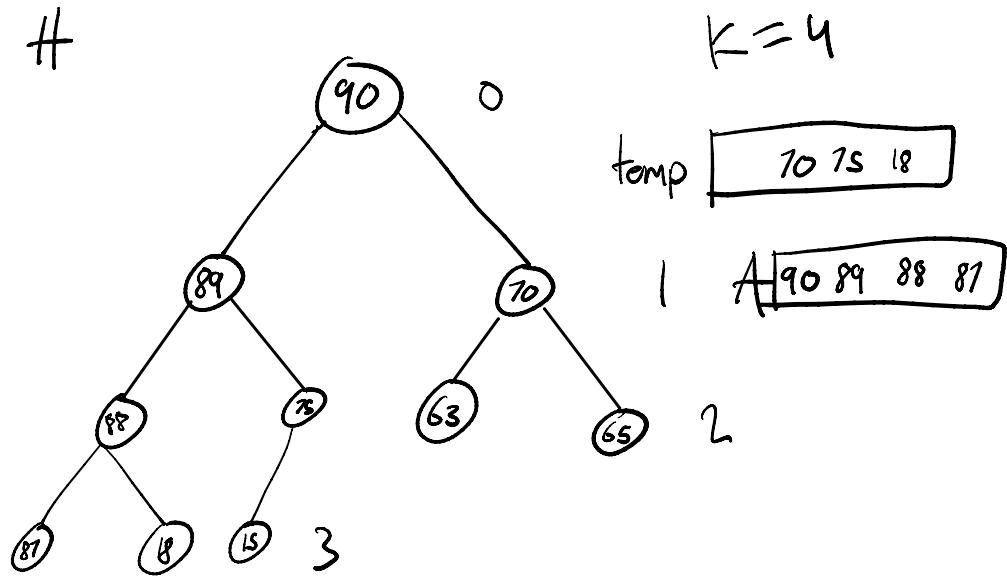


Q5

October 6, 2023 1:13 AM



Thought process:

1. Create a list called `tmp`
2. Start from $i=0$
3. Append i to `tmp`
4. iterate through `temp` and find the largest key
5. Pop off the largest key and append it to `A`
6. Append the children of the popped key in the previous step to `tmp`
7. Repeat from step 4. until there are k elements in `A`

k th Largest Elements (H, k):

1. $\text{tmp} = \text{new list}$
2. $\text{largestIndex} = 0$
3. $A = \text{array of size } k$
4. $\text{tmp.add}(\text{largestIndex})$
5. $\text{for } (i \text{ in range}(k))$
 6. $\text{largestIndex} = \max(\text{tmp}, \text{index with greatest value})$
 7. $\text{largestValue} = H[\text{tmp.pop}(\text{largestIndex})]$
 8. $A[i] = \text{largestValue}$
 9. $\text{left} = \text{leftChild}(\text{largestIndex})$
 10. $\text{right} = \text{rightChild}(\text{largestIndex})$
 11. $\text{if } (\text{left} \neq \text{null}):$
 12. $\text{tmp.add}(\text{left})$
 13. $\text{if } (\text{right} \neq \text{null}):$
 14. $\text{tmp.add}(\text{right})$
15. $\text{return } A$

Proof of correctness:

Loop invariant: At the beginning of each iteration of the loop A contains the i largest node values and tmp contains the indices of the i greatest nodes in H .

Initialization: Before the very first iteration of the loop $i=0$ and A is empty. tmp contains the index of the root and the root of a max-heap is always the largest value. Therefore the loop invariant holds.

Maintenance: During every iteration of the loop the node containing the greatest value is extracted from its index in tmp and its value is added to A . Then the indices of the children of the extracted node are added to tmp if they are not null. Therefore, ensuring that A contains the i greatest values in H and tmp contains the indices of the i greatest nodes.

Termination: the loop terminates when $i = K$, at this point A contains the K greatest values of H and tmp contains the indices of the K values.

Proof of termination:

The for loop iterates from 0 to K and no modifications are made to other than incrementation by the loop. No modifications are made to K or H , therefore termination of the loop occurs when $i = K$.