

a) Algorithm  $\text{alg1}(L[0 \dots n-1])$ if ( $n == 1$ ) return  $L[0] \rightarrow O(1)$ 

else

 $\text{tmp} = \text{alg1}(L[0 \dots n-2]) \rightarrow T(n-1)$ if ( $\text{tmp} \leq L[n-1]$ ) return  $\text{tmp} \rightarrow O(1)$ else return  $L[n-1] \rightarrow O(1)$ 

$$T(n) = T(n-1) + O(1)$$

$$T(n-1) = T(n-2) + O(1)$$

$$T(n-2) = T(n-3) + O(1)$$

 $\vdots$ 

$$+ T(1) = T(0) + O(1)$$

$$T(n) = T(0) + (n-1)$$

$$T(n) = O(n)$$

$$T(n) = O(n)$$

 $O(n)$ 

$$(n-1) \cdot O(1)$$

b) Algorithm  $\text{alg2}(X[1 \dots r])$ if ( $l == r$ ) return  $X[l] \rightarrow O(1)$ 

else

$$flr = \text{floor}((l+r)/2) + O(1)$$

$$\text{tmp1} = \text{alg2}(X[l \dots flr]) \rightarrow T(\frac{n}{2})$$

$$\text{tmp2} = \text{alg2}(X[flr+1 \dots r]) \rightarrow T(\frac{n}{2})$$

if ( $\text{tmp1} \leq \text{tmp2}$ ) return  $\text{tmp1} \rightarrow O(1)$ else return  $\text{tmp2} \rightarrow O(1)$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(1)$$

according to master theorem  $a=2$   $b=2$   $f(n)=O(1)$

$$n^{\log_2 2} = n \quad f(n) \in O(n^{\log_2 2}) \Rightarrow \Theta(n)$$

2) You are given a polynomial  $p(x)$  like

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  you are supposed to write a brute force algorithm for computing the value of the polynomial at given point  $x_0$ .

algorithm (  $P[0 \dots n], n$  )

result = 0

for  $i=n$  to 0

mult = 1

for  $j=1$  to  $n$

mult = mult \*  $x_0$   $O(1)$

result = result +  $P[i] * \text{mult}$   $O(1)$

return result

}  $O(n-1)$  }  $O(n)$

$$T(n) = \sum_{i=n}^0 \sum_{j=1}^n O(1) = ((n-1)+1) \cdot O(1)$$

$$\sum_{i=n}^0 O(n) = n \cdot O(n)$$

$$T(n) = O(n^2)$$

algorithm (points[0...n])

mindist = inf

for i=0 to n  $\rightarrow \Theta(n)$

for j=i+1 to n

if (getDistance(points[i], points[j])  $\Theta(1)$ )  $\left. \begin{array}{l} \text{min} = \text{getDistance}(\text{points}[i], \text{points}[j]) \end{array} \right\} \Theta(n)$

return min

$$T(n) = \sum_{i=0}^n \sum_{j=i+1}^n \Theta(1) = \sum_{i=0}^n (n - (i+1) + 1) \Theta(1) = \Theta(n)$$

$$\sum_{i=0}^n \Theta(n) = \Theta(n) \cdot n = \Theta(n^2)$$

5) Algorithm(stations[0...n])  
list[] most-profit, consecutive;

for i to n  
clear consecutive;

if stations[i].profit > total-profit(most-profit)

clear most-profit

most-profit.add(station[i]);

consecutive.add(station[i]);

for j=i+1 to n

consecutive.add(station[j]);

if (total-profit(consecutive) > total-profit(most-profit))

swap (total-profit, most-profit)

return most-profit

$$T(n) = \sum_{i=0}^n \sum_{j=i+1}^n 2\Theta(n) + \Theta(n)$$

$\downarrow$

swap and total-profits are  $\Theta(n)$  algorithms.  $(n - (i+1) + 1) \Theta(n)$

$$T(n) = \Theta(n^3) \leftarrow (n+1) \cdot \Theta(n) \cdot (n-i) \leftarrow \sum_{i=0}^n \Theta(n)(n-i)$$