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MTH:2310 DISCRETE MATH
QUIZ 2
DUE: FRIDAY SEPT. 19

Complete all of the following problems being as detailed as you can be. You may work in groups, but you must submit your own solutions using your own words. You must submit a physical copy of your solutions to me unless told otherwise. If there is a specific number of problems which seem unsolvable let me know first, I may have made a typo. Even if you cannot completely solve a problem, a good description of key terms and concepts relevant to the problem is sufficient for some partial credit.

Handwriting: *(4 points)*
Your ability to express your solutions is just as important as mathematical accuracy.
You will be penalized up to 2 points for arithmetic errors.
You will be penalized up to 2 points for disorganized or hard to read solutions.

Problem 1. *(8 points) Predicates and Quantifiers*
Consider the following quantified statement.

$$\exists! x(2x^2 - 3x = 2)$$

(a) Translate the expression into a complete sentence.

there exists only one x such that $2x^2-3x=2$

(b) What is the truth value of the statement if x may be any real number? $\exists! x (2x^2 - 3x = 2)$

$(x-2)(2x-1) \quad x = 2, \frac{1}{2}$

as the statement can be satisfied by multiple values of x, the unique existential qualifier makes the statement False

(c) What is the truth value of the statement if x may be any rational number?

the truth value remains false as shown above, as 2 and $\frac{1}{2}$ are both rational numbers.

(d) What is the truth value of the statement if x may be any integer?

the statement is true.

as the only viable value of x within the domain of $\mathbb{Z}$ is 2. this means there is a unique solution, thereby satisfying the qualifier.

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Problem 2. *(8 points) Satisfiability*
Determine whether the following compound proposition is satisfiable. If it is, provide at least one assignment of truth values which "satisfies" the compound proposition.

$$\Big((\sim p) \vee (\sim q) \vee r\Big) \wedge \Big((\sim p) \vee q \vee (\sim s)\Big) \wedge \Big(p \vee (\sim q) \vee (\sim s)\Big)$$
$$\wedge \Big((\sim p) \vee (\sim r) \vee (\sim s)\Big) \wedge \Big(p \vee q \vee (\sim r)\Big) \wedge \Big(p \vee (\sim r) \vee (\sim s)\Big)$$

$(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim s) \wedge (p \vee \sim q \vee \sim s)$

T for all but $p, q \equiv T, r \equiv F$

T for all but $p, s \equiv T, q \equiv F$

T for all but $q, s \equiv T, p \equiv F$

$\wedge (\sim p \vee \sim r \sim s) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim r \vee \sim s)$

T for all but $p, s, r \equiv T$

T for all but $p, q \equiv F, r \equiv T$

T for all but $r, s \equiv T, p \equiv F$

Let $p, q, r, s := F$

$(\sim F \ldots) \wedge (\sim F \ldots) \wedge (F \vee \sim F \ldots) \wedge (\sim F \ldots)$
$\wedge (F \vee F \vee \sim F) \wedge (F \vee \sim F \ldots)$

$\therefore$ $p, q, r, s := F$ satisfies the compound preposition

Problem 3. *(8 points) Predicates and Quantifiers*
Translate the following sentence into an expression using quantifiers, propositional functions, and logic operators. Completely define every variable and function introduced. Determine the statement's truth value if it has one and justify your answer.

"For every real number x, there exists a real number y such that $x^2+y^2 = 4$."

Domain of x, y: $\mathbb{R}$

$P(x,y) := x^2+y^2=4$

$\forall_x(\exists_y(P(x,y)))$

Let $x=3$

$3^2+y^2=4$

$y^2=-5$

$\sqrt{y^2}=\sqrt{-5}$

$y=\sqrt{-5}$

$\sqrt{-5}$ is not in $\mathbb{R}$,

$\therefore$ the statement is false as there exists a value of x for which no satisfactory value of y exists

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Problem 4. *(12 points) Rules of Inference*

(a) *(8 points)* Find the argument form which describes the following argument. Clearly define all variables introduced.

"If the algorithm is correct, it terminates or it is deterministic.
If the algorithm terminates, it is correct or it is efficient.
The algorithm is correct or it did not terminate.
The algorithm is not efficient.
Therefore, the algorithm is correct or deterministic."

$\boxed{\begin{array}{l} p := \text{the algorithm is correct} \\ q := \text{the algorithm terminates} \\ r := \text{the algorithm is deterministic} \\ s := \text{the algorithm is efficient} \end{array}}$

$$((p \to q \vee r) \wedge (q \to p \vee s) \wedge (p \vee \sim q) \wedge \sim s) \to (p \vee r)$$

(b) *(4 points)* Is the argument from part (a) valid or invalid? Justify your answer.

Evaluate w/ $p, q, r, s := F$

$$((p \to q \vee r) \wedge (q \to p \vee s) \wedge (p \vee \sim q) \wedge \sim s) \to p \vee r$$

$$((F \to F \vee F) \wedge (F \to F \vee F) \wedge (F \vee \sim F) \wedge \sim F \to F \vee F$$

$$(T \wedge T \wedge T \wedge T) \to F$$

Since $T \to F$ is not true, the argument is not valid