14. Show that each conditional statement in Exercise 12 is a tautology using the fact that a conditional statement is false exactly when the hypothesis is true and the conclusion is false. (Do not use truth tables.)

a) $[\neg p \wedge (p \vee q)] \rightarrow q$
b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
c) $[p \wedge (p \rightarrow q)] \rightarrow q$
d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

a)

$\sim p \wedge (p \vee q) := T, \quad q := F$

$\sim p \wedge (p \vee F)$

$\sim p \wedge F \equiv F$ ∴ always true

b) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

$p \rightarrow r := F \quad p := T, r := F$

$(T \rightarrow q) \wedge (q \rightarrow F) := T$

because q must be true to satisfy $T \rightarrow q$, and $T \rightarrow F \equiv F$, the preposition cannot be F

c) $(p \wedge (p \rightarrow q)) \rightarrow q$

$q := F$

$p \wedge (p \rightarrow F)$

left side is satisfied by $p := T$, right by $p := F$
∴ always true

d) $(\underbrace{(p \vee q)}_{a} \wedge \underbrace{(p \rightarrow r)}_{b} \wedge \underbrace{(q \rightarrow r)}_{c}) \rightarrow r$

$r := F$

to satisfy b, c $p, q := F$. ∵ $p \vee q$ is then F
∴ tautology

19. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

$p := T$

$\underbrace{\sim q}_{a} \wedge \underbrace{(T \rightarrow q)}_{b}$

a requires $q \equiv F$, ∵ then $b \equiv F$ ∴ tautology

28. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

$(p \rightarrow q) \vee (p \rightarrow r) = p \rightarrow (q \vee r)$

using the distributive prop.

$p \rightarrow (q \vee r)$ becomes $(p \rightarrow q) \vee (p \rightarrow r)$

∴ $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

65. Determine whether each of these compound propositions is satisfiable.

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
b) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

a) $(p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

$\cancel{p, q := T}$ $\quad p, q := F \quad p, q := F$

$p := T, q := F$ $\quad \cancel{p := F, q := T}$

$p, q := F$

$\therefore$ Satisfiable when $p := T, q := F$ / $p, q := F$

b) $(p \to q) \land (p \to \neg q) \land (\neg p \to q) \to (\neg p \to \neg q)$

~~$p, q := T$~~
$p, q := F$
$p := F, q := T$

$p, q := F$
~~$p := T, q := F$~~
$p := F, q := T$

$p := F, q := T$

~~$p := F, q := T$~~

Not Satisfiable

c) $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$

Not satisfiable, $p \neq \neg p$

2. Let $P(x)$ be the statement "The word x contains the letter a." What are these truth values?

a) P(orange) **b)** P(lemon)
c) P(true) **d)** P(false)

a) T
b) F
c) F
d) T

7. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$ **b)** $\forall x(C(x) \land F(x))$
c) $\exists x(C(x) \to F(x))$ **d)** $\exists x(C(x) \land F(x))$

a) everyone who is a comedian is funny
b) everyone is a comedian and is funny
c) there are some people who are comedians, which makes them funny
d) There are some people who are comedians and funny.

12. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$
d) $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$
g) $\forall x \neg Q(x)$

a) T
b) T
c) F
d) T
e) F
f) T
g) F

23. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.
b) Everyone in your class is friendly.
c) There is a person in your class who was not born in California.
d) A student in your class has been in a movie.
e) No student in your class has taken a course in logic programming.

$q(x)$ = a given person, x, is in your class

a)
$P(x)$ = a given person, x, can speak Hindi
$\exists x(P(x))$
$\exists x(P(x) \land q(x))$

b)
$P(x)$ = a given person, x, is friendly

P(x) = a given [illegible] x, is [illegible]

$\forall x(P(x))$

$\forall x(q(x) \rightarrow P(X))$

c)

P(X) = a given person, X, was born in California

$\exists x(\sim P(x))$

$\exists x(q(x) \wedge \sim P(x))$

d)

P(X) = a given person, X, has been in a movie

$\exists x(P(x))$

$\exists x(q(x) \wedge P(X)$

e)

P(X) = a given person, X, has taken a course in logic programming

$\forall x(\sim P(X))$

$\forall x(q(X) \rightarrow \sim P(X))$

36. Express the negation of each of these statements in terms of quantifiers without using the negation symbol.

a) $\forall x(-2 < x < 3)$
b) $\forall x(0 \leq x < 5)$
c) $\exists x(-4 \leq x \leq 1)$
d) $\exists x(-5 < x < -1)$

37. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x(x^2 \geq x)$
b) $\forall x(x > 0 \vee x < 0)$
c) $\forall x(x = 1)$

36

a) $\forall x((X \leq -2) \vee (3 \leq X))$

b) $\forall x((X < 0) \vee (X \geq 5))$

c) $\forall x((X < -4) \vee (X > 1))$

d) $\forall x((X \leq -5) \vee (X \geq -1))$

37

a) [illegible] possible to [illegible]

) None possible, [illegible]

b) $x = 0$

c) $x \neq 1$ e.g. $x = 2$