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Planning for Robust Airline Operations: Optimizing Aircraft Routings and Flight Departure Times to Minimize Passenger Disruptions

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Airlines typically construct their schedules assuming that every flight leg will depart and arrive as planned. Because this optimistic scenario rarely occurs, these plans are frequently disrupted and airlines often incur significant costs in addition to those originally planned. Flight delays and schedule disruptions also cause passenger delays and disruptions. A more robust plan can reduce the occurrence and impact of these delays, thereby reducing costs. In this paper, we present two new approaches to minimize passenger disruptions and achieve robust airline schedule plans. The first approach involves routing aircraft, and the second involves retiming flight departure times.

Because each airplane usually flies a sequence of flight legs, delay of one flight leg might propagate along the aircraft route to downstream flight legs and cause further delays and disruptions. We propose a new approach to reduce delay propagation by intelligently routing aircraft. We formulate this problem as a mixed-integer programming problem with stochastically generated inputs. An algorithmic solution approach is presented. Computational results obtained using data from a major U.S. airline show that our approach can reduce delay propagation significantly, thus improving on-time performance and reducing the numbers of passengers disrupted.

Our second area of research considers passengers who miss their flight legs due to insufficient connection time. We develop a new approach to minimize the number of passenger misconnections by retiming the departure times of flight legs within a small time window. We formulate the problem and an algorithmic solution approach is presented. Computational results obtained using data from a major U.S. airline show that this approach can substantially reduce the number of passenger misconnections without significantly increasing operational costs.

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1. Introduction

A common assumption in airline schedule planning (the process of generating the schedule with the greatest revenue potential) is that flight legs will be operated as planned. Because this optimistic scenario rarely occurs, airline schedules are frequently disrupted resulting in significant additional costs to airlines and passengers. It is estimated that the financial impact of irregularities on the daily operations of a single major U.S. domestic carrier may exceed $440 million per annum in lost revenue, crew overtime pay, and passenger hospitality costs (Clarke and Smith 1999). Additionally, the Air Transport Association estimates that when passenger delay costs are considered, delays cost airlines and consumers about $6.5 billion in 2000 (Air Transport Association 2003).

In 2000, approximately 30% of the flight legs operated by one major U.S. airline was delayed, and about 3.5% of these flight legs were cancelled (Bratu and Barnhart 2002). These delays and cancellations lead to disruptions in aircraft routings, crew schedules, and passenger itineraries. A passenger is considered to be *disrupted* if one or more of the flight legs in his/her itinerary is cancelled, or if a flight leg is delayed beyond the point where the passenger can successfully connect to the next flight leg in his or her itinerary. For the same major U.S. airline, it is estimated that approximately 4% of passengers are disrupted (with about half of them being connecting passengers) resulting in (i) very long delays (for example, on a day with adverse weather conditions, disrupted passengers were delayed on average 419 minutes, compared to 14 minutes for nondisrupted passengers), (ii) significant direct revenue loss to airlines and passenger, and (iii) loss of passenger goodwill.

Given the predicted doubling of air traffic in the next 10 to 15 years, the slow growth in aviation system capacity (Mead 2000), and the findings of the MIT Global Airline Industry Program (1999) and Schaefer et al. (2001) that each 1% increase in air traffic will result in a 5% increase in delays, there will likely be more frequent and serious schedule disruptions if nothing is done to change the way airline schedules are developed. Because aviation is such a critical component of the national transportation infrastructure (providing efficiencies in the movement of both people and cargo), there is a clear need for airline schedules that are less susceptible to delays and cancellations.

There is growing consensus among researchers that schedule robustness can be improved by explicitly considering possible delays and cancellations during the creation of schedule plans. However, building robustness into the schedule in this proactive manner presents a number of challenges. First, robustness is difficult to define. A robust plan might be a plan that yields the minimum cost for the worst case, the minimum expected cost, or minimizes costs given a required level of service. Second, it is difficult to capture in a tractable model the complex operations that result when severe weather conditions exist, especially in hub-and-spoke networks. Third, optimization models capturing stochasticity are often computationally intractable when applied to large-scale airline problems. Last, conventional models for airline schedule planning minimize *planned* costs, while airlines' ultimate goal is to minimize *realized* costs, that is, the sum of planned costs and the costs of delays and disruptions. However, it is difficult to estimate a priori the *realized* costs to include them in a planning model.

In this paper, we make two contributions. First, we propose a new approach to generating aircraft routes that minimize delay propagation. We formulate the problem as a mixed-integer program and develop an algorithmic approach to solve it. We investigate the value of our robust plan over the plan generated by conventional approaches using data from a major U.S. airline. The results show that our approach can reduce delay propagation significantly, improve on-time performance, and reduce the number of passengers missing their connections.

Second, we propose a new approach to minimize the expected total number of passenger misconnections. We formulate the problem as mixed-integer program where flight leg departure times are moved within a small time window, analyze the properties of the model, and develop an algorithmic approach. The computational results obtained using data from a major U.S. airline show that this approach, which has desirable computational properties, can significantly reduce the number of disrupted passengers.

The paper is structured as follows. In §2, we survey the literature in the area of robust planning and then provide alternative definitions of robustness and present a modeling framework for robust airline schedule planning. In §3, we present a robust aircraft maintenance routing model and its associated solution approach. By routing aircraft in different ways, we can reduce the delay propagating throughout the network. We also provide proof-of-concept results using data from a major U.S. airline. In §4, we present the idea of rescheduling each flight leg within a small time window to minimize passenger disruptions. We show various ways to model this problem and analyze the properties of the models. We also present and analyze results using data from a major U.S. airline. Finally, in §5, we discuss possible extensions of our robust airline scheduling models.

2. Robust Airline Schedule Planning

There are at least two ways to deal with schedule disruptions. The typical approach is to reoptimize the schedule after a disruption occurs. A more proactive approach is to build robustness into the schedule in the planning stage. To understand how this might be done, it is important to understand the schedule planning process (§2.1), the conventional approach of schedule recovery (§2.2), and previously proposed approaches to robust planning (§2.3).

2.1. The Airline Schedule Planning Process

The airline schedule planning problem has been studied extensively and numerous models and algorithmic approaches have been developed. Barnhart and Talluri (1997) and Cohn and Barnhart (2003) present structural overviews of this planning process and detailed literature reviews. A brief overview is provided here.

Because the airline schedule planning problem is too large to be solved in a single decision model, the problem is traditionally divided into sequential subproblems defined as (i) schedule generation, (ii) fleet assignment, (iii) maintenance routing, and (iv) crew scheduling.

The solution to the schedule generation problem is a schedule defined by markets, frequencies, and the specific departure and arrival times of each flight leg. Because the schedule affects every operational decision, it has the biggest impact on an airline's profitability.

The solution to the fleet assignment problem is the assignment of a specific aircraft type to each flight leg in the schedule, matching as closely as possible the seat capacity of aircraft to the demand, thereby minimizing operating expenses and lost revenue caused by insufficient capacity. Fleet assignment models have been widely applied in practice, and significant savings have been achieved. For example, Subramanian et al. (1994) report $100 million per year in savings at Delta Airlines.

The solution to the maintenance routing problem is a set of routes, one for each aircraft, in which all aircraft are maintained at the right place and right time. The objective of the maintenance routing problem is to find maintenance feasible routes for each aircraft, given a fleeted schedule and the number of available aircraft of each fleet type. This problem is discussed in detail in §3.1.

The solution to the crew scheduling problem is the assignment of cockpit and cabin crews to flight legs that minimize cost and satisfy regulatory agency requirements and collective bargaining agreements.

2.2. Schedule Recovery

When disruptions occur, airlines typically recover from disruptions in stages (Rosenberger, Johnson, and Nemhauser 2001a). In the first stage, new aircraft routings are created by rerouting aircraft and delaying/canceling flight legs. In the second stage, cockpit and cabin crew are reassigned and where necessary reserve cockpit and cabin crew are called. In the third stage, passengers are reaccommodated. Interested readers are referred to Clarke and Smith (2000) and Rosenberger, Johnson, and Nemhauser (2001a) for a detailed review. Related literature includes Teodorovic and Guberinic (1984); Jarrah et al. (1993); Teodorovic and Stojkovic (1995); Yan and Yang (1996); Mathaisel (1996); Cao and Kanafani (1997); Lettovsky (1997); Luo and Yu (1997); Yan and Tu (1997); Thengvall, Bard, and Yu (2000); and Yu et al. (2003).

2.3. Robust Planning

Although robust airline schedule planning is a relatively new concept, "robust planning" has been studied by many researchers and applied in various fields such as robot design, manufacturing, supply chain management and logistics, telecommunications, economics, ecology, water and environmental management, and portfolio management in finance. For detailed reviews, readers are referred to Zimmermann (1991), Watanabe and Ellis (1993), Birge (1995), Kouvelis and Yu (1997), and the Stochastic Programming Community (2003). The methodologies used include stochastic programming (Birge and Louveaux 1997), scenario planning (Mulvey, Vanderbei, and Zenios 1995; Kouvelis and Yu 1997), and fuzzy optimization (Zimmermann 1991; Sakawa 1993).

In part because airlines have incurred billions of dollars in revenue losses due to unplanned disruptions, researchers are beginning to consider possible delays and disruptions in the planning stage. Ageeva and Clarke (2000) present a robust aircraft maintenance routing model to provide opportunities to swap planes. Chebalov and Klabjan (2002) propose a similar idea for crew scheduling. Rosenberger, Johnson, and Nemhauser (2001b) develop a robust fleet assignment and aircraft rotation model with many short cycles. Schaefer et al. (2001) propose a stochastic extension to the deterministic crew scheduling problem. With simulation, they obtain a linear approximation of expected crew costs and then solve the resulting deterministic crew scheduling problem. Yen and Birge (2001) develop a two-stage stochastic integer programming model to minimize total expected crew costs. Kang and Clarke (2002) propose the idea of a degradable airline schedule where a current airline schedule is partitioned into several schedules in independent layers that are prioritized, with higher priority layers recovered first. Independence of layers ensures that disruptions are isolated within a layer, thus preventing disruptions from propagating throughout the network. For a detailed review, readers are referred to Lan (2003).

3. Robust Aircraft Maintenance Routing

3.1. The Aircraft Maintenance Routing Problem

The goal of the aircraft maintenance routing problem is to determine a sequence of flight legs, called *aircraft routings*, to be flown by individual aircraft such that each flight leg is included in exactly one aircraft routing, and all aircraft are properly maintained. In most optimization models for the aircraft maintenance routing problem, the objective is to maximize *through revenue*, the potential revenue obtained by offering passengers the opportunity to stay on the same aircraft when making a connection at an airport. In practice, this additional revenue is very difficult to determine accurately and the financial impact is relatively small (Klabjan et al. 1999, Cordeau et al. 2000). The aircraft maintenance routing problem can thus be cast as a feasibility problem, providing an opportunity to achieve robustness with minimal cost implications.

The FAA mandates four main categories of airline safety checks: A, B, C, and D checks, varying in scope, duration, and frequency (Clarke et al. 1996). Usually, the maintenance routing problem presented in the literature considers only A checks. Among the four safety checks, A checks are the only checks that need to be performed frequently. A checks are required after every 60 hours of flying, although airlines enforce more stringent maintenance requirements and typically perform A checks after every 40 to 45 hours of flying (about three to four calendar days). Because maintenance requires trained professionals and equipment, these checks are only performed at a limited number of airports.

Recent work in the area of maintenance routing includes Feo and Bard (1989), Kabbani and Patty (1992), Desaulniers et al. (1997), Clarke et al. (1996), Barnhart et al. (1998b), Gopalan and Talluri (1998), and Talluri (1998). These models assume that (1) the fleeted schedule will repeat everyday and (2) aircraft that overnight at a maintenance base have the opportunity to undergo maintenance. It is perhaps most important to note (within the context of operational robustness) that none of these models considers the impact of delays and cancellations.

3.2. Delay Propagation

Flight leg delays may be divided into the following two categories.

- *Propagated delay*: Delay that occurs when the aircraft to be used for a flight leg is delayed on its prior flight leg. This delay is a function of an aircraft's routing. For the major U.S. airline for which we have data, propagated delay is approximately 20% to 30% of total delay.
- *Nonpropagated delay*: Delay that occurs for reasons that are not a function of routing. We also call this independent delay (independent of routing).

Figure 1 illustrates the relationships between departures, arrivals, and delays. The solid arrows represent the original schedule for two flight legs i and j. The dotted arrows represent the actual departures and arrivals of these flight legs. PDT refers to planned departure time, and ADT refers to actual departure time. PAT refers to planned arrival time, and AAT refers to actual arrival time. The turn time is the time between the arrival of the aircraft at the gate and the time this aircraft is ready for the next flight. The minimum turn time is the minimum time required to

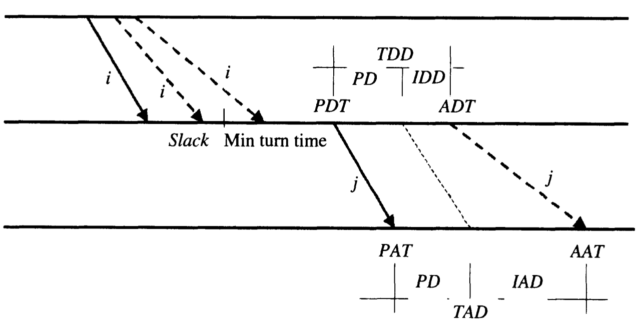

Figure 1 **Departures, Arrivals, and Delays**

deboard, unload baggage, clean, cater, fuel, load baggage, and board an aircraft. If PTT_{ij} is the planned turn time between flight leg i and flight leg j, and MTT is the minimum turn time, then the slack is the difference between planned turn time and minimum turn time, that is,

$$PTT_{ij} = PDT_j - PAT_i, \tag{1}$$

and

$$Slack_{ij} = PTT_{ij} - MTT. \tag{2}$$

TDD refers to total departure delay, comprised of independent departure delay (IDD) and propagated delay (PD). PD_{ij}, the delay propagated from flight leg i to flight leg j if both flight legs are flown by the same aircraft, can be determined as follows:

$$PD_{ij} = \max(TAD_i - Slack_{ij}, 0). \tag{3}$$

TAD, the total arrival delay, is also comprised of two parts, namely propagated delay (PD) and independent arrival delay (IAD).

3.3. Modeling the Robust Aircraft Maintenance Routing Problem

Because each aircraft routing is a sequence of flight legs flown by a single aircraft, an arrival delay will result in a departure delay if there is not enough slack between two consecutive flight legs in that routing. This "delay propagation" often results in delays for downstream flight legs, and delays and disruptions for crews and passengers. This is especially true at hubs where aircraft, crew, and passenger flows are closely interrelated. Given that flight leg delays are due in part to the propagation of delays along aircraft routings, flight leg delay can be reduced if slack is optimally assigned to aircraft routings (that is, at the airports along aircraft routings where slack is needed most). The underlying premise in our modeling approach is that it is possible to reduce propagated delay and overall flight leg delays by intelligently routing the aircraft, allocating slack optimally to absorb the delay propagation.

Figure 2 illustrates the idea. Assume that flight leg f_1 and flight leg f_3 are in the same route (string) s_1, and flight leg f_2 and flight leg f_4 are in the same route (string) s_2. Suppose, based on historical data, we know that flight leg f_1 is delayed, as shown in the figure, on average to the position of f_1'. This delay is longer than the slack between flight leg f_1 and flight leg f_3, causing delay to propagate from flight leg f_1 to flight leg f_3, and causing flight leg f_3 to be delayed or cancelled if the delay is too long. As a result, passengers connecting from flight leg f_3 to other flight legs will likely be disrupted. Our goal is to consider the historical delay data in selecting aircraft routes, so

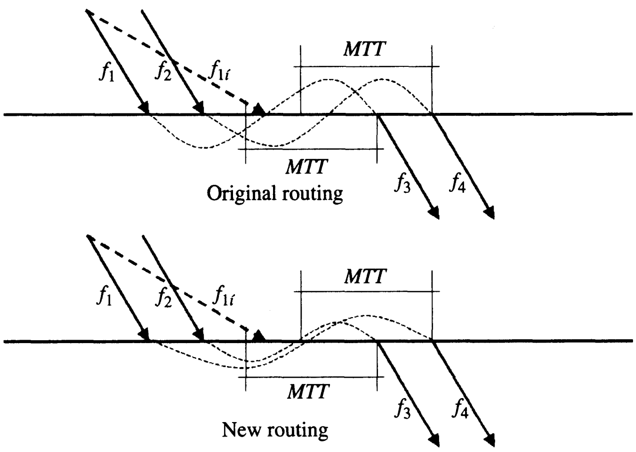

Figure 2 **Rerouting and Delay Propagation**

that the delay and/or cancellation of flight leg f_3 and the resulting passenger disruptions can be reduced. To illustrate, assume that historical data show that on average flight leg f_2 arrives on time. Then, a better way to construct the aircraft routes is illustrated in the "new routing" shown in Figure 2, that is, to put flight leg f_1 and flight leg f_4 in the same route, and flight leg f_2 and flight leg f_3 in another route. The effect is to add more slack after the often-delayed flight leg f_1 to mitigate the downstream effects of its delay.

This problem can be solved separately for each fleet type. Because delays propagate along the aircraft routes, it is difficult to use leg-based models to track delay propagation. Thus, a routing-based model is more appropriate. Such a model, a string-based formulation for robust aircraft maintenance routing with the objective to minimize total *expected* propagated delay, is presented in this section. A string is a sequence of connected flight legs that begins and ends at maintenance stations (possibly different ones).

3.3.1. Determining Delays for Feasible Routes. Both propagated delay and total arrival delay are a function of routing. Thus, while historical values for propagated delay and total arrival delay can be computed for each flight leg in existing routings, no such values are available for routings that have not been previously realized. However, because independent arrival delay is not a function of routing, independent arrival delay can be calculated for each flight leg by tracking actual routings of each individual aircraft. The total arrival delays and propagated delays of flight legs in any routing can then be generated, as described below.

Algorithm 1: Generate Delay Data.

1. Determine propagated delays (PD) for each sequence of flight legs i, j in the historical data: $PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$.
2. Determine independent arrival delays (IAD) for each flight leg from historical data: $IAD_j = TAD_j - PD_{ij}$.
3. Determine total arrival delay (TAD) and PD for each flight leg of any routing, given the independent arrival delay (IAD) for each flight leg:
   - For the first flight leg i on each string, $TAD = IAD$; and
   - For subsequent flight legs j in the routing, in sequence: $PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$ and $TAD_j = IAD_j + PD_{ij}$.

3.3.2. Delay Distribution. We determined the distribution of delay using the Airline Service Quality Performance (ASQP) database. The ASQP database provides flight leg information for all the domestic flight legs of major airlines in the United States (that is, airlines generating revenues of \$1 billion or more annually). This database is available to the general public. ASQP provides the following information for each flight leg: planned departure time and arrival time, actual departure time and arrival time (including wheels-off and wheels-on time, taxi-out and taxi-in time, airborne time), and airplane tail number. For cancelled flight legs, reasons for cancellation and airplane tail number are not available.

The arrival delays are usually strongly asymmetric, with some flight legs arriving early (the arrival delays are negative), but most flight legs arriving on time or late. More specifically, most flight legs arrive around the scheduled arrival time, with very few of them arriving very early (more than 20 minutes), and some arriving very late (more than one hour). Therefore, the natural candidates for the arrival delay distributions are the gamma, log-normal, and Weibull distributions.

SAS was used to estimate the parameters and calculate the test statistics. The χ^2 test and/or the Kolmogorov test were used to determine if the total arrival delays follow a specific distribution. We found the log-normal distribution to be the best fit among the distributions listed above. With a significance level of 0.01, the null hypothesis is accepted for 84% of all flight legs, implying that the actual arrival delays for 84% of the flight legs follow a log-normal distribution. For these flight legs, the shape parameters are usually less than one and location parameters are less than zero. The reader is referred to Lan (2003) for details.

3.3.3. Formulation of the Robust Aircraft Maintenance Routing Model. Let S be the set of feasible strings, F be the set of daily flight legs, F^+ be the set of flight legs originating at a maintenance station, and F^- be the set of flight legs terminating at a maintenance station. We denote the set of ground variables (including the overnight or wraparound arcs to ensure that the flight schedule can repeat daily) as G, the set of strings ending with flight leg i as S_i^-, and the set of strings beginning with flight leg i as S_i^+. We have one binary decision variable x_s for each feasible string s. We have ground variables denoted by y, which are used to count the number of aircraft on the ground at maintenance stations. Let pd_{ij}^s be the delay propagated from flight leg i to flight leg j if flight leg i and flight leg j are in string s. Let a_{is} equal 1 if flight leg i is in string s, and equal 0 otherwise. Ground variables $y_{i,d}^-$ equal the number of aircraft on the ground before flight leg i departs, and ground variables $y_{i,d}^+$ equal the number of aircraft on the ground after flight leg i departs; ground variables $y_{i,a}^-$ equal the number of aircraft on the ground before flight leg i arrives, and ground variables $y_{i,a}^+$ equal the number of aircraft on the ground after flight leg i arrives. r_s is the number of times string s crosses the *count time*, a point in time when aircraft are counted, p_g is the number of times ground arc g crosses the count time, and N is the number of planes available.

The robust aircraft maintenance routing (RAMR) model is written as follows:

$$\min E\bigg(\sum_{s\in S}\bigg(\sum_{(i,j)\in s} pd_{ij}^s\bigg)x_s\bigg) \tag{4}$$

$$\text{subject to} \quad \sum_{s\in S} a_{is}x_s = 1 \quad \forall\, i\in F, \tag{5}$$

$$\sum_{s\in S_i^+} x_s - y_{i,d}^- + y_{i,d}^+ = 0 \quad \forall\, i\in F^+, \tag{6}$$

$$-\sum_{s\in S_i^-} x_s - y_{i,a}^- + y_{i,a}^+ = 0 \quad \forall\, i\in F^-, \tag{7}$$

$$\sum_{s\in S} r_s x_s + \sum_{g\in G} p_g y_g \le N, \tag{8}$$

$$y_g \ge 0 \quad \forall\, g\in G, \tag{9}$$

$$x_s \in \{0,1\} \quad \forall\, s\in S. \tag{10}$$

The objective (4) is to minimize the expected total propagated delay of selected strings. Constraints (5) are cover constraints that ensure each flight leg is in exactly one string. Constraints (6) and (7) are flow balance constraints that ensure the number of aircraft arriving at and departing from a location are equal. Constraint (8) is the count constraint to ensure that the total number of aircraft in use at the count time (and thus at any point in time due to the cyclic, daily nature of the flight schedule) does not exceed the number of aircraft in the fleet. Constraints (9) and (10) force the number of aircraft on the ground to be nonnegative and the number of aircraft assigned to a string to be 0 or 1. Because variable y_g is a sum of binary x variables, the integrality constraints on the y variables can be relaxed, as discussed in Hane et al. (1995).

3.4. Solution Approach

The robust aircraft maintenance routing (RAMR) problem is a stochastic discrete optimization problem. There is extensive literature addressing variants of this problem type. For a detailed literature review, the reader is referred to Kleywegt, Shapiro, and Homem-de-Mello (2001), in which they propose a Monte Carlo simulation-based approach for solving these problems. Their method is particularly applicable when the expected value function in the objective cannot be written in closed form and/or its values cannot be easily calculated. Our model, however, is a stochastic discrete optimization problem without random variables in the constraints, and with an objective function (4) that can be rewritten as:

$$\begin{aligned}\min E\bigg[\sum_s x_s \times \bigg(\sum_{(i,j)\in s} pd_{ij}^s\bigg)\bigg] &= \min \sum_s x_s \times E\bigg[\sum_{(i,j)\in s} pd_{ij}^s\bigg] \\ &= \min \sum_{s\in S}\bigg(x_s \times \sum_{(i,j)\in s} E[pd_{ij}^s]\bigg).\end{aligned} \tag{11}$$

$E[pd_{ij}^s]$ can be computed offline for each pair of successive flight legs i and j using the approach detailed in Lan (2003). Then, RAMR is a deterministic mixed-integer linear program with a large number of 0-1 variables. For realistic problems, the complete generation of the corresponding instance, let alone its solution, requires prohibitive amounts of time and memory. The problem can be solved, however, using a *branch-and-price* approach. Branch-and-price is branch-and-bound with a linear programming relaxation solved using column generation at each node of the branch-and-bound tree.

3.4.1. Solving the LP Relaxation. Column generation is used to solve the linear programming (LP) relaxation of the RAMR problem, because it is impractical to enumerate all feasible strings explicitly. The algorithm's steps are summarized as follows.

Algorithm 2: Solving the LP Relaxation of RAMR.

1. Form the restricted master problem (*RMP*), that is, the *RAMR LP* with only a subset of the variables.
2. Solve the *RMP* to find an optimal primal and dual solution.
3. Using the dual solution of Step 2, solve the pricing problem to identify if one or more variables have negative reduced cost. If so, add them to the *RMP* and go to Step 2; else stop: The *LP* is solved.

3.4.2. The Pricing Problem. Let $d_s = \sum_{(i,j)\in s} E[pd_{ij}^s]$ represent the total propagated delay along string s,

π_i be the dual variable associated with the cover constraint for flight leg i, δ be the dual variable corresponding to the count constraint, and λ_i be the dual variable corresponding to the flow balance constraint for string s beginning or ending with flight leg i. The reduced cost of a string s beginning with flight leg m and ending with flight leg n is

$$\bar{d}_s = d_s - \sum_i a_{is}\pi_i - r_s\delta - \lambda_m + \lambda_n.$$

Barnhart et al. (1998b) show that the pricing subproblem of their string-based maintenance routing model can be cast as a constrained shortest path problem in a connection network. For our model, however, the pricing problem cannot be cast as a shortest path problem. The reason is that d_s $(=\sum_{(i,j)\in s} E[pd_{ij}^s])$ cannot be assigned to each connection arc (the arc connecting the arrival of one flight leg to the departure of another flight leg at an airport) because the propagated delay for each pair of flight legs depends on the string to which they belong. Thus, we solve the pricing problem approximately without explicitly evaluating the reduced cost for each possible string. We construct a connection network by allocating $-\sum_i a_{is}\pi_i - r_s\delta - \lambda_m + \lambda_n$ to the corresponding flight arcs and connection arcs. We then solve shortest path problems for all OD pairs of the network. If the costs for all shortest paths are greater than or equal to zero, then no columns have negative reduced cost, because d_s is greater than or equal to zero, by definition. Thus, no columns will be added and the LP problem has been solved to optimality. For each shortest path with negative cost, we add d_s to its reduced costs and if the resulting total sum is less than zero, then the corresponding column is added to RMP. The augmented RMP is re-solved and the process repeats until a stopping criteria specifying the maximum number of iterations or the minimum objective function improvement is met. Although this method does not guarantee optimality because there might be unidentified paths with negative cost, it is tractable.

3.4.3. IP Solution. An integer solution to the robust aircraft maintenance routing problem can be obtained using a special branching strategy called "branch on follow-ons" (Ryan and Foster 1981, Barnhart et al. 1998b). As proved in Barnhart et al. (1998a), this strategy will generate optimal integer solutions to the problem. This strategy may be summarized as follows.

ALGORITHM 3: BRANCH ON FOLLOW-ONS.

1. If the solution is not fractional, the current maintenance routing problem is solved. If the solution is fractional, identify a fractional string s_1 with $0 < x_{s_1} < 1$. Denote the sequence of flight legs in s_1 as $f_1, f_2, f_3, \ldots, f_{n-1}, f_n$.

Table 1 Characteristics of Four Maintenance Routing Problems

Network	Num of flight legs	Num of strings
N1	20	7,909,144
N2	59	614,240
N3	97	6,354,384
N4	102	51,730,736

2. Identify another string s_2 (one exists) containing flight leg f_i in s_1 but not f_{i+1} in s_1. Define S_L as the set of strings with each string containing flight leg f_i followed by flight leg f_{i+1}.
   - On the left branch, force flight leg f_i to be followed by flight leg f_{i+1} with $\sum_{s\in S_L} x_s = 1$. To ensure the pricing subproblem generates strings satisfying this rule, eliminate from the connection network (1) all arcs connecting flight leg f_i to any flight leg other than flight leg f_{i+1}, and (2) all arcs connecting to flight leg f_{i+1} from any flight leg other than flight leg f_i.
   - On the right branch, do not allow flight leg f_i to be followed by flight leg f_{i+1}, that is, require that $\sum_{s\notin S_L} x_s = 1$. To ensure the pricing subproblem generates only strings satisfying this rule, eliminate from the network all arcs connecting flight leg f_i to flight leg f_{i+1}.

3.5. Proof of Concept

We used the RAMR model and solution algorithm to create routings for four different fleet types operated by a major U.S. network carrier. Because in practice the model will be built using historical data and then applied to future operations, the routings were created using ASQP (flight leg delay and cancellation) data and passenger booking data for July 2000 and then evaluated using the corresponding data for August 2000. Both flight leg delay and passenger disruption statistics were determined.

3.5.1. Underlying Networks. Table 1 presents the characteristics of the four different maintenance routing problems, each representing a different fleet type. The column "Num of strings" represents all possible strings for each network. Although the number of flight legs in each fleet is relatively small, the number of possible strings is very large.

3.5.2. Computational Results. Our solution algorithm was implemented in C++ and CPLEX 6.5 on a HPC 3000 workstation. The results are presented below.

Flight Leg Delay. Flight leg delay statistics are presented in Table 2. Column "Old PD" indicates the propagated delay in minutes in the historical data; column "New PD" indicates the propagated delay in minutes for our routing solution; column "PD reduced" indicates the reduction in propagated

Table 2 Propagated Delays Based on August 2000 Data

Network	Old PD	New PD	PD reduced	% of PD reduced
N1	6,749	4,923	1,826	27
N2	4,106	2,548	1,558	38
N3	8,919	4,113	4,806	54
N4	14,526	9,921	6,940	48
Total	34,300	21,505	15,130	44

Table 3 Distribution for Propagated Delays

P-delay	(0, 30]	(30, 60]	(60, 90]	(90, 120]	>120	>0
Old (%)	4.8	1.8	1.2	0.5	0.7	9.1
New (%)	2.6	0.9	0.7	0.2	0.6	5.0

Table 4 Distribution for Total Delays and On-Time Performance

	Total delay			On-time rates		
	>15 min	>60 min	>120 min	15 min	60 min	120 min
Old (%)	22.3	7.9	2.9	77.7	92.1	97.1
New (%)	20.7	6.9	2.6	79.3	93.1	97.4

Table 5 On-Time Performance Rank for U.S. Major Airlines

Airlines	Northwest	Continental	Delta	TWA	Southwest
On-time rates (%)	79.2	77.7	77.3	76.7	76.2
Rank	1	2	3	4	5

delay minutes resulting from our new routing solution; and column "% of PD reduced" indicates the percentage reduction in propagated delay. On average, the RAMR model reduces total propagated delay in August by 44% compared to the aircraft routings used by the airline.

The distribution of propagated delays (in August 2000) for both the actual aircraft routings and our routings are summarized in Table 3. The notation "$(a, b]$" indicates that the propagated delay is greater than a minutes and less than or equal to b minutes. The row "Old" represents the percentage of flight legs with propagated delay in the specified ranges for the actual routings, and the row "New" represents the percentage of flight legs with propagated delay in the specified ranges for the new routings. As the table shows, the new routing solution reduces the number of delayed flight legs for each possible range.

The distributions of total delays for both the existing routing and new routings, using August 2000 data for the four networks, are summarized in Table 4. The Department of Transportation (DOT) on-time arrival rate (delay less than 15 minutes) increases 1.6%, while the 60- and 120-minute on-time rates (arrival delay less than 60 and 120 minutes, respectively) are also improved. Note that an increase of 1.6% in the on-time performance of any of the airlines listed in Table 5 (Bureau of Transportation Statistics 2003) would have resulted in at least a one-position improvement for that airline. This is of significance to airlines because the DOT on-time ranking is publicly available and often cited as an important indicator of airline performance.

Passenger Disruptions. We investigated the effect of our routing solution on passenger disruptions by comparing the number of disrupted passengers based on existing routings with those for our new routings.

Figure 3 illustrates some concepts related to passenger disruption. In this section, we consider disrupted passengers to be those passengers who miss their connections because of flight leg delays. As defined in §3.2, PDT refers to planned departure time, and ADT refers to actual departure time. PAT refers to planned arrival time, and AAT refers to actual arrival time. MCT refers to the minimum connecting time needed by a passenger to connect to the next flight leg in his or her itinerary. PCT refers to planned connecting time, and ACT refers to the actual connecting time. Slack is the difference between the planned connecting time and the minimum connecting time. The relationships between these terms are summarized as follows:

$$PCT = PDT - PAT, \tag{12}$$

$$Slack = PCT - MCT, \tag{13}$$

and

$$ACT = ADT - AAT. \tag{14}$$

For any connecting passenger, he/she will be disrupted if

$$ACT < MCT. \tag{15}$$

To determine the number of disrupted passengers, we first compute the departure and arrival times for each flight using Algorithm 1 and then determine the departure and arrival times of each flight leg for our new routings using:

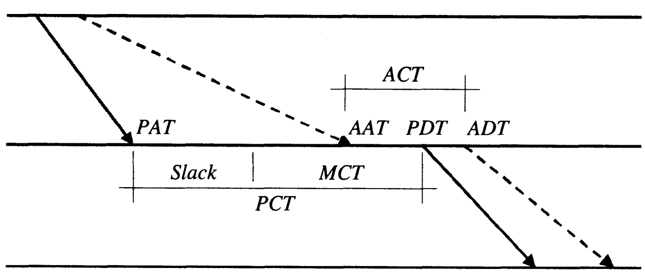

Figure 3 Passenger Disruption

Algorithm 4. $ADT_j = PDT_j + TDD_j$, where ADT_j is the actual departure time of flight j expected if our new routing is utilized; and

$AAT_j = PAT_j + TAD_j$, where AAT_j is the actual arrival time of flight j expected if our new routing is implemented.

The next step, calculating the number of disrupted passengers for a given routing solution, is achieved according to the following rules.

1. If a flight leg is cancelled, all passengers on that flight leg are disrupted.

2. If flight leg A is followed by flight leg B and both flight legs are operated, and $ADT_B - AAT_A < T_{\min}$, where $T_{\min}$ is the minimum connecting time for a passenger, then all passengers connecting from flight leg A to flight leg B are disrupted.

3. For those flight legs without ASQP records (that is, flight legs operated by nonjet aircraft), we do not have the data for the actual departure and arrival times. Therefore, we count only the disrupted passengers with connections for which all flight legs have ASQP records.

4. Passengers are counted as disrupted at most once. If a passenger is disrupted on any flight leg of his/her itinerary, that passenger is not counted as disrupted on any other flight leg.

Using the above rules, we estimate the number of disrupted passengers in August 2000 for both the historical routing and our new routing. The results are summarized in Table 6.

Column "Total num of D-pax" represents the total number of disrupted passengers caused by flight leg delays (not by flight leg cancellations) for the historical routing. Because the number of passengers disrupted by flight cancellations in our experiments is independent of the routings, we do not include them in our analysis. In actuality, routings with less propagated delay might result in fewer cancellations, further reducing the number of disrupted passengers. Column "D-pax reduced" represents the reduction in the number of disrupted passengers using our new routing solution, and column "D-pax reduced (%)" represents the percentage reduction in disrupted passengers. On average, our RAMR approach reduces by about 11% the number of passengers disrupted by flight leg delays.

Table 6 **Results on Disrupted Passengers**

Network	Total num of D-pax	D-pax reduced	D-pax reduced (%)
N1	986	147	14.9
N2	1,070	79	7.4
N3	1,463	161	11.0
N4	3,323	355	10.7
Total	6,842	742	10.8

In summary, our RAMR approach can reduce total propagated delay, improve on-time performance, and reduce the number of disrupted passengers.

4. Flight Schedule Retiming to Reduce Passenger Missed Connections

If connection slack is absorbed by flight leg delay, passengers connecting between two flight legs will be disrupted. Adding more slack can be good for connecting passengers, but can result in reduced productivity of the fleet. The challenge then is to determine where to add this slack so as to maximize the benefit to passengers without requiring additional aircraft to fly the schedule. Moving flight leg departure times provides an opportunity to allocate slack to reduce passenger disruptions and maintain aircraft productivity. In practice, flight leg departure times are adjusted in small time windows beginning several weeks before the flight leg's departure up until the day of departure.

Levin (1971) proposed the idea of adding time windows to fleet routing and scheduling models. Related research can be found in Desaulniers et al. (1997), Klabjan et al. (1999), Rexing et al. (2000), and Stojkovic et al. (2002).

The time window, specifying how much time a given flight leg can be shifted, can be modeled with a simple extension of the basic flight network. By placing copies of a flight arc at specified intervals within that flight's time window and requiring only one of the flight arc copies to be used, we model the choice of flight leg departure time. Because the scheduled time of some flight legs is more flexible than others, the width of each time window is a parameter that can be different for every flight. Moreover, the interval between copies is another parameter, one that can impact the tractability of the model and quality of the solution. To guarantee that flight legs are allowed to depart at any time within the time window, copies should be placed at one-minute intervals. It will be shown, however, in §4.3 that using a narrow interval instead of a broader one causes an explosion in the problem size, but often fails to generate substantially better solutions. We generate *robust schedules* minimizing the number of disrupted passengers by selecting flight leg departure times for specified (relatively short) departure time windows, given the flight schedule, fleet assignment, and aircraft routing decisions.

4.1. Flight Schedule Retiming Models and Their Properties

4.1.1. A Connection-Based Flight Schedule Retiming Model.

Let binary decision variable $f_{i,n}$ for each flight leg i copy n equal one if flight leg i copy

n is selected, and zero otherwise, and let binary variable $x_{i,n}^{j,m}$, representing the connection between flight leg i copy n and flight leg j copy m, equal one if the connection between flight leg i copy n and flight leg j copy m is selected, and zero otherwise. Let $dp_{i,n}^{j,m}$ be the number of disrupted passengers between flight leg i and flight leg j if flight leg i copy n and flight leg j copy m are selected. We denote the set of all flight legs as F, the set of all flight legs to which passengers connect as F^I, and the set of all flight legs from which passengers connect as F^O. Let N_i be the number of copies generated for flight leg i. We denote the set of flight legs with passengers connecting from flight leg i as $C^+(i)$. Similarly, we denote the set of flight legs with passengers connecting to flight leg i as $C^-(i)$. Our objective is to minimize the expected total number of disrupted passengers, subject to the following constraints.

1. For each flight, exactly one copy must be selected.
2. Each selected connection between two flight legs i and j must connect the selected copies of flight legs i and j. For example, if flight leg i copy 2 and flight leg j copy 3 are selected, then the copy of the connection from flight leg i copy 2 to flight leg j copy 3 must be selected, that is, $x_{i,2}^{j,3} = 1$.
3. The current fleeting and routing solutions cannot be altered.

Objective Function. The objective function of our retiming model to minimize the expected number of disrupted passengers can be written as

$$\begin{aligned}\min E\left[\sum_{i\in F^O}\sum_{n\in N_i}\sum_{j\in C^+(i)}\sum_{m\in N_j} dp_{i,n}^{j,m} x_{i,n}^{j,m}\right] \\ = \min \sum_{i\in F^O}\sum_{n\in N_i}\sum_{j\in C^+(i)}\sum_{m\in N_j} x_{i,n}^{j,m} \times E\left[dp_{i,n}^{j,m}\right].\end{aligned}$$

To compute $E[dp_{i,n}^{j,m}]$, we need to know the distribution of $dp_{i,n}^{j,m}$, that is, the number of disrupted passengers connecting from flight leg i copy n to leg j copy m, for all flight legs i and j and all copies n and m. We assume that if the difference between the actual departure time of flight leg j and the actual arrival time of flight leg i is less than the minimum connecting time MCT, all passengers connecting from flight leg i to flight leg j are disrupted. And, if the difference is at least as great as MCT, connecting passengers are not disrupted. Based on this, the distribution of $dp_{i,n}^{j,m}$ is a binary distribution, namely

$$dp_{i,n}^{j,m} = \begin{cases} c_{ij} & \text{with probability } p \\ 0 & \text{with probability } 1-p, \end{cases} \tag{16}$$

where c_{ij} is the number of passengers connecting from flight leg i to flight leg j. Probability p is determined as follows:

$$p = \text{prob}(ADT_{j,m} - AAT_{i,n} < MCT), \tag{17}$$

where $ADT_{j,m}$ is the actual departure time of flight leg j if copy m is selected, and $AAT_{i,n}$ is the actual arrival time of flight leg i if copy n is selected. As discussed in §3, the distribution of ADT and AAT for each flight leg can be determined for any flight schedule, fleeting, and routing. Then, $E[dp_{i,n}^{j,m}]$ can be determined for each connection between any pair of flight legs.

Model Formulation. The connection-based flight schedule retiming (CFSR) model is written as follows:

$$\text{Min} \sum_{i\in F^O}\sum_{n\in N_i}\sum_{j\in C^+(i)}\sum_{m\in N_j} x_{i,n}^{j,m} \times E\left[dp_{i,n}^{j,m}\right] \tag{18}$$

subject to

$$\sum_{n\in N_i} f_{i,n} = 1 \quad \forall i \in F, \tag{19}$$

$$\sum_{m\in N_j} x_{i,n}^{j,m} = f_{i,n} \quad \forall i \in F^O,\ n \in N_i,\ j \in C^+(i), \tag{20}$$

$$\sum_{n\in N_i} x_{i,n}^{j,m} = f_{j,m} \quad \forall j \in F^I,\ m \in N_j,\ i \in C^-(j), \tag{21}$$

$$f_{i,n} \in \{0,1\} \quad \forall i \in F,\ n \in N_i, \tag{22}$$

$$x_{i,n}^{j,m} \in \{0,1\} \quad \forall i \in F^O, n \in N_i, j \in C^+(i), m \in N_j. \tag{23}$$

The objective function (18) minimizes the expected total number of disrupted passengers. Constraints (19) are cover constraints that ensure, given the integrality requirements of each variable f (22), exactly one copy will be selected for each flight leg. Constraints (20) and (21), with (22) and (23), jointly ensure that variables f and x are selected consistently. As we explained above, this problem will be solved after solving the fleet assignment and aircraft maintenance routing problems. Therefore, we need to add constraints to maintain the current fleeting and routing solution, as discussed in the next section.

Enabling Current Routings and Itineraries. To maintain the current fleeting, aircraft routings, and passenger itineraries while selecting flight departure times, we must ensure that (1) the planned turn time for each aircraft always exceeds the minimum turn time, and (2) the planned connection time for each passenger always exceeds the minimum connecting time. For example, in Figure 4, suppose flight legs 1 and 2 are in an aircraft route, or in a passenger itinerary. If the time between the arrival of flight leg copy $f_{1,7}$ and the departure of flight leg copy $f_{2,1}$ is less than the minimum turn time, or alternatively the

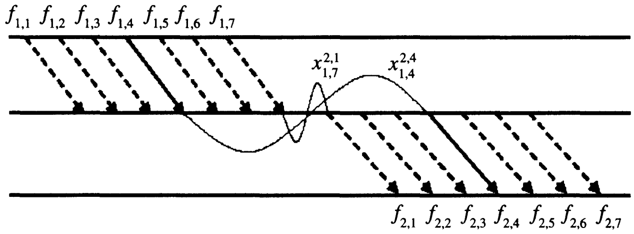

Figure 4 Example: How to Keep Current Routing Solution

minimum connecting time, then flight leg 1 copy 7 and flight leg 2 copy 1 cannot be selected together, implying that $x_{1,7}^{2,1}$ must equal 0. In general, for any pair of flight legs $i-j$ in an aircraft route or passenger itinerary, we can keep the current solution feasible by forcing $x_{i,n}^{j,m}=0$ if the time between the arrival of flight leg copy $f_{i,n}$ and the departure of flight leg copy $f_{j,m}$ is less than the minimum turn or connecting time. This can be implemented easily by setting to zero the upper and lower bounds for each such x variable, or by not including these variables in the model.

Model Properties. In this section, we analyze the CFSR model properties. Specifically, constraints (23) can be eliminated as shown in the following.

THEOREM 1. *The integrality of the connection variables (constraints* (23)) *can be relaxed.*

PROOF. Consider flight legs i_1 and j_1 such that flight leg i_1 is followed by flight leg j_1 in an aircraft routing. Constraints (19) and (22) ensure that, for every flight leg, exactly one copy will be selected. Suppose copy n_1 of flight leg i_1 and copy m_1 of flight leg j_1 are selected, then

$$f_{i_1,n_1}=1; \qquad f_{i_1,n}=0, \quad \forall n\in N_{i_1} \text{ and } n\neq n_1,$$
$$f_{j_1,m_1}=1; \qquad f_{j_1,m}=0, \quad \forall m\in N_{j_1} \text{ and } m\neq m_1.$$

From constraints (20), we have

$$\forall n\in N_{i_1} \text{ and } n\neq n_1, \quad f_{i_1,n}=0=\sum_{m\in N_{j_1}} x_{i_1,n}^{j_1,m}.$$

Because $x\geq 0$, this implies

$$x_{i_1,n}^{j_1,m}=0, \quad \forall n\in N_{i_1} \text{ and } n\neq n_1,\ m\in N_{j_1}.$$

Similarly, from constraints (21), we have

$$x_{i_1,n}^{j_1,m}=0, \quad \forall n\in N_{i_1},\ \forall m\in N_{j_1} \text{ and } m\neq m_1,$$

which implies

$$\sum_{m\in N_{j_1},\, m\neq m_1} x_{i_1,n}^{j_1,m}=0, \quad \forall n\in N_{i_1},$$

and

$$\sum_{m\in N_{j_1},\, m\neq m_1} x_{i_1,n_1}^{j_1,m}=0.$$

Thus, together with constraints (20), we have

$$\begin{aligned} f_{i_1,n_1} &= 1=\sum_{m\in N_{j_1}} x_{i_1,n_1}^{j_1,m} \\ &= x_{i_1,n_1}^{j_1,m_1}+\sum_{m\in N_{j_1},\, m\neq m_1} x_{i_1,n_1}^{j_1,m} \\ &= x_{i_1,n_1}^{j_1,m_1}. \end{aligned}$$

Hence, for any pair of flight legs i_1 and j_1

$$x_{i_1,n_1}^{j_1,m_1}=1 \quad \text{and} \quad x_{i_1,n}^{j_1,m}=0,$$
$$\forall n\in N_{i_1},\ m\in N_{j_1} \text{ and } n\neq n_1 \text{ or } m\neq m_1. \quad \square$$

The CFSR model can thus be rewritten equivalently by replacing (23) with

$$0\leq x_{i,n}^{j,m}\leq 1 \quad \forall i\in F^O,\ n\in N_i,\ j\in C^+(i),\ m\in N_j. \quad (24)$$

Alternative, yet equivalent, formulations of the connection-based flight schedule retiming model can be found in Lan (2003). Lan (2003) proves that the LP relaxation of the CFSR model is at least as strong as those of alternative formulations he considers, and can be stronger in some instances.

4.2. Solution Approach

4.2.1. Overview of the Solution Approach. The CFSR formulation is a deterministic mixed-integer program with a large number of variables (recall that in CFSR, we must consider all flight and passenger connections for all fleet types). For practical problems, complete generation of all variables will require prohibitive amounts of time and memory. Thus, we solve these problems using branch-and-price (see §3.4 for a detailed description).

4.2.2. Branching Strategy. After solving an LP relaxation at a node of the branch-and-bound tree, we branch based on the cover constraints:

$$\sum_{n\in N_i} f_{i,n}=1 \quad \forall i\in F.$$

Building on the results of Hane et al. (1995), we employ special ordered set branching in which we divide the set of variables $f_{i,n}$ for each flight leg i into two sets. We force the sum of the variables in the first set to equal one on one branch, and the sum of the variables in the second set to equal one on the other branch. For fleet assignment problems, Hane et al. (1995) show that this is a more effective branching strategy than branching on individual variables.

4.2.3. Column Generation. At an iteration of the column generation algorithm, let $\pi_{i,n}^{j}$ be the optimal dual variables associated with constraints (20) and

$\pi_i^{j,m}$ be the optimal dual variables associated with constraints (21). Then, the reduced cost for each connection copy between flight legs i and j is

$$\overline{dp}_{i,n}^{j,m} = E[dp_{i,n}^{j,m}] - \pi_{i,n}^{j} - \pi_i^{j,m}. \tag{25}$$

Because the number of columns is just over one million for a typical airline problem, a large but manageable number, the reduced cost for each copy of each connection can be calculated explicitly and all columns with negative reduced costs are added to the restricted master problem at each iteration.

4.3. Proof of Concept

4.3.1. Underlying Networks. For the computational experiments with our retiming model, we combine the four networks (described in Table 1) to form one network with a total of 278 flight legs and four fleet types. Because there are many passengers connecting in this network, we also consider flight legs in the full airline network that form passenger connections with the flight legs in the 278 flight leg network. For these additional flight legs, we fix the current schedule. The total number of flight legs considered in this expanded network is 1,067.

4.3.2. Data and Validation. We use the same July and August 2000 data used in our computational experiments in §3.5. We solve our robust aircraft maintenance routing model (see §3) using July 2000 data to obtain a routing solution, and then compute the corresponding delays for each flight leg. Given these delays, the expected number of disrupted passengers for each connection copy is estimated. The sample average of the number of disrupted passengers is used as an approximation of the mean. Then, we solve our flight leg schedule retiming models on the July 2000 data to determine flight leg departure times for August 2000. Next, using the departure times selected by our model, we calculate the number of disrupted passengers for August 2000. We also compute the number of disrupted passengers for the actual August 2000 schedule.

4.3.3. Computational Results. The results obtained by applying our flight leg schedule retiming models to the network of a major U.S. airline (described in §4.3.1) are presented below. Problems are solved using CPLEX 6.5 on an HPC 3000 machine with 1 GB RAM.

Size and Bound. Using a 30-minute time window allowing flight legs to depart at most 15 minutes earlier or later than originally scheduled, we generate copies for flight arcs every 5 minutes, for a total of 7 copies in each flight leg's time window. The numbers of constraints, variables, and nonzeros in the CFSR model are 7,506, 27,013, and 59,836, respectively. The LP relaxation of the CFSR model is very tight. For this problem instance, an optimal solution is found at the root node of the branch-and-bound tree, requiring only 13 seconds to find an optimal solution.

Table 7 **Effects of Retiming on Numbers of Disrupted Passengers (August 2000 Data)**

Time window	Old D-pax	New D-pax	D-pax reduced	D-pax reduced (%)
±15 min (7 copies)	18,808	11,348	7,460	39.7
±10 min (5 copies)	18,808	12,732	6,076	32.3
±5 min (3 copies)	18,808	15,042	3,766	20.0

Misconnections and Time Window Width. The number of passenger misconnections that can be avoided through retiming is shown in Table 7 for varying time windows. "Time window" indicates the total time (in minutes) flight legs are allowed to shift and the number of copies of flight legs generated in this time window. For example, ±15 min (7 copies) allows each flight leg to depart at most 15 minutes earlier or later than originally scheduled. Because we generate copies for flight arcs every five minutes, there are seven copies in this time window. "Old D-pax" indicates the total number of passenger misconnections in the original schedule, and "New D-pax" indicates the number of passenger misconnections in our new schedule. "D-pax reduced" and "D-pax reduced (%)" indicate the difference in the number (and percentage) of passenger misconnections between the old and new schedules. Note that in our computational experiment, we consider only those passengers whose itineraries have at least 1 flight leg included in the subnetwork with 278 flight legs. The disruption status of all other passengers is unchanged by our retiming solution.

The results obtained by applying our retiming decisions based on July 2000 data to the August 2000 flight network are summarized in Table 7. If flight leg departure times are allowed to shift in a 30-minute time window, about 40% fewer passengers miss their connections, while a 20-minute time window reduces the number of passenger misconnections by over 30%, and a 10-minute time window reduces it by 20%.

Effects of Copy Interval. In Table 8, we provide results of our analysis in which we assumed a minimum connection time of 30 minutes and varied the flight leg copy interval in time windows of various widths. "Increase" indicates the factor increase in the numbers of nonzeros in the model compared to the base case with a five-minute copy interval. In Table 9, "Improve" indicates the percentage reduction in the number of disrupted passengers, again compared to a five-minute copy interval. Generating copies for flight legs every minute results in dramatically increased

Table 8 Comparison of the Problem Sizes (Five-Minute Copy Interval vs. One-Minute Copy Interval)

Time window	Num of constrs	Num of vars	Num of nonzeros	Increase
±15 min (7 copies)	7,506	27,013	59,836	1.0
±15 min (31 copies)	32,514	507,253	1,040,236	17.4
±10 min (5 copies)	5,422	14,085	32,320	1.0
±10 min (21 copies)	22,094	234,213	485,856	15.0
±5 min (3 copies)	3,338	5,325	13,140	1.0
±5 min (11 copies)	11,674	65,373	139,876	10.6

Table 9 Comparison of Numbers of Disrupted Passengers (Five-Minute Copy Interval vs. One-Minute Copy Interval)

Time window	Old D-pax	New D-pax	D-pax reduced	Improve (%)
±15 min (7 copies)	17,459	10,899	6,560 (37.6%)	0.0
±15 min (31 copies)	17,459	10,865	6,594 (37.8%)	0.52
±10 min (5 copies)	17,459	12,070	5,389 (30.9%)	0.0
±10 min (21 copies)	17,459	12,056	5,403 (30.9%)	0.26
±5 min (3 copies)	17,459	14,069	3,390 (19.4%)	0.0
±5 min (11 copies)	17,459	14,058	3,401 (19.5%)	0.28

problem sizes and modest benefit. By placing copies more sparsely, we improve model tractability considerably and obtain solutions that are nearly as good.

Estimating the Impact on Passenger Delays. The passenger delay experienced in August 2000, using historical data, is 419 minutes, with disrupted passengers accounting for 51% of total passenger delay (in minutes). By applying our model (with the minimum connecting time of 30 minutes and flight leg copies generated every 5 minutes within 30-minute time windows), we achieve a reduction of about 40% in the total number of disrupted passengers and a corresponding 20% decrease in total passenger delay. Moving from 30- to 20-minute time windows decreases delay minutes by about 16%, while a 10-minute time window achieves a reduction of roughly 10%.

5. Possible Extensions

5.1. Integrated Robust Aircraft Maintenance Routing and Fleet Assignment

The string-based model proposed by Barnhart et al. (1998b) can solve fleet assignment and maintenance routing problems at the same time. Similarly, one extension for our robust aircraft maintenance routing model is to adopt it to solve integrated fleet assignment and maintenance routing. Adding fleeting decisions results in more feasible strings, potentially leading to improved solutions with reduced delay propagation. When solving integrated fleet assignment and maintenance routing, however, it is inappropriate to minimize delay propagation without considering fleet assignment costs. Applying an idea similar to that proposed by Rosenberger, Johnson, and Nemhauser (2001b), we develop two integrated models for robust aircraft maintenance routing and fleet assignment (see Lan 2003 for details). The first model minimizes total fleet assignment and maintenance routing costs, but constrains total expected propagated delay to a specified threshold value. The second model minimizes total expected propagated delay, and limits fleet assignment and maintenance routing costs to a particular upper bound.

5.2. Robust Aircraft Maintenance Routing with Time Windows

Allowing flight legs to be rescheduled within a small time window and simultaneously determining aircraft routings could lead to a more robust routing solution, one that minimizes delay propagation. To model this problem, the string-based model with copies of each flight leg can be used (see Lan 2003 for details). We can also integrate robust maintenance routing with time windows and fleet assignment to enhance robustness of the plan. Likely, such a model will have tractability issues when solving large-scale problems. Research in this direction should focus on better formulations of the problem and/or new ways to reduce problem size and exploit problem structure.

5.3. Fleet Assignment with Time Window and Passenger Disruption Considerations

Recall in §4, we minimize the number of disrupted passengers by adding a time window for each flight. Integrating this model and the fleet assignment with time windows model allows fleeting decisions to be affected by their impact on passenger disruptions. The difficulty is in determining the costs of passenger disruptions. Passenger disruptions result not only in reaccommodation costs but also in costs associated with loss of goodwill. Thus, similar to what we have done in §5.1, we develop two integrated models for balancing fleet assignment costs with improvements in passenger travel times (see Lan 2003 for details). The first model minimizes the fleet assignment cost but constrains the expected number of disrupted passengers to an upper bound. The second model minimizes the expected number of disrupted passengers and limits fleet assignment costs.

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