README: This is one half of my final year Advanced Econometrics project. It was marked 79/100. I answer the following question(s):

Russia's conflict with Ukraine has increased energy prices and high market volatility of energy prices affecting the economies around the world. Also, the sanctions imposed by the USA and other European countries seem to have fragmented the oil market. In this question you are going to investigate the interactions between various oil markets. If you are a consumer of oil, you would like to diversify your supply. In this exercise we would like to have a mix of various markets (WTI, BRENT, URALS, etc) as our portfolio. (See http://oilprice.com/) Choose four markets and download the daily prices of oils from various markets till the end of January 2023) and answer the following questions:

B1) Check for non-stationarity of the oil prices and then investigate the descriptive statistics (including higher moments) of the returns. How would you compute the confidence intervals for the moments? What is your inference?

The time period selected is from 31st January 2021 to 31st January 2023. Relative to the ongoing Russian-Ukraine war which formally began in February 2022, the sample includes data a year either side. In particular, the start date is drawn a year after the beginning of the Covid-19 pandemic, as prices returned to that of pre-pandemic by this timeframe.

B1.1 Non-stationarity

Stationarity is a property of a time series or stochastic process Y_t that is characterised by the distribution of Y_t being independent of time (Stock and Watson). In particular, this means the joint distribution $(Y_1, Y_2, Y_3, \ldots, Y_t) = (Y_{1+m}, Y_{2+m}, Y_{3+m}, \ldots, Y_{t+m}) \; \forall \, t, m$. In the opposite case, that is *non*-stationarity, time series are typically characterised by either *deterministic* or *stochastic* trends (Stock and Watson).

A time series with a deterministic trend (being *trend stationary*) can be seen in equation (1).

$$Y_t = \alpha + \beta t + u_t \tag{1}$$

Assuming u_t itself is white noise, removing this deterministic trend means the remaining residual series (i.e. u_t) is stationary stochastic.

$$(Y_t - \alpha - \beta t) = u_t$$

A time series that exerts a stochastic trend (being *difference stationary*), can be seen for example in an AR(1) model:

$$Y_t = c + \varphi_1 Y_{t-1} + u_t \tag{2}$$

or in operator form $$(1 - \varphi_1 L) Y_t = c + u_t$$

If $|\varphi_1| < 1$ (i.e. the root $\frac{1}{|\varphi_1|} < 1$), the process is stationary (Stock and Watson, Appendix 15.2). However, if $|\varphi_1| = 1$ then the characteristic equation has a root at L = 1 (a.k.a. having a *unit root*) and this means the process has a stochastic trend and is not stationary. The same condition generalising to the AR(p) case (p $\in \mathbb{Z}^+$), where all the roots of the characteristic must be outside the unit circle for stationarity.

In the case where there are *d* unit roots, it can be shown that differencing the equation *d* times leads to a stationary process (denoted an I(d) series). In the I(1) case:

$$\Delta Y_t = Y_t - Y_{t-1} = c + \delta Y_{t-1} + u_t \qquad \text{(Where } \delta = 1 - \varphi_1\text{)}$$

This form sets up the "Dickey-Fuller" hypothesis test of δ, with hypotheses

$$H0 : \delta = 0 \text{ (unit root)}$$

$$H1 : \delta < 0 \text{ (no unit root, stationarity)}$$

$$\text{As } \delta = 0 \iff |\varphi_1| = 1$$

Using a typical least squares t-test for δ, is problematic (Stock and Watson). Y_{t-1} is itself non-stationary, so asymptotically it does not take a t-distribution meaning the usual critical values are not applicable.

The potential functional form of the series could be beyond that of a simple random walk. (3) allows for both a constant relationship in the difference and a deterministic trend:

$$\Delta Y_t = \mu + \delta Y_{t-1} + \beta t + u_t \tag{3}$$

However, u_t may not be i.i.d./white noise. Including p-1 lags of the dependent variable in the regression 'whitens' the error, correcting for the additional variance in u_t. This sets up the augmented Dicky fuller (ADF) test. This requires simulation for generating critical values rather than typical t-statistics. The value of p (the number of lags) can be estimated using information criterion, specifically AIC (Stock and Watson).

KPSS test complement the ADF (Lee and Schmidt, 1996). The KPSS statistic k_0 is estimated by first detrending/demeaning the series, giving e_t. If there is a unit root, the cumulative variance of e_t will grow large. The null and alternative hypothesis of stationarity are therefore flipped relative to the ADF test, based on the KPSS critical values relative to k_0:

$$k_0 = \frac{T^{-2}\sum_{t-1}^{T} {s_t}^2}{T^{-1}\sum_{t-1}^{T} {e_t}^2}$$

Table 1	*WTI price*	*BRENT price*	*URALS price*	*MURBAN price*
ADF statistics:				
$\boldsymbol{\delta} = 0$	-1.99	-2.12	-2.22	-1.9
$\boldsymbol{\delta} = \mu = 0$	1.91	2.06	2.17	1.69
$\boldsymbol{\delta} = \mu = \boldsymbol{\beta} = 0$	2.68	2.86	3.25	2.34
KPSS statistic	4.74***	4.71***	1.45***	4.53***

Failure to reject any ADF hypotheses implies unit root is present, no drift or time trend is found for all oil prices. The KPSS statistic confirms the unit root presence. This suggests all four returns have stationarity

B1.2 Descriptive Statistics

The log prices and log returns of the four crude oils are presented on the following page. The descriptive statistics (mean, standard deviation, skewness, and kurtosis) of the log returns for each variable are presented in table 2. Log returns of Y_t are given by taking $\log(Y_t) - \log(Y_{t-1})$.

The latter two (higher) moments can be estimated by equation X and X:

$$unbiased\ skew = K3 = \frac{T-1}{\sqrt{T}(T-2)}\left(\frac{\sum_{i=1}^{T}(Y_i - \overline{Y}_i)^3}{s^3}\right)$$

$$unbiased\ kurtosis = K4 = \frac{T-1}{(T-2)(T-3)}\left[(T+1)\left(\frac{\sum_{i=1}^{T}(Y_i - \overline{Y}_i)^4}{s^4}\right) + 6\right] - 3$$

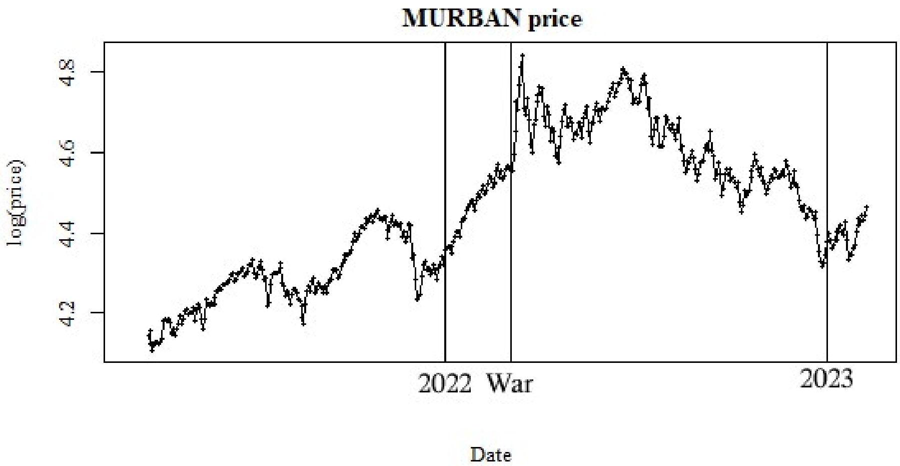
Where K3 is defined in relation to the gaussian kurtosis value of 3, so K3>0 means the sample distribution has excess kurtosis to that of a normal distribution. Given the large sample (T>30), the standard error of skewness and kurtosis can be estimated by $\sqrt{\frac{6}{T}}$ and $\sqrt{\frac{24}{T}}$ respectively. The Jarque-Bera test of joint skewness and kurtosis against a χ^2 is also performed. Kernel Density estimations of the returns are also plotted against a normal distribution, alongside Q-Q plots.

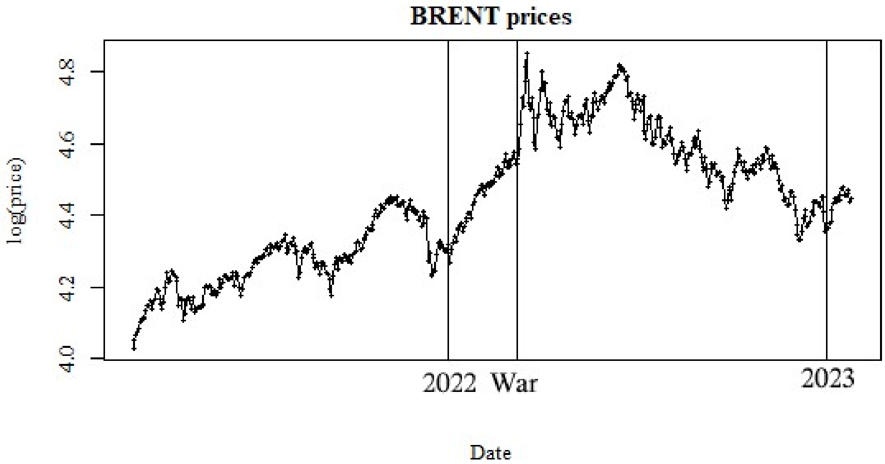
Table 2	***Returns WTI***	***Returns BRENT***	***Returns URALS***	***Returns MURBAN***
Mean	0.00077	0.000799	-0.0000137	0.000676
Min	-0.14	-0.141	-0.131	-0.132
Max	0.0802	0.0843	0.15	0.0763
Standard Deviation	0.0268	0.0253	0.0317	0.0251
Skew	-0.78*** (0.110)	-0.838*** (0.110)	-0.363*** (0.110)	-0.755*** (0.110)
Kurtosis excess	5.75*** (0.219)	6.4*** (0.219)	5.7*** (0.219)	5.53*** (0.219)
Jarque-Bera	209***	312***	169***	170***

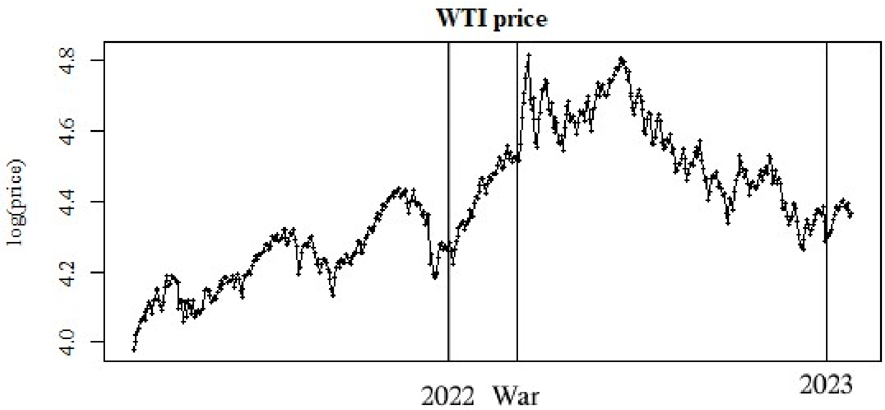
This implies that the returns are non-stationary and non-normal. This is further confirmed by the QQ and kernel density plots on the next page, as one would expect the kernel density to be close to the gaussian pdf line, and for the sample quantiles to match the theoretical quantiles in the QQ plots.

Constructing a confidence interval for the higher moments is straight forward, as T>30 we can utilise the asymptotic relationship that $\frac{K(3,4)}{S(K3,4)} \sim N(0,1)$ for both the Kurtosis and Skew. For a 95% confidence interval, the two tails therefore lie 1.96*(Standard Error) above and below the estimate. However, the assumption of normality for traditional mean and standard deviation confidence intervals is violated with our sample thus this method is not applicable for these lower moments. Taking a bootstrap quantile-based confidence interval removes the assumption of normality as the confidence interval is based on the distribution of the data itself. Bootstrapping involves resampling Q times with replacement to achieve a series of Q bootstrap samples, then calculating the statistics of each sample that gives a distribution of statistics (for instance, the distribution of the Q means across the bootstrap samples). The 95% confidence interval of the mean, for instance, is then given by the 2.5^{th} and 97.5^{th} mean quantile values. The interval can therefore reflect, for example, any non-symmetry of the sample distribution (which we see in the significant skew).

Figure 1-4


MURBAN price
log(price)
4.2
4.4
4.6
4.8
2022 War
2023
Date


BRENT prices
log(price)
4.0
4.2
4.4
4.6
4.8
2022 War
2023
Date


WTI price
log(price)
4.0
4.2
4.4
4.6
4.8
2022 War
2023

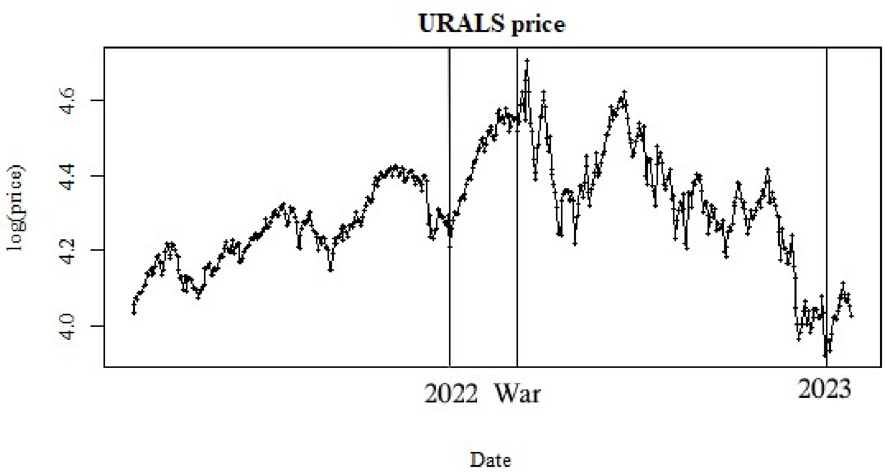
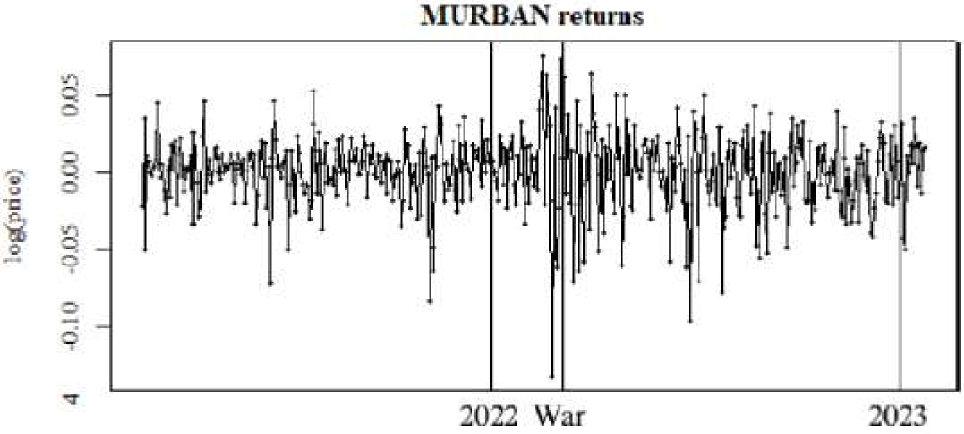
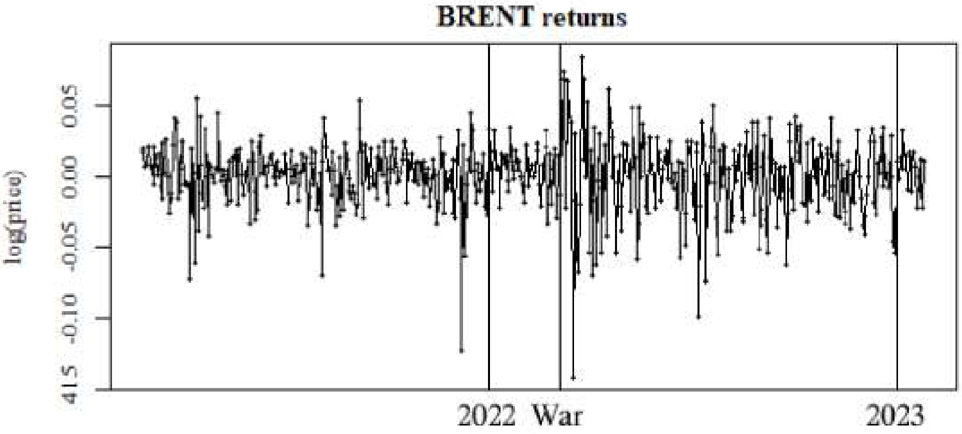
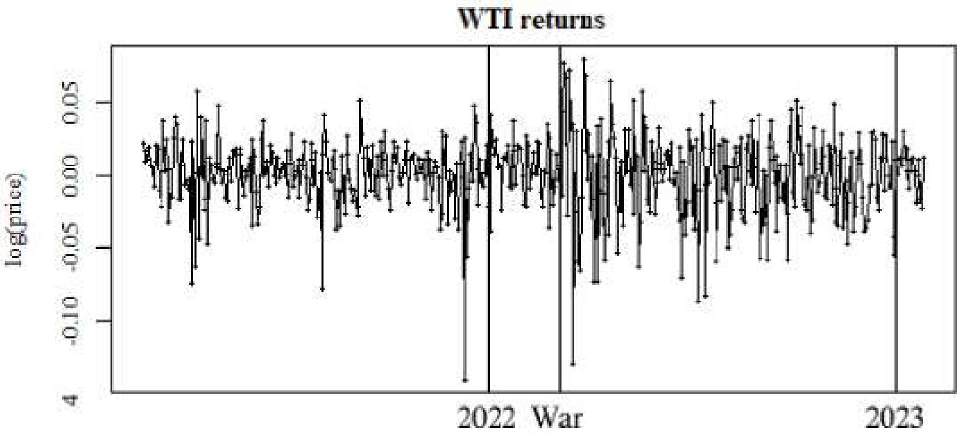

URALS price
log(price)
4.0
4.2
4.4
4.6
2022 War
2023
Date

Figure 5-8


MURBAN returns
log(price)
0.05
0.00
-0.05
-0.10
2022
War
2023


BRENT returns
log(price)
0.05
0.00
-0.05
-0.10
2022
War
2023


WTI returns
log(price)
0.05
0.00
-0.05
-0.10
2022
War
2023

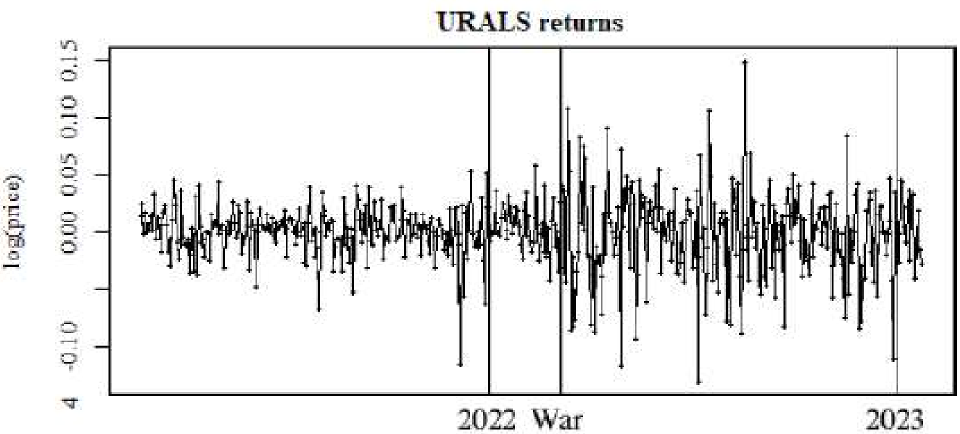

URALS returns
log(price)
0.15
0.10
0.05
0.00
-0.10
2022
War
2023

Figure 9-13

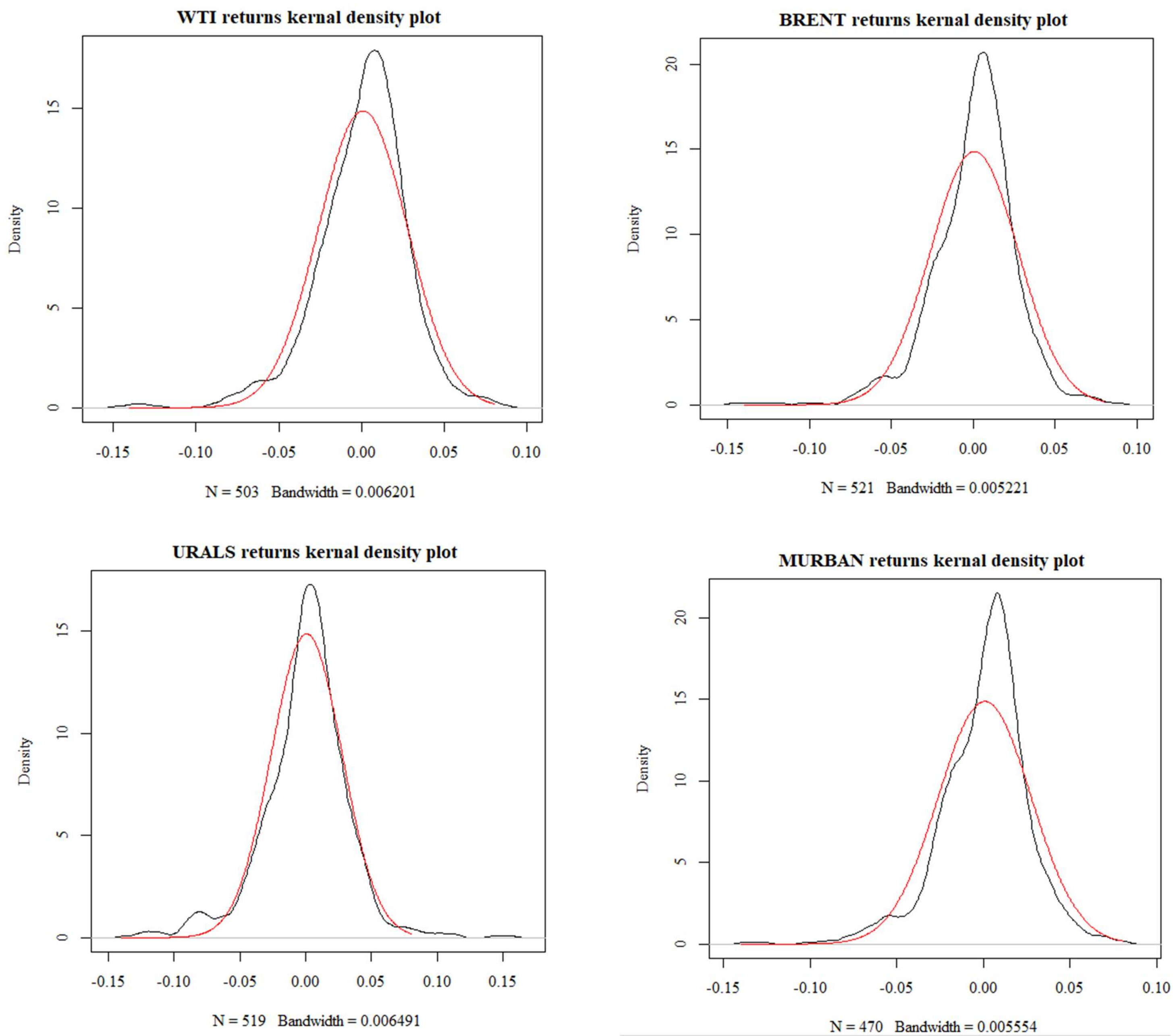
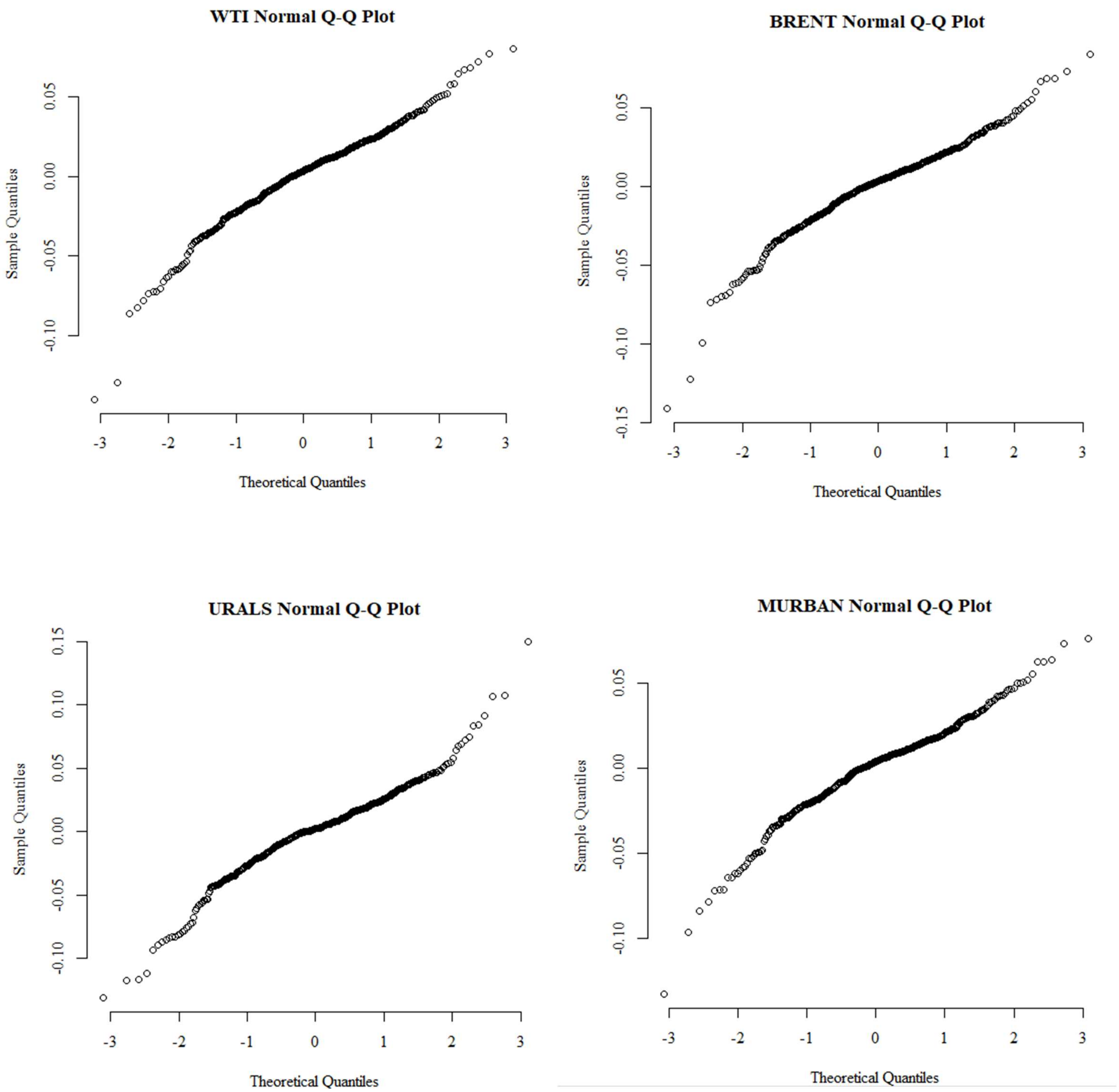

WTI returns kernal density plot
Density
0
5
10
15
-0.15
-0.10
-0.05
0.00
0.05
0.10
N = 503 Bandwidth = 0.006201
BRENT returns kernal density plot
Density
0
5
10
15
20
-0.15
-0.10
-0.05
0.00
0.05
0.10
N = 521 Bandwidth = 0.005221
URALS returns kernal density plot
Density
0
5
10
15
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
N = 519 Bandwidth = 0.006491
MURBAN returns kernal density plot
Density
0
5
10
15
20
-0.15
-0.10
-0.05
0.00
0.05
0.10
N = 470 Bandwidth = 0.005554

Figure 14-17


WTI Normal Q-Q Plot
BRENT Normal Q-Q Plot
URALS Normal Q-Q Plot
MURBAN Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles

B2) Compute the pairwise difference of the prices (also known as spread), estimate the time-series models for the spread and check for stationarity. Also analyse whether the oil prices are pairwise co-integrated. Compare and comment on the results.

To estimate with the appropriate time series model the stationarity of the six different spreads are explored, using the ADF and KPSS tests described as above. Notation: WB represents WTI – BRENT:

Table 3	***Spread WB***	***Spread WU***	***Spread WM***	***Spread BU***	***Spread BM***	***Spread UM***
ADF statistics:						
$\boldsymbol{\delta}=0$	-3.0603	-2.6081	-4.8348***	-2.5051	-6.0436***	-2.8757
$\boldsymbol{\delta}=\mu=0$	3.293	2.6751	7.8151***	2.6359	12.1918***	3.1601
$\boldsymbol{\delta}=\mu=\boldsymbol{\beta}=0$	4.901	3.503	11.7165***	3.3467	18.2871***	4.2712
KPSS statistic	3.2117***	6.0549***	0.1691	5.5962***	0.2093	6.1893***
Diff. ADF $\boldsymbol{\delta}=0$	-20.36***	-21.59***	excluded	-22.46***	-22.16***	excluded

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The two spreads that show stationarity (no presence of a unit root) are WM and BM. The KPSS statistic however is also significant, but of reduced significance.

Combing an AR(p) process as described in B1 with a "Moving Average" process of order q (MA(q)) gives rise to an ARMA process which we can use to estimate time series models. Whilst AR(p) models the relationship between an observation and its lagged *values*, an MA(q) process models the relationship between an observation and its lagged *error terms*. ARMA therefore models processes as a function of both autoregressive and moving average parameters, so is more flexible in capturing the correct fit of the time series (Stock and Watson, 2019). The estimation process requires stationarity: the four I(1) spread processes can be made stationary using first differencing (as seen in table 3), therefore the differenced spread is used in the ARMA process (a.k.a. the ARIMA process with d=1, where ARIMA(p,0,q) is analogous to ARMA(p,q)).

To determine the value of "q" and "p" for each model, the Akaike Information criterion (AIC) is used to find the combination of lags with the "best" trade-off between goodness of fit and model complexity/overfitting. Specifically, minimising the function

$$AIC(p,q) = ln(\sigma^2(p,q)) + \frac{2 \times (p+q)}{T}$$

Where σ^2 is the residual standard deviation. When finding a minimum, the $ln(\sigma^2(p,q))$ term 'rewards' the contribution of more variables (lags) to explaining the relationship whilst the second term 'punishes' the number of additional lags.

The AIC is minimised for all (p,d,q) ∈ Z with $1 < p,q < 7$ and where d = 1 or 0 is predetermined by unit root presence.

Table 4	***Spread WB***	***Spread WU***	***Spread WM***	***Spread BU***	***Spread BM***	***Spread UM***
(p,d,q)	(6,1,5)	(5,1,5)	(2,0,6)	(6,1,6)	(2,0,5)	(4,1,6)
AIC	-3691.825	-2404.223	-2840.197	-2513.941	-3014.145	-2200.759

Model Results

Table 5	***Spread WB***	***Spread WU***	***Spread WM***	***Spread BU***	***Spread BM***	***Spread UM***
ar1	-0.23 0.621	-0.71** 0.033	0.15 0.540	-0.07 0.598	0.24 0.263	0.01 0.985
ar2	0.27 0.386	0.06 0.882	0.78*** 0.001	0.21 0.301	0.45 0.113	-0.36 0.304
ar3	0.18 0.712	0.43 0.141	-	0.64*** 0.000	-	0.46** 0.049
ar4	-0.08 0.785	0.32 0.132	-	0.16 0.366	-	-0.1 0.825
ar5	-0.34** 0.038	-0.20 0.125	-	-0.41** 0.019	-	-
ar6	-0.14** 0.046	-	-	0.21** 0.023	-	-
ma1	-0.02 0.970	0.32 0.330	0.62** 0.016	-0.30** 0.029	0.52** 0.016	-0.35 0.391
ma2	-0.46 0.168	-0.44 0.148	-0.26*** 0.003	-0.33 0.183	0.03 0.901	0.26 0.414
ma3	-0.15 0.780	-0.43 0.171	-0.13 0.108	-0.56*** 0.001	0.00 0.932	-0.60*** 0.001
ma4	0.14 0.659	-0.04 0.803	-0.03 0.692	0.18 0.308	0.12 0.573	0.28 0.576
ma5	0.30 0.139	0.36** 0.033	-0.02 0.756	0.55 0.017	0.14 0.203	-0.02 0.908
ma6	-	-	-0.00 0.947	-0.34 0.001	-	0.12 0.145
constant	-	-	-0.04*** 0.001	-	0.00 0.329	-

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B2.2 Pairwise Cointegration

Two non-stationary time series Y_t and X_t are cointegrated if there exists a real value θ for which the series $(Y_t - \theta X_t)$ is stationary. Similar to the logic of differencing I(1), if the two series 'move together' (i.e. the 'underlying' stochastic trend is the same) then subtracting some multiple of one from the other should 'cancel' out this stochasticity leaving a stationary process.

As all our variables are I(1) or I(0), regressing one on the other across each pair allows for the coefficients θ to be estimated. Subtracting this amount of X_t leaves a residual series that can then be tested for a unit root as before. The 'vicarious' null hypothesis is that of non-cointegration (so failing to reject a unit root (the ADF null) implies non-cointegration) and the converse implies cointegration.

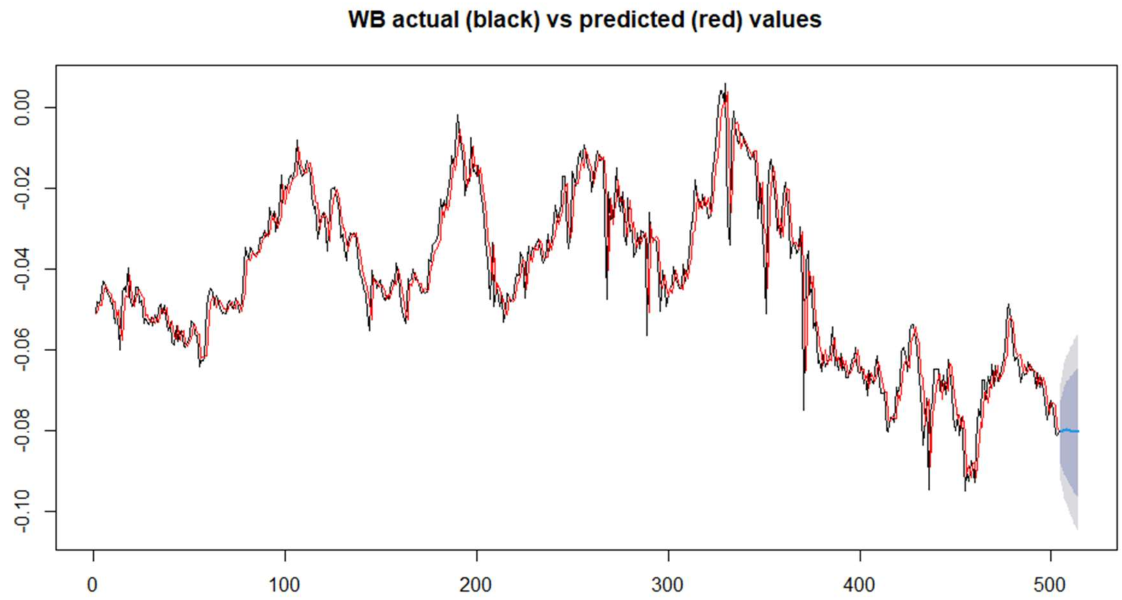
Table 6	***Coint WB***	***Coint WU***	***Coint WM***	***Coint BU***	***Coint BM***	***Coint UM***
EG value	-1.96	-0.75	-3.2374**	-0.6	-5.78 ***	-1.38
P-value	>0.10	>0.10	0.0243	>0.10	<0.01	>0.10

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The two spreads (WM and BM) that showed previously showed stationarity also show cointegration between the prices themselves. The OLS regressions give $\theta_{WM} = 0.9521$ and $\theta_{BM} = 0.9634$, both very close to the unitary difference coefficient of the spreads.

Some of the spread models estimated in table 5 show adequate model fit, with WM BU and UM all having significant coefficients in both the AR and MA components of the model. The other three spread models however show minimal significance in either/both components, implying that despite minimising the Information Criteria, the spreads are not well explained by AR and/or MA processes, and the model is likely overfitted. The predicted values of the model relative to the actual values, as seen in figures 18-25 show a reasonable fit in the first three models mentioned.

Figure 18-19


WB actual (black) vs predicted (red) values
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
0
100
200
300
400
500

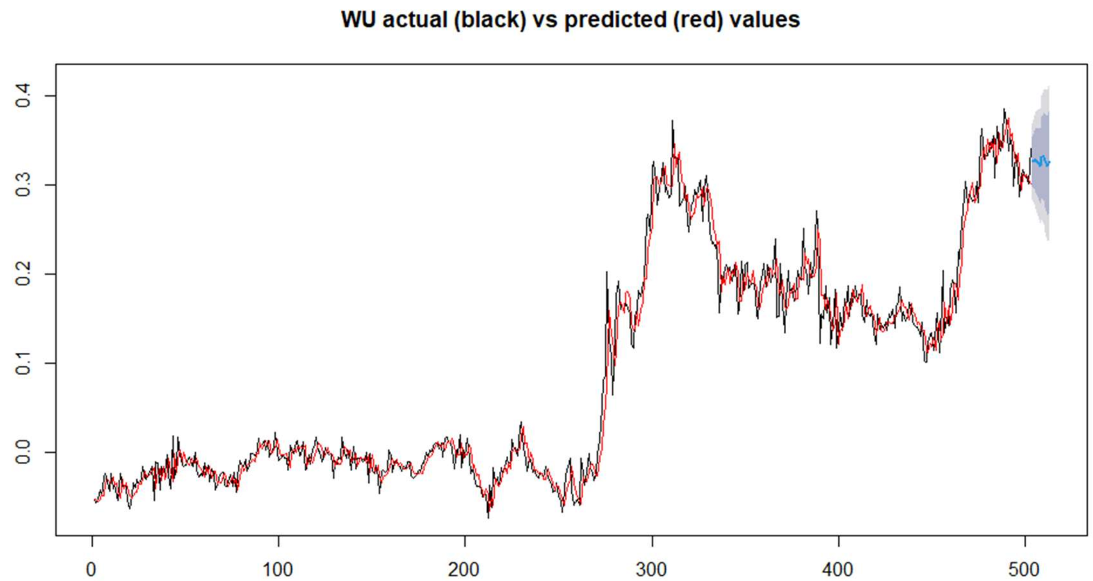
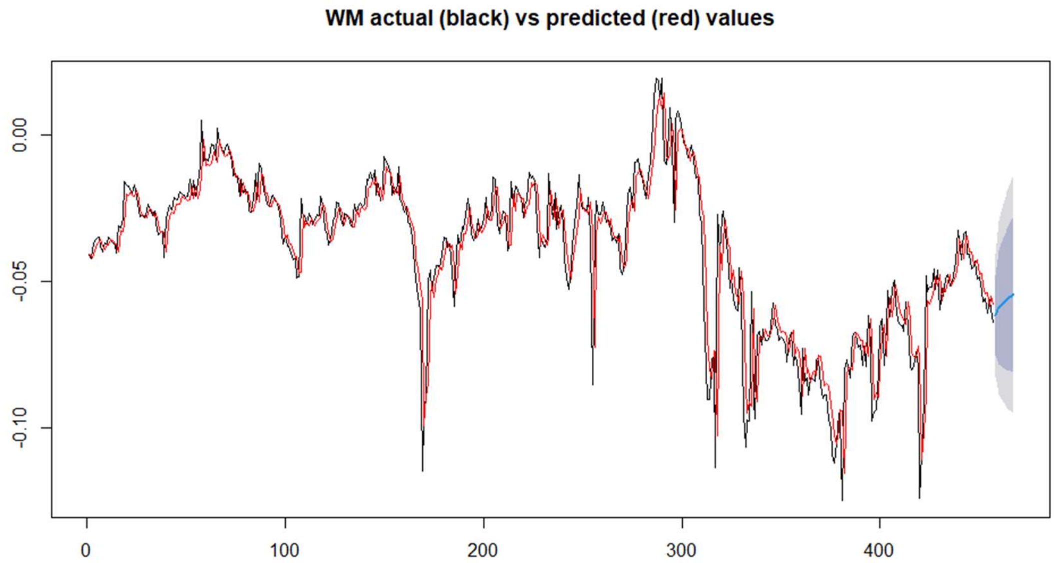
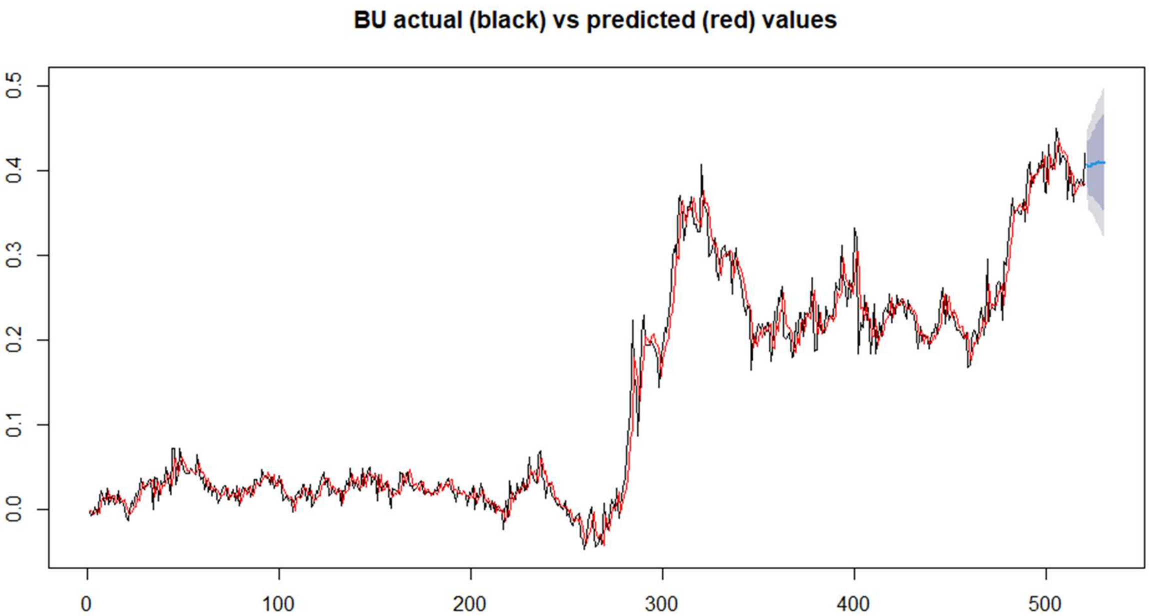

WU actual (black) vs predicted (red) values
0.4
0.3
0.2
0.1
0.0
0
100
200
300
400
500

Figure 20-21

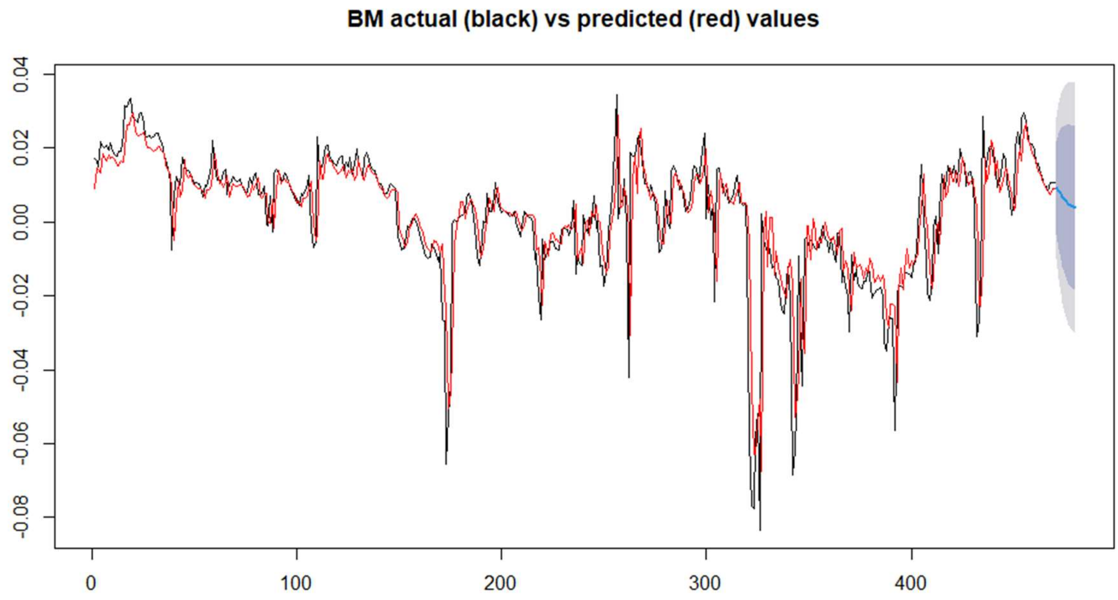


WM actual (black) vs predicted (red) values
0.00
-0.05
-0.10
0
100
200
300
400

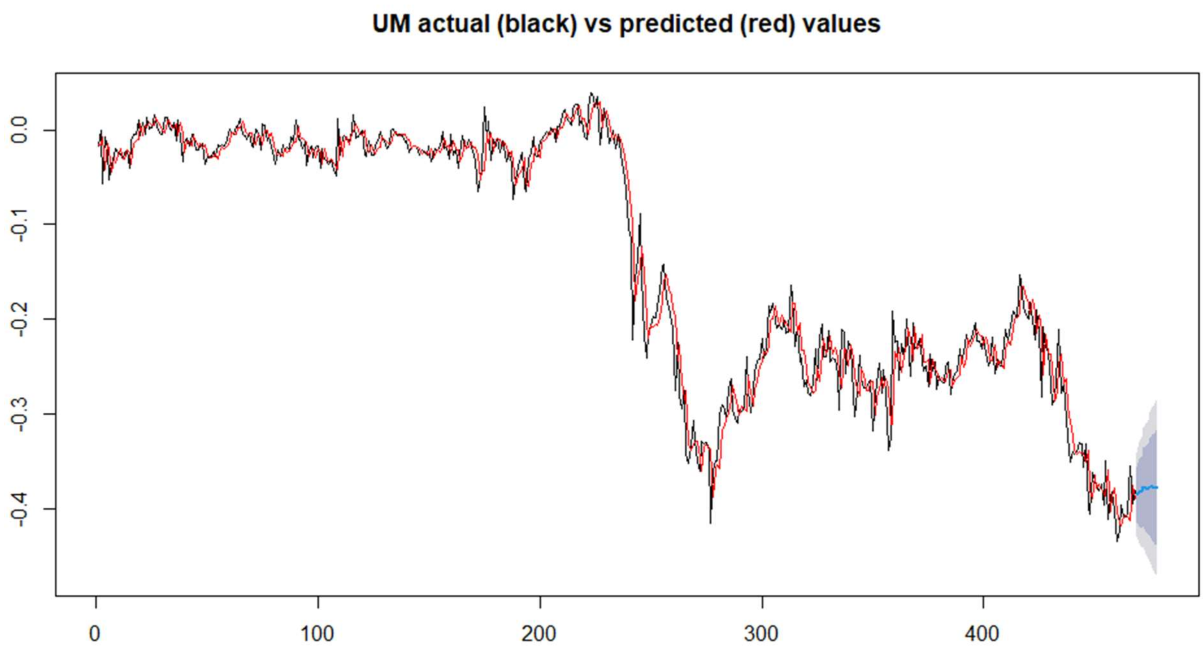


BU actual (black) vs predicted (red) values
0.5
0.4
0.3
0.2
0.1
0.0
0
100
200
300
400
500

Figure 22-23



BM actual (black) vs predicted (red) values
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
0
100
200
300
400



UM actual (black) vs predicted (red) values
0.0
-0.1
-0.2
-0.3
-0.4
0
100
200
300
400

B3) Estimate models for the return's volatility of your chosen oil market for the entire sample period and for the periods before and after the start of the Russian-Ukrainian conflict (24th February 2022). Thus, you need to estimate three models. Compare and comment on your results. Also compare the estimated higher moments for the volatility models with the results from part 1.

The returns showed volatility clustering of across all four oil markets, visible in figure X, and also showed excess higher moments relative to the gaussian distributions seen in part 1. To remedy this, alongside potential conditional heteroskedasticity, I employ the ARMA-EGARCH model which is explained shortly. To test for the presence of these 'ARCH effects', we investigate the squared residuals of the returns employing the Ljung-Box (LB) test alongside the ACF functions of the square residuals.

EGARCH models the conditional variance of the stems from the simpler ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH models. Consider the following demeaned returns $(Y_t - \alpha)$:

$$Y_t - \alpha = u_t$$

Where u_t takes a gaussian distribution, with a mean of 0, but the variance σ_t^2 of u_t is modelled as conditional on past square errors up to lag p (ARCH(p)):

$$\sigma_t^2 = (0^2) + \mathbb{E}\left(u_t^2 \middle| u_{t-1}, u_{t-2}, \dots\right) = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_p u_{t-p}^2$$

This incorporates heteroskedasticity and therefore accounts for volatility clustering into the model. Even if the demeaned returns (u_t) are uncorrelated with previous values of itself, a positive α_i implies if the returns suffer a shock of large magnitude in one period, the following periods are also likely to be subject to large-magnitude shocks. The generalisation of this model, GARCH(p,q), allows for the incorporation of 'q' lagged conditional variances (i.e. lagged values of itself) alongside the lagged squared errors. This allows nonlinear relationships between past shocks and current volatility to be captured.

$$\sigma_t^2 = (0^2) + \mathbb{E}\left(u_t^2 \middle| u_{t-1}, u_{t-2}, \dots\right) = \alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_p u_{t-p}^2 + \varphi_1 \sigma_{t-1}^2 + \dots + \varphi_1 \sigma_{t-q}^2$$

A further extension of this is Exponential GARCH. The EGARCH(1,1) model in equation X assumes that the logarithm of the conditional variance follows the process:

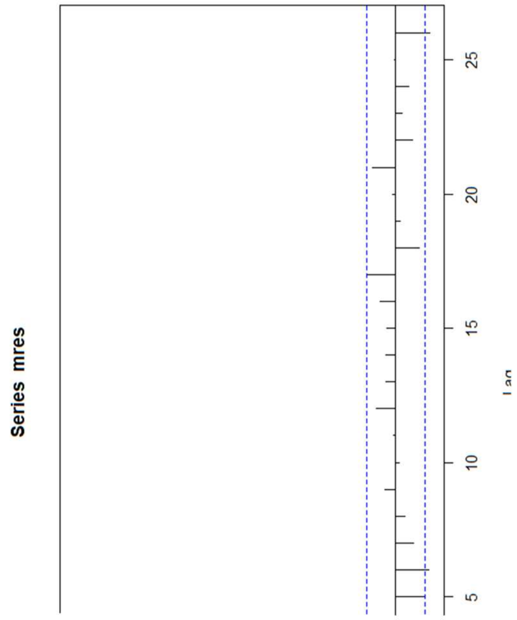
$$\ln(\sigma_t^2) = \alpha_0 + \gamma_1 \frac{|u_{t-1}| + \delta_1 u_{t-1}^2}{\sigma_{t-1}} + \varphi_1 \ln(\sigma_{t-1}^2)$$

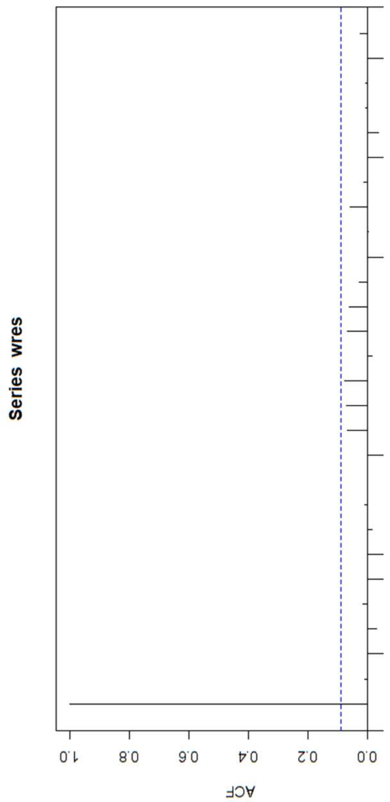
EGARCH improves upon the ARCH/GARCH modelling framework in a few ways. Firstly, without this embedded non-negativity constraint of EGARCH (from taking the logarithm) the estimation of ARCH/GARCH could give a negative conditional variance. Secondly, taking absolute values of the past squared errors allows for an asymmetry in the response of volatility to positive and negative shocks, where-as ARCH/GARCH does not. This is useful as negative shocks tend to have a larger impact on volatility than positive shocks (Engle, 1993).

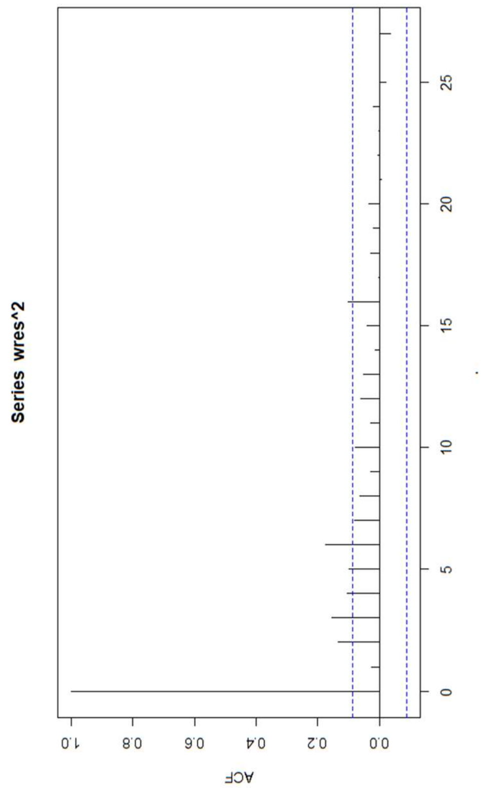
One can combine the aforementioned ARMA(r,s) process to model the conditional mean of the returns and EGARCH(p,q) to model the conditional variance of the returns in an ARMA(r,s)-EGARCH(p,q) model. This is the model I employ. In order to determine the 'best' fitting model I again minimise the AIC for all integers 0-6 over (r,s,p,q).

B3.1 Results

ACF/LB ***F***igure 24-27







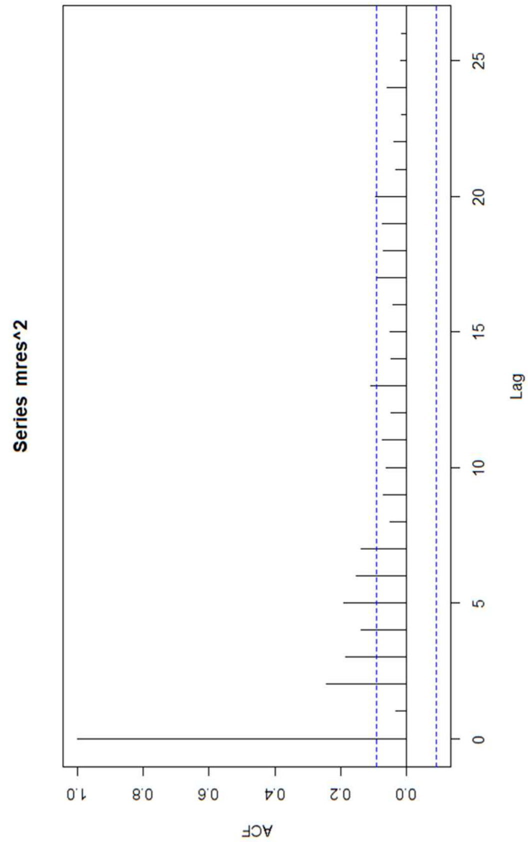
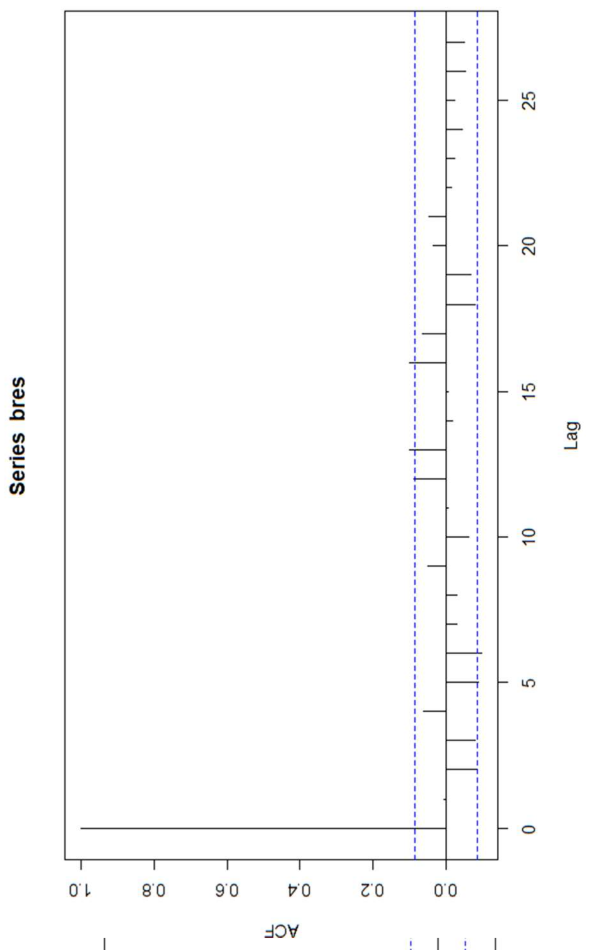


Figure 28-31


Series bres
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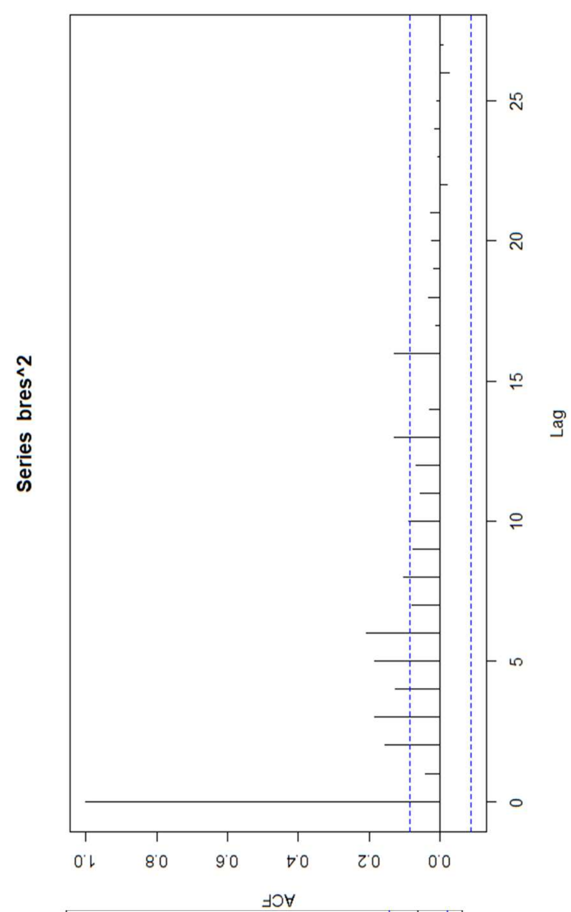
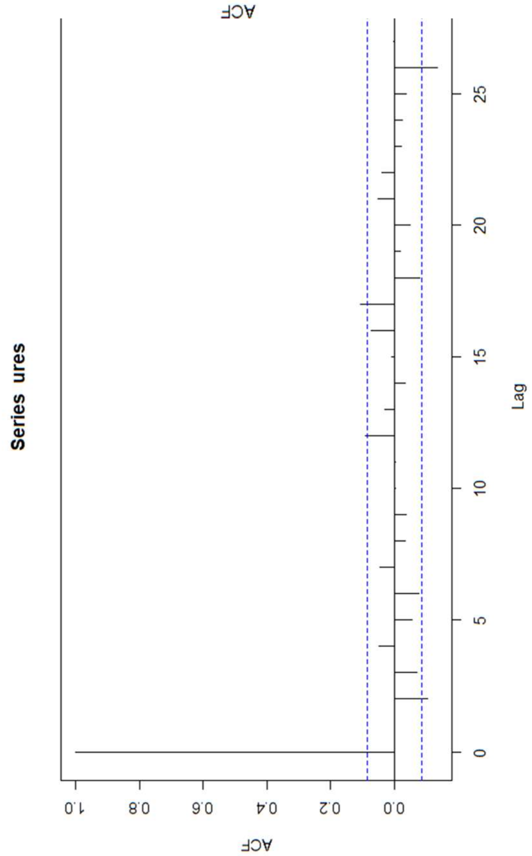
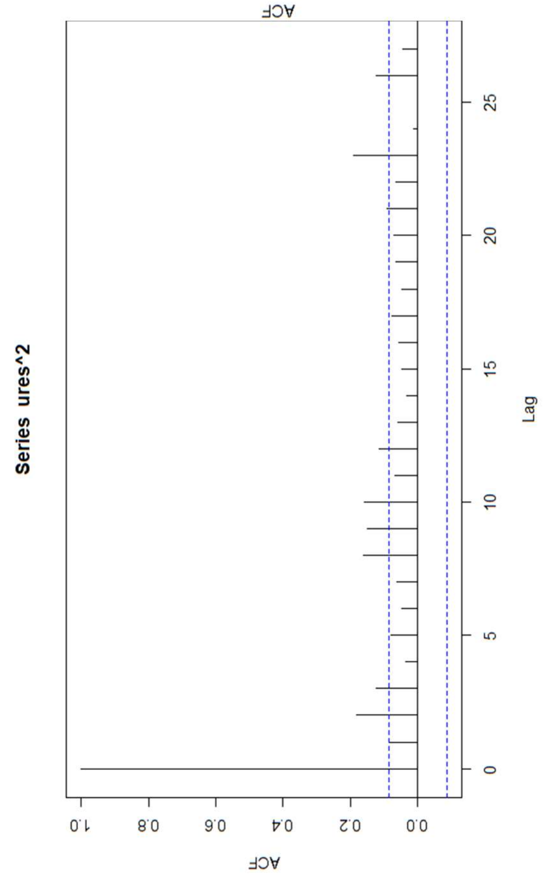

Series bres^2
ACF
Lag


Series ures
ACF
Lag


Series ures^2
ACF
Lag

All the oil markets show correlation in the squared error term, meaning volatility clusters are likely to be present and therefore the EGARCH framework is appropriate in this context. The Box-Ljung tests also all gave p-values below 0.05.

Table 7 ***AIC minimizing (r,s,p,q)***	***WTI***	***BRENT***	***URALS***	***MURBAN***
Full Sample AIC	(4,3,0,3) -4.3837	(2,5,0,0)	(3,3,0,0)	(2,3,5,2)
Pre-Invasion AIC	(4,4,0,1) -4.8644	(2,3,4,3)	(3,3,0,1) -5.0221	(2,1,3,3) -5.013
Post-Invasion AIC	(1,1,3,3) -4.0691	(2,1,0,4)	(3,3,1,2)	(2,4,0,0)

ARMA-EGARCH Estimation

Table 8 ***WTI estimation***	***Full sample***		***Pre-Invasion***		***Post-Invasion***	
	Coef	S.E.	Coef.	S.E.	Coef.	S.E.
mu	0.001424	0.000000	0.002974	0.000003	0.006746	0.000001
ar1		0.000019			-0.279695	0.000017
ar2		0.000025			0.391400	0.000029
ar3		0.000018			0.869620	0.000111
ma1	0.045450	0.000535	-0.0715	0.000051	0.362806	0.000068
ma2	-0.140925	0.000018			-0.404540	0.000025
ma3	-0.085849	0.000017			-1.069678	0.000056
omega	-3.971030	0.000025	-5.8761	0.001857	-0.032597	0.000080
alpha1	-0.107856	0.000017	-0.3542	0.000059	0.067706	0.000066
alpha2	-0.168937	0.000052	-0.2662	0.000053		
alpha3	-0.252456	0.000011	-0.2721	0.000032		
alpha4	-0.176078	0.000078	-0.1592	0.000015		
beta1	-0.418583		-0.3913	0.000030	0.996876	0.000147
beta2	0.080571		0.84627	0.000084		
beta3	0.805592		0.35657	0.000028		
beta4			-0.5439	0.000123		
gamma1	0.226600	0.000026	0.08513	0.000036	0.054985	0.000019
gamma2	0.454661	0.000049	-0.0855	0.000045		
gamma3	0.242988	0.000024	-0.1054	0.000063		
gamma4	0.232966	0.000033	0.24537	0.000029		
All coefficients were statistically significant to the 1% level.						

B3.2 Discussion of WTI results

All components of the model were found to be significant from non-zero, implying the previous values of WTI returns can be used to forecast WTI returns. In particular, the models of all three samples had significant and BIC minimising EGARCH components, implying volatility clustering is present not only across the whole sample (pre and post war) but also within the pre and post war periods themselves. Specifically the gamma γ coefficients being significant implies the asymmetric response of the conditional volatility to shocks: In the full and post-war sample, the positive coefficients imply presence of the 'leverage' effect, whereby negative shocks tend to have a bigger impact on volatility than positive. However, this is not true in the pre-war sample (with vice versa implied), potentially illustrating the increase scepticism as a result of the tense geopolitical climate post invasion.

Estimations of the other three markets are below.

Table 9 ***BRENT estimation***	***Full sample***		***Pre-Invasion***		***Post-Invasion***	
	Coef	S.E	Coef.	S.E.	Coef.	S.E.
mu	0.001224	0.0e+00	0.002503	0.000170	-0.001212	0.000001
ar1			-0.200528	0.007675		
ar2			0.191629	0.005018		
ar3			-0.619297	0.014384		
ar4			-0.082672	0.008344		
ma1			0.135532	0.004249	0.161741	0.000033
ma2			-0.351482	0.006517	-0.188177	0.000051
ma3			0.480022	0.009208	-0.188316	0.000172
ma4					0.030795	0.000007
omega	-0.924815	7.2e-05	-1.733074	0.190500	-0.153807	0.000030
alpha1	-0.053797	5.0e-06	-0.308191	0.086640	-0.111556	0.000042
alpha2	0.054666	5.0e-06	-0.092387	0.063261	0.096658	0.000022^
beta1	0.466297	3.8e-05	0.728039	0.000000	0.980174	0.000151
beta2	-0.245090	2.1e-05	0.156787	0.029393		
beta3	0.979764	7.0e-05	-0.098574	0.023145		
beta4	-0.294372	2.4e-05				
beta5	-0.029867	3.0e-06				
gamma1	0.268489	2.4e-05	-0.503573	0.111265	-0.817587	0.000181
gamma2	0.311001	2.7e-05	0.522962	0.130481	0.711214	0.000146
All coefficients were statistically significant to the 1% level, apart from alpha2^ in post-invasion						

Table 10 ***URALS estimation***	***Full sample***		***Pre-Invasion***		***Post-Invasion***	
	Coef	S.E	Coef.	S.E.	Coef.	S.E.
mu	0.000182	0.000002	0.001946	0.000000	-0.001533	0.000000
ar1					0.900873	0.000077
ma1			-0.060098	0.000023	-0.908439	0.000078
ma2					-0.129443	0.000010
omega	-0.557033	0.001584	-7.516646	0.000843	-6.282292	0.000652
alpha1	-0.112483	0.000052	-0.233568	0.000033	-0.144251	0.000017
alpha2	-0.020387	0.000329	-0.648487	0.000061	-0.136287	0.000013
alpha3	-0.103305	0.000016	-0.505817	0.000066	-0.100650	0.000012
beta1	0.856504	0.001990	-0.999982	0.000099	0.187098	0.000021
beta2	-0.867876	0.000487	0.381114	0.000038	-0.699494	0.000070
beta3	0.932766	0.001981	0.691792	0.000077	0.581941	0.000063
gamma1	0.133443	0.000754	0.345381	0.000055	0.293071	0.000031
gamma2	0.187360	0.000289	-0.173819	0.000037	0.469711	0.000054
gamma3	-0.029621	0.001024	-0.390247	0.000062	0.098518	0.000015
All coefficients were statistically significant to the 1% level.						

Table 11 ***MURBAN estimation***	***Full sample***		***Pre-Invasion***		***Post-Invasion***	
	Coef	S.E	Coef.	S.E.	Coef.	S.E.
mu	0.001850	0.000001	0.002554	0.000000	-0.000489	0.0e+00
ar1	1.469346	0.000089	0.885794	0.000160		
ar2	-1.097591	0.000066	0.626001	0.000098		

ar3	0.205691	0.000017	-0.559344	0.000167		
ar4	-0.097222	0.000008				
ar5	-0.054689	0.000005				
ma1	-1.488549	0.000115	-0.964686	0.000238		
ma2	0.960875	0.000066	-0.760037	0.000204		
ma3			0.786098	0.000139		
omega	-1.954084	0.000270	-0.431031	0.000066	-1.991290	2.4e-04
alpha1	-0.047041	0.000022	-0.093361	0.000084	0.134431	1.5e-05
alpha2	0.113743	0.000017	-0.110832	0.000133	0.069339	8.0e-06
beta1	0.756377	0.000044	0.948792	0.000151	0.041025	8.0e-06
beta2	-0.782658	0.000067			-0.186997	1.9e-05
beta3	0.769885	0.000063			0.234556	2.6e-05
beta4					0.646828	7.2e-05
gamma1	0.148582	0.000031	-0.279538	0.000051	-0.091060	1.9e-05
gamma2	0.562151	0.000067	0.068833	0.000010	0.784206	8.3e-05
All coefficients were statistically significant to the 1% level.						

Focusing on WTI (sorry, only until the Q&A I thought we were estimating all four) we saw statistically significant excess (unconditional) kurtosis and skewness in the returns themselves. The kurtosis of the residuals from the model can be given by

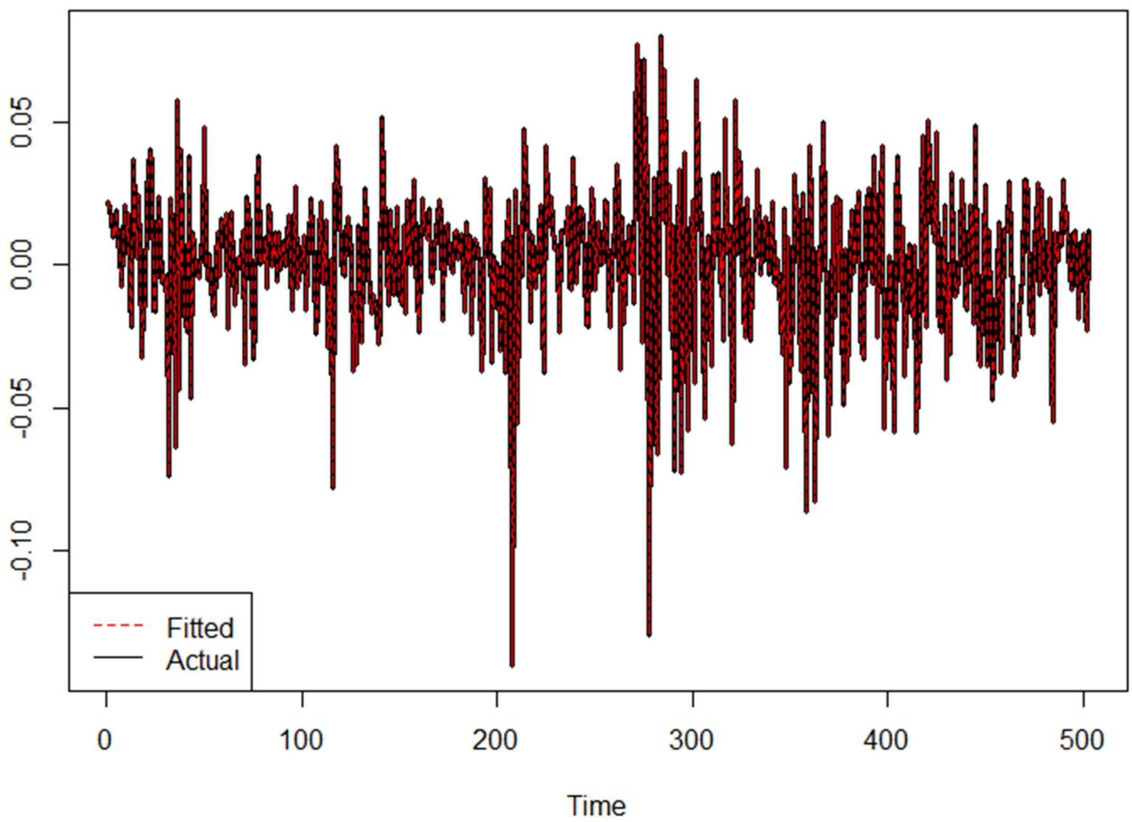
$$k_4 = \frac{3(1-\alpha_1{}^2)}{(1-3\alpha_1{}^2)}$$

Where α_1 is the coefficient of $y_{t-1}{}^2$. If $\alpha_1 = 0$ the kurtosis is 3 i.e. the same as the normal, if α_1 is close to 1/3 the kurtosis is very large.

Table 12 ***AIC minimizing (r,s,p,q)***		***WTI (s.d. in brackets)***
Data (from B1)	***Skewness***	-0.78*** (0.110)
	Kurtosis	5.75*** (0.219)
Full Sample residuals	***Skewness***	-0.8104*** (0.154)
	Kurtosis	5.809451*** (0.309)
Pre-Invasion residuals	***Skewness***	-0.7869099*** (0.154)
	Kurtosis	5.803936*** (0.309)
Post-Invasion residuals	***Skewness***	-0.7433149*** (0.154)
	Kurtosis	5.588742*** (0.309)

This shows that the ARMA-EGARCH model explains a lot of the non-normality of the actual returns. Figure 32 maps the fitted values of the full sample estimation to the returns themselves and suggests the model is a good fit.

Figure 32


0.05
0.00
-0.05
-0.10
Fitted
Actual
0
100
200
300
400
500
Time

B4) How many co-integration relationships do you find among the oil markets? Check for the entire sample and then split the sample at an appropriate point (start of Russian-Ukrainian conflict?) and redo the exercise. Comment on your results.

Whilst I used the ADF cointegration test in question B2 to examine cointegration relationships between the prices, this was in the bivariate 'spread' setting. To evaluate the co-integration relationships of the oil markets, I instead employ the Johansen technique, a multivariate test that can examine the relationships between multiple variables at the same time. Another shortcoming of the ADF test is that one must define one variable as dependent and the other the independent variable, treating the variables asymmetrically - the Johansen technique, circumnavigates this issue:

For a vector y_t of g potentially co-integrated, I(1) variables (in our case, the log of oil prices in each market), one can specify the relationship between each respective variable and the lagged values of the all the variables (including itself) as in the vector equation X:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_k y_{t-k} + u_t$$

Where y_t is g x 1 vector, and A_p is a gxg coefficient matrix where the i-th row of A_p corresponds the i-th variable in y_t to the effects to the p-th lags of all the variables (including itself, the i-th). Hence the vector equation embodies all possible dependent/independent variable combinations so none need to be explicitly specified. Re-writing this as a Vector Error Correction Model (VECM), taking y_{t-1} from both sides, gives equation X:

$$\Delta y_t = \Pi y_{t-k} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t$$

With $\Pi = \left(\sum_{i=1}^{k} \mathrm{A}_i\right) - I_g$ (that describes the long run relationship between the prices)

and $\Gamma_i = \left(\sum_{j=1}^{i} \mathrm{A}_j\right) - I_g$

The number of co-integrated relationships in y (up to a maximum of g-1) is then given by the rank of Π, which can be evaluated by the number of non-zero eigenvalues λ_i. One way to evaluate this is by means of a joint 'trace' test, which involves comparing the eigenvalues to the λ_{trace} test statistic (given by equation X) in order. Under no cointegration, $\lambda_i \approx 0$ $\forall$ i i.e. the rank of Π will be close to zero. However if, for example, there is 1 cointegration relationship present, then (in order) the largest eigenvalue λ_1 must be significantly non-zero whilst all λ_i eigenvalues with i > 1 must be close to zero (Brooks, 2002). The test statistic transforms λ_i to $\ln(1 - \lambda_i)$ but this doesn't affect the circumstances described local to λi = 0. The trace test statistic itself is given by:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{g} \ln\left(1 - \widehat{\lambda_i}\right)$$

Where $\widehat{\lambda_i}$ (NB subscript i, MS word cuts off the dot when a hat is applied…) is the estimate of the i-th eigenvalue (in size order) from Π. 'r' is the index for the null hypothesis that there are less than 'r' cointegration relationships between the variables (Brooks, 2002). In other words, when r <= x has a statistically significant value, we reject the null hypothesis that there are no more than x cointegration relationships present. So for instance, if r=0 is statistically significant, we reject the null hypothesis that there is less than 1 cointegration (so at least 1 is present).

B4.1 Results

Table 13		***Test Value***	***10%***	***5%***	***1%***	***Rank***	***WTI***	***BRENT***	***URALS***	***MURBAN***
Full sample AIC = 3 lags	r<=3	0.99	6.50	8.18	11.65	***WTI***	1	1	1	1
	r<=2	6.39	15.66	17.95	23.52	***BRENT***	-3.3552	-0.6580	-1.0771	-976.86
	r<=1	16.19	28.71	31.52	37.22	***URALS***	-0.2115	-0.1376	0.08732	875.335
	r=0	62.97***	45.23	48.28	55.43	***MURBAN***	2.34238	-0.3003	0.2217	162.203
Post-Invasion AIC = 2	r<=3	2.52	6.50	8.18	11.65	***WTI***	1	1	1	1
	r<=2	9.33	15.6	17.95	23.52	***BRENT***	-1.36049	-7.6930	-0.3060	-0.9401
	r<=1	38.42***	28.71	31.52	37.22	***URALS***	0.17009	-1.4668	-0.2762	0.0560
	r=0	81.5***	45.23	48.28	55.43	***MURBAN***	-0.03995	8.1955	-0.5078	0.1656
Pre-Invasion AIC = 2	r<=3	1.11	6.50	8.18	11.65	***WTI***	1	1	1	1
	r<=2	10.20	15.66	17.95	23.52	***BRENT***	-9.8251	-94.553	-0.85199	-0.4025
	r<=1	39.36***	28.71	31.52	37.22	***URALS***	-0.1495	122.905	-0.02929	-0.3666
	r=0	70.36***	45.23	48.28	55.43	***MURBAN***	8.210	-43.047	-0.17032	-0.5807

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

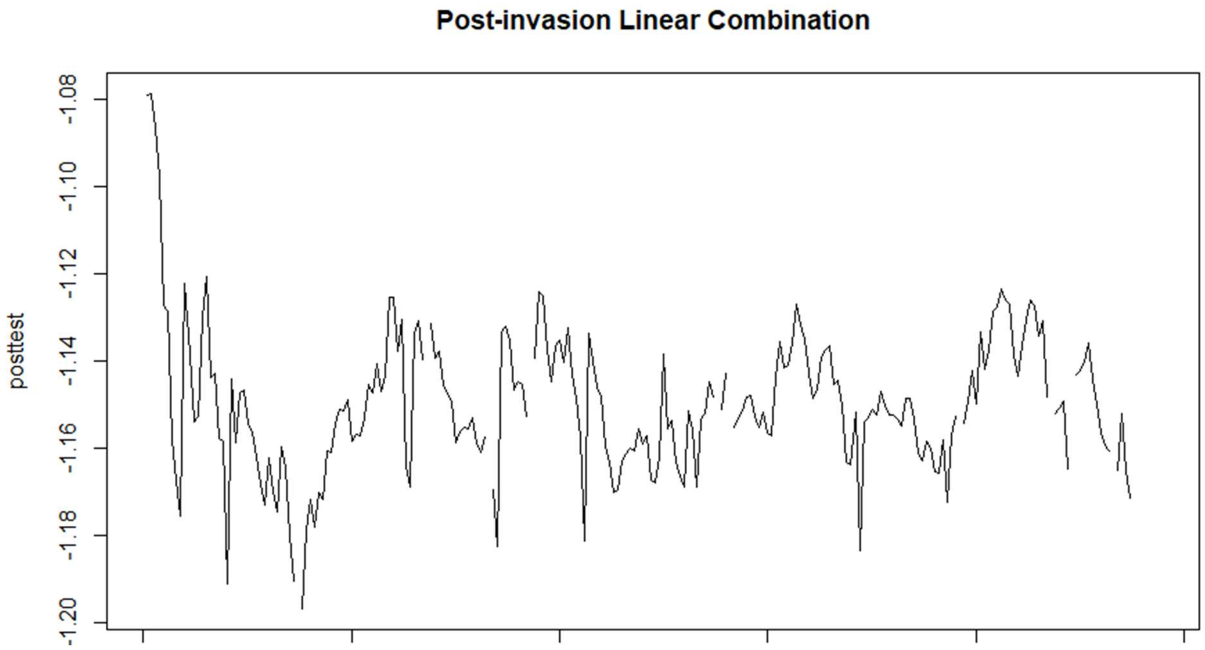
The sample is split at the invasion date to see if the global market dynamics changed significantly, given the geo-political driven economic sanctions and supply infrastructure changes of the oil market caused by the invasion. AIC is used to determine the optimal length of lags within each test. Interestingly, the full sample population only has one significant cointegrated relationship present, whereas both the pre and post invasion samples have two. This sample result aligns with the bivariate cointegration analysis, as MURABN was pairwise cointegrated with both WTI and BRENT.

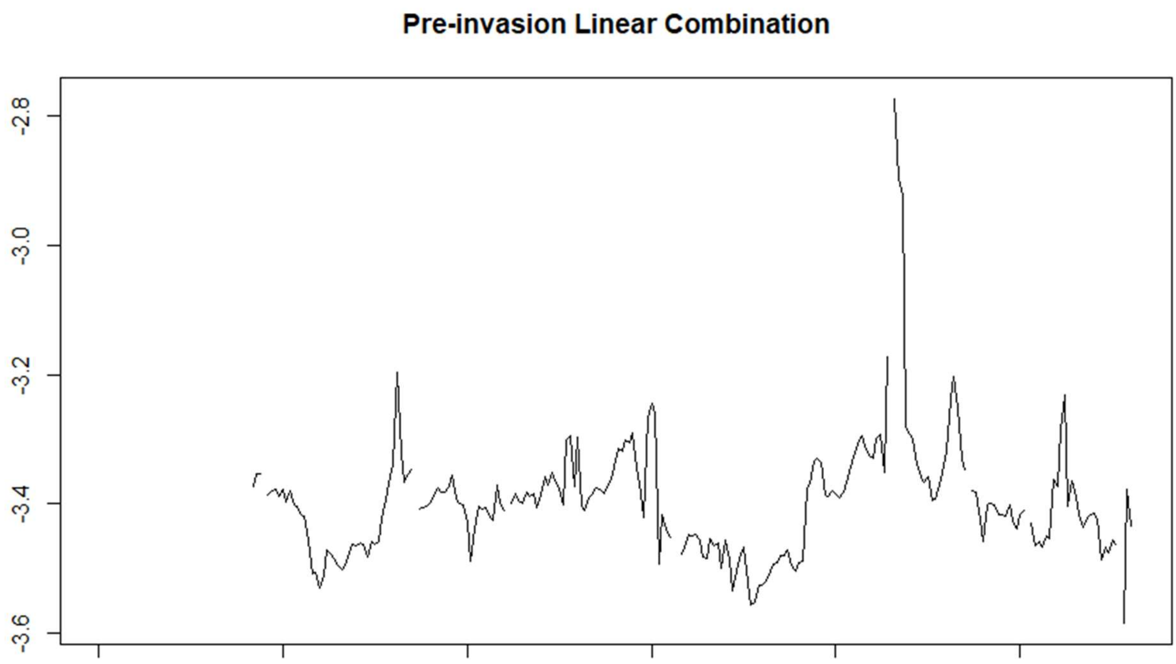
The linear combination of the oil prices, using the components of the eigenvector associated with highest eigenvalue as coefficients, gives a stationary series. For example in the split samples, the series given by the linear combination of the four oil price series (with WTI associated with the largest eigenvalue) is:

$$LinCom_{Pre} = \log(WTI) - 1.36049368 \log(\mathrm{BRENT}) + 0.17009939 \log(\mathrm{URALS}) - 0.03995844 \log(\mathrm{MURBAN})$$

$$LinCom_{Post} = \log(WTI) - 9.825111 \log(\mathrm{BRENT}) - 0.149553 \log(\mathrm{URALS}) + 8.210690 \log(\mathrm{MURBAN})$$

Figure 33-34





The ADF test shows 1% significance for lags 0-5, rejecting the null hypothesis meaning the series shows stationarity with drift for both pre and post samples.

Question B Bibliography

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