# Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

**Market-Basket transactions**

| *TID* | *Items* |
|---|---|
| **1** | **Bread, Milk** |
| **2** | **Bread, Diaper, Beer, Eggs** |
| **3** | **Milk, Diaper, Beer, Coke** |
| **4** | **Bread, Milk, Diaper, Beer** |
| **5** | **Bread, Milk, Diaper, Coke** |

**Example of Association Rules**

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

# Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

| *TID* | *Items* |
|---|---|
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| 5 | Bread, Milk, Diaper, Coke |

# Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example: $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$
- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

| *TID* | *Items* |
|---|---|
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Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

# Association Rule Mining

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

# Mining Association Rules

| *TID* | *Items* |
|---|---|
| **1** | **Bread, Milk** |
| **2** | **Bread, Diaper, Beer, Eggs** |
| **3** | **Milk, Diaper, Beer, Coke** |
| **4** | **Bread, Milk, Diaper, Beer** |
| **5** | **Bread, Milk, Diaper, Coke** |

## Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

## Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

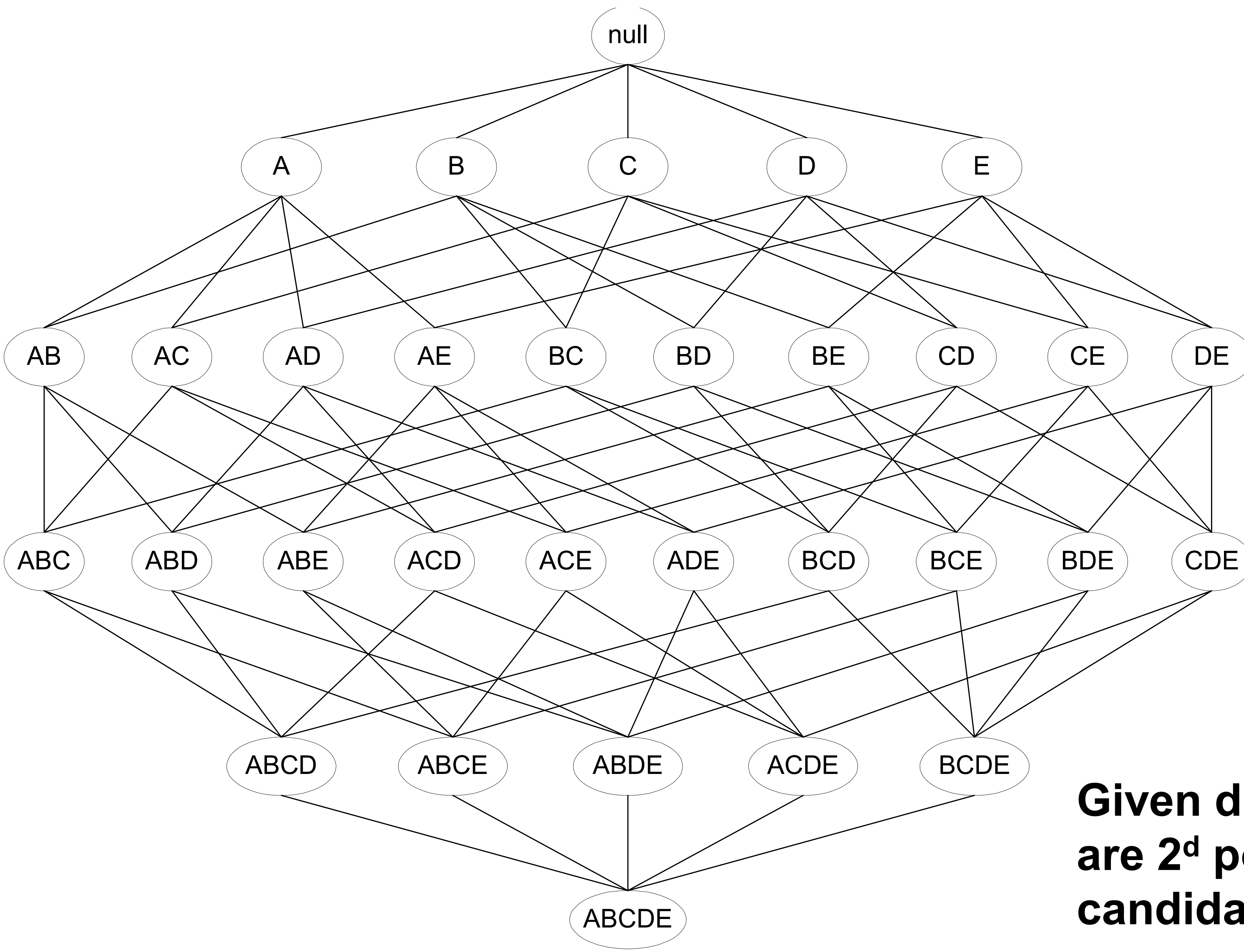
# Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support ≥ minsup

  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

# Frequent Itemset Generation




**Given d items, there are $2^d$ possible candidate itemsets**

# Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

**Transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
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N

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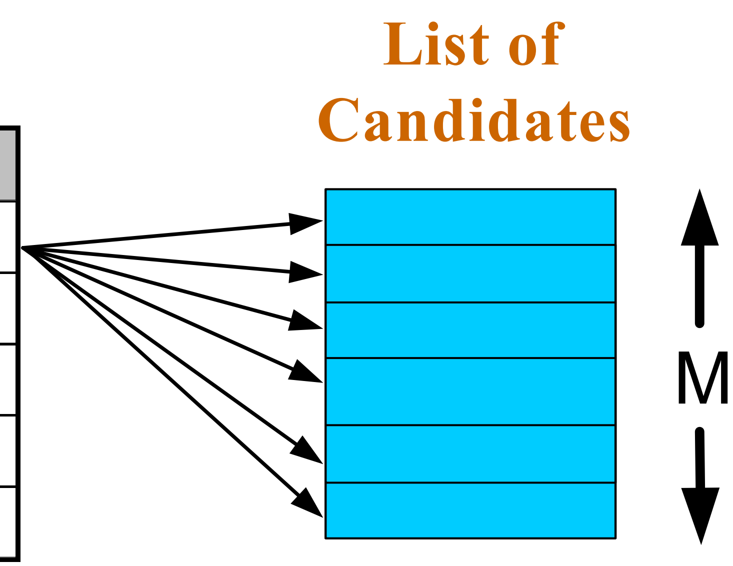
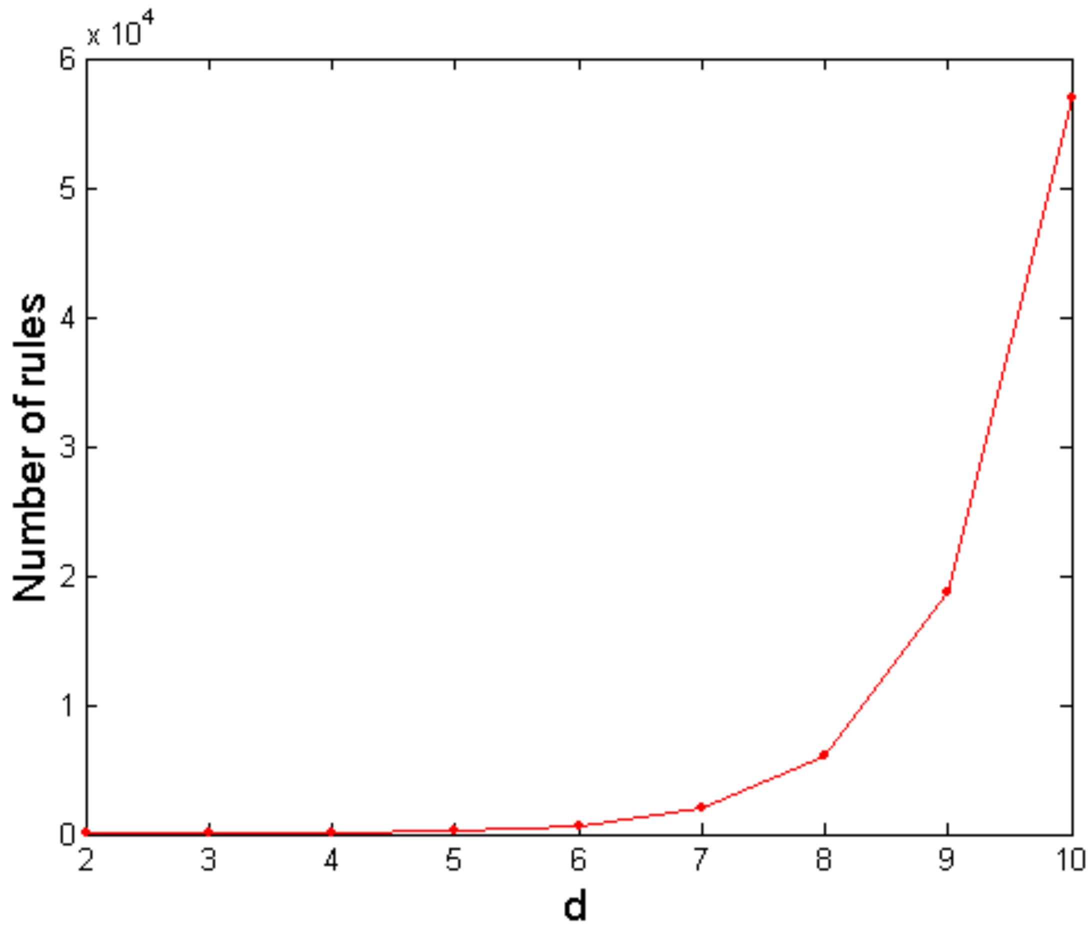



  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$ !!!

# Computational Complexity

- Given d unique items:
  - Total number of itemsets = $2^d$
  - Total number of possible association rules:




$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$

$$= 3^d - 2^{d+1} + 1$$

**If d=6, R = 602 rules**

# Reducing Number of Candidates

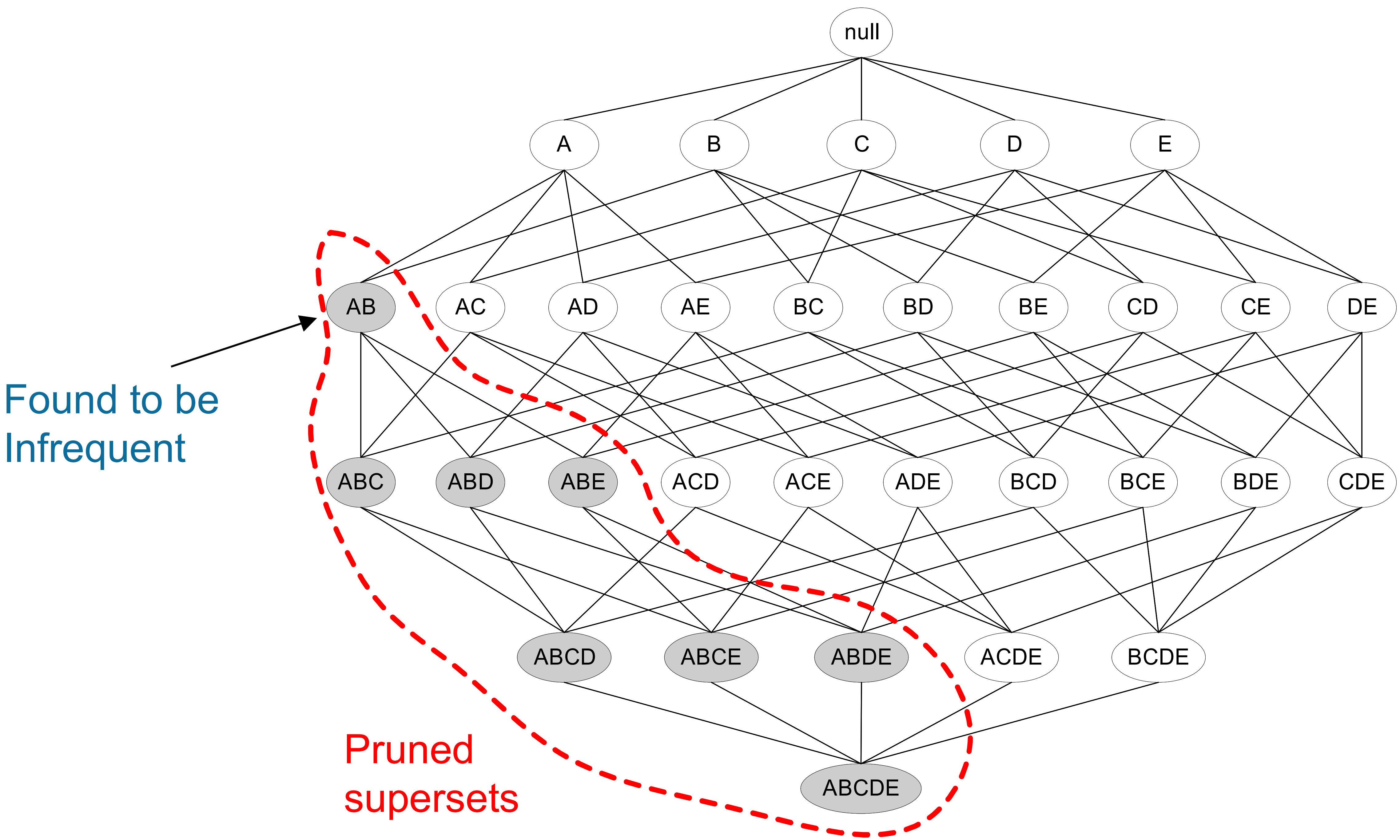
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

# Illustrating Apriori Principle



# Illustrating Apriori Principle

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk}** | **3** |
| **{Bread,Beer}** | **2** |
| **{Bread,Diaper}** | **3** |
| **{Milk,Beer}** | **2** |
| **{Milk,Diaper}** | **3** |
| **{Beer,Diaper}** | **3** |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk,Diaper}** | **3** |

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
$6 + 6 + 1 = 13$

# Apriori Algorithm

- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length (k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subsets of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent

# Recall: Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

# Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

| | | | |
|---|---|---|---|
| $ABC \rightarrow D$, | $ABD \rightarrow C$, | $ACD \rightarrow B$, | $BCD \rightarrow A$, |
| $A \rightarrow BCD$, | $B \rightarrow ACD$, | $C \rightarrow ABD$, | $D \rightarrow ABC$ |
| $AB \rightarrow CD$, | $AC \rightarrow BD$, | $AD \rightarrow BC$, | $BC \rightarrow AD$, |
| $BD \rightarrow AC$, | $CD \rightarrow AB$, | | |

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

# Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

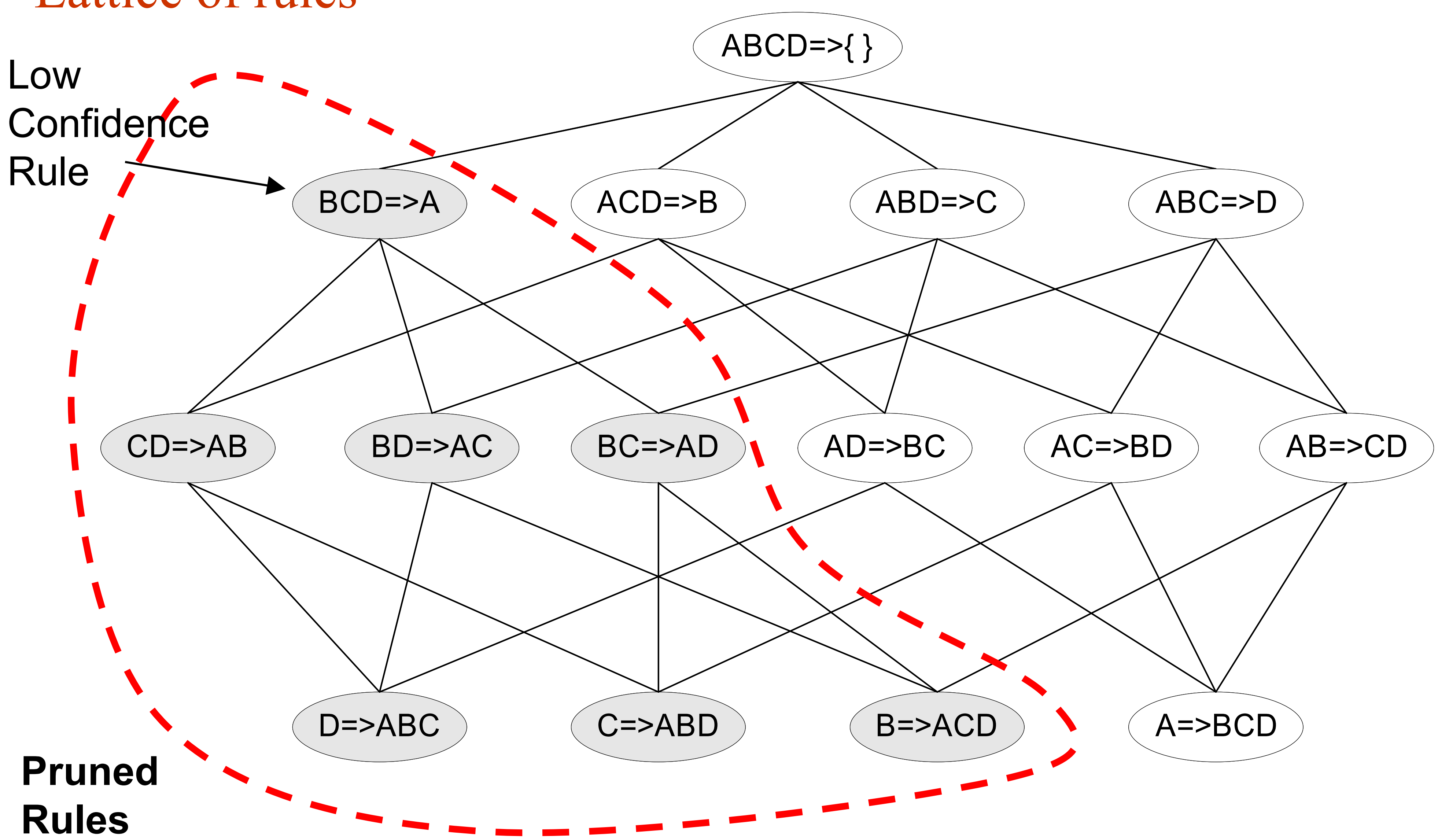
  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

# Rule Generation for Apriori Algorithm

Lattice of rules



# Evaluating Generated Rules

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if {A,B,C} $\rightarrow$ {D} and {A,B} $\rightarrow$ {D} have same support & confidence

- Interestingness measures can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

# Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

| | $Y$ | $\overline{Y}$ | |
|---|---|---|---|
| $X$ | $f_{11}$ | $f_{10}$ | $f_{1+}$ |
| $\overline{X}$ | $f_{01}$ | $f_{00}$ | $f_{o+}$ |
| | $f_{+1}$ | $f_{+0}$ | $\|T\|$ |

$f_{11}$: support of $X$ and $Y$
$f_{10}$: support of $X$ and $\overline{Y}$
$f_{01}$: support of $\overline{X}$ and $Y$
$f_{00}$: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

# Drawback of Confidence

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 15 | 5 | 20 |
| $\overline{\text{Tea}}$ | 75 | 5 | 80 |
| | 90 | 10 | 100 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) =

# Drawback of Confidence

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 15 | 5 | 20 |
| $\overline{\text{Tea}}$ | 75 | 5 | 80 |
| | 90 | 10 | 100 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Although confidence is high, rule is misleading

$\Rightarrow$ P(Coffee|$\overline{\text{Tea}}$) = 0.9375

# Statistical Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)} = \frac{\text{conf.}(X \rightarrow Y)}{\text{supp.}(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

# Example: Lift/Interest

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 15 | 5 | 20 |
| $\overline{\text{Tea}}$ | 75 | 5 | 80 |
| | 90 | 10 | 100 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = 0.75/0.9= 0.8333

# Subjective Interestingness Measure

- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).
- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)