30)(100)(12)(100)(4)(100)}{(8)(6)(6)} = 4000000$

**5.16.** A certain type of wood costs ₹500 per $m^3$. A solid cubical block of this wood costs, ₹108. Find its volume as well as its length.

**Sol:** Volume $= \frac{108}{500} = 0 \cdot 216\ m^3$

As the block is in the shape of a cube, its length $= \sqrt[3]{\text{volume}} = 0.6$ m

**5.17.** A cylinder has a diameter of 36 cm. It has water upto a height of 22 cm. A metal cuboid which has a length of 22 cm and each of whose lateral face is a square of side 18 cm is immersed in it. Find the rise in the height of the water level $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Let the rise in the water level be h cm.

$\frac{22}{7}\left(\frac{36}{2}\right)^2 h = (22)(18^2) \Rightarrow h = 7$

**5.18.** A steel pipe has an external diameter of 1·6 cm and a thickness of 1 mm. Each cubic cm of it weights 8 gms. Find its weight if its length is 70 cm $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** External radius $= \frac{1 \cdot 6}{2} = 0 \cdot 8$ cm

Internal radius = 0·8 – 0·1 = 0·7 cm

Volume $= \frac{22}{7}(70)\,(0 \cdot 8^2 - 0 \cdot 7^2) = 33$

Weight = (33) (8) = 264 gm

**5.19.** The radius as well as the height of a right circular cone increases by 20%. Find the percentage increase in its volume.

**Sol:** Let the radius as well as the height of the cone be 100.

Initial volume $= \frac{1}{3}\pi(100^2)(100) = \frac{1}{3}\pi(100^3)$

New radius = New height = 120

Final volume $= \frac{1}{3}\pi(1 \cdot 2(100))^3$

$= 1.728 \left(\frac{1}{3}\pi(100^3)\right)$

∴ Volume increased by 72·8%

**5.20.** A swimming pool is 200 ft long and 60 ft wide. It is 2 ft deep at the shallow end of the length and is 6 ft deep at the deep end of the length. Find the volume of the water contained in it.

**Sol:** Area of cross section

$= \frac{1}{2}(200)\,(2+6) = 800$ sq.m

Volume = (Area of cross section) (width)
= 48000 cubic m

**5.21.** A metallic solid cylinder has a diameter of 24 cm and a height of 96 cm. It is melted and made into 48 solid spheres of equal size. Find the diameter of each sphere.

**Sol:** Volume of the cylinder $= \frac{22}{7}\left(\frac{24}{2}\right)^2(14)$

As the cylinder is melted and recasted into a 48 indentical solid spheres, the volume of the cylinder is equal to the total volume of all the 48 spheres.

Let the diameter of each sphere be d cm

$48\left[\frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{d}{2}\right)^3\right] = \frac{22}{7}(12)^2\ (96)$

d = 12

**5.22.** A cylinder has its height equal to twice its diameter. If its radius is r cm, find its volume in terms of r.

**Sol:** Height = 2 (diameter)
= 2 (2r) = 4r cm
Volume = πr2 (4r)
= 4πr3 cubic cm

**5.23.** A solid is in the form of a cylinder surmounded by a cone. The diameter of the cone is 14 cm The heights of the cylinder and the cone are 12 cm and 6cm respectively. Find its volume. $\left(\text{take } \pi = \frac{22}{7}\right)$.

**Sol:** Volume = Volume of cylinder + Volume of cone

$= \pi\left(\frac{14}{2}\right)^2(12) + \frac{1}{3}\pi\left(\frac{14}{2}\right)^2(6)$

= 49π (14)

$= (49)\left(\frac{22}{7}\right)(14)$

= 2156 cubic cm.

**5.24.** The area of the base of a right circular cone is 1386 sq.cm. Its height is 20 cm. Find its volume and curved surface area $\left(\text{take } \pi = \frac{22}{7}\right)$.

**Sol:** Let the radius be r cm.

$\frac{22}{7}r^2 = 1386 \Rightarrow r = 21$

Volume $= \frac{1}{3} \times \frac{22}{7}\ (21^2)\ (20)$

= 9240 cubic cm
Let the slant height be l cm.

$l = \sqrt{21^2 + 20^2} = 29$ cm

Curved surface area

$= \frac{22}{7}(21)\,(29) = 1914$ sq.cm.

**5.25.** A conical cup is filled with ice-cream. The ice cream forms a hemispherical shape on its open top. The height of the hemispherical part is 7 cm. The radius of the hemispherical part equals the height of the cone. Find the volume of the ice cream. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** As the radius of the hemispherical part equals the height of the cone, radius of the part = height of the part = height of the cone = 7 cm

Volume of the ice cream

$= \frac{1}{3}\pi\,(7^2)(7) + \frac{2}{3}\pi(7^3)$

$= \frac{22}{7}(7^3) = 1078$ cubic cm

**5.26.** Find the volume of the largest right circular cylinder which can be cut from a cube of side 7 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** The largest right circular cylinder will have its height as well as its diameter equal to the side of the cube.

$\therefore$ Its volume

$= \frac{22}{7}\left(\left(\frac{7}{2}\right)^2\right)(7) = 269\cdot 5$ cubic cm

**5.27.** A ten rupee note measures 15 cm × 8 cm and a bundle of 90 such notes is 1 cm thick. Find the value of the ten rupee notes that can be contained in a box of size 48 cm × 36 cm × 30 cm, if the bundles are tightly packed in it without any empty space.

**Sol:** Volume of the box = (48 × 36 × 30) cubic cm

Volume of each bundle = (15 × 8 × 1) cubic cm

Number of bundles in the box

$= \frac{(48)\,(36)\,(30)}{(15)(8)(1)} = 432$

Value of each bundle = ₹900

$\therefore$ Total value = (900) (432) = ₹388800

**5.28.** The area of the floor of a conical tent having a circular base is 616 sq.m. Find the canvas required for the tent if its height is 48 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Let the radius of the tent be r m.

$\frac{22}{7}r^2 = 616$

r = 14

Let the slant height be l m.

$l = \sqrt{14^2 + 48^2} = 50$

Canvas required = $\frac{22}{7}$ (14) (50) = 2200 sq.m.

**5.29.** A roller has a length of 3 m. Its diameter is 0·7 m. It requires 600 revolutions of the roller to level a road. The cost of usage of the roller is ₹5 per sq.m. Find the total cost of levelling the road. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Curved surface area of the roller

$= 2\left(\frac{22}{7}\right)\left(\frac{0\cdot 7}{2}\right)(3) = 6\cdot 6$ sq.m

Cost of levelling the road

= (600) (6·6) (5) = ₹19800

**5.30.** A sphere and a hemisphere have the same radius. Find the ratio of their

(i) volumes.

(ii) curved surface areas.

(iii) total surface areas.

**Sol:** (i) Ratio of volumes = $\frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 2 : 1$

(ii) Curved surface area of a sphere = Its total surface area = $4\pi r^2$

$\therefore$ Ratio of curved surface areas

$= 4\pi r^2 : 2\pi r^2 = 2 : 1$

(iii) Ratio of total surface areas

$= 4\pi r^2 : (2\pi r^2 + \pi r^2) = 4 : 3$

## Concept Review Questions

***Directions for questions 1 to 50:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

**1.** The sides of a triangle are 4 cm and 6 cm. The angle included between them is 30º. Find the area of the triangle. (in sq cm).
(A) 3 (B) 6 (C) 12 (D) 7.5

**2.** Find the area of a triangle whose inradius is r and semi-perimeter is s.
(A) rs (B) (3/2)rs (C) 2rs (D) (5/2)rs

**3.** Find the area of a triangle whose sides are a, b and c and circumradius is R.
(A) abc/R (B) abc/2R (C) abc/3R (D) abc/4R

**4.** Find the area (in sq cm) of an equilateral triangle whose side is 6 cm.
(A) $3\sqrt{3}$ (B) $4.5\sqrt{3}$ (C) $6\sqrt{3}$ (D) $9\sqrt{3}$

**5.** The perimeter of an isosceles triangle is 36 cm. If one of the equal sides is 13 cm, find the area of the triangle. (in cm$^2$)

**6.** Find the area of a triangle whose sides measure 14 cm, 48 cm and 50 cm.
(A) 672 cm$^2$ (B) 336 cm$^2$ (C) 350 cm$^2$ (D) 700 cm$^2$

**7.** $T_1$ is a triangle. $T_2$ is another triangle formed by joining the midpoints of the sides of $T_1$. Find the ratio of the areas of $T_2$ and $T_1$.
(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 1 : 6

**8.** A rectangular sheet has an area of 1680 sq m and a perimeter of 164 m. Find its length (in m).

**9.** What is the length of the diagonal of a square whose area is equal to twice the area of a rectangle of length 81 m and breadth 25 m?
(A) 90 m (B) $90/\sqrt{2}$ m
(C) $90\sqrt{2}$ m (D) $81\sqrt{2}$ m

**10.** Find the ratio of the area of an equilateral triangle of side 2a units to that of a square whose diagonal is 2a units.
(A) $\sqrt{3}$ : 1 (B) $\sqrt{3}$ : 8 (C) $\sqrt{3}$ : 2 (D) $\sqrt{3}$ : 4

**11.** Find the area (in sq cm) of a rhombus whose diagonals are 80 cm and 18 cm.

**12.** A trapezium has its shorter and the longer sides as 4 cm and 20 cm respectively. Its parallel sides are 5 cm apart. Find its area (in sq cm).
(A) 40 (B) 50 (C) 60 (D) 70

**13.** PQRS is a quadrilateral. PR = 12 cm. The perpendicular distances from Q and S to PR are 6 cm and 4 cm respectively. Find the area of PQRS (in sq cm).
(A) 60 (B) 30
(C) 45 (D) None of these

**14.** If the perimeter of a rhombus is 52 cm and one of its diagonals is 10 cm, then find its area. (in sq. cm)
(A) 154 (B) 168 (C) 120 (D) 175

**15.** A rectangular sheet has an area of 420 sq m and a perimeter of 82 m. Find the length of its diagonal (in m).

**16.** A wire has a length of 264 cm. It is bent to form a rectangle whose adjacent sides are in the ratio 8 : 3. Find the area of the rectangle. (in sq. cm)
(A) 4032 (B) 4230 (C) 3428 (D) 3456

**17.** Find the area of a square whose diagonal is $6\sqrt{6}$ cm. (in cm$^2$)

**18.** Find the height of a trapezium whose parallel sides are 14 cm and 8 cm and area is 154 cm$^2$.
(A) 7 cm (B) 14 cm (C) 10.5 cm (D) 21 cm

**19.** Find the length of the diagonal and the area of a rectangle respectively whose length is 12 cm and breadth is 5 cm.
(A) 13 cm; 60 cm$^2$ (B) 13 cm; 84 cm$^2$
(C) 9.5 cm; 76 cm$^2$ (D) 13.2 cm; 79 cm$^2$

**20.** If a path 3 m wide is laid all round and outside a field of dimensions 25 m $\times$ 15 m, then find the area of the path.
(A) 276 sq.m (B) 256 sq.m
(C) 240 sq.m (D) 266 sq.m

**21.** A trapezium has a height of 12 cm. Its longer and shorter parallel sides are 21 cm and 3 cm respectively. Find its area (in sq cm).

**22.** The circumference of a circle is $21\pi$ cm. Find its area.
(A) $20\pi$ cm$^2$ (B) 276.5 cm$^2$
(C) 214 cm$^2$ (D) 346.5 cm$^2$

**23.** Find the perimeter of a square inscribed in a circle of radius 7 cm.
(A) $14\sqrt{2}$ cm (B) $14/\sqrt{2}$ cm
(C) $28\sqrt{2}$ cm (D) $28/\sqrt{2}$ cm

**24.** The wheel of a cycle covers 1100 m by making 175 revolutions. Find the diameter of the wheel. (in m)

**25.** Find the ratio of the area of the circle inscribed in an equilateral triangle of side a units to that of the circle circumscribing it.
(A) 1 : 2 (B) 1 : 4 (C) 2 : 3 (D) 2 : 5

**26.** The radius of a circular garden is 21 m. If a pathway of width 7 m runs all around and inside the garden, then find its area. (in m$^2$)

**27.** A semicircle has a radius of 14 cm. Find its perimeter (in cm) (Take $\pi$ = 22/7).

$\boxed{\phantom{000}}$

**28.** A sector has a radius of 7 cm and a central angle of 72°. Find its area (in sq cm) (Take $\pi$ = 22/7).
(A) 15.4 (B) 23.1 (C) 30.8 (D) 46.2

**29.** A cyclic quadrilateral has its sides as 6 cm, 7 cm, 8 cm and 9 cm. Find its area (in sq cm).
(A) $18\sqrt{21}$ (B) $24\sqrt{21}$ (C) $36\sqrt{21}$ (D) $12\sqrt{21}$

**30.** A rectangular prism has the length and breadth of its base as 6 cm and 4 cm respectively. If its height is 6 cm, find its lateral surface area (in sq cm).

$\boxed{\phantom{000}}$

**31.** A prism has a square base whose side is 6 cm. Its height is 10 cm. Find its total surface area (in sq cm).
(A) 240 (B) 312 (C) 200 (D) 440

**32.** A prism has a base which is an equilateral triangle of side 6 cm. Its height is 20 cm. Find its volume. (in cubic cm).
(A) $180\sqrt{3}$ (B) $90\sqrt{3}$ (C) $60\sqrt{3}$ (D) $120\sqrt{3}$

**33.** A cuboid has its length, breadth and height as 5 cm, 3 cm and 2 cm respectively. Find its volume (in cubic cm).

$\boxed{\phantom{000}}$

**34.** A cuboid has its length, breadth and height as l, b and h respectively. Find the length of its body diagonal (in cm).
(A) $(lbh)^{1/3}$ (B) $\sqrt{l^2+b^2+4h^2}$
(C) $\sqrt{l^2+4b^2+h^2}$ (D) $\sqrt{l^2+b^2+h^2}$

**35.** A cube has an edge of 6 cm. Find its face diagonal (in cm).

$\boxed{\phantom{000}}\sqrt{2}$

**36.** A cube has an edge of 8 cm. Find its body diagonal (in cm).
(A) $8\sqrt{2}$ (B) $4\sqrt{2}$ (C) $8\sqrt{3}$ (D) $4\sqrt{3}$

**37. (i)** A cuboid has its length, breadth and height as 6 cm, 5 cm and 4 cm respectively. Find its lateral surface area (in sq cm).
(A) 88 (B) 108
(C) 100 (D) None of these

**(ii)** In the previous question, find the total surface area of the cuboid (in sq cm).
(A) 136 (B) 144 (C) 156 (D) 148

**38. (i)** A cube has an edge of 10 cm. Find its lateral surface area (in sq cm).
(A) 400 (B) 360 (C) 440 (D) 420

**(ii)** In the previous question, find the total surface area of the cube. (in sq cm)
(A) 480 (B) 600 (C) 720 (D) 660

**39.** A solid hemisphere has a radius of 6 cm. Find its total surface area (in sq cm).

$\boxed{\phantom{000}}\ \pi$

**40. (i)** A sphere has a radius of 12 cm. Find its volume (in cubic cm).
(A) $1152\pi$ (B) $576\pi$ (C) $2304\pi$ (D) $4608\pi$

**(ii)** In the previous question, find the surface area of the sphere (in sq cm).
(A) $576\pi$ (B) $2304\pi$ (C) $1152\pi$ (D) $4608\pi$

**41.** If the radius of a solid hemisphere is 6 cm, find the curved surface area of the hemisphere (in sq cm).

$\boxed{\phantom{000}}\ \pi$

**42.** If the radius of a solid hemisphere is 6 cm, find the volume of the hemisphere (in cublc cm).
(A) $72\pi$ (B) $144\pi$ (C) $108\pi$ (D) $180\pi$

**43.** A prism and a pyramid have the same base as well as height. Find the ratio of their volumes.
(A) 2 : 1 (B) 3 : 1 (C) 2 : 3 (D) 3 : 2

**44.** Two adjacent sides of a parallelogram have lengths of 8 cm and 10 cm. The angle between them is 30°. Find the area (in sq cm) of the parallelogram.

$\boxed{\phantom{000}}$

**45. (i)** A pyramid has a slant height of 8 cm and a square base of side 4 cm. Find its lateral surface area (in sq cm).
(A) 48 (B) 54 (C) 60 (D) 64

**(ii)** In the previous question, find the total surface area of the pyramid (in sq cm).
(A) 72 (B) 80 (C) 88 (D) 96

**46. (i)** A frustum of a cone has its top radius, base radius and its slant height as 6 cm, 8 cm and 5 cm respectively. Find its lateral surface area (in sq cm).
(A) $65\pi$ (B) $68\pi$ (C) $72\pi$ (D) $70\pi$

**(ii)** In the previous question, find the total surface area of the frustum (in sq cm).
(A) $85\pi$ (B) $115\pi$ (C) $140\pi$ (D) $170\pi$

**47.** A regular hexagon has a side of 4 cm. Find its area (in sq cm).
(A) $24\sqrt{3}$ (B) $18\sqrt{3}$ (C) $12\sqrt{3}$ (D) $15\sqrt{3}$

**48.** A right circular cone is cut parallel to its base at half its height. Find the ratio of the volume of the frustum formed and that of the original cone.
(A) 1 : 8 (B) 7 : 8 (C) 1 : 4 (D) 1 : 3

**49.** In the previous question, find the ratio of the curved surface areas of the smaller cone and the original cone.
(A) 1 : 4 (B) 1 : 3 (C) 1 : 8 (D) 1 : 7

**50.** A right square pyramid is cut parallel to its base at half of its height. Find the ratio of the volumes of the smaller pyramid and the frustum formed.
(A) 1 : 8 (B) 1 : 7 (C) 1 : 4 (D) 1 : 3

## *Exercise – 5(a)*

***Directions for questions 1 to 35:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

**1.** An isosceles triangle has a perimeter of 72 cm. Each of its equal sides is 6 cm longer than its base. Find its area (in sq cm).
(A) 240 (B) 180 (C) 210 (D) 270

**2.** In a triangle, the sum of the squares of two of the sides is not more than four times its area. If the product of these sides is 12, find its area. (in sq units)

**3.** A rhombus can be divided into two congruent triangles of perimeter 36 cm each, by drawing one diagonal. It can be divided into four congruent triangles of perimeter 24 cm each, by drawing both the diagonals. Find the side of the rhombus.
(A) 10 cm (B) 13 cm (C) 15 cm (D) 12 cm

**4.** The area of a trapezium is 98 $cm^2$ and its height is equal to the shorter of the two parallel sides. If the longer of the two parallel sides is of 21 cm length, find the height of the trapezium. (in cm)

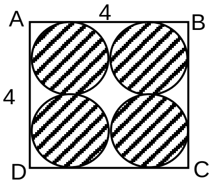
**5.** Let PQRSTU be a regular hexagon. The area of the triangle formed by joining any three alternate vertices of the hexagon is K times the area of the hexagon. Find K.
(A) 1/3 (B) 1/4 (C) 1/6 (D) 1/2

**6.** Find the area of a parallelogram whose adjacent sides are 20 cm and 10 cm, the angle between them being 45°.
(A) $300\sqrt{2}$ sq.cm
(B) $150\sqrt{2}$ sq.cm
(C) $200\sqrt{2}$ sq.cm
(D) $100\sqrt{2}$ sq.cm

**7.** There are three thin wires. One of them is bent to form a circle, another is bent to form an equilateral triangle and the third is bent to form a square. If all the resulting figures enclose the same area, which of the following statements is / are true?
I. The wire bent to form a circle will have the least length.
II. The wire bent to form a triangle will have the greatest length
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**8.** The perimeter of the sector of a circle of radius 42 cm is 108 cm. Find the area of the sector. (in sq cm)

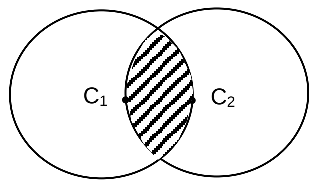
**9.** In the figure below, ABCD is a square of area 16 sq. units and all the four circles have equal radius. What is the ratio of the area of the shaded region to that of the unshaded region?




(A) $2\pi : \sqrt{3}$ (B) $\pi : (1 - \pi)$
(C) $\pi : (\sqrt{2} - \pi)$ (D) $\pi : (4 - \pi)$

**10.** Three congruent circles are drawn in such a way that exactly one circle passes through the centres of the other two circles, which touch each other externally. If the radius of each circle is r, then find the total area of the region, common to any two circles.
(A) $\frac{r^2}{2}\left(\pi - \sqrt{3}\right)$ (B) $2r^2\left(\pi - 3\sqrt{3}\right)$
(C) $2r^2\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$ (D) $\frac{3}{2}r^2\left(\pi - \sqrt{3}\right)$

**11.** $C_1$ and $C_2$ are centers of the two circles with same radius 8 cm. What is the area of the shaded region in square centimeters?




(A) $\frac{128\pi}{3} - 32\sqrt{3}$ (B) $116\pi - 16\sqrt{3}$
(C) $\frac{118\pi}{3} - 24\sqrt{3}$ (D) $45\pi - 32\sqrt{3}$

**12.** A circle is divided into five sectors. The central angle of the $i^{th}$ sector where $2 \le i \le 5$ is twice the central angle of the (i – 1)th sector. If the radius of the circle is 2 cm, find the area of the smallest sector. (in sq. cm).
(A) $\frac{4\pi}{15}$ (B) $\frac{6\pi}{63}$ (C) $\frac{4\pi}{127}$ (D) $\frac{4\pi}{31}$

**13.** Find the area of a cyclic quadrilateral of sides 2 cm, 4 cm, 6 cm and 8 cm.
(A) $16\sqrt{15}$ $cm^2$ (B) $2\sqrt{90}$ $cm^2$
(C) $2\sqrt{93}$ $cm^2$ (D) $8\sqrt{6}$ $cm^2$

**14.** An equilateral triangle has a circle inscribed in it and is circumscribed by a circle. There is another equilateral triangle inscribed in the inner circle. Find the ratio of the areas of the outer circle and the inner equilateral triangle.
(A) $\frac{16\pi}{3\sqrt{3}}$ (B) $\frac{8\pi}{2\sqrt{3}}$ (C) $\frac{24\pi}{3\sqrt{3}}$ (D) $\frac{20\pi}{3\sqrt{3}}$

**15.** Find the ratio of the areas of the regular hexagons inscribed in and circumscribed around a circle of radius 10 cm.
(A) 3 : 1 (B) 1 : 3 (C) 3 : 4 (D) 4 : 3

**16.** Inside a rectangular plot of dimensions 70 ft × 40 ft, a pit of dimensions 10 ft × 5 ft is dug to a depth of 27.5 ft. If the earth dug out is uniformly spread in the remaining part of the plot, find the rise in the level of the plot. (in ft)

**17.** A cylindrical vessel of base radius 4 cm is filled with water to a height of 6 cm. Lead shots each of radius 2 mm are dropped into it and the water level rises to 8.50 cm. Find the number of lead shots dropped.
(A) 500 (B) 3750 (C) 1000 (D) 1500

**18.** The sum of the radius of the base and the slant height of a cone is 25 cm. If the total surface area of the cone is $200\pi$ cm², then find the curved surface area of the cone. (in cm$^2$)
(A) $156\pi$ cm² (B) $126\pi$ cm²
(C) $136\pi$ cm² (D) $164\pi$ cm$^2$

**19.** Find the volume of the largest right circular cone that can be cut out of a cuboid of dimensions 56 cm × 21 cm × 14 cm.
(A) 16248 cm$^3$ (B) 1048 cm$^3$
(C) 1617 cm$^3$ (D) 2874.67 cm$^3$

**20.** The model of an elevated dome is in the form of a cylinder surmounted by a hemisphere. If the diameter of the cylinder is 14 cm and the overall height is 21 cm, find the difference between the curved surface areas of the cylinder and the hemisphere (in cm$^2$).

**21.** The radius of a cone is r cm and its height is h cm. The change in volume when the height is decreased by x cm is the same as the change in volume when the radius is decreased by x cm. Find the relation between x, r and h.
(A) $x = \dfrac{2rh - r^2}{h}$ (B) $x = \dfrac{2rh + r^2}{h}$
(C) $x = \dfrac{r^2 - 2rh}{h}$ (D) $x = 2r + r^2$

**22.** The thickness of a pipe is 2 mm and its external diameter is 0·8 cm. If 1 cm$^3$ of the material of the pipe weighs 15 gm, then find the weight of the pipe of length 28 cm. (in gm)

**23.** A reservoir in the shape of a cuboid is of dimensions 50 m × 30 m × 20 m. How long will it take to fill it with water flowing at 10 km/hr through a pipe of inner Cross-sectional area 25 cm$^2$? (in hrs)

**24.** The radii of the top and bottom cross sections of a bucket are 21 cm and 7 cm respectively. If the capacity of the bucket is $2548\pi$ cm$^3$, find the height of the bucket.
(A) 9 cm (B) 12 cm (C) 18 cm (D) 15 cm

**25.** There is a closed rectangular shed with dimensions 28 m × 8 m inside a field. A cow is tied to one corner of this shed with a rope 12 m long. What is the area that the cow can graze in the field whose dimensions are large enough to allow the cow to graze the maximum area possible? (in sq m)

**26.** There are two identical cubes ($C_1$ and $C_2$). $C_1$ is perfectly cut into N identical small cubes. A sphere is inscribed in each of these cubes. A sphere is also inscribed in $C_2$. The total volume occupied by the spheres in $C_1$ is $V_1$. The volume occupied by the sphere in $C_2$ is $V_2$. Find $V_1 : V_2$.
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 3

**27.** $R_1$ and $R_2$ are identical rectangles. S is a square having the same area as either of these rectangles. $R_1$ is rolled along its length so that the opposite breadths coincide to form a cylinder $C_1$ of volume $C_a$. $R_2$ is rolled along its breadth so that the opposite lengths coincide to form a cylinder $C_2$ of volume $C_b$. S is folded along one of its sides so that the two sides perpendicular to that side coincide to form a cylinder $C_3$ of volume $C_c$. Which of the following holds true?
(A) $C_a > C_b > C_c$ (B) $C_b > C_a > C_c$
(C) $C_a > C_c > C_b$ (D) $C_b > C_c > C_a$

**28.** The length, breadth and height of a room are in the ratio 7 : 4 : 5. If the volume of the room is 30240 m$^3$, find the difference in the costs of covering the walls with paper at ₹5 per sq.m and with paper at ₹5.50 per. sq.m. (in ₹)

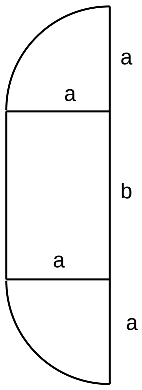
**29.** The radius and the height of a cylinder are equal to the radius of a sphere. If the ratio of the numerical values of the curved surface area of the cylinder and the volume of the sphere is 1 : 3, find the volume of the sphere.
(A) $\dfrac{243\pi}{2}$ (B) $\dfrac{216\pi}{7}$ (C) $\dfrac{144\pi}{7}$ (D) $108\pi$

**30.** There are two tanks. One of them is in the shape of a cuboid and the other is in the shape of a hemisphere. The base of the cuboidal tank is a square. Both tanks have equal base perimeter and equal heights. The volume of the hemispherical tank is less than that of the cuboidal tank by approximately ______.
(A) 11% (B) 15% (C) 17% (D) 24%

**31.** PQRS is a square. T and U are points on PQ and QR respectively such that TQ = UR = $\dfrac{PQ}{4}$. Find the ratio of the areas of the triangle TUS and PQRS.
(A) 17 : 64 (B) 13 : 32 (C) 13 : 64 (D) 17 : 32

**32.** The sides of a quadrilateral are 15, 45, 15, 63. If the sides 45 and 63 are parallel, then area of the quadrilateral is

**33.** The perimeter of the figure is 160.
If the area of the figure is A, find the value of $\frac{A + a^2}{a}$.




**34.** The base of a regular pyramid is a square of area A. The height of the pyramid is one-third times the semi-perimeter of the square. The area of any of the triangular regions of the pyramid is S. If A = Ks, what is the value of k?.

**35.** A closed cuboidal box is inscribed in a sphere whose diameter is $50\sqrt{2}$. The box has a total surface area of 9400. The sum of the lengths of all the edges of the box is .

## *Exercise – 5(b)*

***Directions for questions 1 to 45:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

**1.** The perimeter of a right-angled triangle is 90 cm and its area is 270 cm$^2$. Find the length of the hypotenuse.
(A) 39 cm (B) 37 cm (C) 38 cm (D) 40 cm

**2.** Find the area of a triangle whose sides are 34 cm, 50 cm and 52 cm (in cm)

**3.** If the altitude of an equilateral triangle ABC is $6\sqrt{3}$ cm, find its area.
(A) $36\sqrt{3}$ cm$^2$ (B) $48\sqrt{3}$ cm$^2$
(C) $60\sqrt{3}$ cm$^2$ (D) $72\sqrt{3}$ cm$^2$

**4.** What is the inradius of the triangle whose sides are 7 cm, 4 cm and 9 cm?
(A) $\frac{2\sqrt{5}}{7}$ cm (B) $\frac{6\sqrt{5}}{7}$ cm
(C) $\frac{3\sqrt{5}}{5}$ cm (D) $\frac{6\sqrt{5}}{5}$ cm

**5.** In triangle ABC, AB = 10 cm and AC = 15 cm. If $\angle A = 60°$ and AD is the bisector of $\angle A$, then find its length, (in cm).
(A) $12\sqrt{3}$ (B) $7\sqrt{3}$ (C) $8\sqrt{3}$ (D) $6\sqrt{3}$

**6.** A quadrilateral was formed by joining the midpoints of the successive sides of a square of side 4 m. A circle was inscribed in the quadrilateral. An equilateral triangle was inscribed in the circle. Find the perimeter of the triangle (in m).
(A) $2\sqrt{3}$ (B) $3\sqrt{3}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{3\sqrt{3}}{4}$

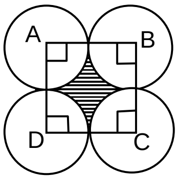
**7.** Find the length of the line joining the midpoints of the oblique sides of an isosceles trapezium if the height is 10 cm and area 150 sq.cm.
(A) 20 cm (B) 30 cm (C) 45 cm (D) 15 cm

**8.** If the area of quadrilateral ABCD is 309 cm$^2$, find the area of ACD if AB, BC, AC and CD measure 24 cm, 7 cm, 25 cm and 36 cm respectively. (in cm$^2$)

**9.** The length of the diagonal BD of a trapezium ABCD is 30 cm. If the altitudes from A and C to the diagonal BD differ by 5 cm and the area of the trapezium is 210 sq.cm, find the sum of the lengths of the perpendiculars from A and C to the diagonal BD.
(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

**10.** The cost of paving a floor with square tiles at ₹7 per sq.m is ₹2240. If the length of the floor is twice the breadth, find the perimeter of the floor.
(A) $64\sqrt{10}$ m (B) $84\sqrt{10}$ m
(C) $8\sqrt{10}$ m (D) $24\sqrt{10}$ m

**11.** Two circles touch each other internally. The distance between the centres of the circles is 7 cm and the difference of their areas is 1078 sq.cm. Find the sum of the radii of the two circles. (in cm)

**12.** 


The diagram shown above has four circles of 7 cm radius each with centres at A, B, C and D. If the quadrilateral ABCD represents a square, find the area of the shaded region.

(A) 42 sq.cm (B) 21 sq.cm
(C) 63 sq.cm (D) 84 sq.cm

**13.** Find the area of a cyclic quadrilateral whose sides are 3 cm, 6 cm, 9 cm and 12 cm (in sq cm).

(A) $12\sqrt{6}$ (B) $18\sqrt{6}$ (C) $24\sqrt{6}$ (D) $36\sqrt{6}$

**14.** The hour hand of a clock is 6 cm long. Find the area swept by it between 11:20 a.m. and 11:55 a.m. (in $cm^2$)

**15.** The perimeter of a square is equal to the perimeter of an equilateral triangle. Find the ratio of the side of the equilateral triangle to the diagonal of the square.

(A) $40\sqrt{2} : 4$ (B) $2\sqrt{2} : 9$
(C) $3 : 4\sqrt{2}$ (D) None of these

**16.** The areas of a circle, a square and an equilateral triangle are equal. If the perimeters of the circle, the square and the triangle are C, S and T respectively, which of the following holds true?

(A) C < T < S (B) S < T < C
(C) C < S < T (D) T < C < S

**17.** Find the area of a regular hexagon whose side equals the side of a square whose perimeter is 24 cm (in sq cm).

(A) $54\sqrt{3}$ (B) $36\sqrt{3}$ (C) $72\sqrt{3}$ (D) $96\sqrt{3}$

**18.** The curved surface area of a cylinder B is 300% more than the curved surface area of a cylinder A. Both cylinders have the same radius. If the height of A is x% less than that of B, find x.

**19.** The radius and the height of a cylinder are equal to the radius of a sphere. If the ratio of the numerical values of the curved surface area of the cylinder and the volume of the sphere is 1 : 3, find the volume of the sphere.

(A) $\frac{243\pi}{2}$ (B) $\frac{216\pi}{7}$ (C) $\frac{144\pi}{7}$ (D) $108\,\pi$

**20.** A rectangle has a length of 60 cm and a breadth of 40 cm. If four squares each of side 4 cm are cut from the four corners of the rectangle and the resulting figure is made into a cuboid, find the volume of the cuboid. (in $cm^3$)

**21.** In a cuboid, the sum of the squares of the three dimensions equals half its total surface area. Its volume is 729 $cm^3$. Find its lateral surface area. (in sq cm).

(A) 576 (B) 900 (C) 324 (D) 648

**22.** If a spherical balloon is inflated in such a way that its radius becomes thrice, by how many times will its surface area increase, when compared to its original surface area?

**23.** The inside of a well thirty feet deep was given a finishing of brick-work. If the inner diameter of the well after the brick-work, which was six inches thick, was completed was ten feet, what was the volume of the brick-work?

(A) $167.5\pi$ cubic feet (B) $157.5\pi$ cubic feet
(C) $187.5\pi$ cubic feet (D) $207.5\pi$ cubic feet

**24.** The radius of a roller is 49 cm and its length is 160 cm. If it takes 600 complete revolutions to move once over a level field, find the area of the field.

(A) 2956.8 sq.m (B) 3157.6 sq.m
(C) 4284.2 sq.m (D) 5186.4 sq.cm

**25.** A conical vessel has its radius equal to one third its height. Due to mechanical defect in the callipers used for measuring, if 1 cm is taken to be 1.01 cm, find the percentage error in the calculated total surface area.

%

**26.** A horse is tied at an outer corner of a rectangular shed of outer dimensions 14 m × 7 m using a 21 m long rope. Find the area outside the shed, (in sq m), over which it can graze. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(A) 1204 (B) 1230 (C) 1260 (D) 1232

**27.** The longest rod which can be placed in a cylindrical room is 29 m long. If the curved surface area of the room is 2640 sq m, find its height (in m.)
(take $\pi = 22/7$)

(A) 18 (B) 21
(C) 24 (D) Cannot be determined

**28.** The total surface area of a cuboid is 432 sq $cm^2$. The areas of two of its adjacent faces are 96 sq. $cm^2$ and 48 $cm^2$. Find its volume (in cubic cm).

(A) 324 (B) 576 (C) 432 (D) 384

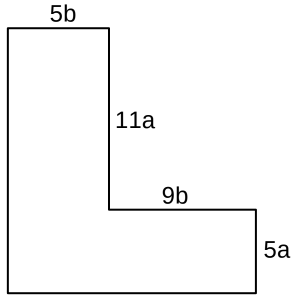
**29.** The radius as well as the height of a cylinder are 5 cm each. If the radius increases by x cm, the volume of the cylinder increases by y cubic cm. If the height increases by 3x cm, the volume increases by y cubic cm. Find x, if x > 0.

**30.** The volume of a cuboid is 140 cu.cm. The areas of two of its faces are 28 $cm^2$ and 20 $cm^2$. Find the sum of the edges of the cuboid. (in $cm^3$)

(A) 140 (B) 160
(C) 180 (D) None of these

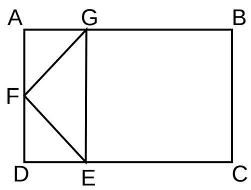
**31.** The area of a rectangle is 247 cm². If the length decreases by 3 cm and the breadth increases by 3 cm, the rectangle becomes a square. Find the perimeter of the original rectangle. (in cm)

**32.** The perimeter of the given figure is 136 and seven times the area of the figure is pa – qa$^2$. Find the value of p + q.




**33.** Raja mowed the grass on a rectangular lawn of dimensions 40 m by 30 m. He mowed using a 1m wide strip. He started mowing from one of the corners of the lawn and moved around it towards its centre. How many times would he go round before he completed mowing half of the lawn?

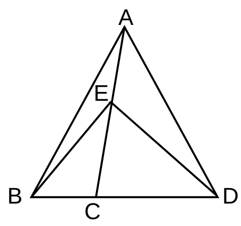
**34.**




ABCD is a rectangle. If F is the midpoint of AD and G and E are points on AB and CD such that DE = $\frac{1}{3}$ DC and AG = $\frac{1}{3}$ AB, then find the ratio of the area of triangle EFG to the area of rectangle ABCD.

(A) 4 : 15 (B) 1 : 6 (C) 2 : 7 (D) 3 : 7

**35.**




BC : CD = 2 : 3 and AE : EC = 3 : 4. Find the ratio of the area of ΔECD to the area of ΔAEB.

(A) 2 : 1 (B) 2 : 3 (C) 3 : 5 (D) 4 : 3

**36.** The area of two adjacent lateral faces of a cuboid are 60 cm$^2$ and 40 cm$^2$. If the volume of the cuboid is 480 cm$^3$, then find the length of the longest diagonal of the cuboid.

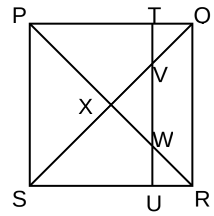
(A) $\sqrt{213}$ cm (B) $\sqrt{233}$ cm

(C) $\sqrt{253}$ cm (D) $\sqrt{264}$ cm

**37.** The perimeter of an equilateral triangle equals that of a rectangle. One of the dimensions of the rectangle equals the side of the triangle. Find the ratio of the areas of the triangle and the rectangle.

(A) $2 : \sqrt{3}$ (B) $\sqrt{3} : 2$ (C) $4 : \sqrt{3}$ (D) $\sqrt{3} : 4$

**38.** In the figure below, PQRS is a square of side a. PTUS is a rectangle. If QV = b, find the ratio of the areas of the triangles XVW and XQR.



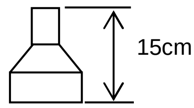

(A) $(a-\sqrt{2}b)^2 : a^2$

(B) $a^2 : (a-\sqrt{2}b)^2$

(C) $4(a-\sqrt{2}b)^2 : a^2$

(D) $a^2 : 4(a-\sqrt{2}b)^2$

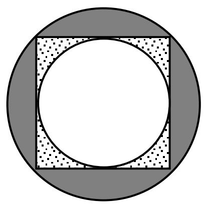
**39.**




A bottle is shaped as shown in the figure above. Its top part is a cylinder, its middle part is a frustum of a cone and its bottom a cylinder. The top and the bottom radii of the frustum are 4 cm and 6 cm respectively. The heights of the upper and the lower cylinders are 8 cm and 4 cm respectively. Find the volume of the bottle. (in cm$^3$)

π

**40.**



In the figure, a square circumscribes a circle and is inscribed in another circle. Find the ratio of the area of the shaded region to that of the dotted region.

(A) 2(π – 1) : (π – 2) (B) 2(π – 2) : (4 – π)

(C) 2(π – 1) : (4 – π) (D) 2(π – 2) : (π – 2)

**41.** The base of a regular pyramid is a square of perimeter P. The height of the pyramid is thrice the diagonal of the square. Find the area of each of the triangular regions of the pyramid.

(A) $\frac{\sqrt{19P^2}}{32}$ (B) $\frac{\sqrt{19P^2}}{16}$

(C) $\frac{\sqrt{73P^2}}{32}$ (D) $\frac{\sqrt{73P^2}}{64}$

**42.** A rectangle R is inscribed in a circle of radius 6 cm. Which of the following statements is/are true?

I. The maximum area of R is 72 sq cm.

II. The least perimeter of R is $24\sqrt{2}$ units.

(A) Only I (B) Only II

(C) Both I and II (D) Neither I nor II

**43.** A cylinder has a total surface area of 440 sq.cm. The sum of its radius and its height is 10 cm. Find its volume. (Take $\pi = 22/7$) (in $cm^3$)

**44.** Two identical circles are centered at E and F and they intersect at G and H. I is a point on the first circle and outside the second circle such that $\angle GIH = 30°$. The ratio of the area of EGFH to the are a of the region common to both the circles is _____.

(A) $3\sqrt{3} : (\pi - 3\sqrt{3})$ (B) $2\sqrt{3} : (\pi - 2\sqrt{3})$

(C) $3\sqrt{3} : (2\pi - 3\sqrt{3})$ (D) $2\sqrt{3} : (2\pi - 2\sqrt{3})$

**45.** Two pipes made of different materials have the same weight. The ratio of the outer diameters of the pipes is 5 : 4. The ratio of the densities of the respective materials is 1 : 2. The ratio of the thickness of the pipes is 5 : 4. Find the ratio of the lengths of the pipes.

(A) 32 : 25 (B) 25 : 32 (C) 8 : 25 (D) 25 : 8

***Directions for questions 46 to 50:*** Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.

Mark (B) if the question can be answered using either statement alone.

Mark (C) if the question can be answered using I and II together but not using I or II alone.

Mark (D) if the question cannot be answered even using I and II together.

**46.** What is the perimeter of a rhombus?
I. The area of the rhombus is 24 $cm^2$.
II. One of the diagonals of the rhombus is 6 cm.

**47.** Find the length of the diagonal of a cube.
I. Total surface area of the cube is 96 sq.cm.
II. Volume of the cube is 64 cc.

**48.** What is the volume of the sphere?
I. The surface area is 120 sq. cm.
II. The sphere can be reformed into a cuboid of a total surface area of 120 sq. cm. by melting.

**49.** What is the volume of a right circular cone?
I. The height is equal to half the radius of the base.
II. The radius is equal to one side of a square of area 4 $cm^2$.

**50.** What is the ratio of the volumes of right circular cylinders A and B?
I. The ratio of the heights of the right circular cylinders A and B is 1 : 2.
II. The ratio of the radii of the bases of A and B is 1 : 4.

## *Key*

### *Concept Review Questions*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | B | 9. | A | 17. | 108 | 25. | B | 33. | 30 | 39. | 108 | 45. | (i) D |
| 2. | A | 10. | C | 18. | B | 26. | 770 | 34. | D | 40. | (i) C | | (ii) B |
| 3. | D | 11. | 720 | 19. | A | 27. | 72 | 35. | 6 | | (ii) A | 46. | (i) D |
| 4. | D | 12. | C | 20. | A | 28. | C | 36. | C | | | | (ii) D |
| 5. | 60 | 13. | A | 21. | 144 | 29. | D | 37. | (i) A | 41. | 72 | 47. | A |
| 6. | B | 14. | C | 22. | D | 30. | 120 | | (ii) D | 42. | B | 48. | B |
| 7. | C | 15. | 29 | 23. | C | 31. | B | 38. | (i) A | 43. | B | 49. | A |
| 8. | 42 | 16. | D | 24. | 2 | 32. | A | | (ii) B | 44. | 40 | 50. | B |

### *Exercise – 5(a)*

| | | | | | |
|---|---|---|---|---|---|
| 1. A | 7. C | 13. D | 19. D | 25. 352 | 31. B |
| 2. 6 | 8. 504 | 14. A | 20. 308 | 26. A | 32. 648 |
| 3. A | 9. D | 15. C | 21. A | 27. C | 33. 80 |
| 4. 7 | 10. C | 16. 0.5 | 22. 158.4 | 28. 1980 | 34. 2.4 |
| 5. D | 11. A | 17. B | 23. 1200 | 29. A | 35. 480 |
| 6. D | 12. D | 18. C | 24. B | 30. B | |

### *Exercise – 5(b)*

| | | | | | |
|---|---|---|---|---|---|
| 1. A | 10. D | 19. A | 28. B | 37. B | 46. C |
| 2. 816 | 11. 49 | 20. 6656 | 29. 5 | 38. A | 47. B |
| 3. A | 12. A | 21. C | 30. D | 39. 348 | 48. A |
| 4. C | 13. B | 22. 8 | 31. 64 | 40. B | 49. C |
| 5. D | 14. 5.5 | 23. B | 32. 5250 | 41. D | 50. C |
| 6. C | 15. D | 24. A | 33. 5 | 42. A | |
| 7. D | 16. C | 25. 2.01 | 34. B | 43. 462 | |
| 8. 225 | 17. A | 26. D | 35. A | 44. C | |
| 9. C | 18. 75 | 27. D | 36. B | 45. A | |