

Now that the simulation is behaving as anticipated, I will explore the question of initial values. For this study, I consider the following DGP:

$$y_{it} = \alpha_i \beta_t + \eta_i \gamma_t e_{it} \quad (3)$$

Where  $\eta_i, \gamma_t \sim \chi_1^2$  to satisfy  $\eta_i \gamma_t > 0$ . I fit a QFM using the same procedure as before, but with a change in the initialization step. Instead of beginning with PCA to guess the initial values, I add noise from a  $\mathcal{N}(0,1)$  to the initial PCA estimate. This is then rotated according to the errors-in-variables rotation I have used previously. The model is fit from these new initial values, and some interesting patterns appear. Consider the examples in tables 1, 2, and 3.

TABLE 1: QFM FITS WITH DIFFERENT NOISE - BAD SECOND FACTOR FIT

| Noise Seed | First Factor Fit | Second Factor Fit | Iterations | Objective Function Value |
|------------|------------------|-------------------|------------|--------------------------|
| No Noise   | 0.9921           | 0.1600            | 14         | 3909.9828                |
| 134935     | 0.9912           | 0.7791            | 33         | 3595.2488                |
| 363439     | 0.9924           | 0.7245            | 18         | 3645.0155                |
| 880628     | 0.9927           | 0.1601            | 24         | 4005.6856                |
| 252318     | 0.9954           | 0.7844            | 84         | 3677.4222                |
| 344982     | 0.9965           | 0.7391            | 16         | 3591.6591                |
| 702677     | 0.9920           | 0.8051            | 54         | 3555.9745                |
| 116075     | 0.9910           | 0.1600            | 19         | 3915.6519                |
| 100298     | 0.9934           | 0.7755            | 25         | 3667.7547                |
| 178700     | 0.9943           | 0.1599            | 28         | 3940.8150                |
| 310893     | 0.9903           | 0.7956            | 28         | 3564.4116                |
| True       | 0.9950           | 0.8008            | 47         | 3525.9113                |

**Notes:** This table reports the  $R^2$  value for a regression each of the true factors on the estimated factors, as well as the value of the objective function evaluated at the estimate. Each observation is a different seed for random noise added to initial values generated by PCA.

In table 1, the fit based on the PCA starting values is quite poor for the second factor. Using different starting values, I am able to get a better fit for the second factor which is mirrored with a large decrease in the value of the objective function. Adding noise can also make the fit worse in terms of objective function value.

In table 2, the fit based on the PCA starting values is quite poor for both factors. Using different starting values, the fit is improved dramatically with the first factor being fit well and the second factor being fit decently.

In table 3, the fit based on the PCA starting values is very good. However, different starting values can produce a lower value of the objective function.

TABLE 2: QFM FITS WITH DIFFERENT NOISE - POOR PCA FIT

| Noise Seed | First Factor Fit | Second Factor Fit | Iterations | Objective Function Value |
|------------|------------------|-------------------|------------|--------------------------|
| No Noise   | 0.0119           | 0.0639            | 3          | 4548.2392                |
| 32851      | 0.9983           | 0.4868            | 91         | 2921.2252                |
| 305155     | 0.9985           | 0.1999            | 28         | 3004.2602                |
| 530302     | 0.9979           | 0.0911            | 26         | 3056.5396                |
| 53515      | 0.9965           | 0.4573            | 21         | 2967.1183                |
| 965115     | 0.9975           | 0.0176            | 100        | 3185.0630                |
| 201177     | 0.0703           | 0.3880            | 95         | 4157.0256                |
| 380261     | 0.9975           | 0.0422            | 11         | 3015.5472                |
| 406115     | 0.9982           | 0.0828            | 100        | 3128.7703                |
| 445781     | 0.9974           | 0.3452            | 15         | 2956.7660                |
| 912360     | 0.9981           | 0.3322            | 72         | 2969.2764                |
| True       | 0.9971           | 0.5003            | 7          | 2901.3943                |

**Notes:** This table reports the  $R^2$  value for a regression each of the true factors on the estimated factors, as well as the value of the objective function evaluated at the estimate. Each observation is a different seed for random noise added to initial values generated by PCA.

TABLE 3: QFM FITS WITH DIFFERENT NOISE - GOOD PCA FIT

| Noise Seed | First Factor Fit | Second Factor Fit | Iterations | Objective Function Value |
|------------|------------------|-------------------|------------|--------------------------|
| No Noise   | 0.9969           | 0.8077            | 68         | 3119.1333                |
| 688017     | 0.9968           | 0.8088            | 48         | 3112.0576                |
| 7268       | 0.9971           | 0.7998            | 20         | 3155.9562                |
| 934064     | 0.9969           | 0.8006            | 25         | 3164.4899                |
| 343860     | 0.9974           | 0.7982            | 18         | 3141.5936                |
| 798375     | 0.9965           | 0.7849            | 19         | 3050.5442                |
| 631366     | 0.9967           | 0.7847            | 71         | 3257.3209                |
| 124501     | 0.9957           | 0.7886            | 38         | 3184.5473                |
| 163844     | 0.9959           | 0.7902            | 51         | 3116.7062                |
| 673964     | 0.9975           | 0.7865            | 45         | 3170.8194                |
| 272234     | 0.9973           | 0.7840            | 19         | 3144.6454                |
| True       | 0.9992           | 0.8096            | 10         | 2989.2456                |

**Notes:** This table reports the  $R^2$  value for a regression each of the true factors on the estimated factors, as well as the value of the objective function evaluated at the estimate. Each observation is a different seed for random noise added to initial values generated by PCA.

Next, I consider starting estimation with the true factor levels and loadings. In each of the 3 examples above, using the true levels and loadings improves the fit and lowers the objective function value. I think this can be thought of as the ideal scenario, at least for the angle from which I am approaching the problem. The trouble we have with PCA is that it is based on the mean, and so is uninformative of the true value of the second factor because at the median, the second factor doesn't contribute. We want the initial value to be close to the true value so that we are close to the global optimum in the objective function. This is then a benchmark against which methods can be compared.