Pairing Algorithm: Calculating birth and death

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Basic terminology review

- Boundary ∂, Cycle c and Homology Group H_p
- Birth and Death

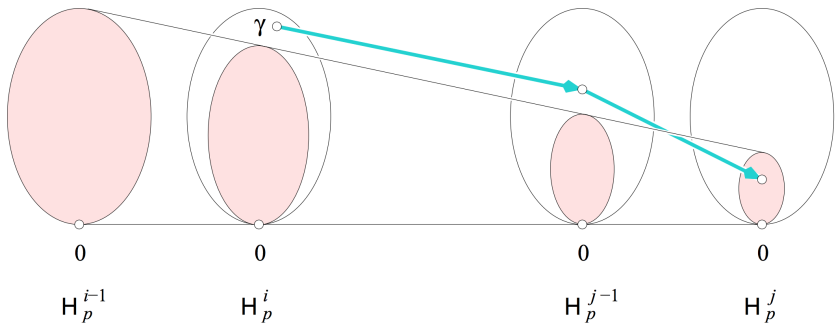


Figure 1: Birth and Death

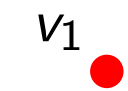
Positive and Negative

- Positive: simplex which create a non-boundary p-cycle
- Negative: simplex which kill an existing $(p-1)$-cycle

Flitration

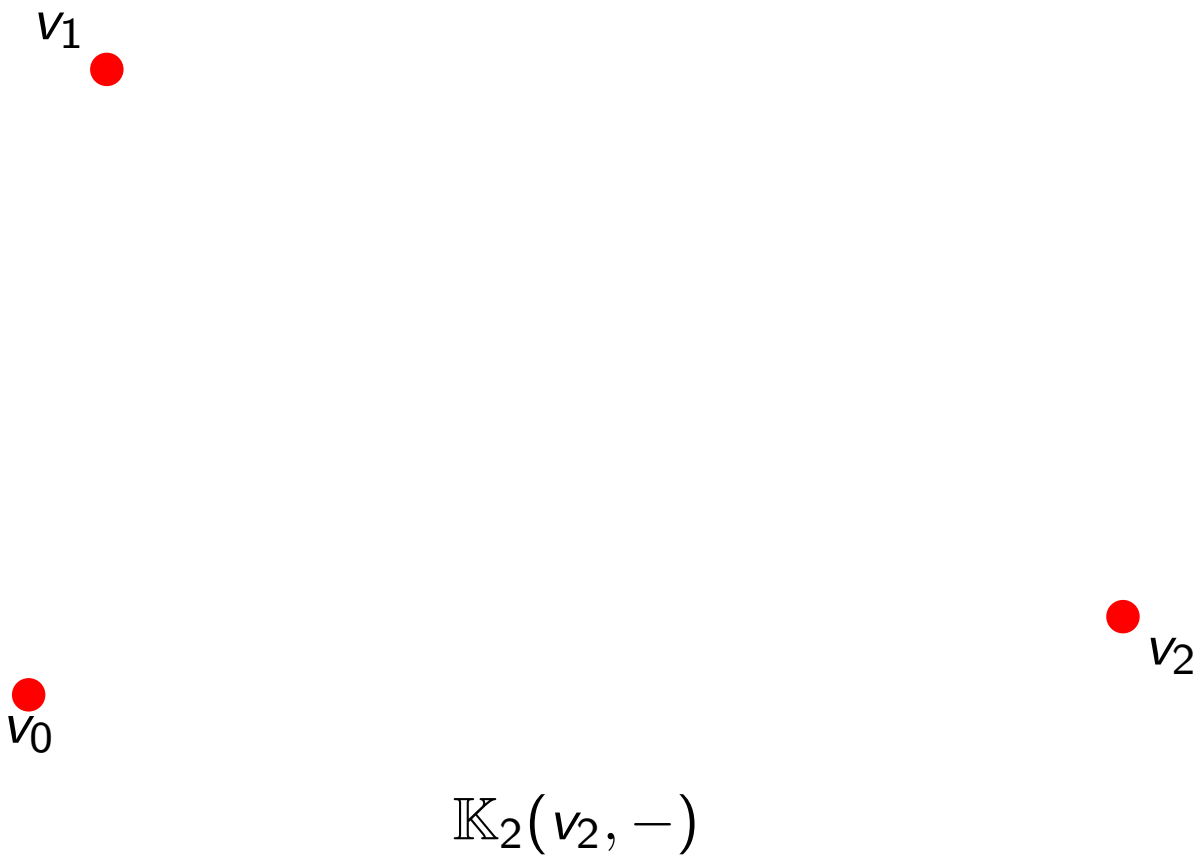
$$\mathbb{K}_0(v_0, -)$$

Flitration



$$\mathbb{K}_1(v_1, -)$$

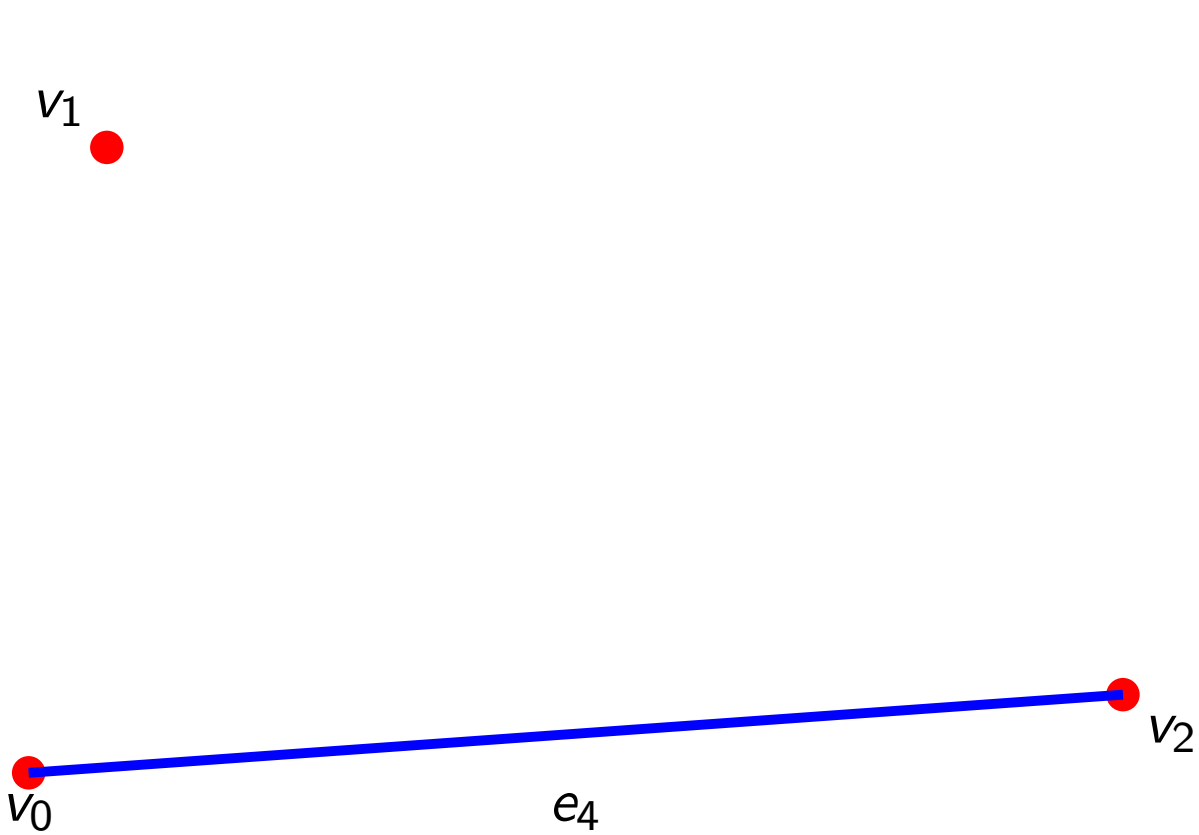
Flitration



Flitration

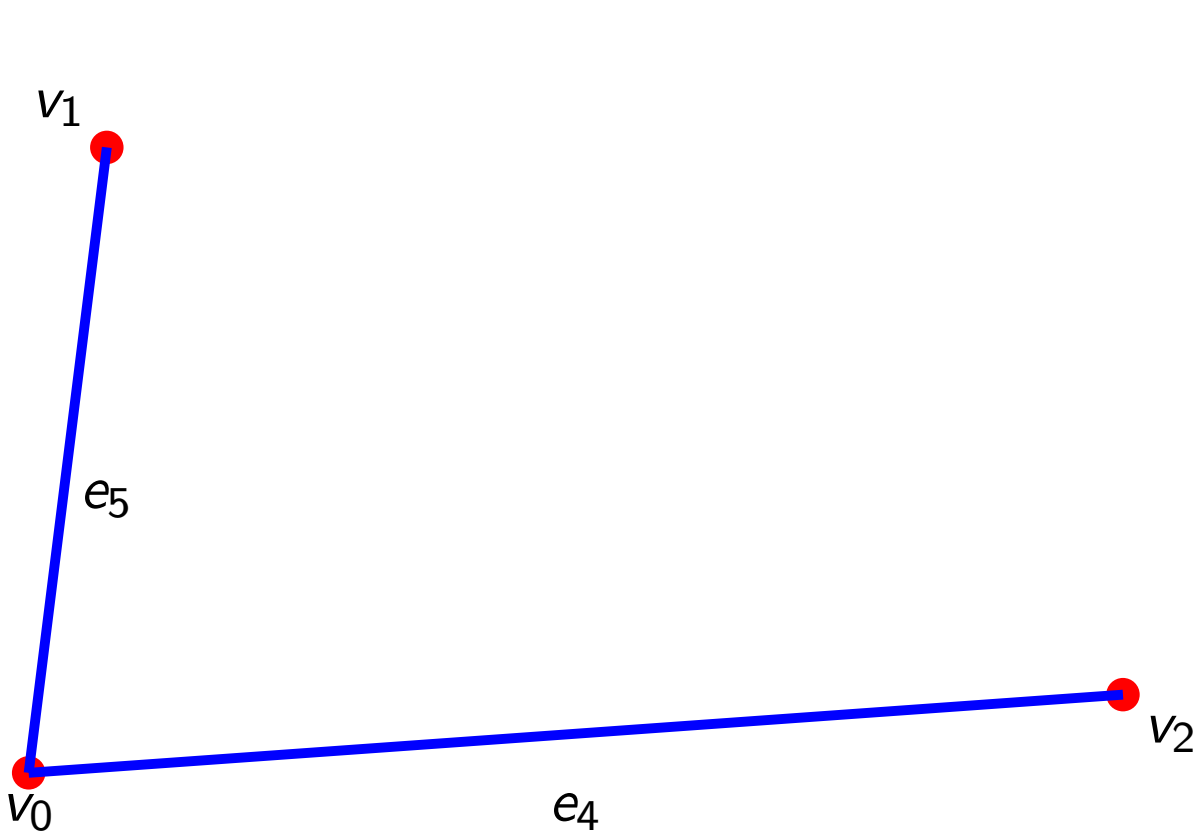
$\mathbb{K}_3(v_3, -)$

Flitration



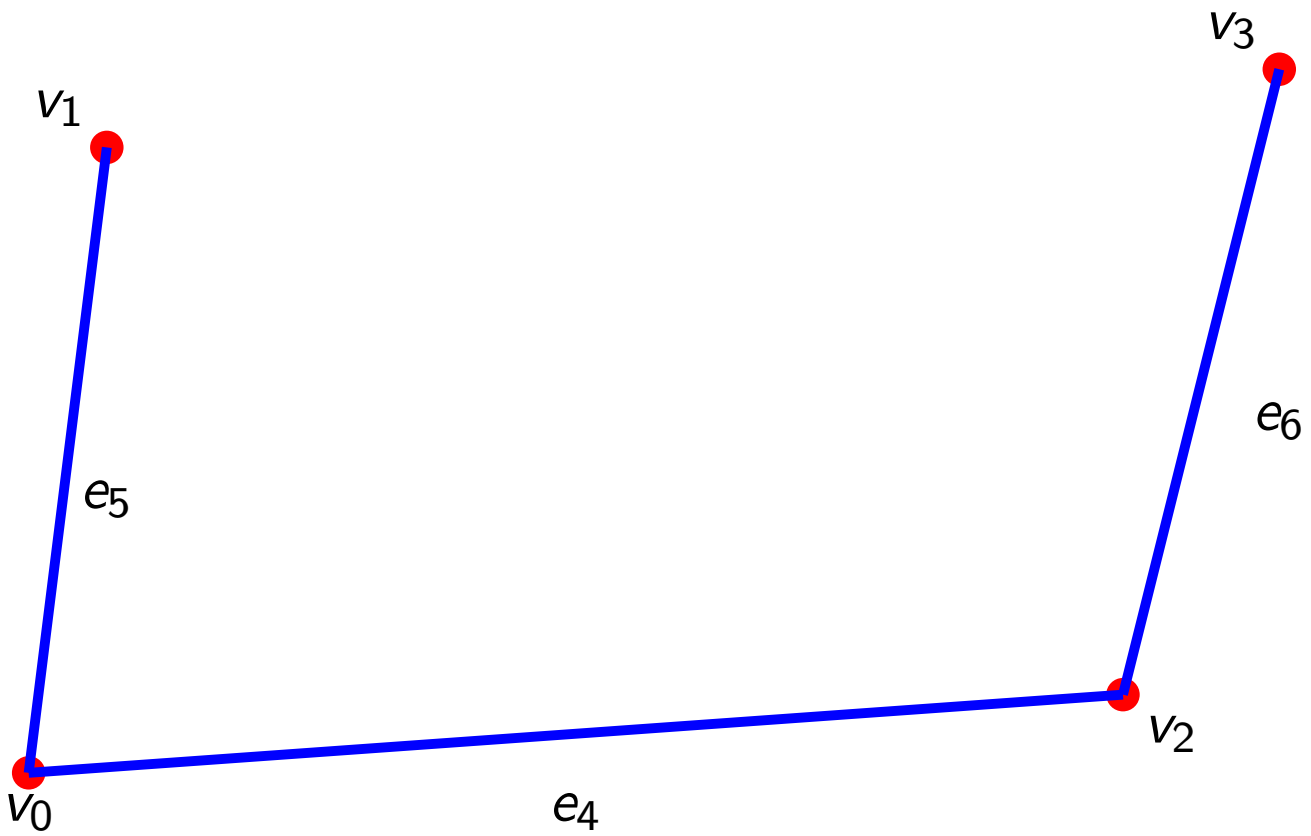
$$\mathbb{K}_4(v_2, e_4)$$

Flitration



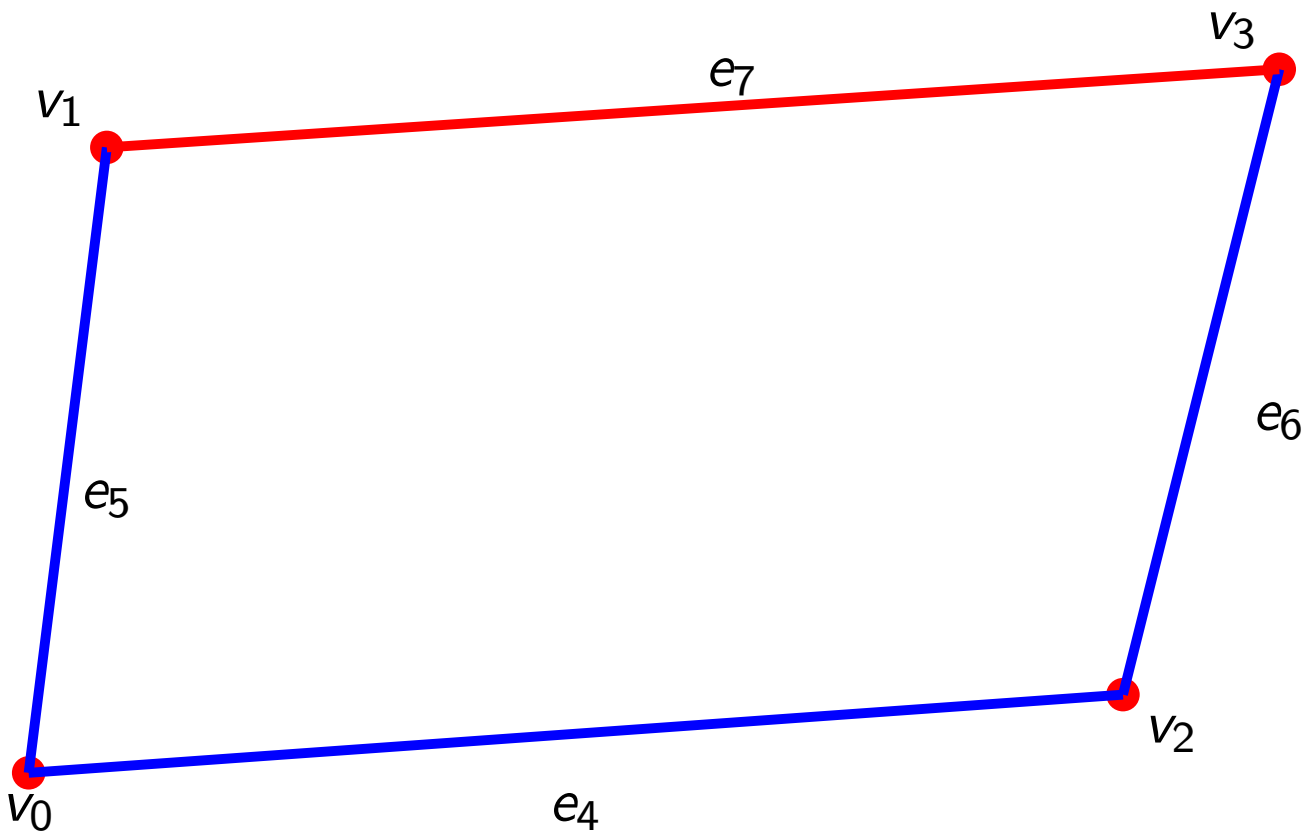
$\mathbb{K}_5(v_1, e_5)$

Flitration



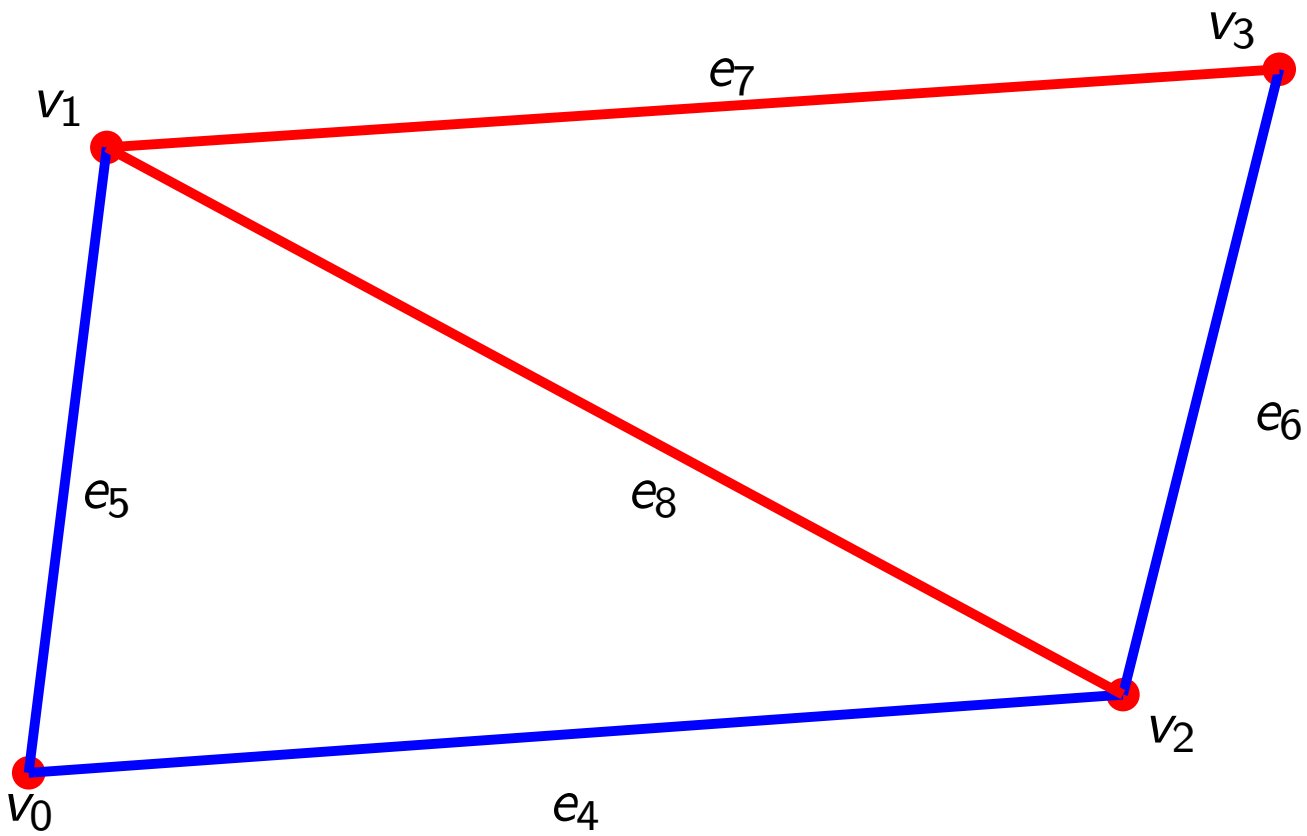
$\mathbb{K}_6(v_3, e_6)$

Flitration



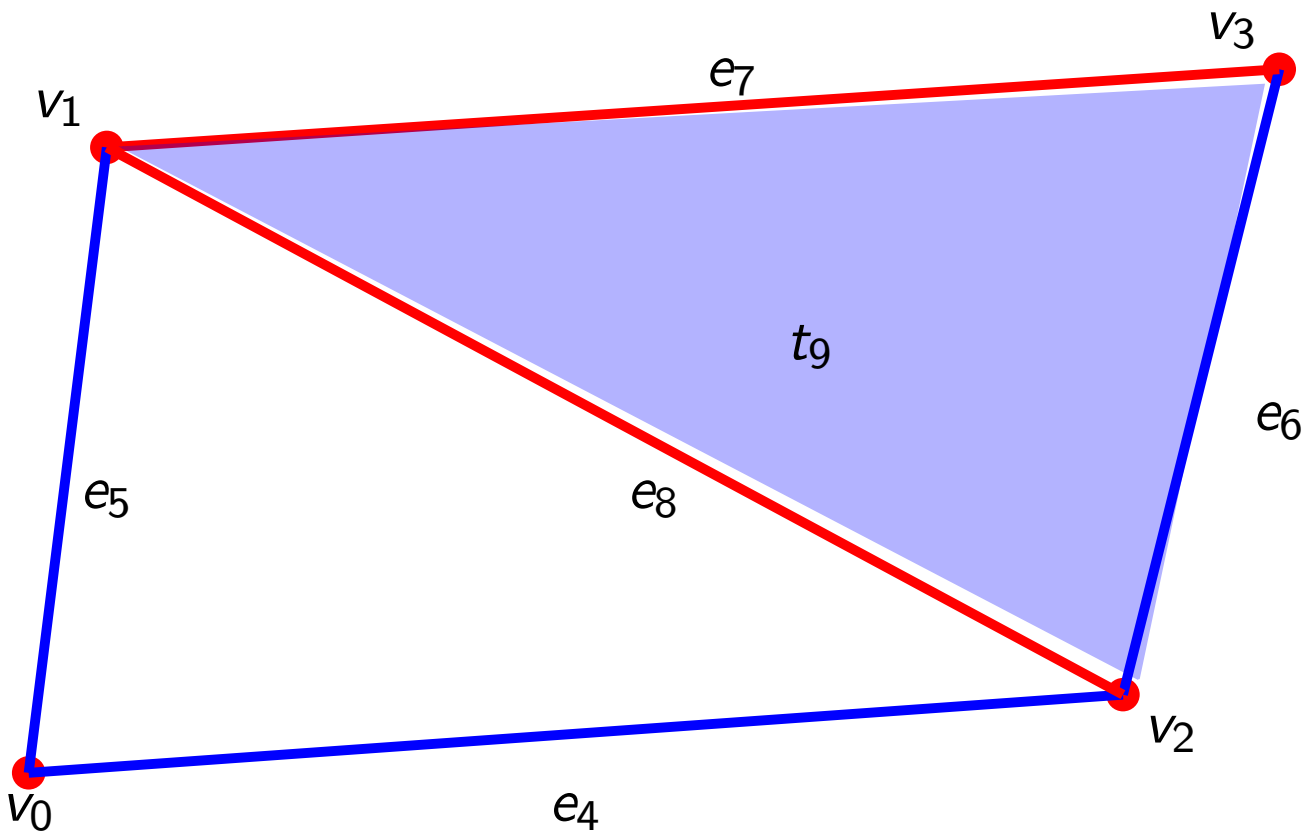
$$\mathbb{K}_7(e_7, -)$$

Flitration



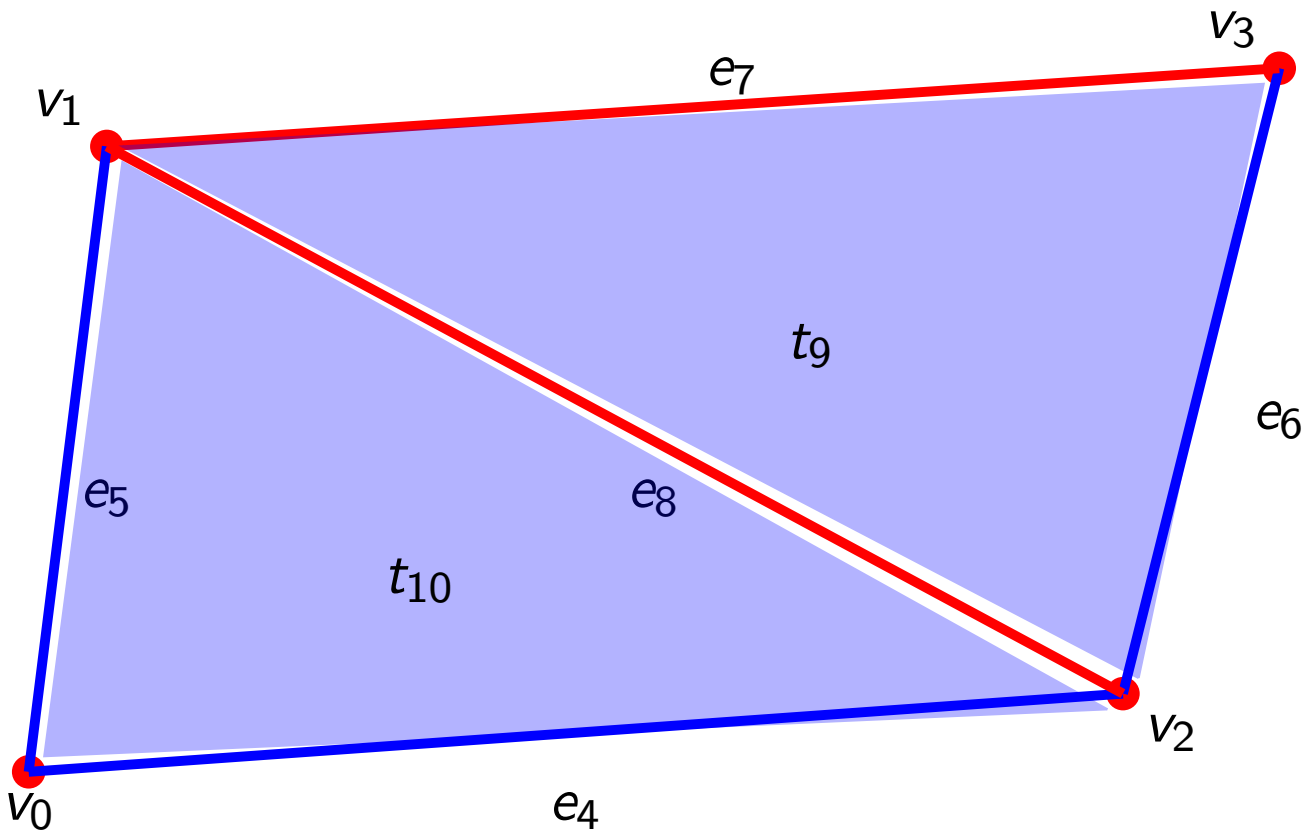
$$\mathbb{K}_8(e_8, -)$$

Flitration



$$\mathbb{K}_9(e_8, t_9)$$

Flitration



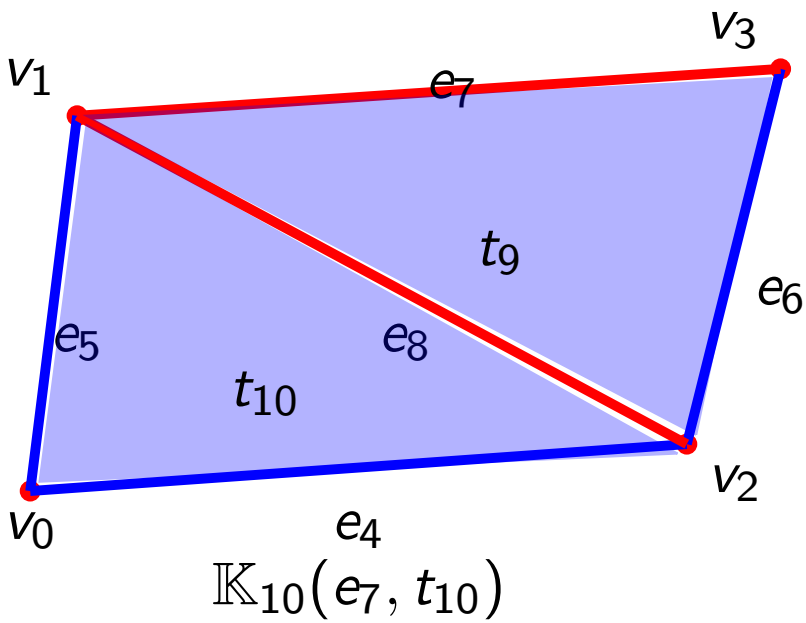
$$\mathbb{K}_{10}(e_7, t_{10})$$

Pairing Algorithm

Algorithm 1 PAIR(σ)

1: $c = \partial_p \sigma$
2: $d \leftarrow$ youngest positive $(p-1)$-simpliex $\in c$
3: **while** d is paired and c is not empty **do**
4: $\quad c'$ be the cycle killed by the simplex paired with d
5: $\quad c = c + c' \mod 2$
6: $\quad d \leftarrow$ youngest positive $(p-1)$-simpliex $\in c$
7: **end while**
8: **if** c is not empty **then**
9: $\quad \sigma$ is a negative p-simplex paired with d
10: **else**
11: $\quad \sigma$ is a positive p-simplex
12: **end if**

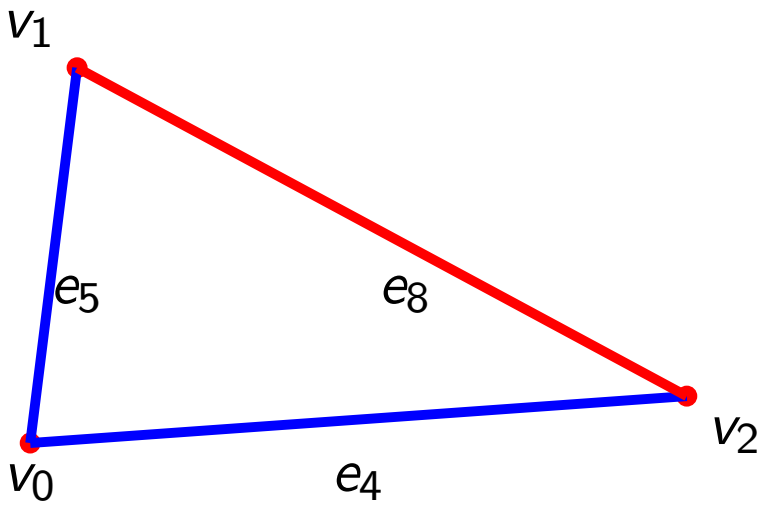
Pairing Algorithm: Example



$\mathbb{K}_{10}(e_7, t_{10})$

- $c = \partial_p(t_{10}) = e_5 + e_4 + e_8$

Pairing Algorithm: Example



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- $d = e_8$, the red edge

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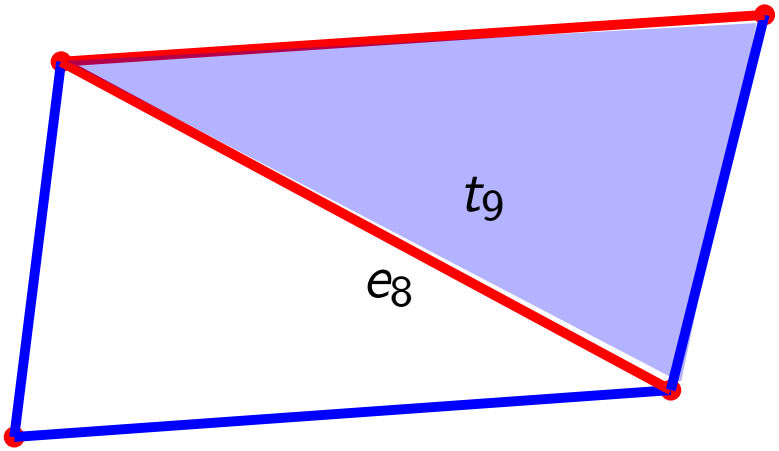
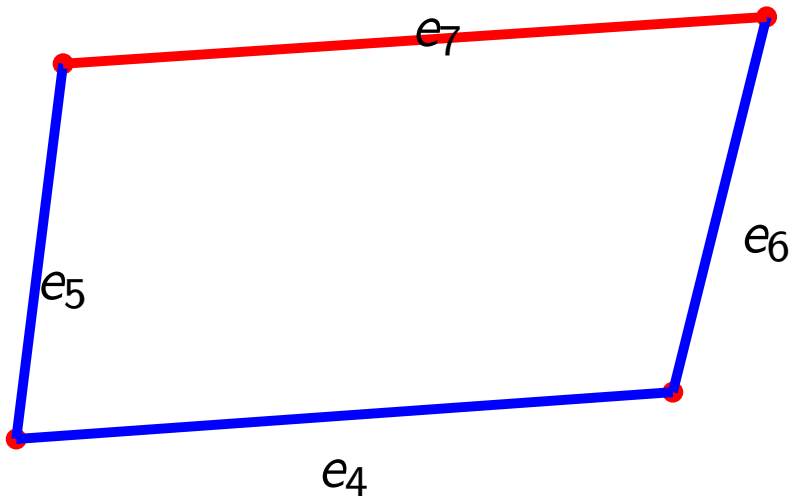


- $c = \partial_p(t_{10}) = e_5 + e_4 + e_8$
- $d = e_8$, the red edge
- Into while-do loop since e_8 pair with t_9

Pairing Algorithm: Example



- $c = \partial_p(t_{10}) = e_5 + e_4 + e_8$
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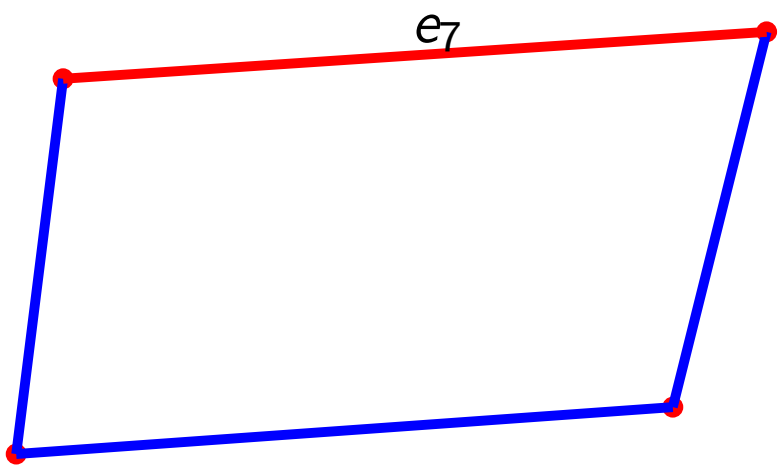
- Into while-do loop since e_8 pair with t_9
- Update c

$$\begin{aligned} c &= e_5 + e_4 + e_8 + \partial_p(t_9) \\ &= e_4 + e_5 + e_6 + e_7 \end{aligned}$$

Pairing Algorithm: Example



$\mathbb{K}_7(e_7, -)$

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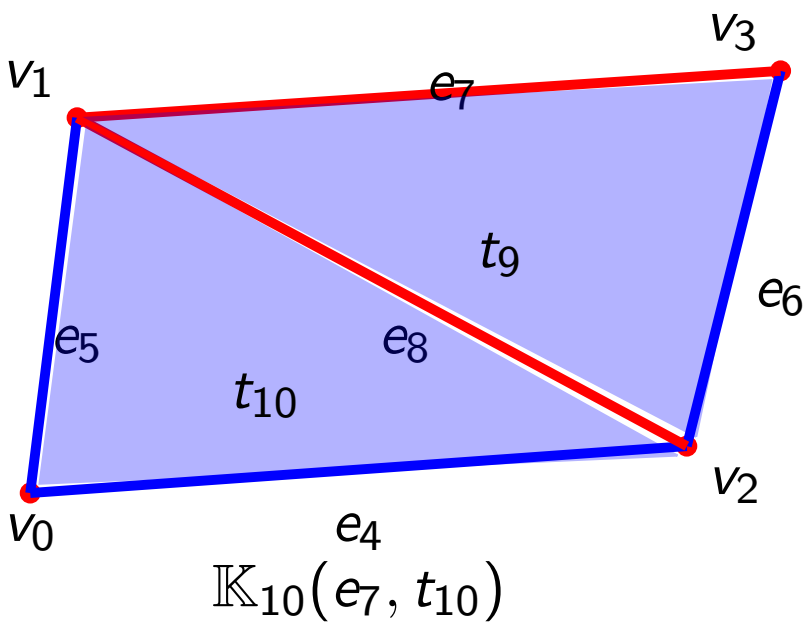


$\mathbb{K}_{10}(e_7, t_{10})$

- $c = \partial_p(t_{10}) = e_5 + e_4 + e_8$
- $d = e_8$, the red edge
- Into while-do loop since e_8 pair with t_9
- Update c

$$c = e_5 + e_4 + e_8 + \partial_p(t_9)$$
$$= e_4 + e_5 + e_6 + e_7$$

- $d = e_7$,Not Paired
- t_{10} is negative paired with e_7

Implementation by Python

Data structure (dict): Input

```
1  K={x:[[],[]] for x in range(11)}
2  sigma = {0:[0],1:[1],2:[2],3:[3],...,10:[0,1,2]}
```

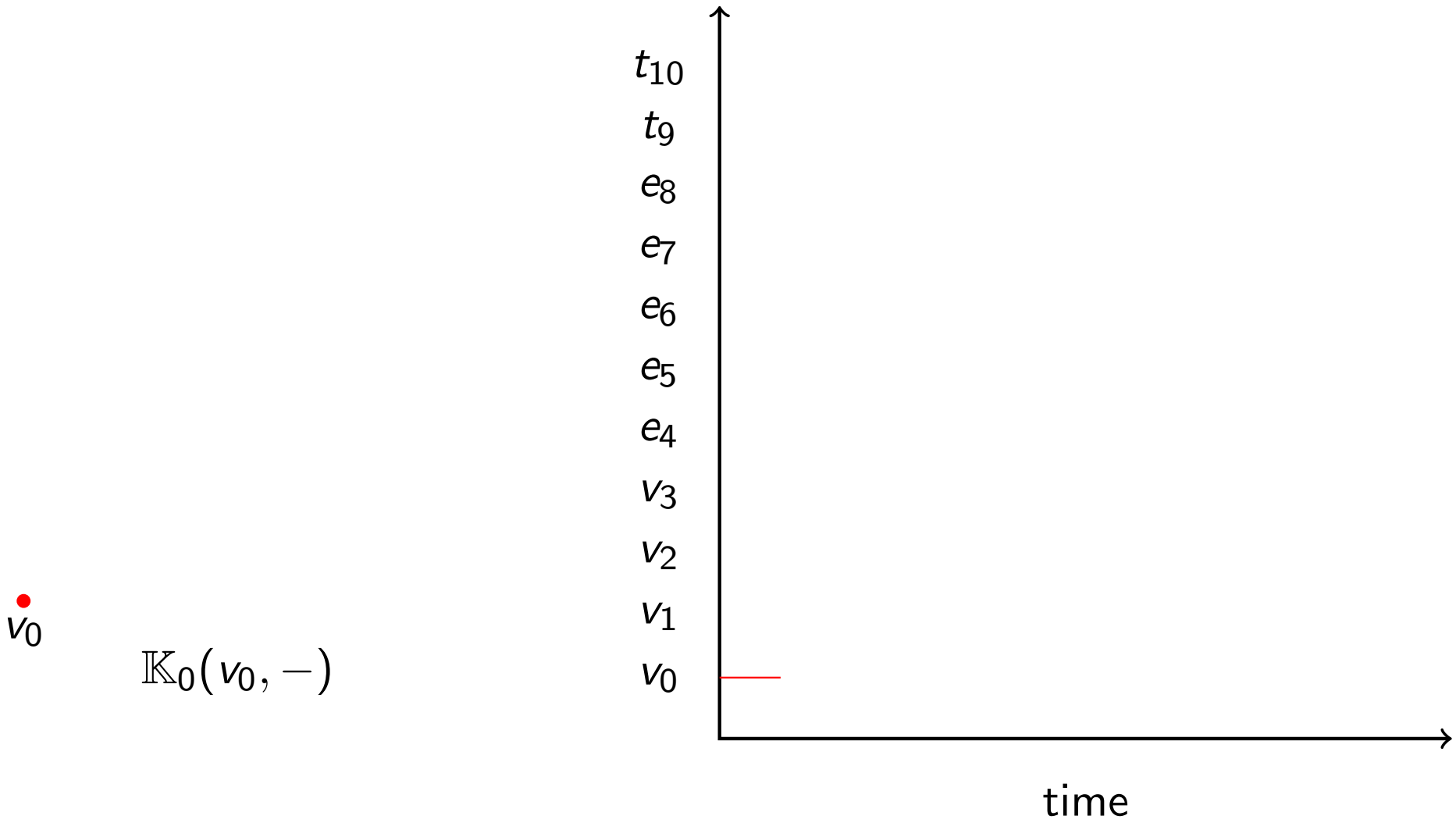
K is a dictionary, the key is the index of filtration, the corresponding value is a list : [positive, negative],
sigma is a dictionary, the key is the index of filtration, the corresponding value is a list: for 0-simplex:[a], 1-simplex: [a,b], 2-simplex: [a,b,c],etc.

Implementation by Python

Data structure (dict): Output

```
{0: [[0], 'unpaired'],
 1: [[1], 'unpaired'],
 2: [[2], 'unpaired'],
 3: [[3], 'unpaired'],
 4: [[2], [4]],
 5: [[1], [5]],
 6: [[3], [6]],
 7: [[7], 'unpaired'],
 8: [[8], 'unpaired'],
 9: [[8], [9]],
 10: [[7], [10]]}
```

Birth and Death



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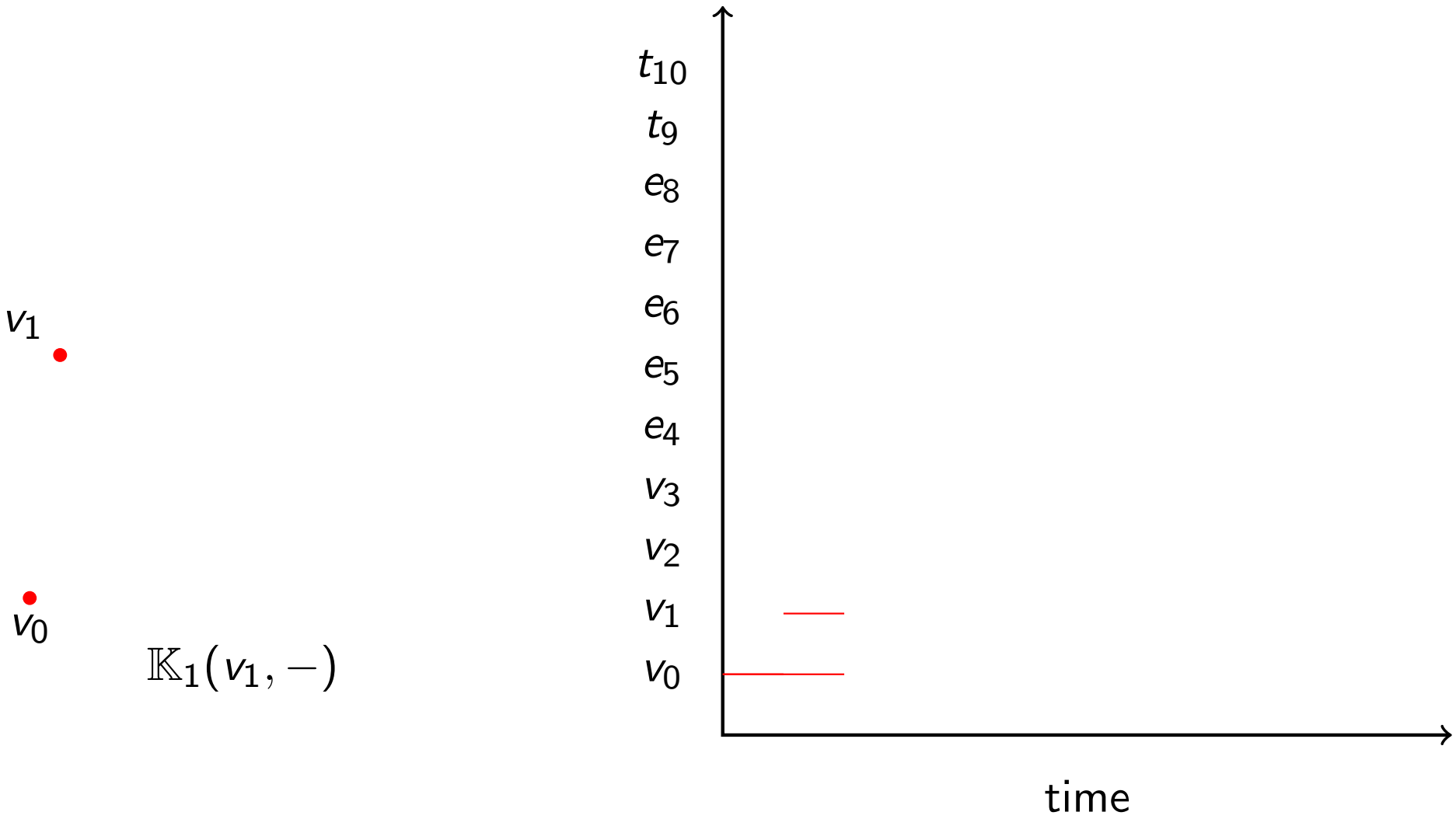

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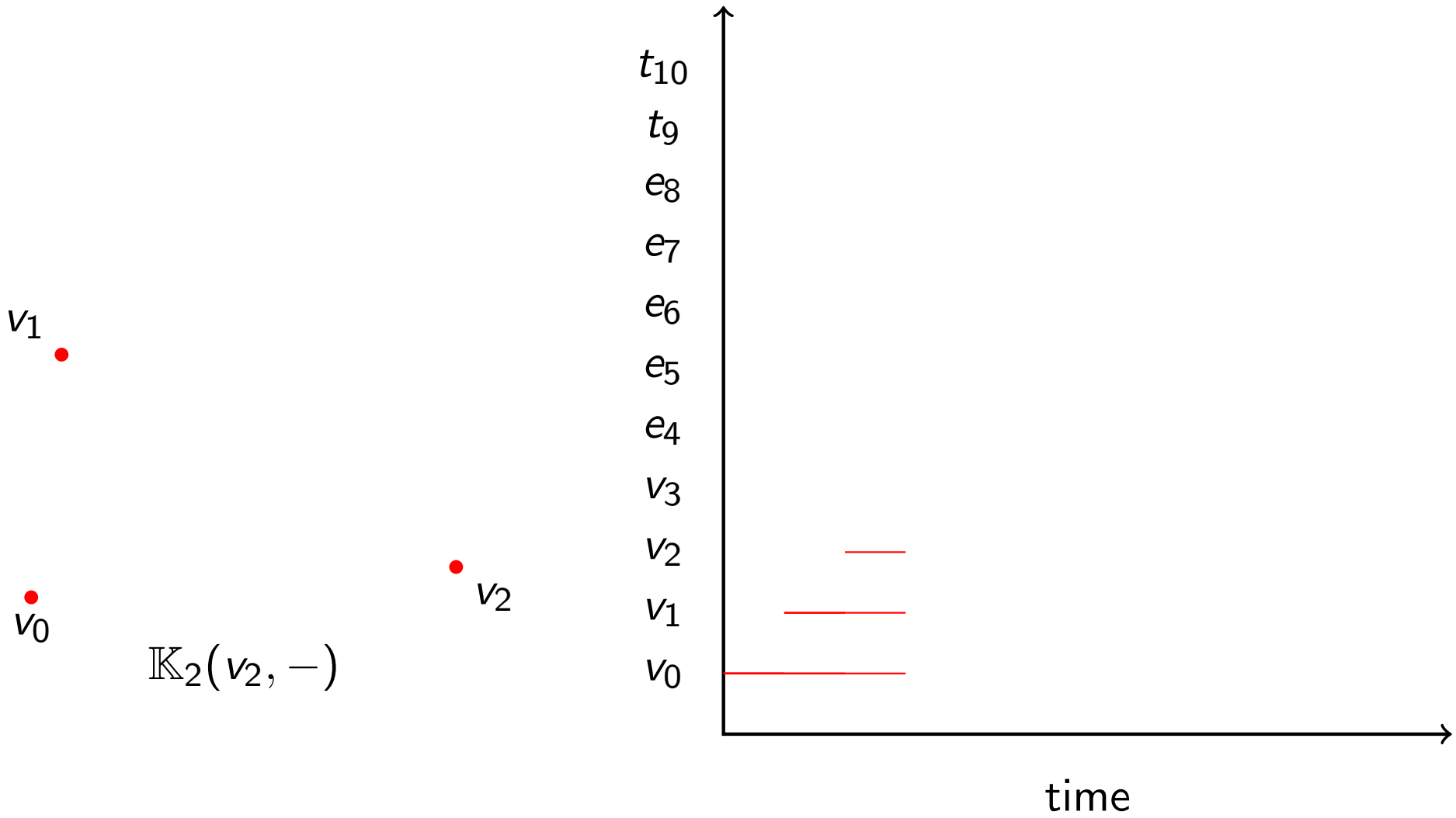

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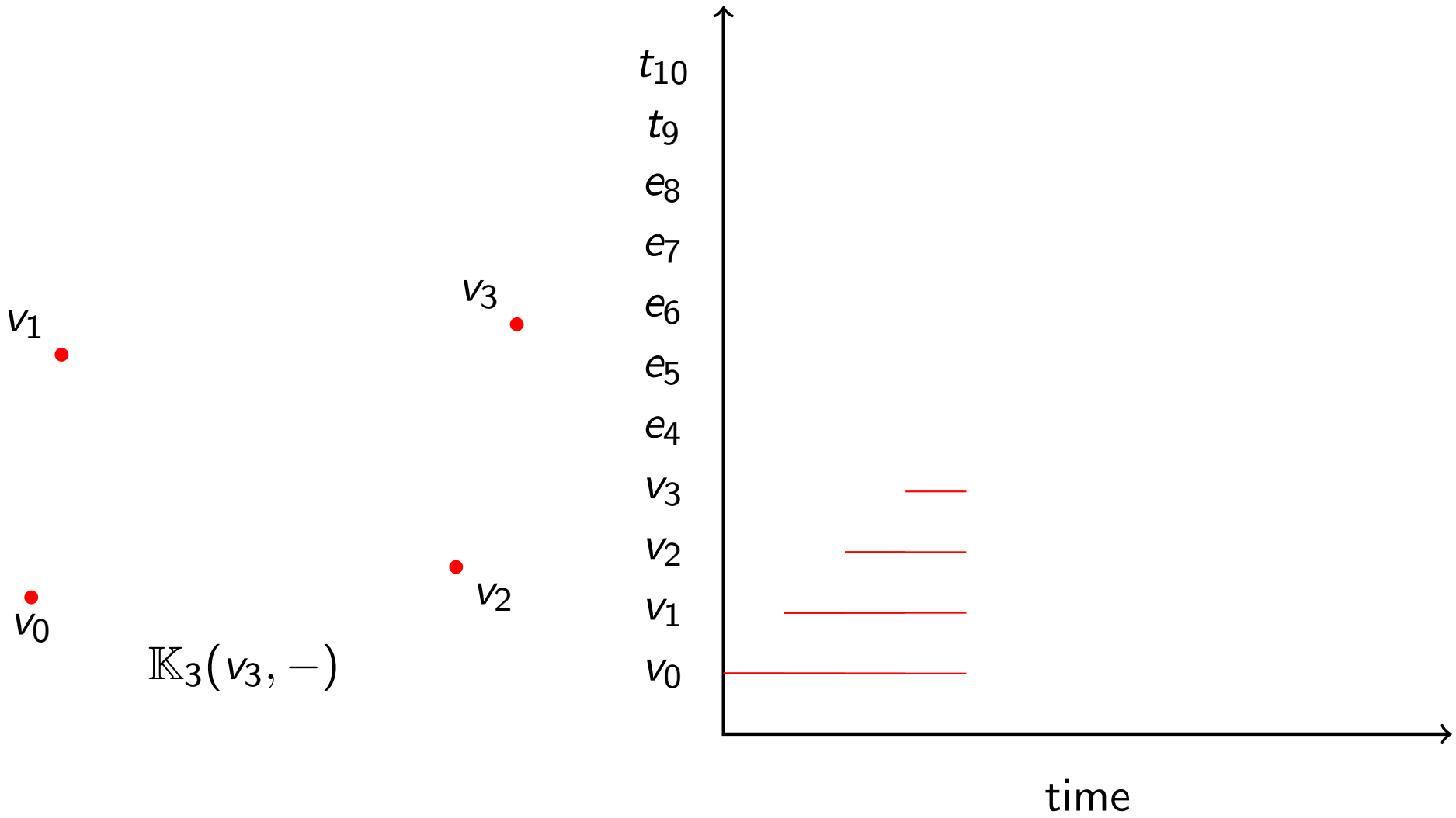

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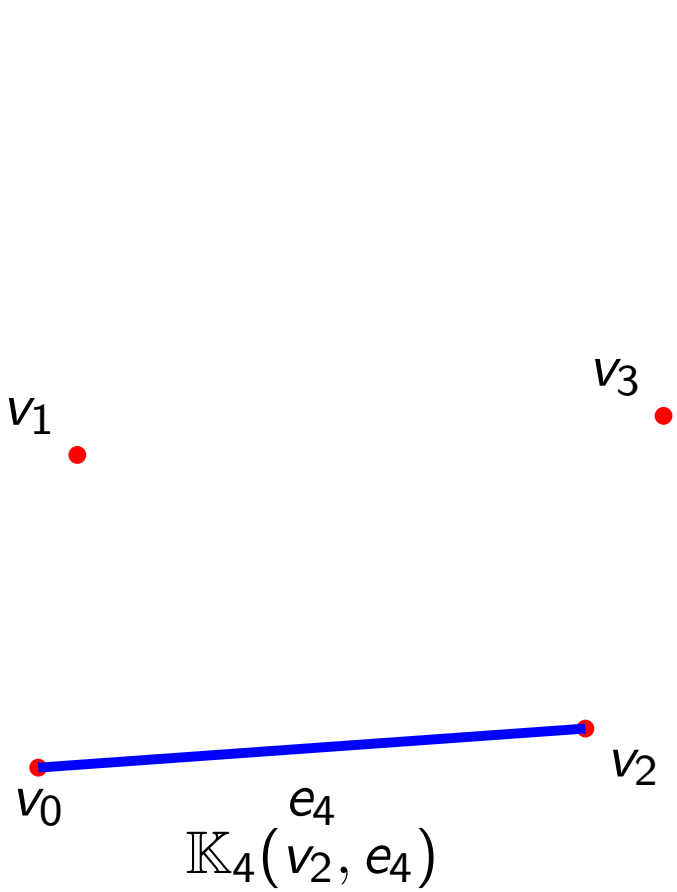

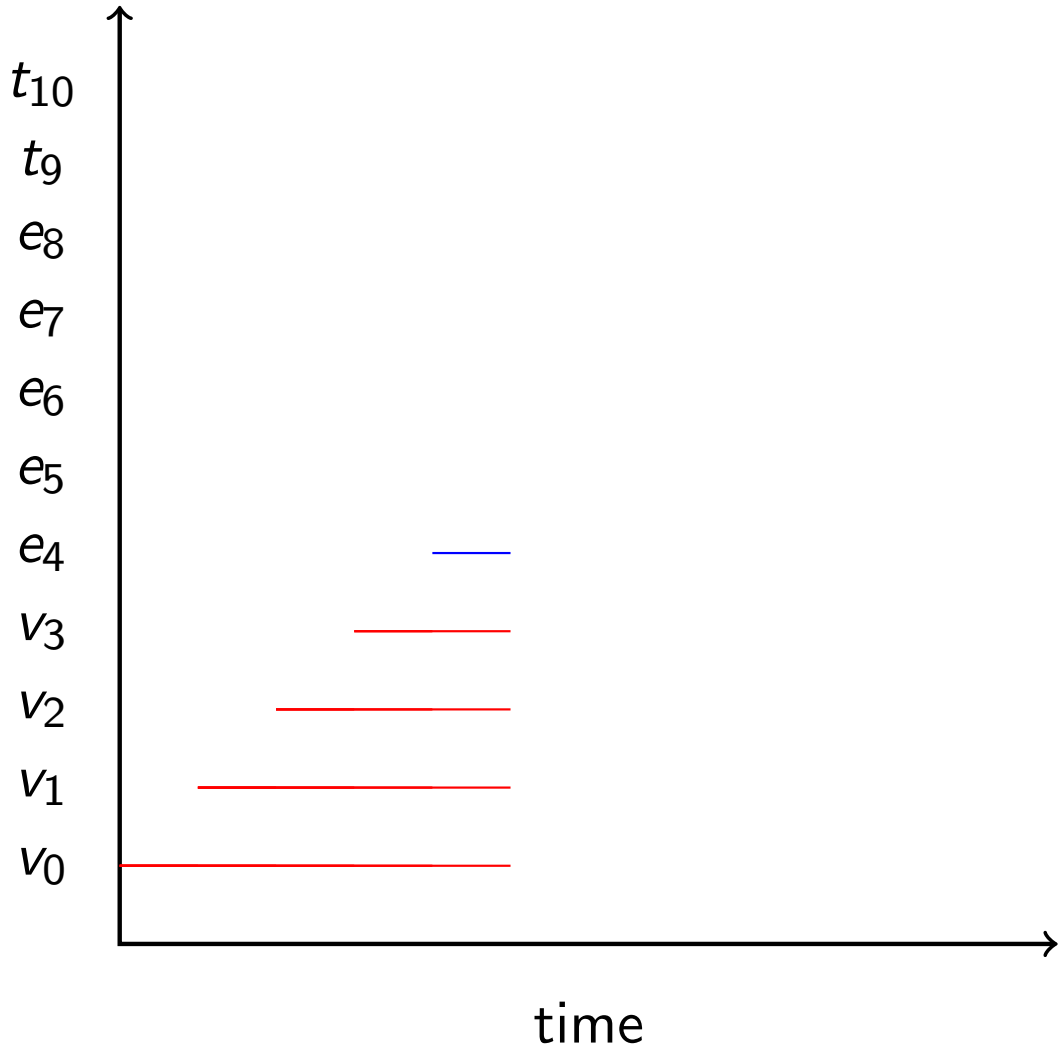

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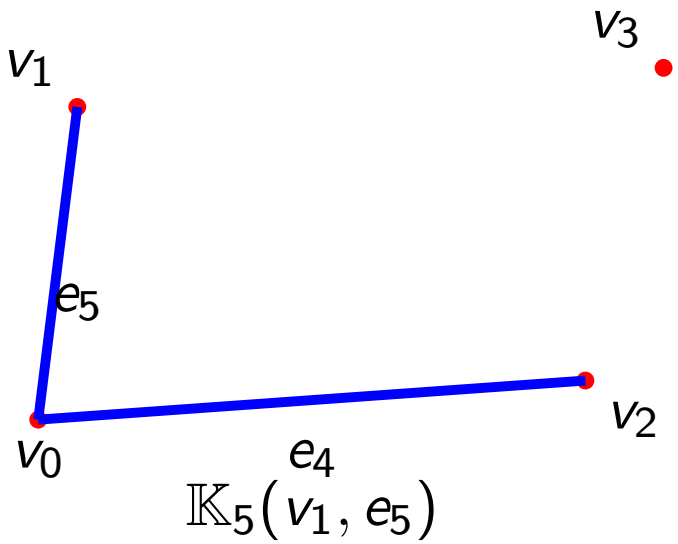

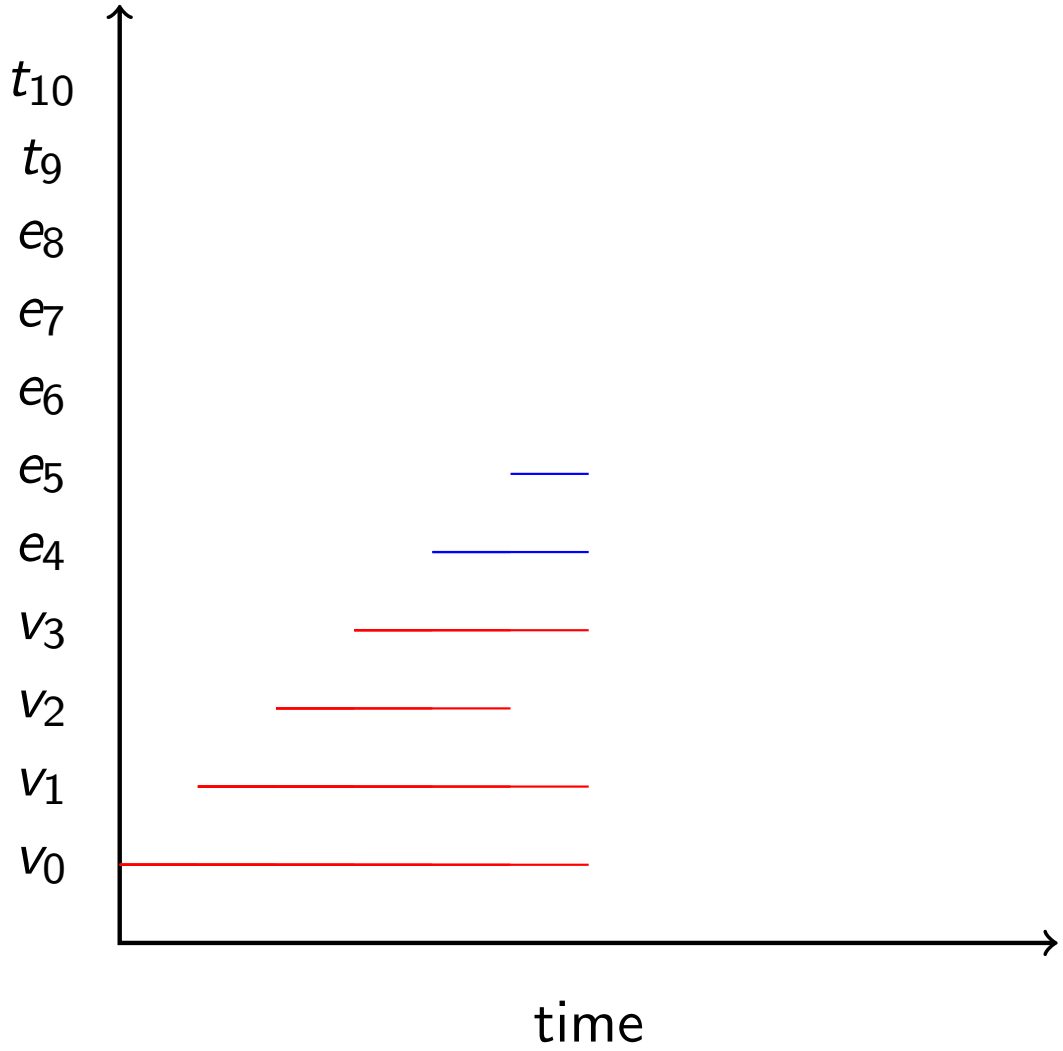

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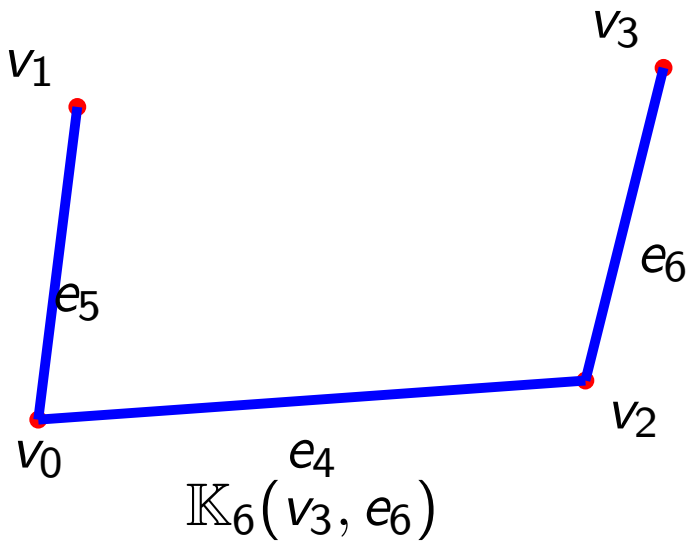

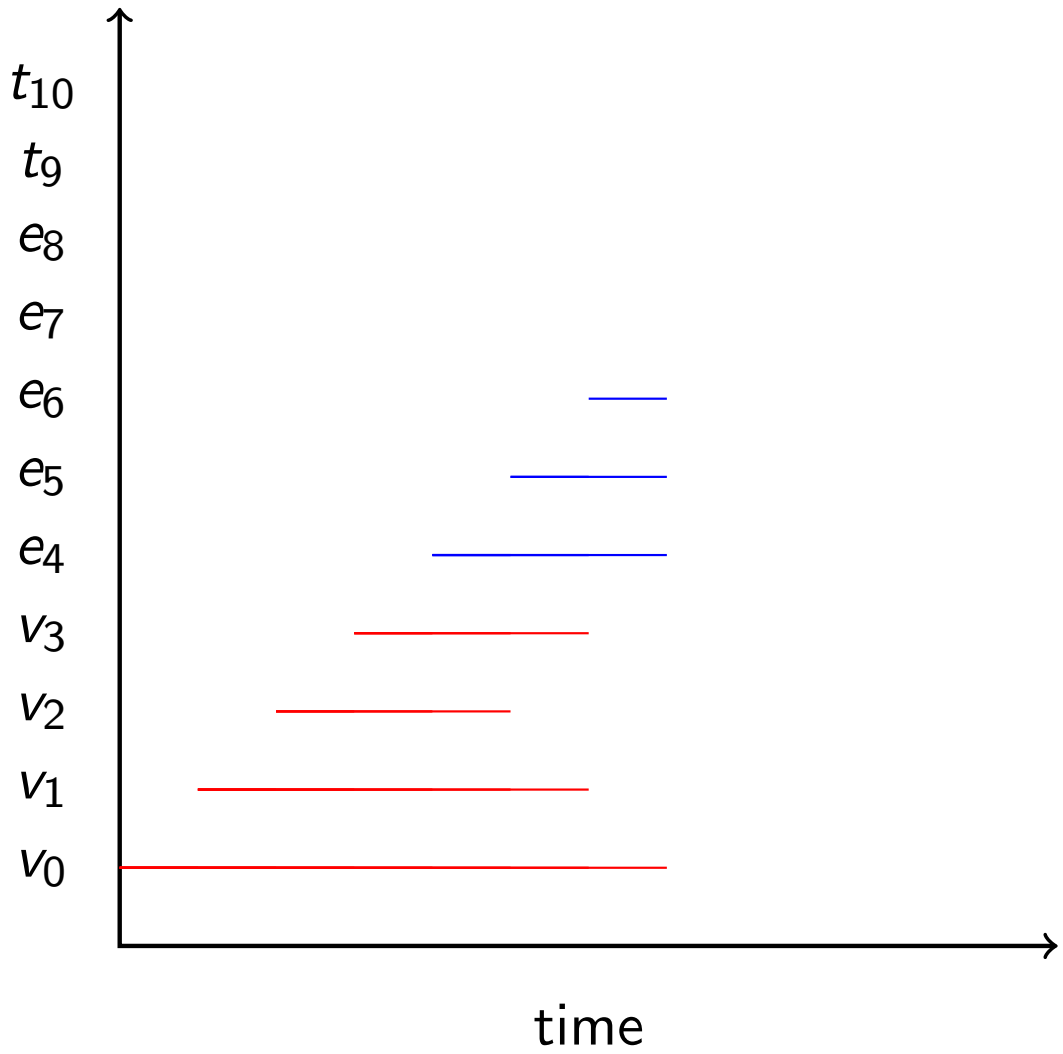

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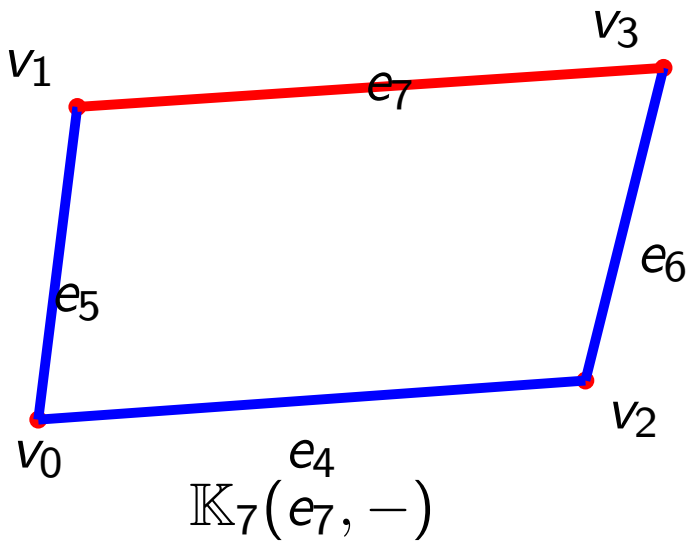

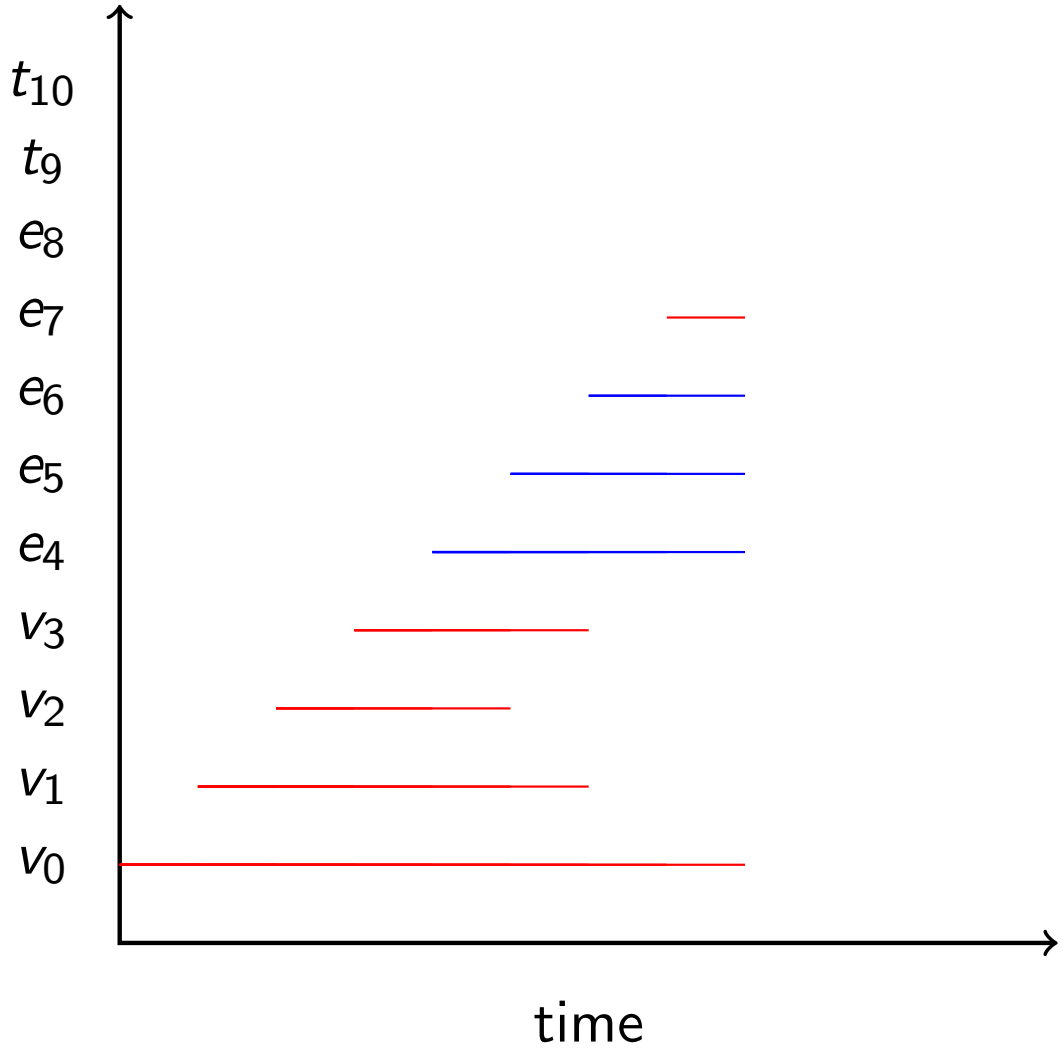

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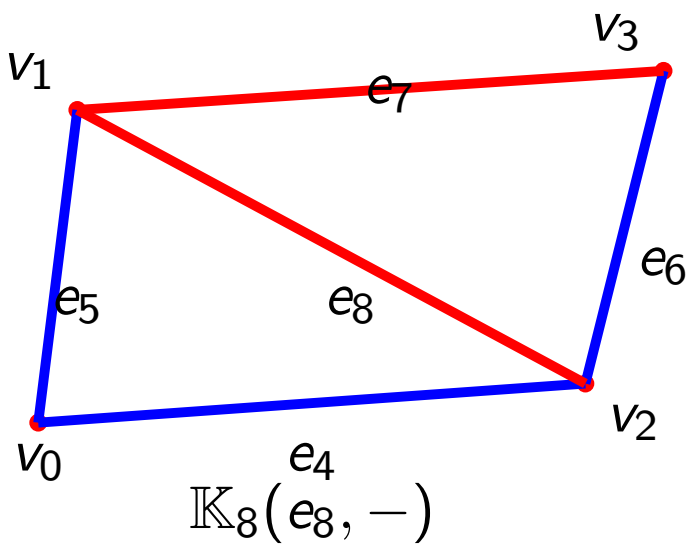

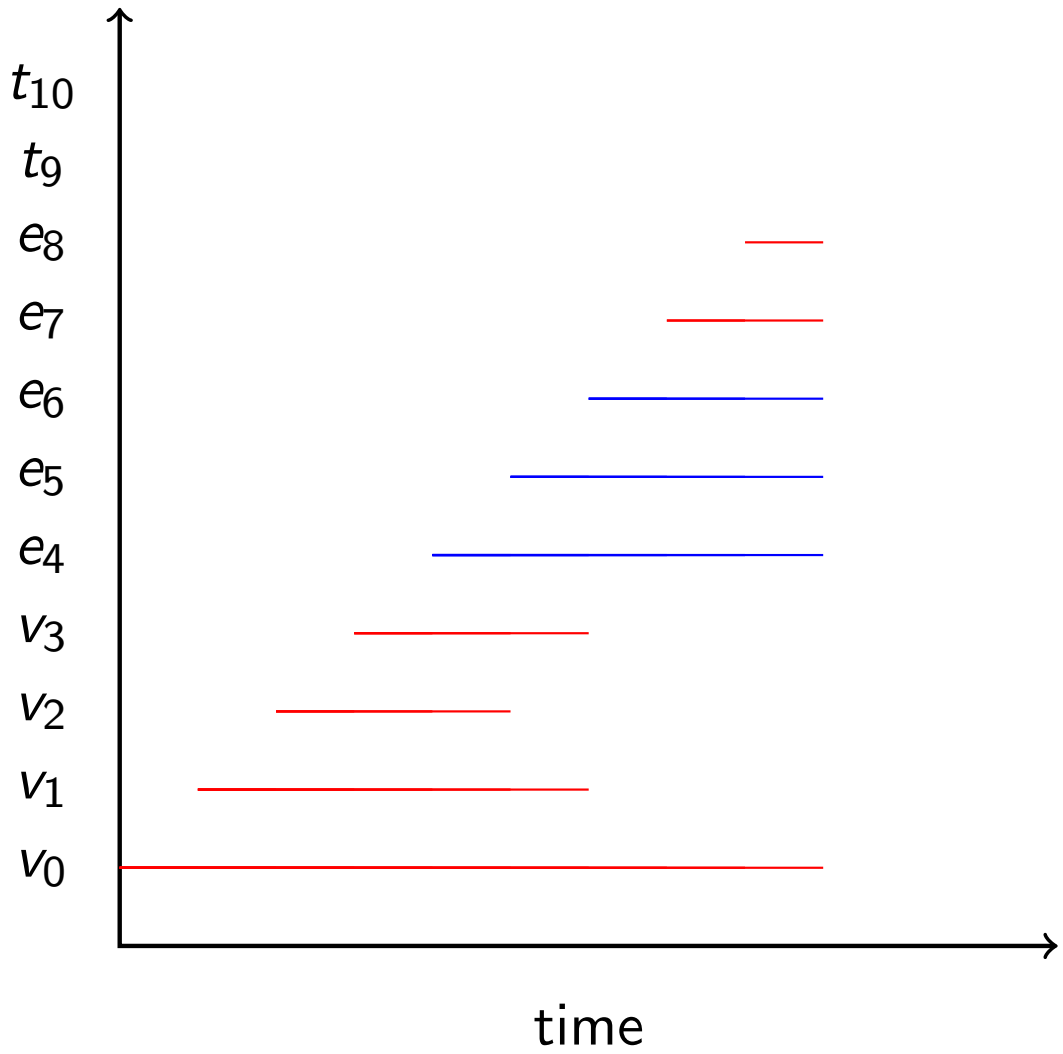

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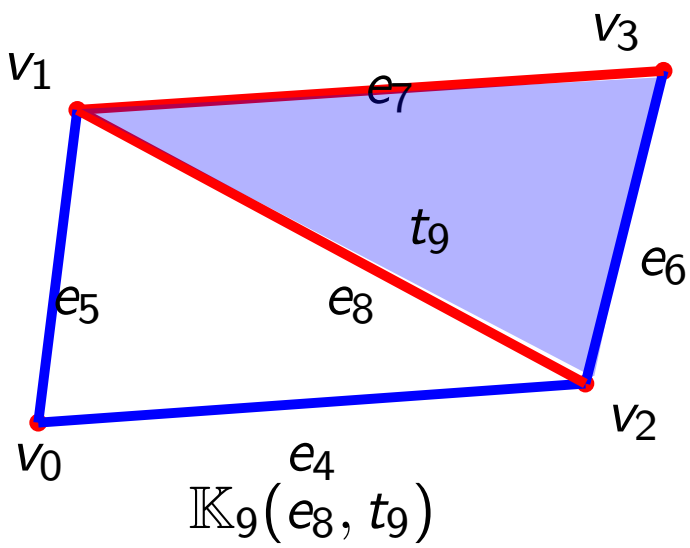

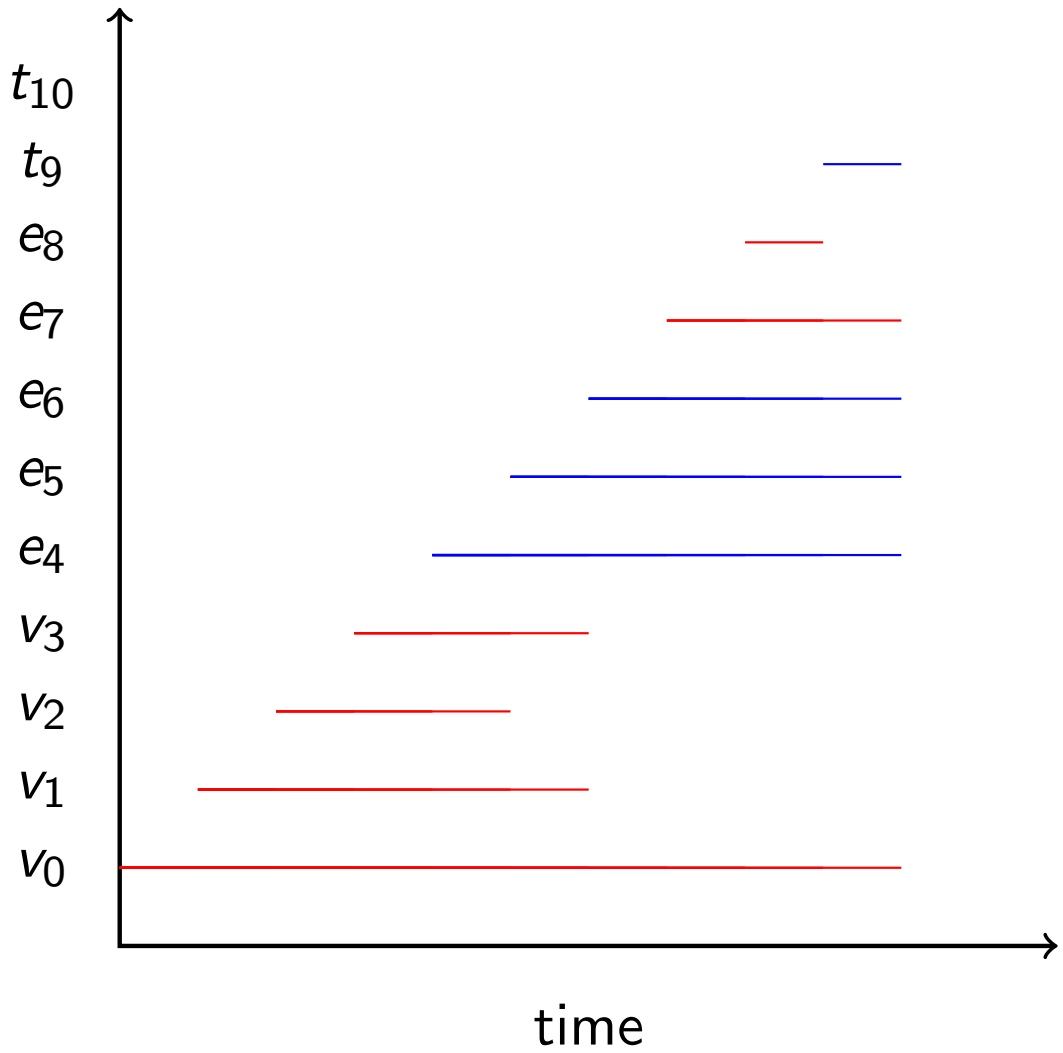

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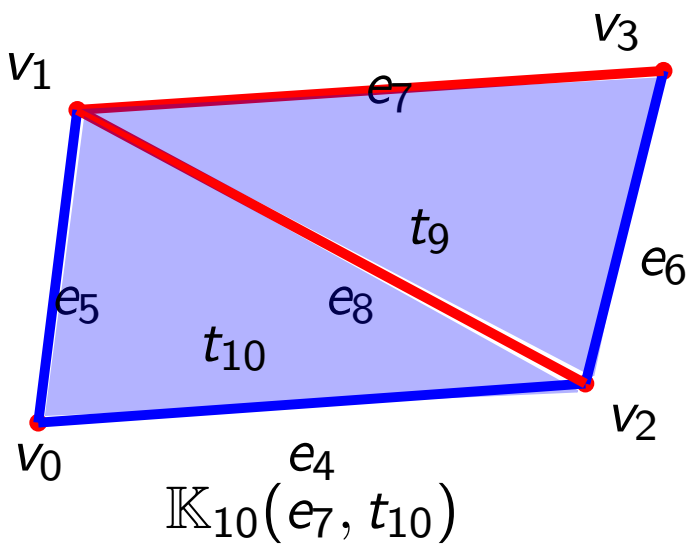

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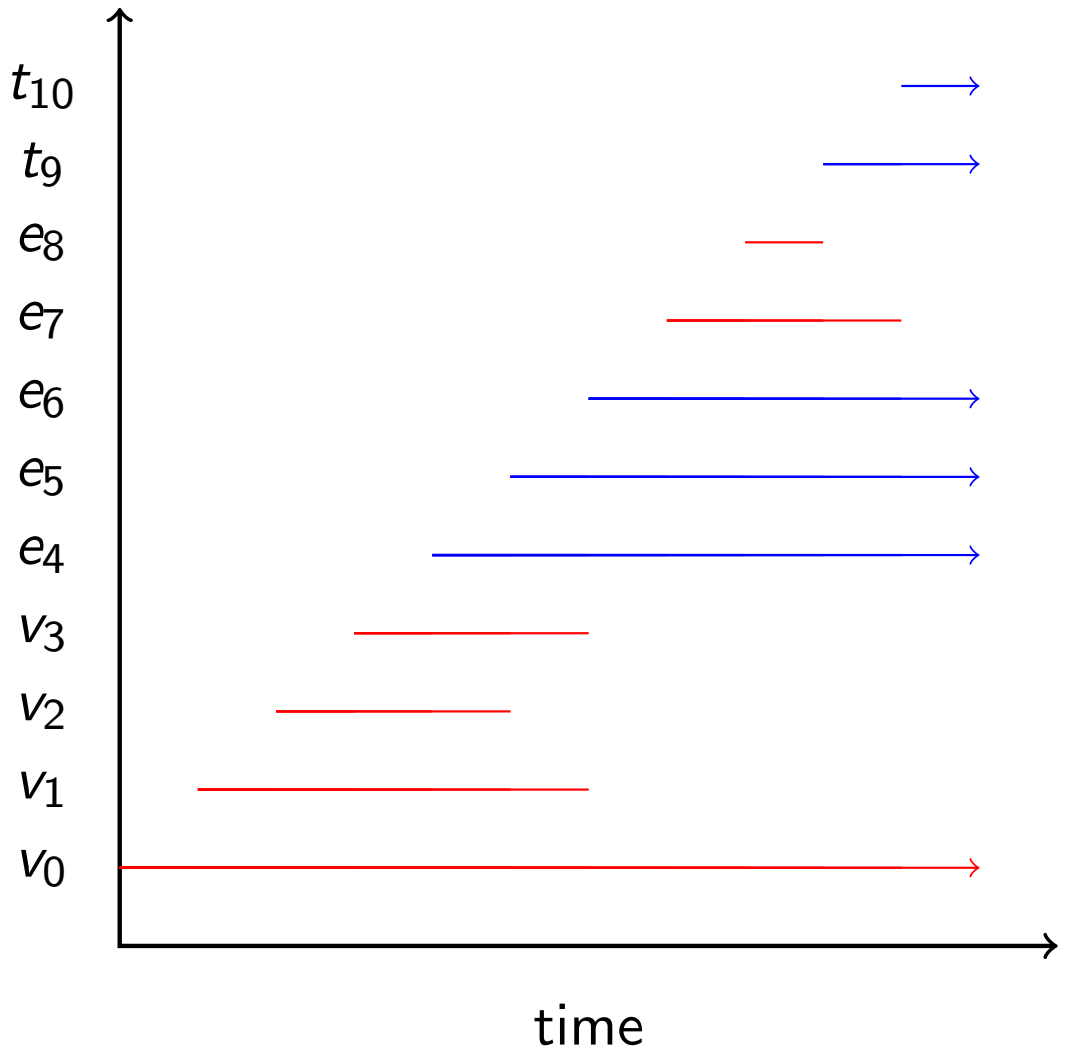

Homology Group