CIS PROGRAMMING ASSIGNMENT 4

REPORT

Work Done By:

1. Samer Aslan (saslan1@jh.edu)
2. Ajay Gawade (agawade1@jh.edu)

Note: This is technically a continuation of PA3 so most of the part is taken from PA3 report.

Objective:

- **Finding closest point on the mesh of triangles**
- **Adding iterations to implement full fledge ICP algorithm**

1) Overview of Program Structure

The purpose of this assignment was to create a complete iterative-closest point registration algorithm. The problem involved a 3D triangular surface mesh of a bone found in CT coordinates and two rigid bodies (LED markers).

One rigid body is rigidly attached to the bone and one to be used as a pointer. LED markers were attached to the two rigid bodies so that the coordinates could be determined in optical coordinates. The closest point on the triangular mesh to several points where the tip of the pointer contacted the bone was found using our find closest point algorithm.

The mesh data was parsed to get the vertices of each known triangle. Then ICP registration could be used to find the point on the mesh that was closest to the tip of rigid body A. First, the transformation from the CT mesh coordinates to the rigid body B coordinates was assumed to be the identity matrix. Then, sample points were found by multiplying the transformation from CT mesh coordinates to rigid body B coordinates by the tip of the pointer A in rigid body coordinates. Now, these sample points were used where points on the CT mesh were closest to the given transformation.

The **find_closest_point** function was implemented in which the nearest point to the sample points on the CT mesh was calculated for every triangle in the mesh. The error between the two points for each triangle was calculated by taking the **2-point norm** (which is used to find the Euclidean distance)) between the points and the smallest error corresponded to the nearest point on the mesh to the pointer tip A.

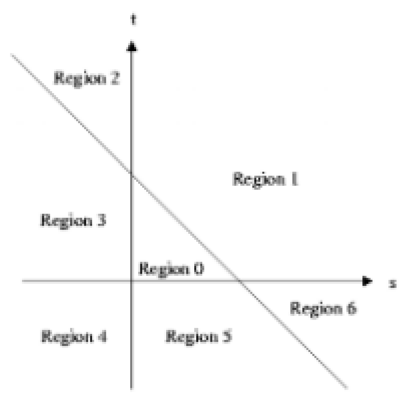
The driver function calls all the helper functions. For every file A-K, it first reads all the data into the necessary structures. Then, for every frame k, it does the necessary frame transformations to get s_k, and then finds the closest points to the mesh by calling find_closest_point_mesh_linear. The mesh linear function calls the find_closest_point function for each triangle. Using the ICP from the slides, we were able to do multiple iterations of this in order to increase the accuracy of our final output. Finally, this is all outputted into the corresponding file. The mathematical/algorithmic approach is further outlined below.

2) Mathematical and Algorithmic Approach:

Find Closest Point:

In PA3, we used the mu and lambda least squares method to find the closest point given some point a and some triangle t. Although this mostly produced correct output, we wanted to ensure that all of our find_closest_point output was correct for this programming assignment, since it isn't the main purpose of this portion. We used a report on the find closest point algorithm (referenced at the bottom) in order to implement this.

Figure 1. Partitioning of the st-plane by triangle domain D.



The diagram above shows how the paper approached this find closest point method. The geometric intuition for this is as follows: there is a level curve of Q that just touches the triangle and for which the region inside the triangle and the region inside the level curve do not overlap. The level curve does one of the following.

1. Touches the edge for which $s + t = 1$.
2. Touches the corner at $(0, 1)$.
3. Touches the edge for which $s = 0$.

Accounting for these three cases according to the paper, we are able to find the closest point c to some point a on triangle t. The paper outlines some pseudocode which we used to implement this algorithm in python.

Registration Frame:

Frame registration is an output of a Transformation function of one frame to another frame. To register a Frame, we need 2 co-ordinate systems and a Transformation function (2D-3D rigid transformation).

We have used Rotation and Translation Matrix for the Frame Composition.

The transformation function is to compute the registration frame transformation from the coordinates of the mesh triangles and Rigid bodies.

We are using point cloud registration for rigid transformation of 3D point-to-surface model.

ICP Algorithm:

Outline of a practical ICP code

Step 1: (matching)

$\mathtt{A} \leftarrow \varnothing;\ \mathtt{B} \leftarrow \varnothing$

For $k \leftarrow 1$ step 1 to N do

begin

$bnd_k = \left\| \mathbf{F}_n \bullet \vec{\mathbf{q}}_k - \vec{\mathbf{c}}_k \right\|$

$\left[\vec{\mathbf{c}}_k, i, d_k \right] \leftarrow \text{FindClosestPoint}\left(\mathbf{F}_n \bullet \vec{\mathbf{q}}_k, \vec{\mathbf{c}}_k, i_k, bnd_k, \mathtt{T} \right);$

// Note: develop first with simple

// search. Later make more

// sophisticated, using T

if ($d_k < \eta_n$) then { put $\vec{\mathbf{q}}_k$ into A; put $\vec{\mathbf{c}}_k$ into B; };

// See also subsequent notes

end

We used the above outline as well as other surrounding slides in order to write our iterative closest point algorithm. First, the algorithm does the registration part (PA3) for the given file that we are running. Next, we set the threshold, maximum number of iterations and the initial guess for F_reg. The threshold and maximum number of iterations were determined by running the ICP algorithm. We wanted to get the lowest possible magnitude error between sk and ck within a reasonable number of iterations. The more iterations, the more we get diminishing returns on reducing error. Thus, we found that a good threshold to compare to was 10^-5. For each iteration, our termination check essentially looks at whether the difference between the current frame and previous frame is below our threshold. This is carried out by the less than check. If all values in this absolute value frame check difference are below our set threshold, we break from the iterations loop and output our sk/ck. Lastly, we have a check using to make sure that we keep tightening nu_naut without getting stuck at a local minimum. This is explained in the code comments.

3) Results:

Debugging:

It is worth mentioning that throughout the algorithmic development process, we used unit tests to validate whether the algorithms were working as intended. This was done thoroughly on both find closest point and find closest point mesh (test_find_closest_point), as they are the core of our project.

For example, we gave sample triangle coordinates and gave point locations that we knew the correct triangle mapping to and used assert statements to ensure that our functions gave this correct output. Additionally, print statements were used throughout in order to ensure that input/output was correct.

Validation of sk/ck using debugging data:

Each debug sample readings test file was inputted, eventually generating output as described in the assignment guidelines. We compared the (sk-ck) norm from the answer text files to our generated output text files in order to determine the accuracy of our algorithms. When we found the error values between our norm (sk – ck) and answer norm (sk - ck), the %error was < 1% for almost all sample frames. When we were debugging our results, we printed to the output file for every iteration and saw that the error values were getting smaller and smaller each iteration. This was a good indication to us that the ICP algorithm was doing its job. The debug overall percent error below was calculated by getting the percent error between the debug answer sk, ck, magnitude values and our sk, ck, magnitude values.

Debug A:

Overall %Error: 0.008%

Answer Debug ck, mag	**Our Debug ck, mag**

-4.77	20.38	13.36	0.000	-4.77	20.38	13.36	0.000
-6.13	17.36	12.30	0.000	-6.13	17.36	12.30	0.001
-0.25	4.91	-21.88	0.000	-0.26	4.91	-21.88	0.003
-30.48	-21.34	-44.17	0.000	-30.48	-21.33	-44.18	0.006
13.62	15.10	-16.42	0.000	13.62	15.11	-16.42	0.001
24.37	14.77	29.17	0.000	24.37	14.76	29.17	0.001
1.85	22.90	11.21	0.000	1.85	22.91	11.21	0.002
26.14	-9.23	3.20	0.000	26.14	-9.23	3.20	0.001
1.28	-3.92	-26.18	0.000	1.28	-3.93	-26.18	0.000
-5.97	0.12	51.09	0.000	-5.97	0.11	51.09	0.002
23.24	15.57	32.08	0.000	23.23	15.58	32.08	0.003
-29.31	4.93	-40.32	0.000	-29.31	4.93	-40.32	0.001
-5.29	24.20	30.19	0.000	-5.29	24.20	30.19	0.001
7.58	23.72	22.85	0.000	7.58	23.72	22.85	0.003
-19.83	2.41	-9.82	0.000	-19.83	2.40	-9.81	0.004
21.00	-16.87	-10.54	0.000	20.99	-16.87	-10.54	0.003
-8.02	-0.58	26.40	0.000	-8.02	-0.59	26.40	0.001
15.84	-12.66	3.37	0.000	15.84	-12.66	3.37	0.004
23.61	8.87	42.85	0.000	23.61	8.86	42.85	0.000
-43.55	-14.44	-26.35	0.000	-43.55	-14.44	-26.35	0.001
-40.59	0.90	-28.05	0.000	-40.59	0.90	-28.05	0.004
-8.14	6.74	32.84	0.000	-8.14	6.74	32.84	0.002
-15.39	-31.25	-32.32	0.000	-15.39	-31.25	-32.32	0.001
-32.82	-15.01	-7.88	0.000	-32.81	-15.02	-7.88	0.002

Debug B:

Overall %Error: 0.007%

Answer Debug ck, mag				**Our Debug ck, mag**			
6.68	-4.80	63.42	0.000	6.68	-4.79	63.42	0.002
-37.20	-17.16	-41.07	0.000	-37.19	-17.17	-41.07	0.000
28.12	19.26	12.82	0.000	28.12	19.26	12.82	0.001
-32.92	-28.12	-19.42	0.000	-32.92	-28.12	-19.42	0.000
23.57	-3.61	-26.38	0.000	23.57	-3.60	-26.38	0.001
18.22	23.91	15.84	0.000	18.22	23.91	15.84	0.004
-4.35	-10.85	7.34	0.000	-4.35	-10.85	7.34	0.000
-27.49	11.10	-23.72	0.000	-27.48	11.11	-23.72	0.002
-9.96	16.47	14.87	0.000	-9.96	16.47	14.87	0.001
-28.21	-31.23	-30.93	0.000	-28.21	-31.23	-30.93	0.000
38.72	4.35	-11.65	0.000	38.72	4.35	-11.65	0.000
34.63	-2.54	-20.18	0.000	34.63	-2.54	-20.18	0.002
37.34	-1.87	4.62	0.000	37.34	-1.87	4.62	0.002
-31.26	-29.71	-31.44	0.000	-31.25	-29.71	-31.44	0.002
22.03	25.55	-4.66	0.000	22.03	25.55	-4.66	0.003
-6.89	-1.90	31.30	0.000	-6.89	-1.89	31.30	0.000
-4.37	-9.64	13.64	0.000	-4.37	-9.64	13.64	0.003
-8.42	21.79	35.55	0.000	-8.42	21.79	35.55	0.001
-21.74	-5.93	-4.67	0.000	-21.74	-5.93	-4.67	0.000
-20.99	-29.50	-14.68	0.000	-20.99	-29.50	-14.68	0.002
-30.47	8.58	-32.78	0.000	-30.47	8.58	-32.78	0.001
-1.14	18.45	46.66	0.000	-1.14	18.45	46.66	0.001
8.90	-6.82	-21.85	0.000	8.90	-6.81	-21.85	0.001
-26.13	-22.38	-7.90	0.000	-26.12	-22.38	-7.90	0.001

Debug C:

Overall %Error: 0.008%

Answer Debug ck, mag	**Our Debug ck, mag**

3.15	-7.02	52.60	0.000	3.14	-7.02	52.60	0.000
30.05	12.94	16.46	0.000	30.05	12.94	16.46	0.001
-33.86	-23.66	-13.48	0.000	-33.86	-23.66	-13.48	0.001
2.23	-12.96	1.39	0.000	2.23	-12.96	1.39	0.005
-9.40	1.90	22.72	0.000	-9.40	1.90	22.72	0.001
7.83	21.66	-5.44	0.000	7.83	21.65	-5.44	0.001
-6.26	24.39	30.96	0.000	-6.26	24.39	30.96	0.003
-7.05	7.54	-21.57	0.000	-7.05	7.54	-21.57	0.005
38.15	-0.49	-4.97	0.000	38.14	-0.49	-4.97	0.002
-5.21	-8.19	-42.35	0.000	-5.21	-8.19	-42.35	0.001
-0.16	-6.31	57.69	0.000	-0.16	-6.31	57.69	0.002
11.75	-5.03	63.32	0.000	11.75	-5.02	63.32	0.001
-23.82	10.27	-35.05	0.000	-23.82	10.27	-35.05	0.000
2.26	0.49	63.44	0.000	2.26	0.49	63.44	0.001
7.08	2.10	-20.17	0.000	7.08	2.10	-20.17	0.001
-10.31	-0.99	16.05	0.000	-10.31	-0.99	16.05	0.004
-40.90	-15.63	-30.97	0.000	-40.90	-15.63	-30.97	0.002
-38.39	-23.03	-19.60	0.000	-38.39	-23.03	-19.60	0.001
-40.74	-2.28	-17.47	0.000	-40.74	-2.28	-17.47	0.002
-42.43	-14.06	-19.75	0.000	-42.43	-14.06	-19.75	0.002
-3.53	10.75	-3.11	0.000	-3.53	10.75	-3.11	0.004
25.88	11.68	28.56	0.000	25.88	11.69	28.56	0.001
-22.08	7.12	-15.97	0.000	-22.08	7.12	-15.97	0.003
-6.52	1.90	48.42	0.000	-6.52	1.91	48.42	0.000

Debug D:

Overall %Error: 0.011%

Answer Debug ck, mag				Our Debug ck, mag			
4.64	10.64	-11.34	0.000	4.64	10.65	-11.33	0.003
-33.65	-26.59	-17.16	0.000	-33.66	-26.59	-17.16	0.002
3.95	19.37	0.63	0.000	3.96	19.38	0.63	0.003
-14.55	6.26	-40.59	0.000	-14.56	6.27	-40.59	0.002
16.82	-17.28	-6.21	0.000	16.82	-17.28	-6.21	0.008
39.18	5.12	-9.55	0.000	39.18	5.13	-9.55	0.004
39.66	7.81	0.29	0.000	39.66	7.82	0.29	0.009
20.03	-13.50	-1.83	0.000	20.03	-13.50	-1.83	0.004
-14.29	17.29	33.75	0.000	-14.30	17.29	33.75	0.003
-29.72	-16.98	-7.53	0.000	-29.72	-16.97	-7.53	0.001
4.37	20.27	43.60	0.000	4.37	20.27	43.60	0.004
13.75	-6.59	25.25	0.000	13.74	-6.59	25.25	0.004
-22.05	-32.76	-26.67	0.000	-22.06	-32.76	-26.68	0.005
-3.94	-16.37	-40.14	0.000	-3.94	-16.37	-40.14	0.006
27.67	8.16	25.23	0.000	27.67	8.15	25.23	0.003
10.01	26.24	7.40	0.000	10.01	26.24	7.40	0.009
11.28	-9.14	13.65	0.000	11.28	-9.14	13.65	0.002
19.26	-17.38	-10.03	0.000	19.26	-17.38	-10.02	0.001
-19.73	-20.87	-46.76	0.000	-19.72	-20.87	-46.76	0.001
6.83	-7.51	25.61	0.000	6.83	-7.51	25.61	0.010
36.10	-2.63	-4.28	0.000	36.10	-2.64	-4.28	0.005
-21.47	8.10	-17.04	0.000	-21.47	8.10	-17.05	0.003
29.48	16.16	14.34	0.000	29.48	16.16	14.34	0.004
-8.55	-0.38	24.12	0.000	-8.55	-0.38	24.12	0.002

Debug E:

Overall %Error: 0.081%

Answer Debug ck, mag	Our Debug ck, mag

-1.55	24.11	21.64	0.162	-1.52	24.11	21.65	0.143
-16.94	6.15	-13.21	0.040	-16.92	6.17	-13.22	0.069
6.52	16.62	-6.00	0.043	6.53	16.65	-5.99	0.061
11.13	-6.57	56.97	0.004	11.11	-6.58	56.98	0.008
5.10	7.49	63.34	0.015	5.10	7.50	63.34	0.029
32.21	13.86	12.44	0.025	32.22	13.84	12.44	0.047
5.12	15.56	63.14	0.010	5.12	15.56	63.14	0.007
4.25	-7.03	39.31	0.101	4.24	-7.03	39.32	0.096
31.00	16.72	11.54	0.018	31.01	16.70	11.55	0.037
11.67	21.95	36.20	0.007	11.67	21.95	36.21	0.015
-14.74	13.50	19.91	0.002	-14.75	13.51	19.91	0.026
-4.50	13.24	5.68	0.089	-4.49	13.26	5.69	0.090
-31.35	-29.59	-31.76	0.111	-31.37	-29.58	-31.77	0.076
-10.93	23.13	19.95	0.069	-10.91	23.15	19.97	0.057
-2.92	15.10	57.42	0.082	-2.92	15.10	57.44	0.078
16.99	-3.87	52.30	0.142	16.98	-3.87	52.32	0.146
-8.79	-22.89	-11.94	0.043	-8.79	-22.88	-11.93	0.066
-34.47	-27.50	-21.48	0.088	-34.50	-27.49	-21.47	0.052
-28.81	-17.63	-7.36	0.017	-28.81	-17.59	-7.37	0.013
7.71	22.56	-3.74	0.125	7.72	22.58	-3.73	0.110
-0.19	-8.31	26.32	0.143	-0.19	-8.31	26.33	0.157
17.70	18.99	46.18	0.069	17.70	18.99	46.20	0.059
-38.25	-17.07	-13.21	0.021	-38.27	-17.02	-13.20	0.038
-2.65	-20.04	-35.85	0.091	-2.63	-20.02	-35.84	0.075

Debug F:

Overall %Error: 0.105%

Answer Debug ck, mag				**Our Debug ck, mag**			
-3.44	4.74	-23.88	0.049	-3.43	4.72	-23.90	0.020
-16.36	10.65	-33.98	0.057	-16.34	10.64	-34.00	0.029
5.51	20.46	43.47	0.028	5.49	20.46	43.45	0.044
11.74	25.80	11.49	0.005	11.74	25.80	11.49	0.010
-27.20	-31.22	-32.86	0.009	-27.18	-31.24	-32.84	0.034
10.89	17.67	-11.24	0.048	10.91	17.64	-11.24	0.041
-9.96	-26.26	-15.41	0.041	-9.98	-26.27	-15.41	0.009
-5.42	19.88	39.71	0.172	-5.44	19.89	39.69	0.181
-6.28	9.81	50.51	0.025	-6.27	9.81	50.51	0.010
-6.20	-1.72	36.14	0.111	-6.21	-1.70	36.14	0.137
-10.45	6.97	3.06	0.129	-10.46	6.96	3.05	0.124
-3.13	-18.75	-10.31	0.002	-3.14	-18.75	-10.30	0.026
-38.22	-23.40	-32.02	0.040	-38.20	-23.43	-32.01	0.053
-42.09	-11.25	-29.05	0.025	-42.07	-11.28	-29.05	0.018
30.50	-2.72	-26.14	0.086	30.51	-2.76	-26.14	0.061
-2.13	16.69	49.04	0.117	-2.14	16.71	49.01	0.084
0.29	10.82	-7.75	0.073	0.29	10.82	-7.76	0.053
-31.93	-3.35	-9.98	0.071	-31.94	-3.37	-9.97	0.052
-9.05	-29.09	-28.17	0.084	-9.07	-29.11	-28.14	0.112
-26.81	-0.11	-45.52	0.089	-26.81	-0.14	-45.54	0.109
-14.95	12.82	29.81	0.050	-14.97	12.84	29.80	0.048
-7.18	1.73	32.97	0.360	-7.18	1.75	32.97	0.337
6.36	10.34	-12.04	0.070	6.36	10.34	-12.05	0.043
21.31	-3.71	-24.65	0.029	21.30	-3.76	-24.65	0.010

Discussion of Debug Results:

Our results were very similar to the answer results. In most cases, they were practically identical. This is highlighted by the low error values above, and can be seen in the magnitude side-by-side view of select data for each debugging case (can’t show all <200 values for each).

Testing on Unknown Data:

Part of our output for each unknown data file is found below. The magnitude error for each of these unknown data files is very similarly low to that of the previous test files. Therefore, it seems likely that our ICP algorithm also works on these unknown datasets.

Unknown G:

-31.40	2.95	-13.76	-31.40	2.95	-13.76	0.002
-41.16	-3.39	-17.37	-41.15	-3.39	-17.37	0.001
-8.87	-20.46	-43.13	-8.86	-20.46	-43.13	0.003
-7.55	8.85	-0.39	-7.55	8.86	-0.39	0.005
11.73	17.86	-26.64	11.73	17.86	-26.64	0.002
-6.72	8.86	46.43	-6.72	8.86	46.43	0.002
19.12	20.34	35.42	19.12	20.34	35.42	0.005
19.28	-8.37	-23.04	19.28	-8.38	-23.05	0.005
-21.93	-14.19	-5.68	-21.93	-14.19	-5.68	0.002
16.26	22.29	-28.72	16.26	22.28	-28.72	0.005
-29.12	-2.89	-46.00	-29.12	-2.89	-46.01	0.005
-40.43	-20.51	-30.71	-40.42	-20.51	-30.71	0.006
-11.74	23.40	20.75	-11.74	23.40	20.75	0.000
-7.96	3.51	29.76	-7.96	3.51	29.76	0.003
-7.48	4.15	32.65	-7.47	4.15	32.65	0.007
-0.58	-0.47	-25.72	-0.58	-0.46	-25.73	0.004
-10.08	-24.40	-12.73	-10.08	-24.39	-12.73	0.001
19.07	21.72	26.41	19.08	21.72	26.42	0.006
-42.40	-14.65	-20.14	-42.41	-14.65	-20.14	0.005
9.93	24.41	-5.00	9.93	24.41	-5.00	0.004
-5.65	11.76	50.41	-5.66	11.76	50.41	0.000
39.82	4.93	1.73	39.82	4.93	1.73	0.000
1.86	16.68	1.38	1.86	16.68	1.38	0.006
0.24	18.55	5.76	0.24	18.55	5.76	0.003

Unknown H:

-43.06	-6.12	-30.09	-43.06	-6.12	-30.09	0.000
5.44	23.97	20.28	5.44	23.98	20.28	0.006
-37.60	-14.74	-11.31	-37.60	-14.74	-11.31	0.000
35.47	10.44	-17.03	35.47	10.44	-17.04	0.002
2.91	-9.32	19.74	2.91	-9.32	19.74	0.004
-7.11	-22.79	-39.19	-7.11	-22.79	-39.19	0.001
-5.20	9.65	-3.68	-5.20	9.65	-3.68	0.000
15.57	24.01	18.01	15.57	24.00	18.01	0.013
11.27	14.06	-11.26	11.27	14.06	-11.26	0.000
-3.43	-12.59	-40.08	-3.43	-12.59	-40.08	0.001
-3.11	-21.65	-34.86	-3.12	-21.65	-34.86	0.005
21.43	20.56	25.09	21.42	20.55	25.09	0.006
-7.29	3.85	33.37	-7.29	3.85	33.37	0.001
-2.39	17.16	8.12	-2.39	17.16	8.11	0.001
-16.05	2.06	-44.71	-16.05	2.06	-44.71	0.000
-27.81	-30.45	-34.12	-27.81	-30.45	-34.12	0.004
17.51	-17.25	-11.30	17.50	-17.25	-11.30	0.003
8.55	-13.02	4.52	8.55	-13.01	4.51	0.007
1.83	10.18	-10.84	1.83	10.18	-10.83	0.002
-4.59	-23.41	-16.98	-4.59	-23.41	-16.98	0.003
-12.53	10.11	-21.06	-12.53	10.12	-21.06	0.003
-25.05	-31.45	-18.96	-25.05	-31.45	-18.96	0.001
15.05	19.68	-30.59	15.05	19.68	-30.59	0.001
-1.26	-16.22	-5.56	-1.26	-16.22	-5.56	0.003

Unknown J:

3.09	23.85	11.19	3.15	23.77	11.21	0.098
8.89	20.82	52.54	8.84	20.98	52.56	0.170
-1.38	-11.02	11.51	-1.36	-10.92	11.50	0.105
-2.70	-22.83	-19.85	-2.62	-22.91	-19.82	0.116
-4.69	13.54	53.33	-4.63	13.50	53.32	0.073
32.06	-6.74	6.58	32.05	-6.72	6.57	0.028
-39.84	-21.98	-24.04	-39.92	-22.03	-24.03	0.099
15.90	-6.60	23.02	15.89	-6.53	23.01	0.071
-3.17	19.54	40.93	-3.20	19.69	41.00	0.166
-6.18	2.65	60.71	-6.29	2.64	60.73	0.109
-5.12	12.66	5.99	-5.22	12.78	5.93	0.160
-10.89	0.41	-43.18	-10.88	0.41	-43.19	0.012
15.32	-11.66	-19.00	15.32	-11.64	-18.98	0.024
24.74	-2.60	25.33	24.64	-2.50	25.30	0.142
30.13	0.14	18.02	30.05	0.16	17.96	0.105
-21.61	-8.55	-48.90	-21.61	-8.53	-48.93	0.030
-0.40	-18.76	-31.71	-0.45	-18.75	-31.70	0.046
28.30	6.95	23.34	28.40	6.96	23.37	0.101
-1.14	-21.51	-27.99	-1.11	-21.53	-28.00	0.035
-28.73	-30.02	-18.68	-28.69	-29.91	-18.74	0.132
-27.68	-23.50	-9.21	-27.69	-23.53	-9.17	0.056
-13.89	-26.10	-12.18	-13.89	-26.12	-12.15	0.034
-10.23	-25.02	-13.36	-10.25	-24.98	-13.39	0.056
-16.66	5.73	-41.82	-16.67	5.81	-41.91	0.119

Unknown K:

-30.54	9.03	-20.13	-30.56	9.06	-20.13	0.036
-9.09	3.56	25.64	-9.01	3.58	25.62	0.093
34.05	-6.45	-12.95	34.07	-6.47	-12.95	0.030
-7.67	5.63	-30.39	-7.68	5.62	-30.39	0.012
-9.15	4.27	25.44	-9.20	4.25	25.45	0.058
14.98	-7.51	16.65	14.98	-7.52	16.65	0.009
-1.76	-6.31	-31.17	-1.72	-6.30	-31.19	0.042
-2.97	-6.04	36.20	-2.94	-5.95	36.18	0.101
19.31	-10.61	-21.06	19.32	-10.61	-21.05	0.009
12.58	24.93	13.93	12.56	25.21	13.99	0.290
-31.14	5.71	-38.32	-31.15	5.73	-38.33	0.025
-23.56	-10.61	-5.23	-23.59	-10.61	-5.19	0.052
-11.60	-30.10	-31.24	-11.63	-30.05	-31.23	0.061
-7.85	-27.36	-32.40	-7.79	-27.46	-32.42	0.115
-14.50	11.10	-22.88	-14.50	11.08	-22.88	0.025
9.34	-14.35	1.81	9.35	-14.27	1.78	0.086
16.46	25.94	-9.82	16.45	25.98	-9.83	0.037
31.70	-8.30	-11.08	31.68	-8.27	-11.09	0.032
18.48	-1.59	58.06	18.47	-1.58	58.06	0.018
31.99	-6.88	6.37	31.97	-6.86	6.36	0.024
-4.19	16.30	46.22	-4.24	16.34	46.23	0.065
26.64	3.10	-30.04	26.65	3.10	-30.03	0.008
12.31	-16.88	-8.40	12.31	-16.89	-8.40	0.006
-31.55	8.96	-21.93	-31.55	8.96	-21.93	0.002

4) Individual Contributions:

Samer: Primarily Coding, Algorithm development (Coding, testing and Validation)

Ajay: Primarily Mathematical approach behind the algorithm, results comparison (the Math Part)

Citations:

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