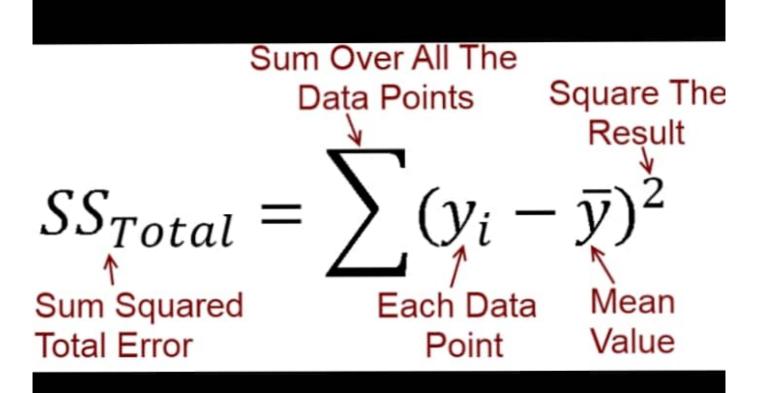Lets take the previous example of 4 types of medicine. Each medicine have their two attribute or features. Our goal is to group this objects into K=2 groups of medicine based on the two features(pH and Weight Index).

Object	Attribute 1(X):Weight Index	Attribute 2(Y):pH
Medicine A	1	1
Medicine B	2	1
Medicine C	4	3
Medicine D	5	4

$$SS_{Total} = \sum (y_i - \bar{y})^2$$

Sum Over All The Data Points

Square The Result

Sum Squared Total Error

Each Data Point

Mean Value

Elbow Method

Total Squared Error

(Cluster =1)

Attribute 1 (x). Weight Index	X − Xmean	$(x - Xmean)^2$
1	1−3 = −2	$(-2)^2 = 4$
2	2−3 = −1	$(-1)^2 = 1$
4	4−3 = 1	$(1)^2 = 1$
5	5−3 = 2	$(2)^2 = 4$

n = 4

Total Squared Error = 10

$$Xmean = \frac{1+2+4+5}{4} = 3$$

Attribute 2 (Y) PH	Y − Ymean	$(Y - Ymean)^2$
1	1 − 2.25 = −1.25	$(-1.25)^2 = 1.5625$
1	1 − 2.25 = −1.25	1.5625
3	3 − 2.25 = 0.75	$(0.75)^2 = 0.5625$
4	4 − 2.25 = 1.75	$(1.75)^2 = 3.0625$

n = 4

Total Sqr Err = 6.75

$$Ymean = \frac{1+1+3+4}{4} = 2.25$$

So, If we put the dataset into 1 Cluster then the Total Squared Error will be $(10 + 6.75) = \boxed{16.75}$

(Cluster = 2)

Cluster 1

X	X − Xmean	(X − Xmean)^2
1	1 − 1.5 = − 0.5	0.25
2	2 − 1.5 = 0.5	0.25
n=2		∑ = 0.5

$$Xmean = \frac{1+2}{2} = 1.5$$

Y	Y − Ymean	(Y − Ymean)^2
1	1 − 1 = 0	0
1	1 − 1 = 0	0
n=2		∑ = 0

$$Ymean = \frac{1+1}{2} = 1$$

Cluster 2

X	X − xmean	(X − Xmean)^2
4	4 − 4.5 = −0.5	0.25
5	5 − 4.5 = 0.5	0.25
n=2		∑ = 0.5

$$xmean = \frac{4+5}{2} = 4.5$$

Y	Y − Ymean	(Y − Ymean)^2
3	3 − 3.5 = 0.5	0.25
4	4 − 3.5 = 0.5	0.25
n=2		∑ = 0.5

$$Ymean = \frac{3+4}{2} = 3.5$$

So, If we put the data set into 2 clusters then the Total Squared Error will be —

$$0.5 + 0.5 + 0.5 = \boxed{1.5}$$

(Cluster = 3)

Cluster 1

X	X − Xmean	$(X-Xmean)^2$
1	−0.5	0.25
2	0.5	0.25

$\Sigma = 0.5$

Y	Y − Ymean	$(Y-Ymn)^2$
1	1−1 =0	0
1	1−1 =0	0

$\Sigma = 0$

Cluster 2

X	X − Xmean	$(X-Xmn)^2$
4	4−4 =0	0

$\Sigma = 0$

Y		
3	3−3 =0	0

$\Sigma = 0$

Cluster 3

X	X − Xmean	$(X-Xn)^2$
5	5−5 =0	0

$\Sigma = 0$

Y	Y − Ymean	$(Y-Yn)^2$
4	4−4 =0	0

$\Sigma = 0$

So, If we put the dataset into 3 clusters then the Total Square Error will be —

$$0.5 + 0 + 0 + 0 + 0 + 0 = \boxed{0.5}$$

Elbow Plot