Q1

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2-2

$n=5$

1	2	3	4	5
5	9	10	4	6

```
BUBBLESORT(A, n)
1 for i = 1 to n - 1
2     for j = n downto i + 1
3         if A[j] < A[j - 1]
4             exchange A[j] with A[j - 1]
```

a) We must also show that the array A' is a permutation of A meaning that A' contains all the Same elements as A but in Sorted order.

Because no modifications are made to A other than Swapping elements we can prove that A' contains all the Same elements of A.

b) Loop invariant: Before each iteration of the for-j loop on lines 2-4 $A[j]$ is the Smallest element of the Subarray $A[j \dots n]$.

Initialization: Initially $j=n$ and $A[n \dots n] = A[n]$, therefore $A[n]$ is a Single element and thus it is also the smallest element so the invariant is Vacuosly true.

Maintenance: Let $j=K$. Therefore $A[K]$ is the Smallest element of $A[K \dots n]$. Now if $A[K] < A[K-1]$ then $A[K]$ and $A[K-1]$ are Swapped on line 4, thus making $A[K-1]$ the Smaller element in the Subarray. If no swap occurs then $A[K-1]$ was already Smaller than $A[K]$ and j is then decrementing to preserve the loop invariant.

Termination: Termination occurs when $j=i$ and $A[i]$ is the Smallest element of $A[i \dots n]$.

c)

Loop Invariant: At the start of every iteration of the for-i loop of lines 1-4 the Subarray $A[1 \ldots i-1]$ Consists of the Smallest elements of $A[1 \ldots n]$ and in Sorted order.

Initialization: Before the initial iteration $i=1$ and thus $A[1 \ldots i-1]$ is empty therefore the loop invariant is vacuously true.

Maintenance: From the loop invariant from part b) we know that after the execution of the inner loop $A[i]$ is the smallest element of Subarray $A[i \ldots n]$. Thus, at the start of the outer for loop $A[1 \ldots i-1]$ contains elements in sorted order that are less than the elements of $A[i \ldots n]$. Therefore, once execution of the outer loop is complete the Subarray $A[1 \ldots i]$ contains elements that are smaller than the elements contained in $A[i+1 \ldots n]$ and in sorted order.

Termination: The outer for-i loop terminates when $i=n$. Thus, $A[1 \ldots n]$ will contain all elements of A and in Sorted order.

d)

The outer for-i loop will run n times and will cause the inner for-j loop to run n-i times, thus resulting in a worst-case running time of $\Theta(n^2)$.

Insertion Sort also has a worst case running time of $\Theta(n^2)$ which is the same as bubble sort however the best case running time of insertion sort is $\Theta(n)$ which is better than bubble sort's best-case running time of $\Theta(n^2)$.