Group Problem Set 6: One and Two Dimensional Motion

In group problem set 3 and 4, we investigated a special case of 1-D motion under constant acceleration where a ball went straight up in the air, reached a maximum height and then fell to the ground in a free fall (under no additional acceleration). The kinematic equations for motion under constant acceleration look identical to the same equations for one dimension.

x-component

$$v_f(x) = a_x t + v_{ox}$$
$$\Delta x = \frac{1}{2}(v_{0x} + v_x)t$$
$$\Delta x = v_{ox}t + \frac{1}{2}a_x t^2$$
$$\Delta x = v_x t - \frac{1}{2}a_x t^2$$
$${v_x}^2 = {v_{0x}}^2 + 2a_x\Delta x$$

y-component

$$v_f(y) = a_y t + v_{oy}$$
$$\Delta y = \frac{1}{2}(v_{0y} + v_y)t$$
$$\Delta y = v_{oy}t + \frac{1}{2}a_y t^2$$
$$\Delta y = v_y t - \frac{1}{2}a_y t^2$$
$${v_y}^2 = {v_{0y}}^2 + 2a_y\Delta y$$

A special case of 2-D motion is projectile motion. Projectile motion occurs when an object is launched with an initial velocity and experiences constant acceleration due to gravity alone. We can characterize several parameters associated with the initial projectile launch, the acceleration of the object, the shape of the path the object takes (it's trajectory).

- Projectile motion starts with an object launched with an initial velocity in the $\vec{x}$ and $\vec{y}$ directions.
- The only acceleration applied to the object during its flight is a constant acceleration due to gravity.
- Projectiles launched in the air take the shape of a parabola.
- The initial velocity and launch angle govern the max height the object travels.
- The initial velocity and launch angle govern the total displacement of the object.
- Air resistance can be ignored.

The kinematic equations of 2-D motion can be simplified for projectile motion.

x-component

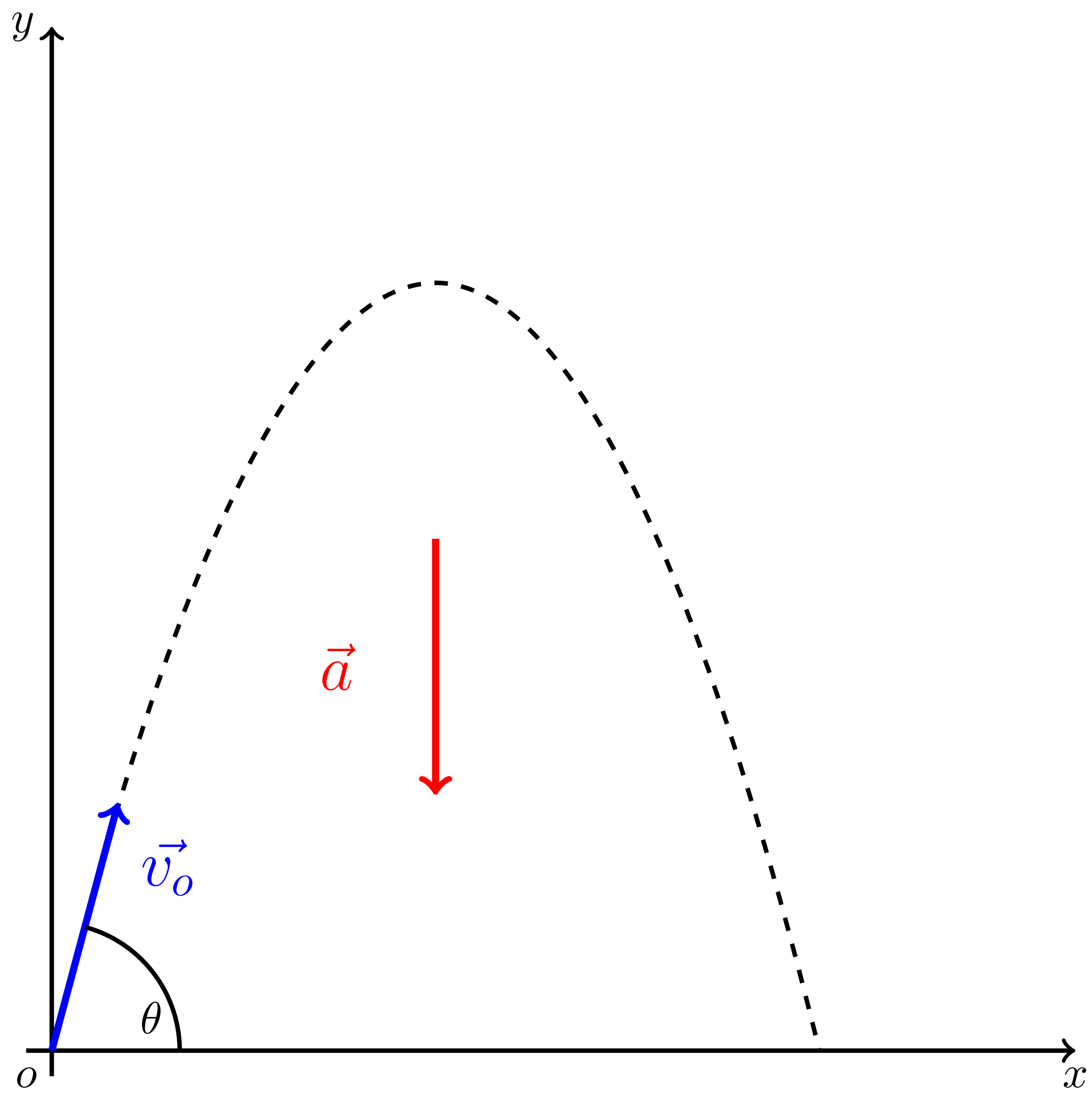
$$a_x = 0$$
$$v_x = v_{xo} = constant$$
$$x = v_{ox}t + x_o$$

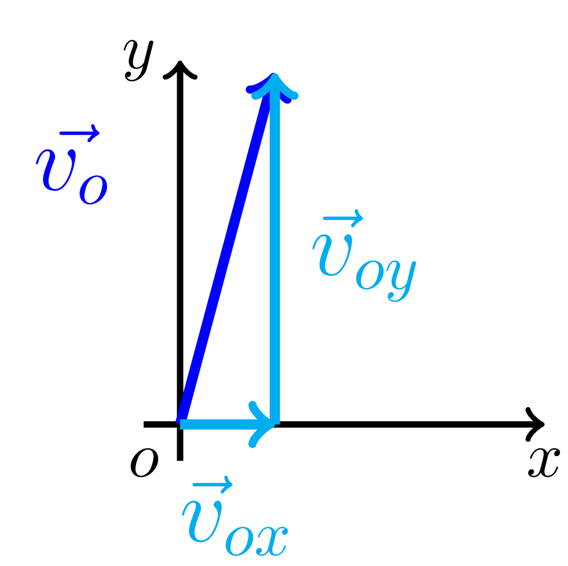
y-component

$$a_y = constant, a_y = -g = -9.8m/s^2$$
$$v_y = v_{yo} + a_y t$$
$$y = y_o + v_{yo}t + \frac{1}{2}a_y t^2$$
$$v_y^2 = v_{yo}^2 + 2a_y(y - y_o)$$

Notice that because the acceleration in the $\vec{x}$ direction is 0, the equations of motion simplify dramatically!

Galileo Galilei was the first to describe the parabolic trajectory of projectile motion. He was also able to characterize some consequences of this trajectory. Galileo made the observation that velocity has the same magnitude at the same height along the parabola. This is pretty impressive as standard practices for keeping time were not accurate to the second and wouldn't be for at least another 100 years (Apparently, Galileo had his assistant whisper in his ear to keep time)! These observations make 2D kinematic problem solving easier.



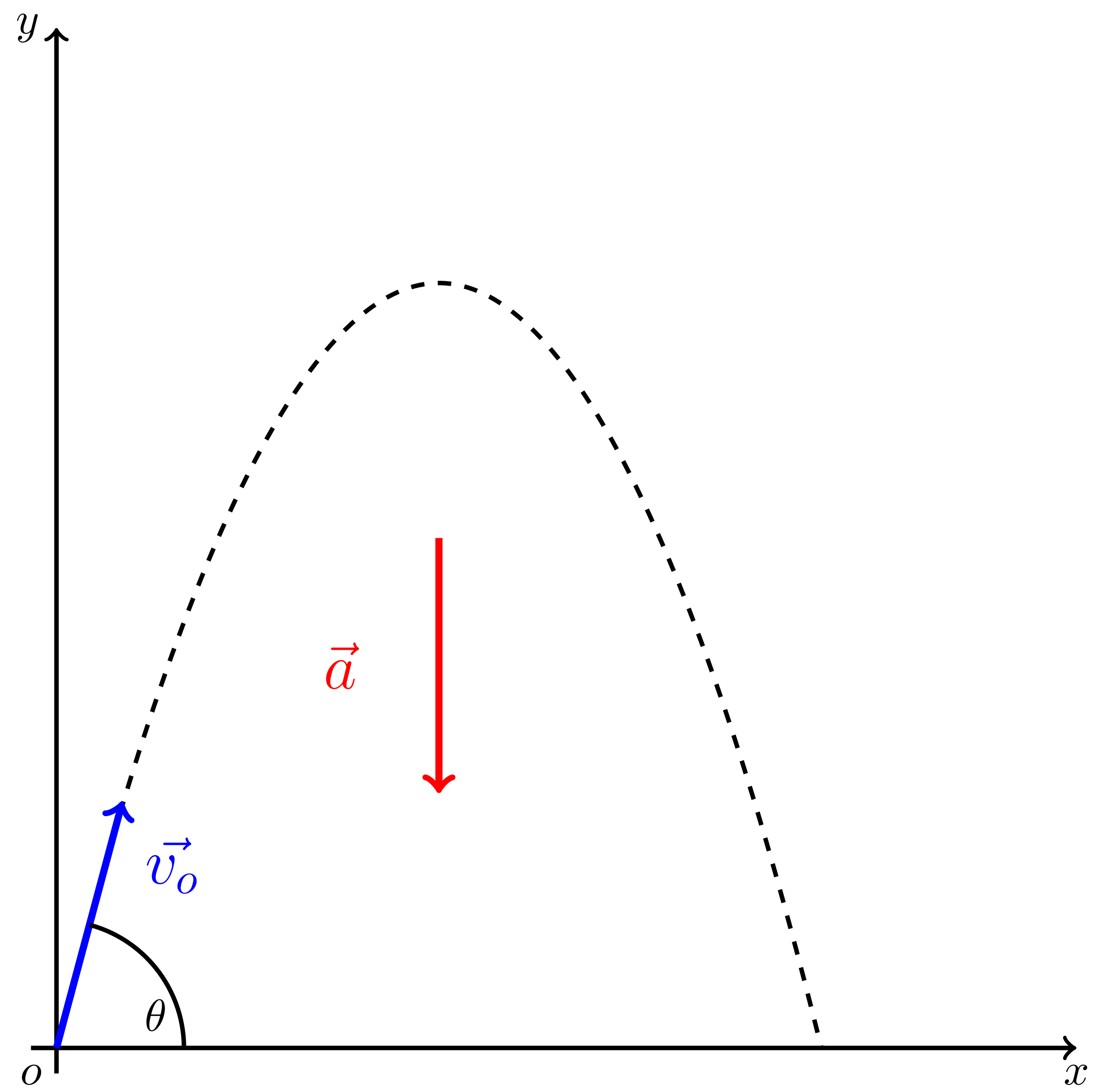


$$\vec{v}_{ox} = v_o cos\theta\hat{x}$$
$$\vec{v}_{oy} = v_o sin\theta\hat{y}$$
$$\vec{a} = -9.8m/s^2$$
$\vec{v}_y = 0$ at the height of the parabola

$\vec{v}_{ox} = \vec{v}$ throughout the flight

$\Delta\vec{v}$ is dependent on $\vec{v}_y$

$$\vec{v}(y)_{up} = -\vec{v}(y)_{down}$$



To calculate the time of flight from launch to landing, use the fact that acceleration is $-g$ in the $\hat{y}$ direction and $\Delta y = 0$:

$$\Delta y = v_{yo} sin\theta t - \frac{1}{2} g t^2 = 0,$$

Solve for t,

$$t = \frac{2 v_o sin\theta}{g}$$

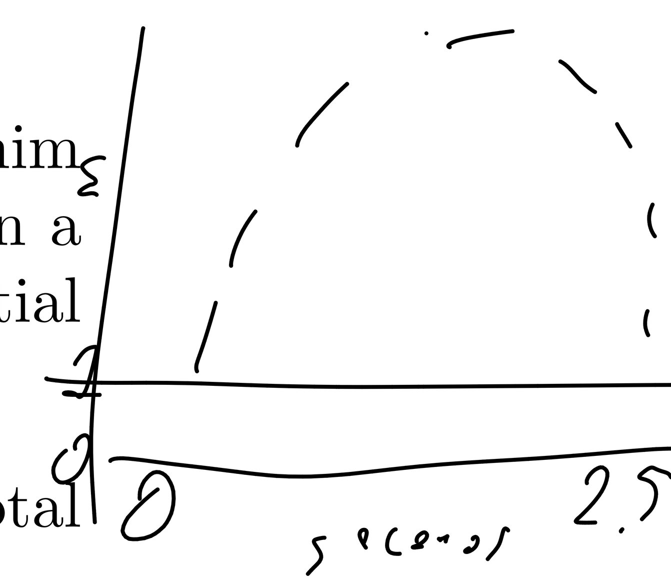
You can use this time to determine the total distance traveled in the x-direction:

$$\Delta x = v_o cos\theta t = v_o cos\theta (\frac{2 v_o sin\theta}{g})$$

combining trigonometric functions:

$$\Delta x = \frac{2 v_o^2}{g} (sin\theta cos\theta) = \frac{v_o^2}{g} (2 sin\theta cos\theta) = \frac{v_o^2}{g} sin 2\theta$$

In Season 5, Episode 14 (*Stress Relief Part 1* https://www.youtube.com/watch?v=ZOavX5qYRTQ), Angela is forced to throw her cat, Bandit, through the ceiling to save him from a potential fire (see link to refresh your memory). The follow problem will analyze the Bandit's trajectory and evaluate what Angela should do to guarantee a more desirable outcome (saving Bandit).



1. Based on the above video clip, Bandit's time of flight is 2.5 s from Angela projecting him through the ceiling at a starting height of 1m off the ground to a final resting location on a desk, 1m off the ground and 3 meters from the initial launch site. Draw diagram of the initial launch and Bandit's trajectory.
2. How long does it take for Bandit to obtain a $\vec{v}_y = 0$? (HINT: You've been given his total time of flight) $\frac{2.5}{2} = 1.25$ s
3. What is Bandit's initial velocity in the y direction? 12.5 m/s
4. How high above the desk does Bandit travel? 7.8125
5. Knowing the entire time of flight and the distance Bandit travels in the x direction, what is his initial and final velocity in the x direction? $V_{0x}, V_{fx} = 1.2$ m/s
6. What is the x component of Bandit's velocity at the apex of his trajectory? 1.2 m/s
7. What is the x-component of Bandit's velocity as he comes down through the ceiling? 1.2 m/s
8. What is the y-component of Bandit's velocity when he hits the desk? −12.5
9. At what angle does Angela toss Bandit? (use your velocity vector)
10. Knowing that standard office ceilings are 3m, what is Bandit's the y-component of his velocity when he enters the ceiling (be sure to take into account that he starts at 1m off the ground)?
11. What is Bandit's velocity when he comes back out of the ceiling?
12. Assuming that Angela can repeatedly propel Bandit with the same initial velocity, at what angle should she release Bandit so that he travels furthest away from danger (HINT: where is $sin\theta = 1$?)? How far and how high will he travel?
13. The range equation relates a total distance traveled in the x-direction dependent on the acceleration in the y-direction ($\Delta x = \frac{v_o^2}{g} sin2\theta$). How would you change the acceleration in the y-direction, if you wanted to make the total distance Bandit travels larger?
14. The acceleration due to gravity on the surface of Mars is approximately $3.71m/s^2$. If Angela tossed Bandit, assuming the same initial velocity and angle of release, would she expect Bandit to travel further in the x-direction? further in the y-direction?
15. If Angela found herself on Mars and wanted to toss Bandit the same distance with the same initial velocity, what should she change to accomplish this? Show your work.