Online E-Companion:

Robust Aircraft Routing

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EC.1. Detailed Computational Results in Section 4.3

Figures EC.1 - EC.9 depict the performance in flight network N_1, and Figures EC.10 - EC.18 depict the performance in flight network N_2. Specifically, Figures EC.1 - EC.3, EC.10 - EC.12 present the relative performance ratio $100 \cdot (\mathrm{DFW} - \mathrm{RAR})/\mathrm{DFW}$ when the mean of the testing data under three different distributions deviate from the training data for two flight networks. Figures EC.4 - EC.6, EC.13 - EC.15 show the relative performance ratio $100 \cdot (\mathrm{DFW} - \mathrm{RAR})/\mathrm{DFW}$ when the standard deviation of the testing data under three different distributions deviate from the training data for two flight networks. Figures EC.7 - EC.9, EC.16 - EC.18 show the relative performance ratio $100 \cdot (\mathrm{DFW} - \mathrm{RAR})/\mathrm{DFW}$ when the correlation structure of the testing data under three different distributions deviate from the training data for two flight networks. Performance is evaluated in three criteria: (1) average total propagated delay, (2) standard deviation of total propagated delay, (3) maximum total propagated delay. The long dashed line indicates 0% relative performance ratio, which denotes the region where DFW and RAR have the same performance.

Figure EC.1 Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_1)

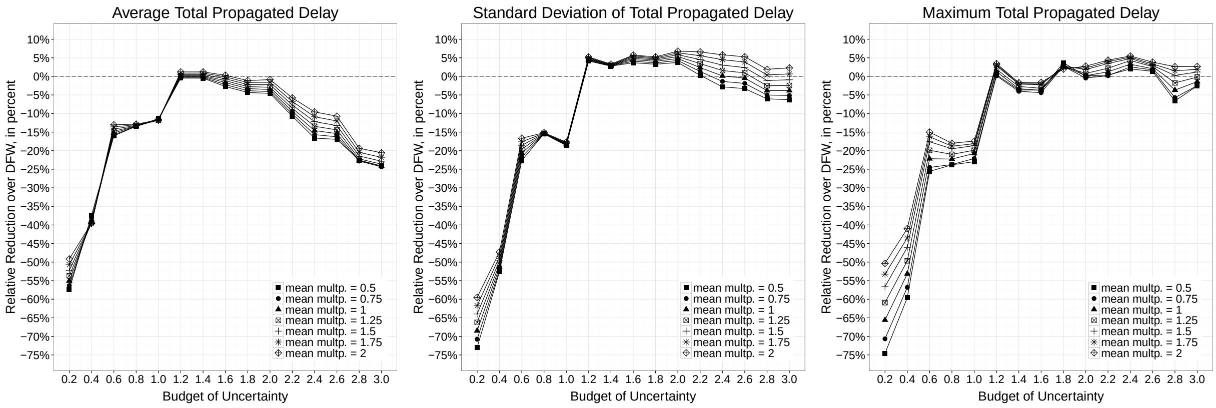


Figure EC.2 Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_1)

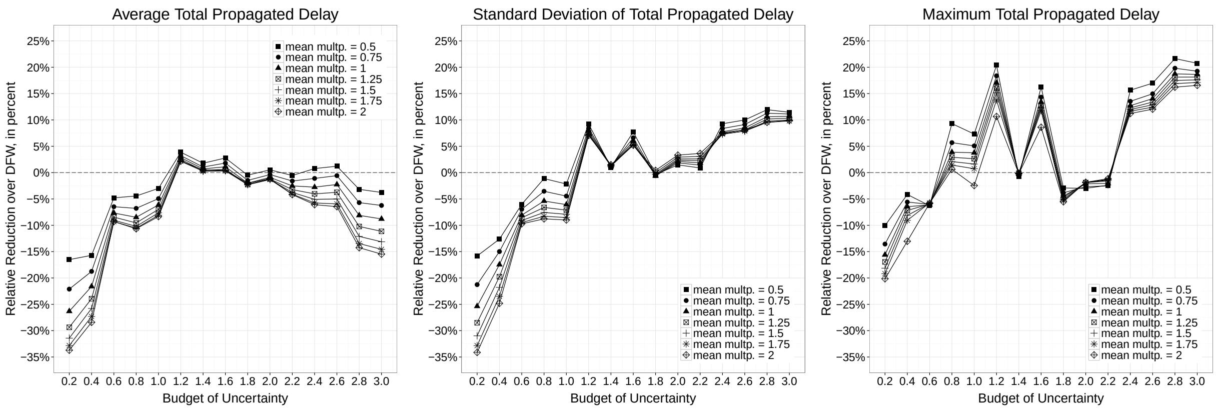


Figure EC.3 Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_1)

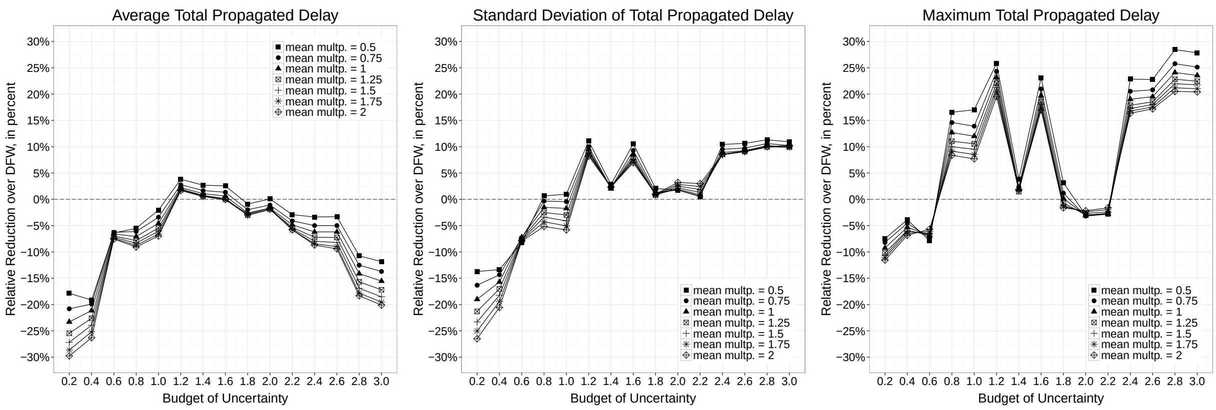


Figure EC.4 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_1)

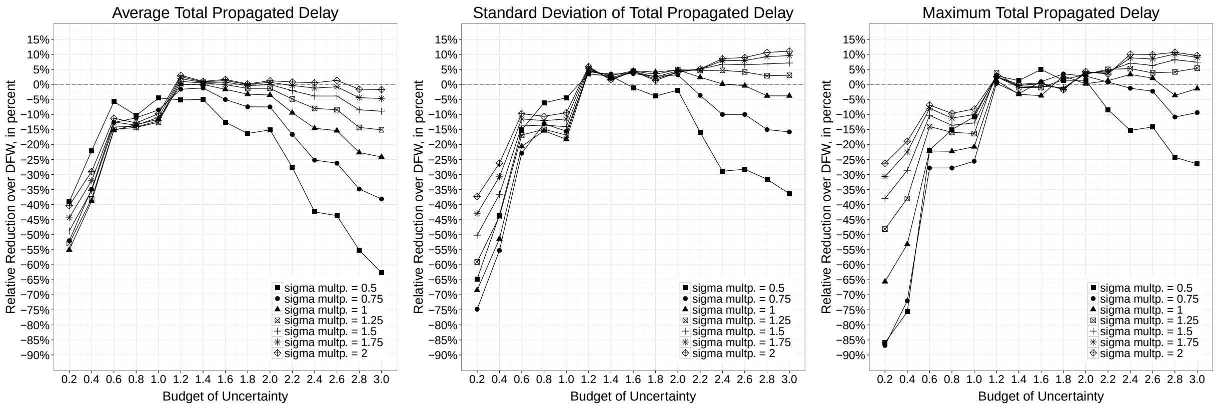


Figure EC.5 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_1)

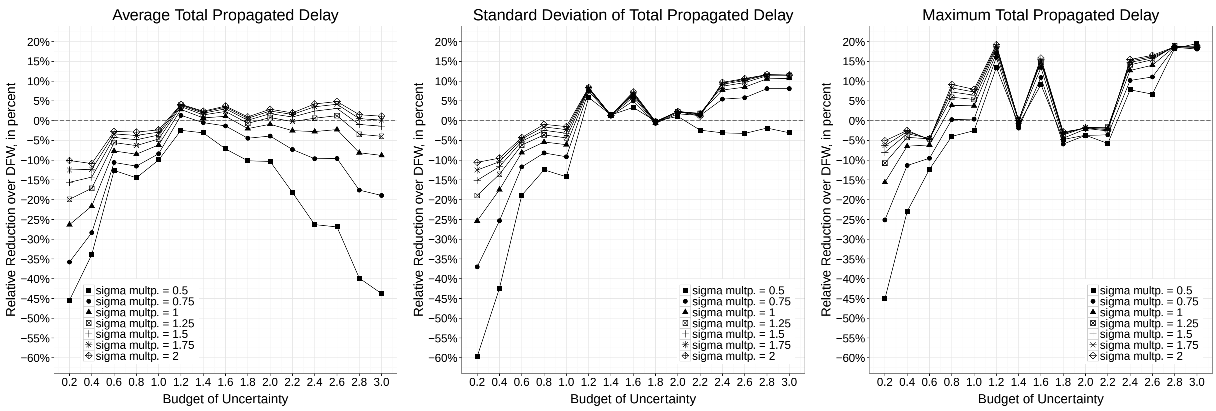


Figure EC.6 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_1)

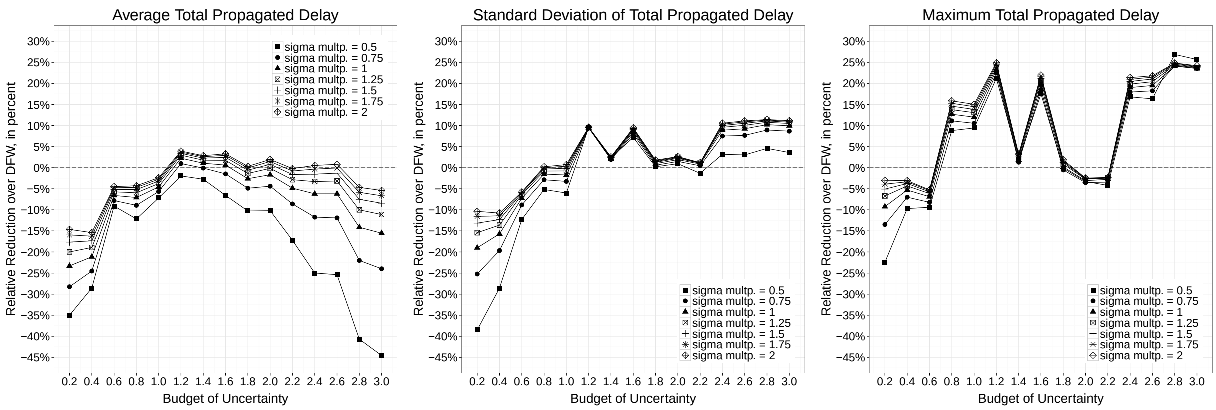


Figure EC.7 Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_1)

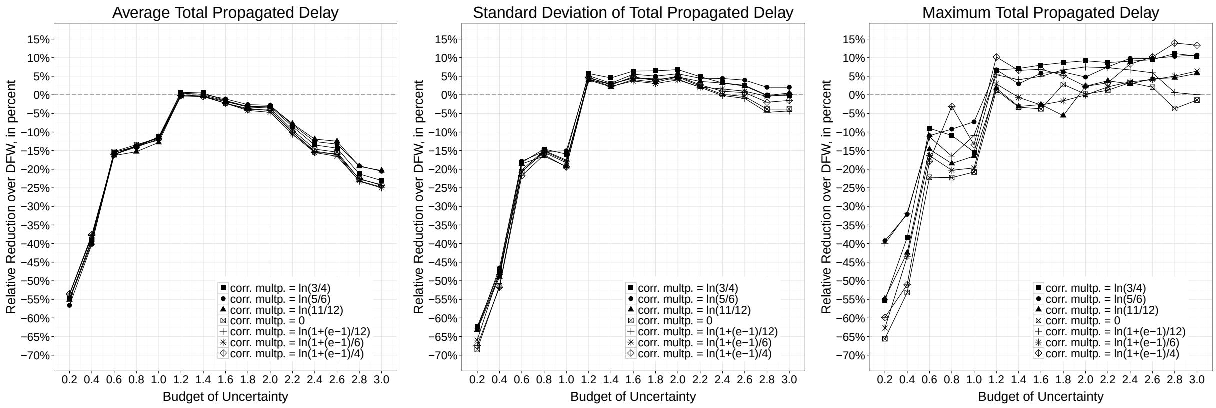


Figure EC.8 Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_1)

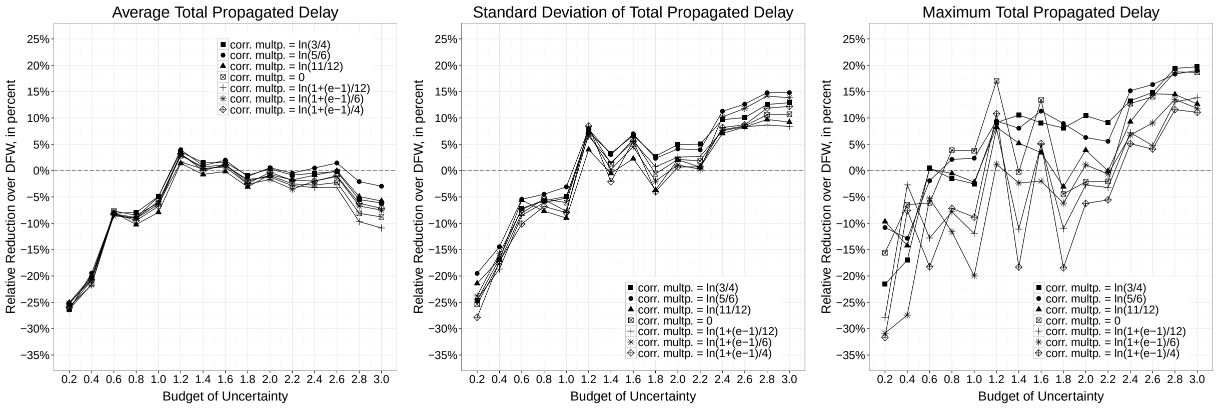


Figure EC.9 Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_1)

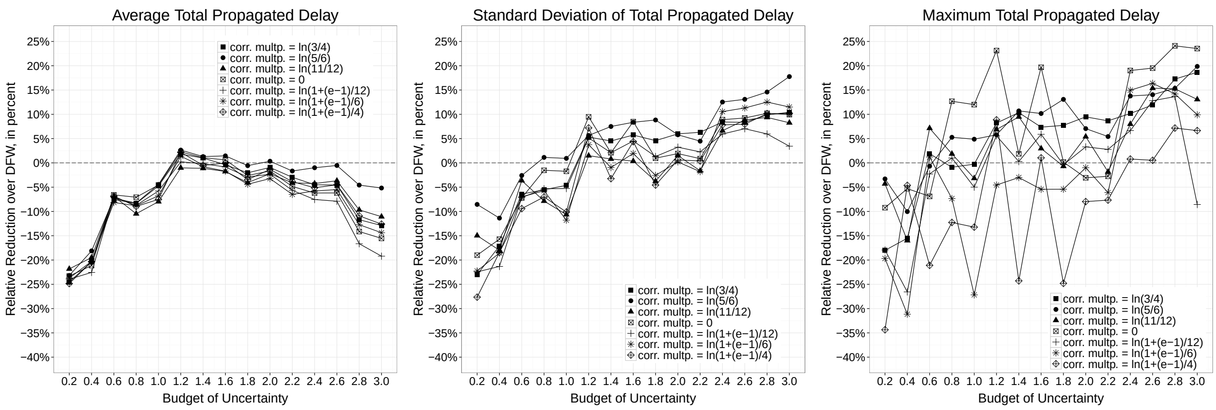


Figure EC.10 **Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_2)**

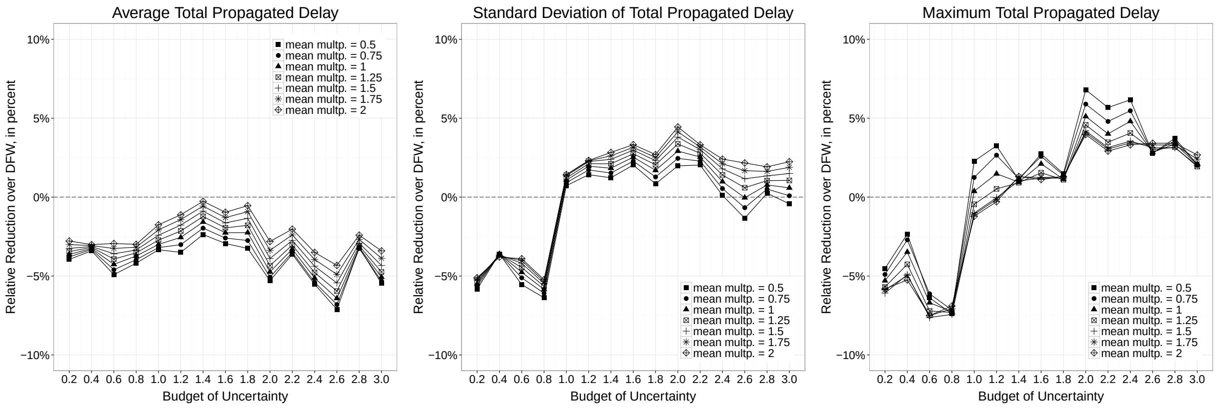


Figure EC.11 **Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_2)**

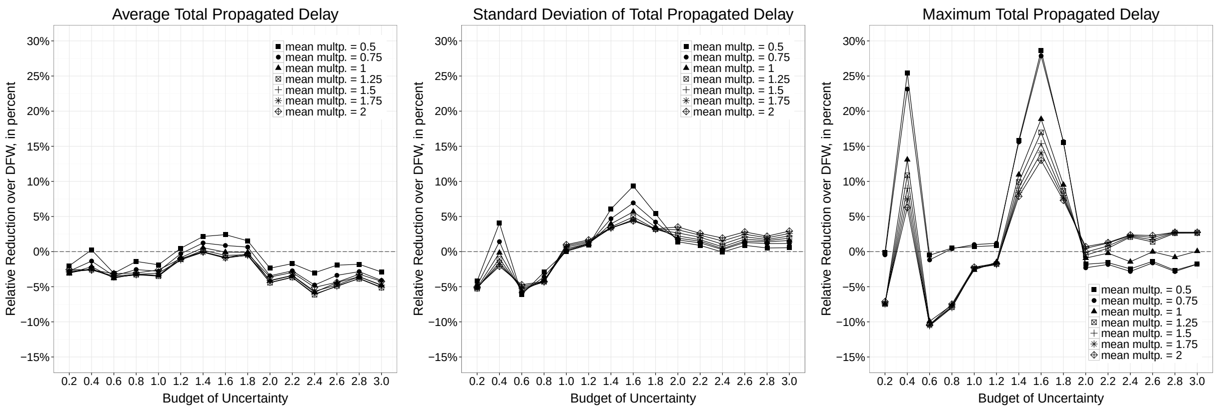


Figure EC.12 **Impact of Deviation in Mean (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_2)**

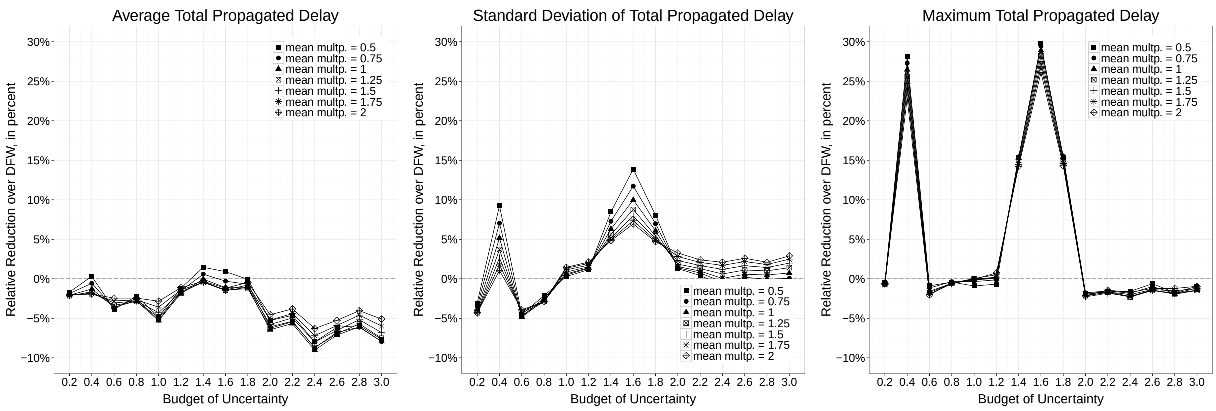


Figure EC.13 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_2)

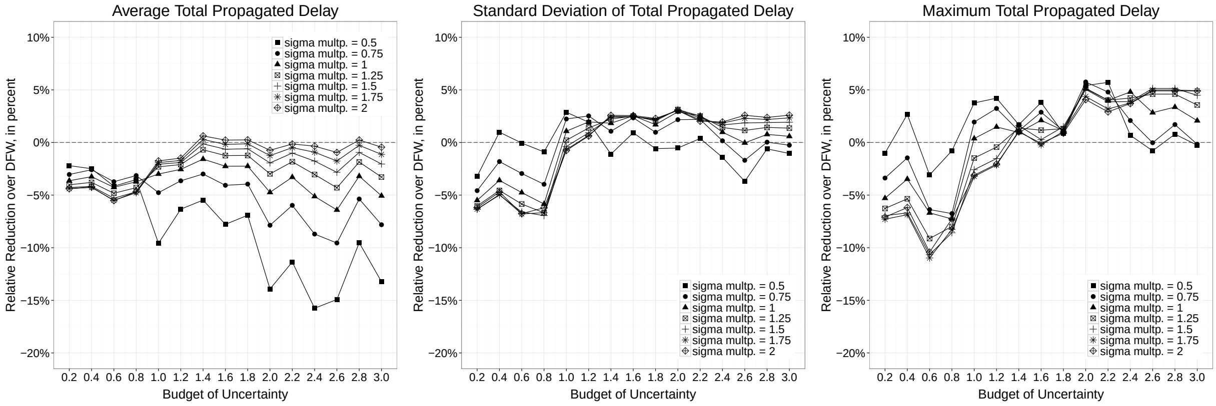


Figure EC.14 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_2)

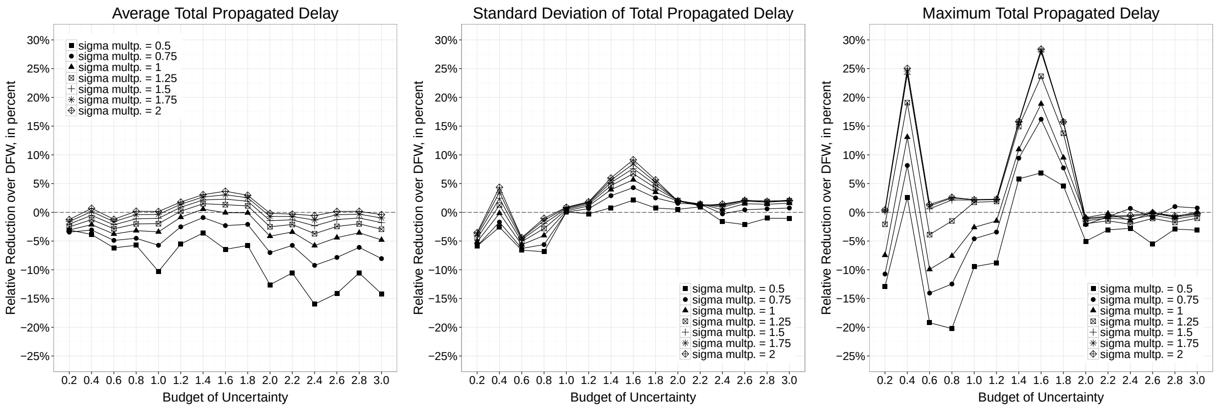


Figure EC.15 Impact of Deviation in Standard Deviation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_2)

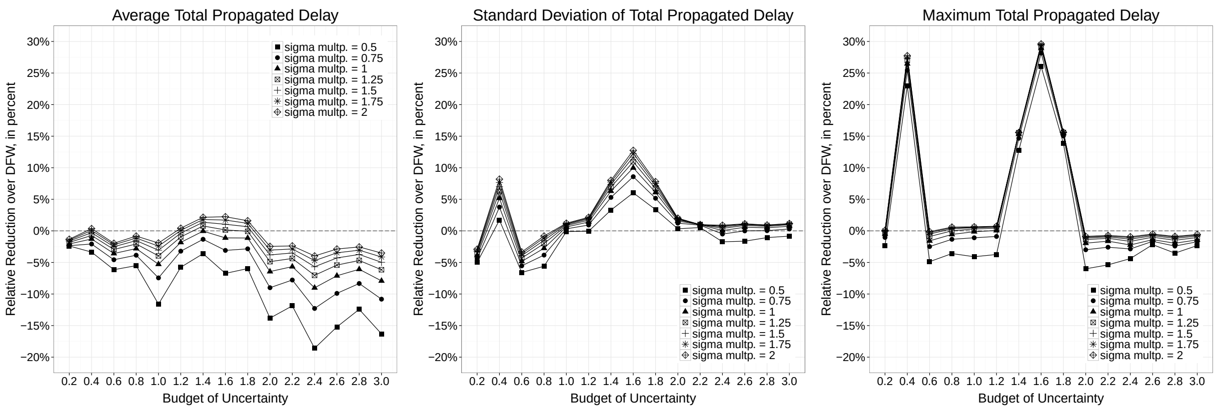


Figure EC.16 **Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Truncated Normal Distribution / Testing Flight Network: N_2)**

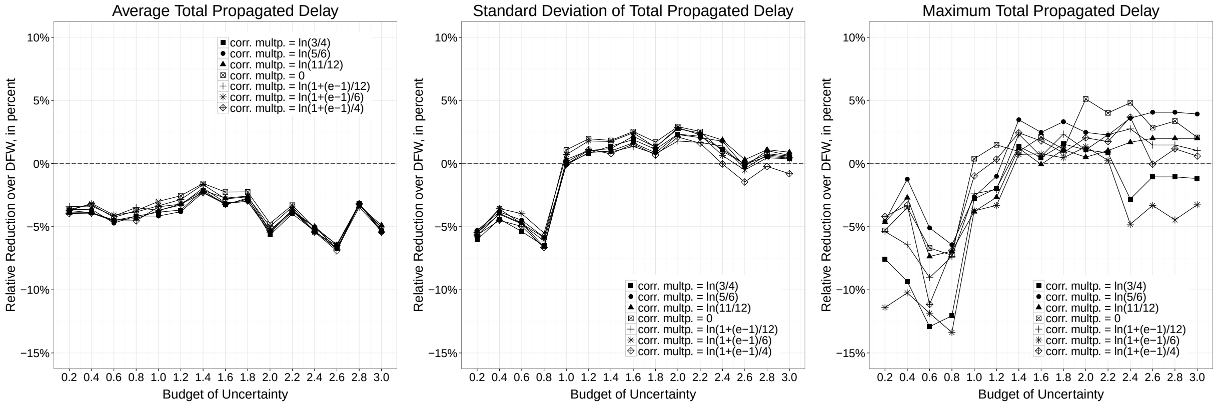


Figure EC.17 **Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Gamma Distribution / Testing Flight Network: N_2)**

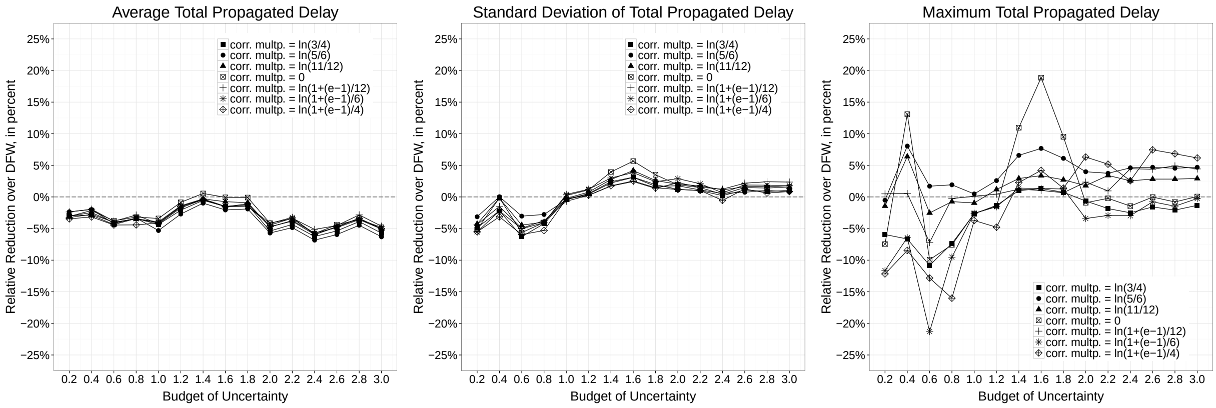
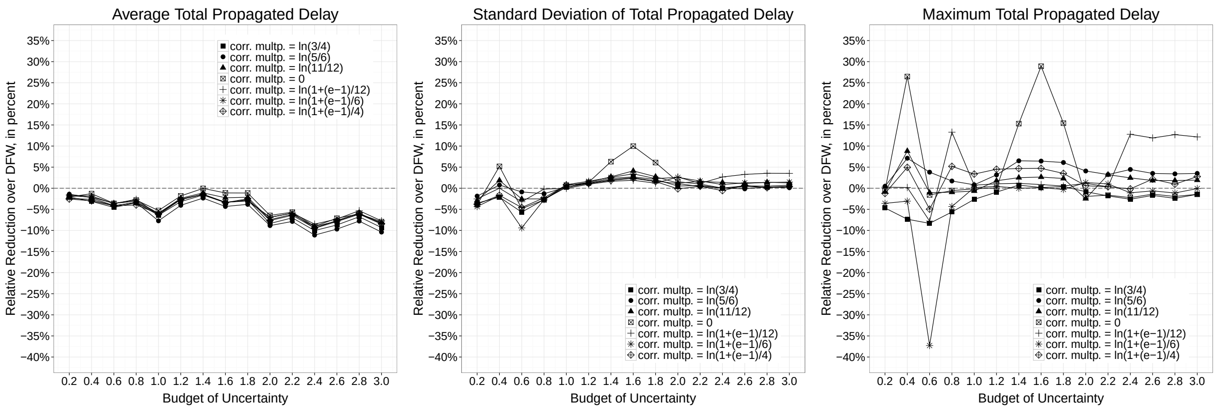


Figure EC.18 **Impact of Deviation in Correlation (Training Delay Data: Truncated Normal Distribution / Testing Delay Data: Log-normal Distribution / Testing Flight Network: N_2)**



EC.2. Method to Perturb Spearman's Rank Correlation Coefficient Matrix

The method is inspired by Galeeva et al. (2007) where they provide a way to generate correlation matrices around a base correlation matrix by perturbing its eigenvalues. The detailed procedure we use is as follows,

1. Apply eigenvalue decomposition on the Spearman's rank correlation coefficient matrix ρ^{train} for the training data set,

$$\rho_{i,j}^{\text{train}} = \sum_{k,l=1}^{|\mathcal{F}|} V_{i,k}\Lambda_{k,l}V_{l,j},$$

where $\Lambda_{k,l} = \lambda_k \delta_{k,l}$. $\delta_{k,l} = 1$ if $k = l$; 0, otherwise. $\lambda_1 > \lambda_2 > \cdots > \lambda_{|\mathcal{F}|}$ are the $|\mathcal{F}|$ eigenvalues for ρ^{train}. V is the eigenvectors.

2. Create variables $\sigma_1, \sigma_2, \cdots, \sigma_{|\mathcal{F}|}$ for each eigenvalue, which satisfy the following set of equations

$$\lambda_1 e^{\sigma_1} = \lambda_2 e^{\sigma_2} = \cdots = \lambda_{|\mathcal{F}|} e^{\sigma_{|\mathcal{F}|}}$$

There are multiple solutions for the set of equations above. We fix $\sigma_1 = 1$, and the remaining values of σ can be calculated accordingly. Since λ_1 is the largest eigenvalue, it can be seen easily that $\sigma_2, \cdots, \sigma_{|\mathcal{F}|} > 1$.

3. Perturb these eigenvalues with a parameter α. The perturbed eigenvalues $\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_{|\mathcal{F}|}$ will be calculated as $\hat{\lambda}_i = \lambda_i e^{\sigma_i \alpha}, \ \forall i = 1, \cdots, |\mathcal{F}|$.

4. Normalize the perturbed eigenvalues to make $\sum_{i=1}^{|\mathcal{F}|} \hat{\lambda}_i = |\mathcal{F}|$.

5. Construct matrix ρ' with the perturbed eigenvalues and the eigenvectors for $\rho_{i,j}^{\text{train}}$,

$$\rho'_{i,j} = \sum_{k,l=1}^{|\mathcal{F}|} V_{i,k}\hat{\Lambda}_{k,l}V_{l,j},$$

where $\hat{\Lambda}_{k,l} = \hat{\lambda}_k \delta_{k,l}$.

6. Construct Spearman's rank correlation coefficient matrix by normalizing matrix ρ',

$$\rho_{i,j}^{\text{test}} = \frac{\rho'_{i,j}}{\sqrt{\rho'_{i,i}\rho'_{j,j}}}$$

The normalizing step makes sure that $-1 \leq \rho_{i,j}^{\text{test}} \leq 1$ and $\rho_{i,i}^{\text{test}} = 1, \ \forall i = 1, \cdots, |\mathcal{F}|$.

To understand the meaning of parameter $\alpha \in (-\infty, 1]$:

- if $\alpha = 1$, $\hat{\lambda}_1 = \hat{\lambda}_2 = \cdots = \hat{\lambda}_{|\mathcal{F}|} = 1$, thus $\hat{\Lambda} = I$ and $\rho' = V\hat{\Lambda}V^T = VV^T = VV^{-1} = I$. This leads to $\rho^{\text{test}} = I$, which means the testing data will have independent primary delays.

- if $\alpha = 0$, then $\hat{\lambda}_i = \lambda_i$, $\forall i = 1, \cdots, |\mathcal{F}|$. This means the testing data will have the same Spearman's rank correlation coefficient matrix as the training data.

- for the case $\alpha \to -\infty$, we have

$$\rho_{i,j}^{\text{test}} = \frac{\rho'_{i,j}}{\sqrt{\rho'_{i,i}\rho'_{j,j}}} = \frac{\sum_{k,l=1}^{|\mathcal{F}|} V_{i,k}\hat{\Lambda}_{k,l}V_{l,j}}{\sqrt{\sum_{k,l=1}^{|\mathcal{F}|} V_{i,k}\hat{\Lambda}_{k,l}V_{l,i}}\sqrt{\sum_{k,l=1}^{|\mathcal{F}|} V_{j,k}\hat{\Lambda}_{k,l}V_{l,j}}} = \frac{\sum_{k=1}^{|\mathcal{F}|} V_{i,k}\hat{\lambda}_k e^{\sigma_k \alpha} V_{k,j}}{\sqrt{\sum_{k=1}^{|\mathcal{F}|} V_{i,k}\hat{\lambda}_k e^{\sigma_k \alpha} V_{k,i}}\sqrt{\sum_{k=1}^{|\mathcal{F}|} V_{j,k}\hat{\lambda}_k e^{\sigma_k \alpha} V_{k,j}}}$$

Divide both the numerator and denominator by e^{α}, we have

$$\rho_{i,j}^{\text{test}} = \frac{V_{i,1}\hat{\lambda}_1 V_{1,j} + \sum_{k=2}^{|\mathcal{F}|} V_{i,k}\hat{\lambda}_k e^{(\sigma_k - 1)\alpha} V_{k,j}}{\sqrt{V_{i,1}\hat{\lambda}_1 V_{1,i} + \sum_{k=2}^{|\mathcal{F}|} V_{i,k}\hat{\lambda}_k e^{(\sigma_k - 1)\alpha} V_{k,i}}\sqrt{V_{j,1}\hat{\lambda}_1 V_{1,j} + \sum_{k=2}^{|\mathcal{F}|} V_{j,k}\hat{\lambda}_k e^{(\sigma_k - 1)\alpha} V_{k,j}}}$$

Since $\sigma_2, \cdots, \sigma_{|\mathcal{F}|} > \sigma_1 = 1$, as $\alpha \to -\infty$ we have

$$\lim_{\alpha \to -\infty} \rho_{i,j}^{\text{test}} = \frac{V_{i,1}\hat{\lambda}_1 V_{1,j}}{|V_{i,1}||\hat{\lambda}_1||V_{1,j}|} = \frac{V_{i,1}V_{1,j}}{|V_{i,1}V_{1,j}|} = \begin{cases} 1, & V_{i,1}V_{1,j} > 0 \\ -1, & V_{i,1}V_{1,j} < 0 \end{cases}$$

This means the testing data will be perfectly correlated. Whether it is positively or negatively correlated depends on the eigenvectors of ρ^{train}.

References

Galeeva, Roza, Jiri Hoogland, Alexander Eydeland, Morgan Stanley. 2007. Measuring correlation risk. Tech. rep.