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1)

a) $2^n + 3^n \in O(4^n)$

$f(n) \le c \cdot g(n)$, $c>0$, $n \ge n_0$ → for $O(n)$

$2^n + 3^n \le c \cdot 4^n$

$n_0 = 1$ → $\underbrace{2+3}_{5} \le 4 \cdot c$
$5 \le 4 \cdot c$ → c must be positive so $2^n + n^3 \in O(4^n)$ ✓

b) $\sqrt{10n^2+7n+3} \in \Omega(n)$

$f(n) \ge c \cdot g(n)$, $c>0$, $n \ge n_0$, $c>0$, $n_0>0$ → for $\Omega\, g(n)$

$\sqrt{10n^2+7n+3} \ge c \cdot n$ $\quad n \ge n_0$

$n \ge 1$ → $\sqrt{10n^2+7n+3} \ge c \cdot n$ $\quad n_0 = 1$, $c = \sqrt{20}$, $c>0$ → $\sqrt{10n^2+7n+3} \in \Omega(n)$ ✓

c) $f(n) < c \cdot g(n)$, $n \ge n_0$, $c>0$, $n_0>0$ $\quad f(n) \in o(g(n))$

$n^2+n < c \cdot n^2$ $\quad n \ge n_0$ for $c=1$, there is no n value that can be satisfy condition. X

d) $3\log_2^2 n \in \Theta(\log_2 n^2)$

$k_1 \cdot g(n) \le f(n) \le k_2 \cdot g(n)$, $k_1, k_2 > 0$, $f(n) \in \Theta(g(n))$

$k_1 \cdot \log_2^2 n^2 \le 3 \cdot \log_2^2 n \le k_2 \cdot \log_2^2 n^2$ → $2k_1 \le 3\log_2 n \le 2k_2$

k_1 and $k_2 = 2$, $n=2$ for this arguments, condition is not satisfied X

e) $(n^3+1)^6 \in O(n^3)$

$f(n) \le c\, g(n)$, $n \ge n_0$, $c>0$, $n_0>0$

$(n^3+1)^6 \le c \cdot n^3$ $\quad$ for positive c values there is no n values which satisfied condition.

2 -

a) $2n \log(n+2)^2 + (n+2)^2 \cdot \log \frac{n}{2}$

$4n \log(n+2) + (n+2)^2 \cdot \log(\frac{n}{2})$

$$\lim_{n \to \infty} \frac{4\cancel{n} \overbrace{\log(n+2)}^{\cancel{\log n}}}{\cancel{n} \cancel{\log n}} \to = 4 \qquad 4n \log(n+2) \in \Theta(n \cdot \log n)$$

$$\lim_{n \to \infty} \frac{(n+\cancel{2})^2 \cdot \log \frac{n}{\cancel{2}}}{n^2 \cdot \log n} = \frac{n^2 \log n}{n^2 \cdot \log n} = 1 \quad (n+2)^2 \cdot \frac{\log n}{2} \in \Theta((n+2)^2 \log)$$

b) $0.0001 \cdot n^4 + 3n^3 + 1$

$$\lim_{n \to \infty} \frac{0.0001 \cdot n^4}{n^4} \to 0.001 \qquad = 0.001 n^4 \in \Theta(n^4)$$

$$\lim_{n \to \infty} \frac{3n^3}{n^3} = 3 \to \qquad 3n^3 \in \Theta(n^3)$$

$$\Theta(n^4) + \Theta(n^3) = \Theta(n^4)$$

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3)

a) $\log n$, $n^{\log n}$, $n^{1.5}$

$$\lim_{n\to\infty} \frac{\log n}{n^{\log n}} = \frac{\infty}{\infty} \to \frac{d}{dx} \to \lim_{n\to\infty} \frac{1}{n \cdot 2n^{\ln(n)-1} \cdot \ln(n)} = \frac{1}{\infty} = \underline{0}$$

$\log(n) \in O(n^{\log n})$

$$\lim_{n\to\infty} \frac{\log n}{n^{1.5}} = \frac{\infty}{\infty} \to \frac{d}{dx} \to \lim_{n\to\infty} \frac{1}{1.5 n \cdot \sqrt{n}} = \frac{1}{\infty} = 0$$

$\log(n) \in O(n^{1.5})$

$$\lim_{n\to\infty} \frac{n^{1.5}}{n^{\log n}} = \frac{\infty}{\infty} \to$$ result of the d'hopital $= 0$, $n^{1.5} \in O(n^{\log n})$

$\underline{n^{\log n} > n^{1.5} > \log(n)}$

b) $n!$, 2^n , n^2 $\qquad n! \cong \sqrt{2\pi n} \cdot \left[\frac{n}{e}\right]^n$

$$\lim_{n\to\infty} \frac{\sqrt{2\pi n} \cdot (\frac{n}{e})^n}{n^2} \to \frac{d}{dx} \to \lim_{n\to\infty} \sqrt{2\pi} \cdot n^{-1.5} \cdot (\frac{n}{e})^n = \infty$$

$n^2 \in O(n!)$

$$\lim_{n\to\infty} \frac{\sqrt{2\pi n} \cdot (\frac{n}{e})^n}{2^n} \to \frac{d}{dx} \to \sqrt{2\pi n} \cdot \left(\frac{n}{2e}\right)^n = \infty$$

$2^n \in O(n!)$

$$\lim_{n\to\infty} \frac{n^2}{2^n} \to \frac{d}{dx} = 0$$ $n^2 \in O(2^n)$ $\to \underline{n! > 2^n > n^2}$

c) $n \log n$, $\sqrt{n}$

$\lim_{n \to \infty} \frac{n \cdot \log n}{\sqrt{n}} \to \frac{d}{dx} \to \sqrt{n} \cdot \log n \to \infty$ $\quad \sqrt{n} \in O(n \log n)$

$n \log n > \sqrt{n}$

d) $n \cdot 2^n$, 3^n

$\lim_{n \to \infty} \frac{n \cdot 2^n}{3^n} \to \frac{d}{dx} = n\left(\frac{2}{3}\right)^n \to 0 = n \cdot 2^n \in O(3^n), 3^n > n \cdot 2^n$

e) $\sqrt{n+10}$, n^3

$\lim_{n \to \infty} \frac{\sqrt{n+10}}{n^3} \xrightarrow{\sqrt{n}} \frac{d}{dx} = \frac{1}{n^{2.5}} = 0$ $\quad \sqrt{n+10} \in O(n^3)$ $n^3 > \sqrt{n+10}$

4)
```
algorithm1(B[0...n-1, 0...n-1])
    for i=0 to n-2 do
        for j=i+1 to n-1 do
            if B[i,j] != B[j,i]
                return false

    return true
```

a- comparison is the basic operation.

b- $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-(i+1)+1) = \sum_{i=0}^{n-2} (n-1)-i = \frac{2 \cdot n-1 \cdot n-1}{2} - \frac{(n-2)\cdot(n-1)}{2} = \frac{n \cdot n-1}{2} = \frac{n^2-n}{2}$

c) $T(n) = \frac{n^2-n}{2} \in O(n^2)$

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5) algorithm2 (A[0..n-1, 0..n-1], B[0..n-1, 0..n-1])

```
for i=0 to n-1 do
    for j=0 to n-1 do
        C[i,j] = 0.0
        for k=0 to n-1 do
            C[i,j] = C[i,j] + A[i,k].B[k,j]

return C
```

a) arithmetic operations are the basic operation.

b) $$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} n^2 = n^3 \rightarrow$$

c) $T(n) = n^3 \in O(n^3)$

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6)

```
solution ( A[0,.... n-1], int target)
    for i=0 to n-1 do
        for j = i+1 to n do
            if( target == A[i]*A[j]
                print A[i] and A[j]
```

$$\sum_{i=0}^{n-1} \sum_{j=i+1}^{n} 1 = \sum_{i=0}^{n-1} n-(i)+1 = \sum_{i=0}^{n-1} n-i+1 = \quad Tn(n^2) \in O(n^2)$$