

StarBench: The D-type expansion of HII regions

Thomas G. Bisbas, Thomas Haworth, Alex C. Raga,
Jonathan Mackey, Pascal Tremblin, and Robin J.R. Williams

1 Objectives

- To reproduce the equation of motion of the ionization front
- To reproduce the equation of motion of the shock front.
- To measure the time evolved mass of the ionized medium.

2 Theoretical Background

Consider a spherically symmetric cloud of radius R_{cl} , of total mass M_{cl} and of uniform density ρ_{cl} consisting of pure atomic hydrogen. Let us suppose we place an exciting source at the centre of the cloud which defines the centre of a Cartesian co-ordinate system. The source emits $\dot{\mathcal{N}}_{\text{LyC}}$ Lyman continuum ionizing photons per unit time. The ionizing photons are monochromatic with energy $h\nu = 13.6$ eV although this choice of energy is not important to the resulting solution. We consider a mean photoionization cross-section $\bar{\sigma} = 6.3 \times 10^{-18} \text{ cm}^2$ and we use the case-B recombination co-efficient, α_{B} , into excited stages only by invoking the *on-the-spot* approximation (Osterbrock, 1974; Whitworth, 2000). Assuming an isothermal HII region at $T_{\text{i}} = 10^4 \text{ K}$ the case-B recombination co-efficient is taken to be $\alpha_{\text{B}} \simeq 2.7 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$. Hereafter, the indices ‘i’ and ‘o’ shall denote the ionized and the neutral medium respectively.

As the ionizing photons interact with the medium, they ionize a region around the exciting source changing its temperature. The temperature of the ionized gas tends to be much higher than the temperature of the surrounding neutral gas. Strömgren (1939) was the first to show that the transition zone of these two regions occurs over a very short distance compared to the dimensions of the HII region. This transition zone is the *ionization front* and we can treat it as a sharp discontinuity. Whitworth (2000) showed that the distance over which the degree of ionization changes from 90% to 10% is given by

$$\Delta R_{\text{St}} \simeq \frac{20m_{\text{p}}}{\rho_{\text{o}}\bar{\sigma}} \simeq 1.72 \times 10^{-4} \text{ pc} \left(\frac{\rho_{\text{o}}}{10^{-20} \text{ g cm}^{-3}} \right)^{-1}, \quad (1)$$

where m_{p} represents the proton mass.

Kahn (1954) studied in detail the propagation of an ionization front into a neutral medium when an ionizing star suddenly switches on. At early times most of the Lyman continuum photons ionize additional gas beyond the instantaneous position of the ionization front. Thus the HII

region expands rapidly at highly supersonic speed relative to the sound speed of the ionized gas. In this first phase the ionization front is called *R-type* (R=Rarefied). As shown by Strömgren (1939), the $\dot{\mathcal{N}}_{\text{LyC}}$ photons emitted by the central source will ionize a spherical region of radius

$$R_{\text{St}} = \left(\frac{3\dot{\mathcal{N}}_{\text{LyC}} m_{\text{p}}^2}{4\pi\alpha_{\text{B}}\rho_{\text{o}}^2} \right)^{1/3}, \quad (2)$$

where $\rho_{\text{o}} = \rho_{\text{cl}}$, $m = m_{\text{p}}/X$ with m_{p} the proton mass and X the fraction by mass of hydrogen, and Hereafter, we refer to the spherical region of radius R_{St} as the “initial Strömgren sphere” and the radius R_{St} as the “initial Strömgren radius”. The R-type phase of expansion terminates once the ionization front reaches this initial Strömgren radius. The timescale for this first phase is of order the recombination time (Whitworth, 2000)

$$t_{\text{D}} = \frac{m_{\text{p}}}{\alpha_{\text{B}}\rho_{\text{o}}} \simeq 19.6 \text{ yrs} \left(\frac{\rho_{\text{o}}}{10^{-20} \text{ g cm}^{-3}} \right)^{-1}. \quad (3)$$

The large temperature difference between the two regions results in a large difference in thermal pressure, and the HII region expands. In this second phase which is called *D-type* (D=Dense), the ionization front propagates at subsonic speed relative to the ionized gas but at supersonic speed relative to the neutral gas. Therefore it is preceded by a *shock front* which sweeps up a dense shell of neutral gas. The shell is bounded on its inside by the ionization front and on its outside by the shock front.

2.1 Differential equations of motion

Within the ionized region we assume that at all times there is a balance between the ionizing photons produced by the star and the recombination events. Therefore

$$\dot{\mathcal{N}}_{\text{LyC}} = \frac{4\pi}{3} \left\{ \frac{\rho_{\text{i}}(t)}{m_{\text{p}}} \right\}^2 \alpha_{\text{B}} R_{\text{IF}}^3(t) \quad (4)$$

which when combined with Eqn.(2) gives

$$\rho_{\text{i}}(t) = \rho_{\text{o}} \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{3/2}. \quad (5)$$

As the HII region expands, it ionizes more neutral gas and eventually its mass increases in time. The time-dependent mass of the ionized region, $M_{\text{i}}(t)$ is given by:

$$M_{\text{i}}(t) = \frac{4\pi}{3} \rho_{\text{o}} R_{\text{St}}^{3/2} R_{\text{IF}}^{3/2}(t). \quad (6)$$

The thermal pressure of the ionized gas in an HII region -which drives the expansion- matches approximately with the thermal pressure of the neutral gas in the shell between the ionization front and the shock front. As pointed out by Raga et al. (2012a), assuming the thin shell approximation and by equating the pressure of the neutral gas in the shell between the ionization front and the shock front with the ram pressure of the undisturbed neutral gas as it is swept up by the shock front we obtain:

$$V_{\text{s}} = \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{3/4} c_{\text{i}} \quad (7)$$

where V_s is the shock front velocity which is the summation of the velocity of the ionization front and the post-shock velocity:

$$V_s = \dot{R}_{\text{IF}}(t) + V_{\text{ps}}. \quad (8)$$

The above equation is known as the ‘piston relation’. The post shock velocity is always greater than the velocity of the ionization front thus the shock front gets thicker.

Assuming that the shock front is isothermal, the post-shock velocity in the rest frame of the neutral medium is

$$V_{\text{ps}} = \frac{V_s}{\mathcal{M}_0^2}, \quad (9)$$

where \mathcal{M}_0 is the Mach number and \mathcal{M}_0^2 describes the compression of the shock. However, $\mathcal{M}_0 = V_s^2/c_o^2$ therefore

$$V_{\text{ps}} = \frac{c_o^2}{V_s}. \quad (10)$$

Combining the above equation with Eqns.(7) and (8) we obtain (Raga et al., 2012a):

$$\frac{1}{c_i} \frac{dR_{\text{IF}}(t)}{dt} = \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{3/4} - \frac{\mu_i T_o}{\mu_o T_i} \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{-3/4}, \quad (11)$$

where μ is the mean molecular weight of the gas. The above equation is the first equation of motion that we will study further on later.

However, a different approach describing the expansion of an HII region was provided by Hosokawa & Inutsuka (2006) who derived the position of the ionization front in time directly from the equation of motion of the expanding shell. Individually, Raga et al. (2012b) argued that the differential Eqn.(11) does not incorporate the inertia of the shocked neutral gas which is created during the expansion of the HII region. To include the inertia, we may write the equation of motion of the mass M within the shocked shell as

$$\frac{d}{dt}(M\dot{R}) = 4\pi R^2(P_i - P_o) \quad (12)$$

where P is the thermal pressure of the gas. The term $P_i - P_o$ corresponds to the net thermal pressure taking into account the pressure acting from the neutral gas onto the ionized gas. Using Eqn.(5) and where $R_{\text{IF}}(t)$ represents now the position of the shocked shell R , we obtain the second-order non-linear differential equation

$$\ddot{R} + \left(\frac{3}{R} \right) \dot{R}^2 = \frac{3R_{\text{St}}^{3/2} c_i^2}{R^{5/2}} - \frac{3c_o^2}{R}. \quad (13)$$

We examine below the two cases of the expansion of an HII region based on Eqns.(11) and (13).

2.2 Expansion based on Equation 11

The ratio $\mu_i T_o / \mu_o T_i$ presented on the rhs of Eqn.(11) is generally small ($\sim 200^{-1}$). At early times the term $\frac{\mu_i T_o}{\mu_o T_i} \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{-3/4}$ can be neglected, therefore Eqn.(11) leads to the so-called

Spitzer solution:

$$R_{\text{IF}}(t) = R_{\text{St}} \left(1 + \frac{7}{4} \frac{c_i t}{R_{\text{St}}} \right)^{4/7}. \quad (14)$$

However, at later times the term $\frac{\mu_i T_o}{\mu_o T_i} \left\{ \frac{R_{\text{St}}}{R_{\text{IF}}(t)} \right\}^{-3/4}$ increases and eventually the HII region stagnates at $t = t_{\text{STAG}}$ which is defined by $\dot{R}_{\text{IF}}(t_{\text{STAG}}) = 0$. By this time the HII region is in pressure equilibrium, thus it does not expand further. The stagnation radius is

$$R_{\text{STAG}} = \left(\frac{c_i}{c_o} \right)^{4/3} R_{\text{St}} \quad (15)$$

the density of the ionized gas is

$$\rho_i = \rho_o \left(\frac{c_o}{c_i} \right)^2 \quad (16)$$

and the total ionized mass is

$$M_i(t_{\text{STAG}}) = \frac{4\pi}{3} R_{\text{St}}^3 \rho_o \left(\frac{c_o}{c_i} \right)^2. \quad (17)$$

From Eqn.(7) we obtain:

$$V_s = \dot{R}_{\text{SF}}(t) = \frac{1}{2} \left\{ \dot{R}_{\text{IF}}(t) + \sqrt{\dot{R}_{\text{IF}}(t)^2 + 4c_o^2} \right\} \quad (18)$$

This is the velocity of the shock front. One can observe that at $t = t_{\text{STAG}}$, $V_s(t_{\text{STAG}}) = c_o$, so the shock front is detached and propagates freely in the interstellar medium as a sound wave at a constant speed of c_o . Integrating the above equation gives the position of the shock front.

Alternatively, using the chain rule we may write

$$\frac{dR_{\text{SF}}}{dR_{\text{IF}}(t)} = \frac{dR_{\text{SF}}(t)}{dt} \frac{dt}{dR_{\text{IF}}(t)} \quad (19)$$

and using Eqns.(7) and (11) we obtain

$$\int_{R_{\text{St}}}^{R_{\text{SF}}} dR_{\text{SF}} = \int_{R_{\text{St}}}^{R_{\text{IF}}} \frac{dR_{\text{IF}}}{1 - \left(\frac{\mu_i T_o}{\mu_o T_i} \right) \left(\frac{R_{\text{IF}}}{R_{\text{St}}} \right)^{3/2}}. \quad (20)$$

If we define as $\eta \equiv R_{\text{IF}}/R_{\text{St}}$ then

$$R_{\text{SF}} = R_{\text{St}} \left\{ 1 + \int_1^{R_{\text{IF}}/R_{\text{St}}} \frac{d\eta}{1 - \frac{\mu_i T_o}{\mu_o T_i} \eta^{3/2}} \right\} \quad (21)$$

This is the equation for the motion of the expanding shell. Note that the integral in the above equation has an analytical solution, although rather extended to present it here.

We direct the reader to Raga et al. (2012a) for a complete discussion on the analytical solution of Eqn.(11).

2.3 Expansion based on Equation 13

The second-order non-linear differential Eqn.(13) represents the equation of motion of the expanding shell when its inertia is taken into account. We solve this equation by defining

$$\dot{R} \equiv \psi; \quad \ddot{R} = \psi' \psi; \quad \psi' \equiv \frac{d\psi}{dR} \quad (22)$$

Thus Eqn.(13) becomes

$$\psi' + \psi f(R) = g(R) \psi^{-1} \quad (23)$$

where

$$f(R) = \frac{3}{R}; \quad g(R) = \frac{3c_i^2 R_{\text{St}}^{3/2}}{R^{5/2}} - \frac{3c_o^2}{R} \quad (24)$$

Equation 23 is the Bernoulli differential equation. To solve it we set $\nu = \psi^2$ thus $\nu' = 2\psi\psi'$

$$\nu' + 2f(R)\nu = 2g(R) \quad (25)$$

which has as analytical solution

$$\nu(R) = \frac{\int \mu(R) 2g(R) dR}{\mu(R)}; \quad \mu(R) = e^{\int 2f(R) dR} \quad (26)$$

where $\mu(R)$ represents an integrating factor. The above leads to

$$\dot{R}_{\text{IF}}(t) = c_i \sqrt{\frac{4}{3} \frac{R_{\text{St}}^{3/2}}{R_{\text{IF}}^{3/2}(t)} - \frac{\mu_i T_o}{\mu_o T_i}} \quad (27)$$

Following the analysis of §2.2 at early times we may neglect the $\mu_i T_o / \mu_o T_i$ term. This in fact corresponds to the assumption of $P_o = 0$ in Eqn.(12). Equation 27 has therefore as analytical solution

$$R_{\text{IF}}(t) = R_{\text{St}} \left(1 + \frac{7}{4} \sqrt{\frac{4}{3}} \frac{c_i t}{R_{\text{St}}} \right)^{4/7}. \quad (28)$$

This equation was first presented by Hosokawa & Inutsuka (2006) and it differs from the known Spitzer expression (Eqn.14) by a $\sqrt{4/3}$ factor. This factor arises from our inclusion of the inertia of the shocked gas due to its own movement leading to a slightly ‘faster’ expansion than that obtained from Eqn.(14).

At later times R_{IF} becomes large, and therefore the two terms contained in the square root of Eqn.(27) become comparable. Eventually at $t = t_{\text{STAG}}$ in which $\dot{R}_{\text{IF}}(t_{\text{STAG}}) = 0$ we obtain the stagnation radius

$$R_{\text{STAG}} = R_{\text{St}} \left(\frac{4}{3} \right)^{2/3} \left(\frac{c_i}{c_o} \right)^{4/3} \quad (29)$$

the density of the ionized medium

$$\rho_i = \rho_o \left(\frac{4}{3} \right)^{2/3} \left(\frac{c_o}{c_i} \right)^{4/3} \quad (30)$$

and the total ionized mass

$$M_i(t_{\text{STAG}}) = \frac{4\pi}{3} \rho_o R_{\text{St}}^3 \left(\frac{4}{3} \right)^{2/3} \left(\frac{c_o}{c_i} \right)^{4/3}. \quad (31)$$

3 The Benchmarking Test

For the purposes of the test we will use very simplified isothermal equation of state for both the ionized and the neutral medium. The treatment of the region where the two media meet is left to each coders' method, however please specify exactly how you do it. The test is purely hydrodynamical i.e. it includes no gravity. Due to the nature of Eqns.(11) and (13) we will run two tests: i) to examine the early phase of the D-type expansion and ii) to examine the later phase of the D-type expansion.

3.1 Early phase

For the early phase expansion test we consider a spherical cloud containing pure hydrogen $X = 1$ and an exciting source emitting $\dot{N}_{\text{LyC}} = 10^{49}$ photons per second placed in the centre of the spherical cloud. The position of the centre defines the centre of a Cartesian co-ordinate system. The density of the sphere is taken to be $\rho_o = 5.21 \times 10^{-21} \text{ g cm}^{-3}$ and the total mass is taken to be $M_{\text{cl}} = 640 M_{\odot}$. The radius of the cloud is therefore $R_{\text{cl}} = 1.257 \text{ pc}$ and the initial Strömgren radius is $R_{\text{St}} = 0.314 \text{ pc}$, or $R_{\text{St}} = 4^{-1} R_{\text{cl}}$.

The temperature of the ionized gas is taken to be $T_i = 10^4 \text{ K}$; the mean molecular weight $\mu_i = 0.5$; and therefore the sound speed $c_i = 12.85 \text{ km/s}^{-1}$. The temperature of the neutral gas is taken to be $T_o = 10^2 \text{ K}$; the mean molecular weight $\mu_o = 1$; and therefore the sound speed $c_o = 0.91 \text{ km/s}^{-1}$.

We evolve the simulation for $t_{\text{end}} = 0.141 \text{ Myr}$. At this time the ionization front has reached the boundaries of the cloud, therefore $R_{\text{IF}}(t_{\text{end}}) \simeq R_{\text{cl}}$.

The left panel of Fig.1 shows the positions of ionization and shock fronts for the early phase based on the equations described above.

3.2 Late phase

Equation 15 gives the stagnation radius as a function of the initial Strömgren radius. We will choose a slightly bigger cloud radius than the stagnation radius to avoid the effect of an expanding shell in vacuum when $t = t_{\text{STAG}}$. Let g be this additional factor determining the extra size of the cloud radius. Then:

$$R_{\text{cl}} = g R_{\text{STAG}} = g \left(\frac{c_i}{c_o} \right)^{4/3} R_{\text{St}}, \quad (32)$$

which therefore leads to $f = g \left(\frac{c_i}{c_o} \right)^{4/3}$. The density of the cloud at $t = 0 \text{ Myr}$ is:

$$\rho_o = g^3 \left(\frac{c_i}{c_o} \right)^4 \frac{\dot{N}_{\text{LyC}} m_p^2}{M_{\text{cl}} \alpha_B} = 5.21 \times 10^{-21} \text{ g cm}^{-3}. \quad (33)$$

Thus the cloud has mass $M_{\text{cl}} = 8 \times 10^3 M_{\odot}$, initial Strömgren radius $R_{\text{St}} = 0.314 \text{ pc}$, $R_{\text{STAG}} = 2.31 \text{ pc}$ and $R_{\text{cl}} = 2.91 \text{ pc}$. The stagnation radius is obtained at $t_{\text{STAG}} \sim 2.2 \text{ Myr}$. The simulation should terminate at $t_{\text{END}} = 3 \text{ Myr}$.

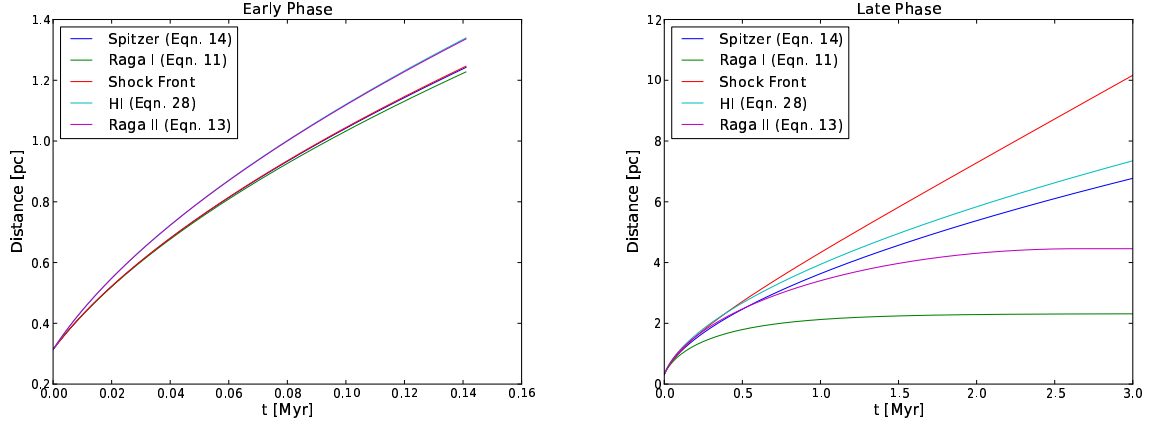


Figure 1: Position of ionization and shock fronts as given by Eqns.(11),(13),(14),(28) and the integrated form of Eqn.(18) for the early phase test (left panel) and the late phase test (right panel). Note that in the right panel, the shock front is detached from the expanding HII region at about $t = 0.5$ Myr and propagates in the ISM at the speed of c_o .

The right panel of Fig.1 shows the positions of ionization and shock fronts for the late phase based on the equations described above.

4 SPH setup

Let us suppose that $R_{cl} = fR_{st}$, where R_{cl} is the radius of the spherical cloud, R_{st} is the Strömgren radius and $f > 1$ is a user-defined factor. f describes the size of the Strömgren sphere in comparison with the size of the spherical cloud. The density at $t = 0$ Myr is constant throughout the cloud. Therefore

$$\rho_o = \frac{3M_{cl}}{4\pi R_{cl}^3} = \frac{3M_{cl}}{4\pi f^3 R_{st}^3} = \frac{3M_{cl}4\pi\alpha_B\rho_o^2}{4\pi f^3 \dot{N}_{LyC} m^2} \quad (34)$$

which in turn gives

$$\rho_o = \frac{f^3 \dot{N}_{LyC} m^2}{M_{cl} \alpha_B} \quad (35)$$

Let N_{SPH}^{St} be the number of SPH particles with mass m_{SPH} consisting the Strömgren sphere. Then

$$\rho_o = \frac{3M_{st}}{4\pi R_{st}^3} = \frac{3N_{SPH}^{St} m_{SPH}}{4\pi R_{st}^3} \quad (36)$$

and

$$\rho_o = \frac{3M_{cl}}{4\pi R_{cl}^3} = \frac{3N_{SPH}^{cl} m_{SPH}}{4\pi R_{cl}^3} \quad (37)$$

Cobmining the above, we obtain $N_{\text{SPH}}^{\text{cl}} = f^3 N_{\text{SPH}}^{\text{St}}$. This equation gives the total number of SPH when we specify the size and the resolution of Strömgren sphere.

In all cases we will use a pre-settled particle distribution in order to form a ‘glass’ structure which minimizes (but does not eliminate) the numerical noise.

4.1 Early phase

For this test we will use $f = 4$ and $N_{\text{SPH}}^{\text{St}} = 10^4$. Therefore the total number of SPH particles is taken to be $N_{\text{SPH}}^{\text{cl}} = 6.4 \times 10^5$ particles. We also use $m_{\text{SPH}} = 10^{-3} M_{\odot}$ which therefore implies $M_{\text{cl}} = 640 M_{\odot}$.

4.2 Late phase

The total number of SPH particles is:

$$N_{\text{SPH}}^{\text{cl}} = g^3 \left(\frac{c_i}{c_o} \right)^4 N_{\text{SPH}}^{\text{St}}. \quad (38)$$

The factor $\left(\frac{c_i}{c_o} \right)^4$ is in general big which can lead to prohibitively high total number of SPH particles. We will therefore adopt a ‘neutral’ temperature of $T_o = 10^3$ K while keeping $\mu_o = 1$ implying a sound speed $c_o = 2.87$ km/s. Using $g = 1.26$ (because at this value $g^3 \simeq 2$) and $N_{\text{SPH}}^{\text{St}} = 10^4$ we obtain $N_{\text{SPH}}^{\text{cl}} = 8 \times 10^6$ total SPH particles. We also use $m_{\text{SPH}} = 10^{-3} M_{\odot}$ which therefore implies $M_{\text{cl}} = 8 \times 10^3 M_{\odot}$.

5 Grid setup

5.1 Early phase

The radiation source is placed at the origin, as before. The gas density is set to $\rho_0 = 5.21 \times 10^{-21} \text{ g cm}^{-3}$, initially neutral with temperature $T = 100$ K. The assumed equation of state is isothermal, for which case $\gamma = 1.0001$. The recommended spatial resolution is 128^3 grid zones per octant, so if the full space is simulated 256^3 is needed, but in principle each octant should be identical (modulo noise). If only one octant is simulated, then reflecting boundary conditions should be used. An octant runs from $\{x, y, z\} = 0$ to 3.874×10^{18} cm, corresponding to $4R_{\text{St}}$ ($R_{\text{St}} = 0.9685 \times 10^{18} \text{ cm} = 0.314 \text{ pc}$). Within $r < R_{\text{St}}$, the gas is ionised and its temperature is set to $T = 10^4$ K. This gives a pressure ratio of 200 between the ionised and neutral gas, for the pure hydrogen gas composition we are using. One simulation with this resolution (fixed uniform grid) should be performed if possible, but contributors may also take advantage of code features such as non-uniform, non-Cartesian, and/or adaptive grids to try to obtain better results. For these subsequent calculations the total number of grid zones should be $\lesssim 10^7$, so that the data files don’t become too large.

5.2 Late Phase

This problem is set up in a similar way, but with the maximum simulation extents now set to $1.26R_{\text{STAG}} = 2.91 \text{ pc}$ from the origin, to agree with the SPH setup (Sec. 3.2). The ISM density is the same, but the neutral gas temperature is 1000 K instead of 100 K, so the pressure ratio between ionised and neutral gas is reduced to 20 in the initial conditions.

6 Output format

The output format should contain three columns and should be of the form (consider this as a FORTRAN example):

```
write(FILEUNIT,'(9E16.7E3)') x, y, z, vx, vy, vz, rho, T, chi
```

where the positions (**x,y,z**) are in pc, velocities (**vx,vy,vz**) are in km/s, density (**rho**) is in g/cm^3 , and temperature (**T**) is in K. **chi** is the ionization fraction given by:

$$\chi = \frac{T_{\text{sim}} - T_{\text{o}}}{T_{\text{i}} - T_{\text{o}}} \quad (39)$$

where T_{sim} is the temperature of the SPH particle/cell. This will return the format (the first four columns are only shown here) i.e.:

```
XX0.1000000E-100XX0.1000000E-100XX0.1000000E+102XX0.1000000E+102
```

where X is one space. Snapshots should be taken at the following times:

Early phase

0.005, 0.01, 0.02, 0.04, 0.08, 0.14 (Myr)

Late phase

0.05, 0.1, 0.2, 0.4, 0.8, 1.6, 3.0 (Myr)

7 Code description

Please send a description of the code in Latex format (.tex file) with no more than 400 words once compiled. Please state also all relevant references.

8 Deadlines & Sending your results

Once you have finished your calculations as described above, please send an email to both

`t.bisbas@ucl.ac.uk` [Thomas Bisbas] | `thaworth@ast.cam.ac.uk` [Thomas Haworth]

informing us that your data is ready. Please do not attach your data here. We will then let you know where and how to upload your results.

Deadline for submitting your results is Friday 28 March 2014. Please respect this deadline as we plan to post-process your results as soon as we receive them and which will trigger the discussion. Happy modelling!

References

Hosokawa, T., & Inutsuka, S.-i. 2006, ApJ, 646, 240

Kahn, F. D. 1954, BAN, 12, 187

Osterbrock, D. E. 1974, Research supported by the Research Corp., Wisconsin Alumni Research Foundation, John Simon Guggenheim Memorial Foundation, Institute for Advanced Studies, and National Science Foundation. San Francisco, W. H. Freeman and Co., 1974. 263 p.,

Raga, A. C., Cantó, J., & Rodríguez, L. F. 2012, MNRAS, 419, L39

Raga, A. C., Cantó, J., & Rodríguez, L. F. 2012, RMxAA, 48, 149

Spitzer, L. 1978, New York Wiley-Interscience, 1978. 333 p.,

Strömgren, B. 1939, ApJ, 89, 526

Whitworth, A. 2000, Private communication