

## *Predicting the First Visibility of the Lunar Crescent*

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We observe the first visibility of the lunar crescent on the western horizon shortly after sunset, and it is the beginning of the month in the Islamic calendar. There are numerous techniques for predicting the day of the first sight of the Moon, but our interest is in the physical method, which began with the investigations of Samaha, Assad, and Mikhail (1969) and Bruin (1977).

To predict when we will observe the first crescent Moon, we need:

1. The topocentric altitude of the Moon; azimuth difference between the centers of the Sun and the Moon ( $DZ$ ); Sun depression ( $d$ ); Earth-Moon distance and topocentric phase angle (or selenocentric angle between the observer's position and the center of the Sun).
2. The luminance of the Moon without atmospheric absorption for the phase angle; (we understand luminance as the luminous flux per unit of the luminous area perpendicular to the observation direction and per unit of solid angle).
3. The atmospheric extinction coefficient for the place of observation, which determines the luminance of the Moon  $B_m$  at the surface of the Earth.
4. The twilight sky luminance  $B_s$ , as a function of  $d$  and  $DZ$ .
5. The threshold contrast or the threshold illuminance for viewing the Moon in the twilight sky; (we define the contrast by  $C = B_m/B_s$ , and the illuminance is the luminous flux that reaches the observer per unit area normal to the observation direction).

Photometric measurements of the Moon at large phase angle are difficult since its observation has to be made at a low altitude above the horizon and therefore, is highly affected by atmospheric attenuation; also, the observation has to be done with twilight light, therefore the Moon's own illumination is added to the illumination of the sky, and finally, it must be added

that the Moon is rarely observed (and for a very short time) with a phase angle greater than  $170^\circ$ , because the brightness twilight sky masks the light emitted by the Moon.

To find the luminance of the Moon some researchers (Sultan, 2006) (Segura, 2021) have extrapolated the empirical formula of the magnitude of the Moon as a function of the phase angle (Allen, 1973, p. 144). Others (Schaefer, 1991) have applied Hapke's (1984) lunar photometric theory. However, we do not have experimental data to confirm that these methods give the true luminance of the Moon, which at large phase angles is highly influenced by macro and micro-shields and by the libration.

The extinction coefficient and the luminance of the sky are easy to measure, but it is impossible to anticipate their value, not even one day in advance. The luminance of the sky does not critically affect the view of the crescent, but a small variation in the extinction coefficient significantly affects the prediction of when the Moon we will see for the first time.

Blackwell (1946) did extensive research on the human eye's sensitivity to see a luminous object against a lit background. His results have been applied to the visibility of the crescent Moon but without being correctly understood.

Blackwell's experiment shows that threshold vision is a probabilistic process. We distinguish three zones of visibility of a luminous object on a bright background. The first area is when the contrast is very high, the probability of vision is 100%; we always see the object. Another is the zone of zero visibility; due to the small contrast, the observer never sees the luminous object. The third is the critical zone of visibility, with an intermediate contrast, where there is a probability of seeing the object. Under the same conditions, the same observer sometimes sees the object and sometimes not, with a certain probability given by Blackwell's results.

Blackwell's experiment showed that the probability of vision in the critical zone does not depend on the luminance of the background and is almost independent of the size of the object, depending exclusively on the threshold contrast. Blackwell gave the threshold contrast of his experiment for a probability of vision of 50%; that is, the observer sees the object half of the times he observes it. With a contrast lower than the threshold, the probability of vision is lower and vice versa.

Researchers have not noticed that Blackwell's experiment gives: 1) the threshold contrast as a function of image size and background luminance, and 2) the probability of vision for the ratio  $C/C_{th}$ ;  $C$  is the contrast of the object and  $C_{th}$  the threshold contrast according to Blackwell's tables.

Other researchers (Knoll, Tousey & Hulburt, 1946) devised experiments to find the threshold contrast for 100% probability. Comparison of the results of the various visual sensitivity experiments give similar results, but they are highly dependent on the experimental procedure

(pupil size, if there is an artificial pupil; exposure time of the luminous object; its shape and orientation; the proportion of its dimensions; chromaticity; visual acuity of the observer;...)

The human eye's retina responds to the number of photons that reach it per unit area and unit of time, that is, the retinal illuminance ( $E_r$ ). When the luminous object is greater than the resolving power of the eye, which we estimate to be one arc minute, the retinal illuminance is proportional to the object's luminance. However, when the object has an angular size smaller than that of the resolution,  $E_r$  is proportional to the illuminance that reaches the eye's pupil (Segura, 2021).

For the situations of interest, the Moon has a smaller width than that of optical resolution, which means that we must adapt the experimental results of Blackwell and other researchers and find the threshold illuminance, which only depends on the luminance of the background and not on the size of the Moon.

To solve the visibility of the crescent, we use the phenomenon discovered by Danjon (1932 and 1936), according to which there is a shortening of the lunar horns that increases with the phase angle. For this reason, we only have to analyze the visibility of the central part of the Moon, which is the last that will be visible, and we assume to be circular and therefore applicable the Blackwell's results.

First we find the luminance of the central part of the Moon according to the phase angle, and then we find the illuminance of the Moon ( $E$ ). From Blackwell's results, we find the threshold illuminance  $E_{th}$  knowing the luminance of the sky. Using the ratio  $E/E_{th}$  and the Blackwell probability distribution, we find the probability of seeing the crescent Moon. If  $E = E_{th}$ , there is a 50% probability; if  $E > E_{th}$ , the probability will be higher, and lower if  $E < E_{th}$ . Except for extreme situations, we will always have a probability different from 100% of observing the crescent Moon, which will increase if there is more than one observer.

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