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Monte Carlo option pricing with asymmetric realized volatility dynamics

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Abstract

What are the advances introduced by realized volatility models in pricing options? In this short paper we analyze a simple option pricing framework based on the dually asymmetric realized volatility model, which emphasizes extended leverage effects and empirical regularity of high volatility risk during high volatility periods. We conduct a brief empirical analysis of the pricing performance of this approach against some benchmark models using data from the S&P 500 options in the 2001–2004 period. The results indicate that as expected the superior forecasting accuracy of realized volatility translates into significantly smaller pricing errors when compared to models of the GARCH family. Most importantly, our results indicate that the presence of leverage effects and a high volatility risk are essential for understanding common option pricing anomalies.

Keywords: Realized volatility; Option pricing; Volatility of volatility; Leverage effects

1. Introduction

The advent of high frequency stock market data and the subsequent introduction of realized volatility measures represented a substantial step forward in the accuracy with which econometric models of volatility could be evaluated and allowed for the development of new and more precise parametric models of time varying volatility. Several researchers have looked into the properties of *ex post* volatility measures derived from high frequency data and developed time series models that invariably outperform latent variable models of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) or stochastic volatility family of models [3] in forecasting future volatility, to the point that the comparison has been dropped altogether in recent papers.

Recent contributions to the realized volatility modeling and forecasting literature are exemplified by Andersen et al. [3], the HAR (heterogeneous autoregressive) model of Corsi [10], the MIDAS (mixed data sample) approach of Ghysels et al. [15] and the unobserved ARMA component model of Refs. [18,23]. Martens et al. [20] develop

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a nonlinear (ARFIMA) model to accommodate level shifts, day of the week, leverage and volatility level effects. Andersen et al. [2] and Tauchen and Zhou [26] argue that the inclusion of jump components significantly improves forecasting performance. McAleer and Medeiros [21] extend the HAR model to account for nonlinearities. Hillebrand and Medeiros [17] also consider nonlinear models and evaluate the benefits of bootstrap aggregation (bagging) for volatility forecasting. Ghysels et al. [15] argue that realized absolute values outperform square return-based volatility measures in predicting future increments in quadratic variation. Scharth and Medeiros [22] introduce multiple regime models linked to asymmetric effects. Liu and Maheu [19] derive a bayesian averaging approach for forecasting realized volatility. Bollerslev et al. [8] propose a full system for returns, jumps and continuous time for components of price movements using realized variation measures.

Despite these successes in modeling the conditional mean of realized volatility, empirical evaluations of this class of models outside the realm of short run forecasting is limited. Fleming et al. [14] examine the economic value of volatility timing using realized volatility. Bandi et al. [5] evaluate and compare the quality of several recently proposed realized volatility estimators in the context of option pricing and trading of short term options on a stylized setting. Stentoft [25] derives an appropriate return and volatility dynamics to be used for option pricing purposes in the context of realized volatility and perform an empirical analysis using stock options for three large American companies.

We emphasize two main empirical regularities that are potentially very relevant for option pricing purposes. First, realized variation measures constructed from high frequency returns reveal a large degree of time series unpredictability in the volatility of asset returns. Even though returns standardized by (*ex post*) quadratic variation measures are nearly gaussian, this unpredictability brings substantially more uncertainty to the empirically relevant (ex-ante) distribution of returns. In this setting carefully modeling the stochastic structure of the time series disturbances of realized volatility is fundamental. Second, there is evidence of very large leverage effects; large falls (rises) in prices being associated with persistent regimes of high (low) variance in the index returns.

In this paper we propose an options pricing framework based on a new realized volatility model that captures all the relevant empirical regularities of the realized volatility series of the S&P 500 index, the dually asymmetric realized volatility (DARV) model of Allen et al. [1]. In this setting returns display conditional volatility, skewness and kurtosis. The main new feature of this model is to recognize that volatility is itself more volatile and more persistent in high volatility periods. Contrary to "peso problem" considerations, we show that when volatility is (nearly) observable it is not necessary to rely on rare realizations on past return data to learn about the tails of the return distribution, an unexplored and large modeling gain enabled by high frequency data.

We conduct a brief empirical analysis of the pricing performance of this approach against some benchmark models using data from the S&P 500 options in the 2001–2004 period. This exercise supplements the extensive empirical analysis of this model conducted in that paper. The results indicate that as expected the superior forecasting accuracy of the proposed realized volatility model translates into significantly smaller pricing errors when compared to models of the GARCH family. More significantly, our results indicate that modeling leverage effects and the volatility of volatility are paramount for reducing common pricing anomalies.

2. Data and stylized facts of realized volatility

2.1. Realized volatility and data

Suppose that at day t the logarithmic prices of a given asset follow a continuous time diffusion:

$$dp(t+\tau) = \mu(t+\tau) + \sigma(t+\tau)dW(t+\tau), \quad 0 \leq \tau \leq 1, \quad t = 1, 2, 3 \ldots \tag{1}$$

where $p(t+\tau)$ is the logarithmic price at time $t+\tau$, is the drift component, $\sigma(t+\tau)$ is the instantaneous volatility (or standard deviation), and $dW(t+\tau)$ is a standard Brownian motion. Andersen et al. [3] (and others) showed that the daily compound returns, defined as $r_t = p(t) - p(t+1)$, are Gaussian conditionally on $\mathcal{F}_t = \sigma(p(s), s \leq t)$, the σ-algebra (information set) generated by the sample paths of p, such that

$$r_t|\mathcal{F}_t \sim N(\int_0^1 \mu(t-1+\tau)d\tau, \int_0^1 \sigma^2(t-1+\tau)d\tau) \tag{2}$$

The term $IV_t = \int_0^1 \sigma^2(t-1+\tau)d\tau$ is known as the integrated variance, which is a measure of the day *t ex post* volatility. In this sense, the integrated variance is the object of interest. In practical applications prices are observed at discrete intervals. If we set $p_{i,t}$, $i = 1, \ldots, n$ to be the *i*th price observation during day *t*, realized variance is defined as $\sum_{i=1}^{n} r_{i,t}^2$. The realized volatility is the square-root of the realized variance and we shall denote it by RV_t. Ignoring the remaining measurement error, this *ex post* volatility measure can modeled as an "observable" variable, in contrast to the latent variable models.

In real data, however, high frequency measures are contaminated by microstructure noise such as bid-ask bounce, asynchronous trading, infrequent trading, price discreteness, among others. In this paper, we turn to the theory developed by Barndorff-Nielsen et al. [6] and implement a realized kernel estimator based on one minute returns and the modified Tukey–Hanning kernel, which is consistent in the presence of microstructure noise.

The empirical analysis focuses on the realized volatility of the S&P 500 index (SPX) and the S&P 500 options traded the Chicago Board Options Exchange (CBOE). The raw intraday data was obtained from the Taqtiq/SIRCA (Securities Industry Research Centre of Asia-Pacific) database. To calculate the realized volatility series we use tick-by-tick open to close quotes originated in the E-Mini S&P500 futures market of the Chicago Mercantile Exchange.[1] The period of analysis for the realized volatility starts in January 2, 1996, and ends in November 28, 2008. For the options, we record daily intraday option prices close to the 14:30 time mark between January 2, 2001, and ends in March 15, 2004. The data for the risk-free rate is obtained from the three month T-Bill.

2.2. Stylized facts

We begin by recalling the most important and generally accepted features of returns and realized volatility that have been identified in the literature and will inform the rest of our analysis:

1. The unconditional distribution of daily returns exhibits excess kurtosis;
2. the time variation in the conditional volatility of daily returns does not fully account for this excess kurtosis;
3. daily returns are not autocorrelated (except for the first order, in some cases);
4. daily returns standardized by realized volatility measures are almost Gaussian;
5. the unconditional and conditional distributions of realized variance and volatility are distinctly non-normal and extremely right-skewed;
6. there is strong evidence of long memory in volatility;
7. negative returns are associated with higher subsequent volatility (asymmetric effects);
8. realized volatility series are very volatile and this volatility risk is strongly positively related to the level of volatility.

Fig. 1 displays the time series of returns, realized volatility and log realized volatility. For a full discussion the reader is referred to AMS1. We emphasize three essential elements for our realized volatility model. The leading property observed in realized volatility which served as the main foundation for models that have been developed has been that of very high persistence observed in those series, which consistently exhibit empirical autocorrelation functions with hyperbolic decaying patterns; this empirical feature can be seen in Fig. 2 and is referred to in a broad sense as long memory or long range dependence.

The presence of asymmetric effects is illustrated by Fig. 3, which plots the time series of realized volatility and monthly returns (rescaled). Virtually all episodes of (persistently) high volatility are associated with streams of negative returns; once the index price recovers the realized volatility tends to quickly fall back to average levels. Fig. 4, which is based on an estimation of HAR-GARCH model for realized volatility, shows the close positive relation between volatility risk and the level of volatility.

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[1] These fully electronic contracts feature among the most liquid derivatives contracts in the world, therefore closely tracking price movements of the S&P 500 index.

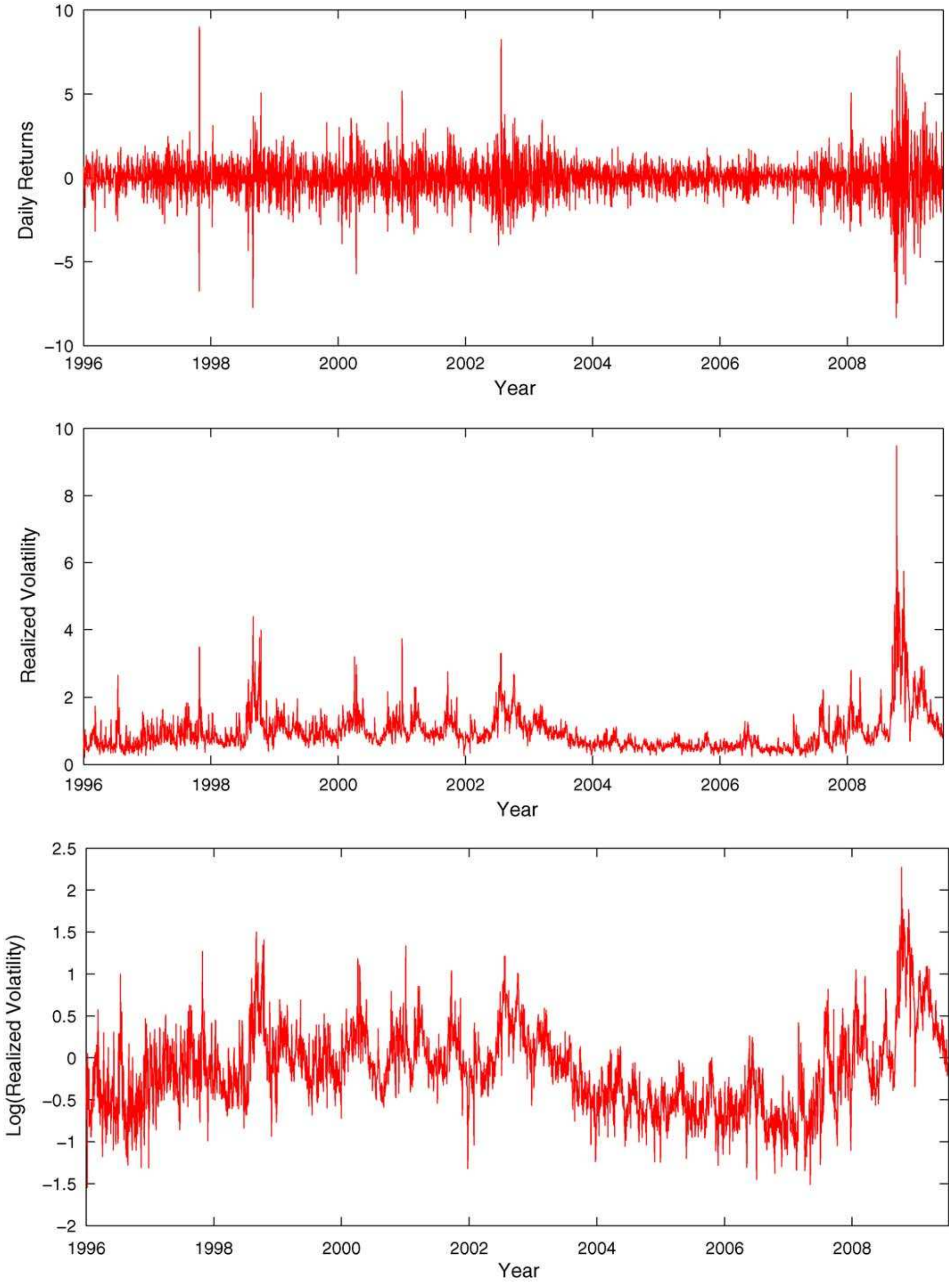

Fig. 1. Time series of returns (top), realized volatility (middle) and log realized volatility (bottom) for the S&P 500 index.

3. The dually asymmetric realized volatility model

The dually asymmetric realized volatility (DARV) model introduced by Allen et al. [1] is a first step in analyzing and incorporating the modeling qualities of a more realistic setting for the volatility risk within a standard realized volatility model. The dual asymmetry in the model comes from leverage effects (as seen in the last section) and the positive relation between the level of volatility and the degree of volatility risk. The fundamental issue that

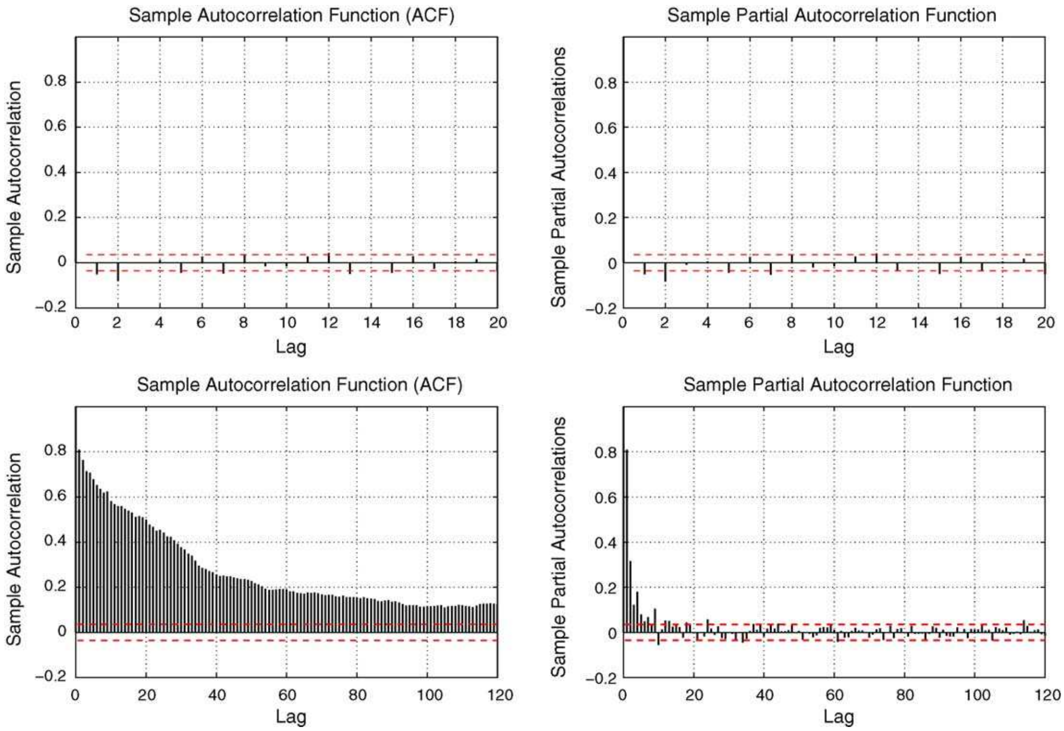

Fig. 2. Sample autocorrelations and partial autocorrelations of returns (top) and realized volatility (bottom). The dashed lines give the standard Bartlett 95% confidence interval.

arises in specifying the model is how to specify the relation between volatility level and the volatility risk in the model.

We directly model the time series of realized volatility (RV_t). Let the conditional variance of the residuals be denoted by h_t^2. In this paper we choose the specification $h_t^2 = \theta_0 + \theta_1 VL_t^2$, where VL_t (the volatility level) is the conditional mean of volatility ($E(RV_t|\mathcal{F}_{t-1})$), where $\mathcal{F}_{t-1}$ is the information set at end of the previous day. Another option would be to directly allow for the asymmetry of positive or negative shocks in volatility in a GARCH model, but we have found our simpler specification to be superior. An advantage of our method is that it is an straightforward way of accounting for the possibility of long memory in the volatility of realized volatility.

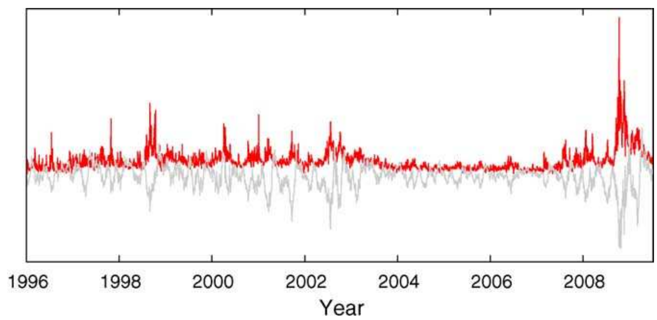

Fig. 3. Realized volatility (top) and monthly returns (bottom) for the S&P 500 index.

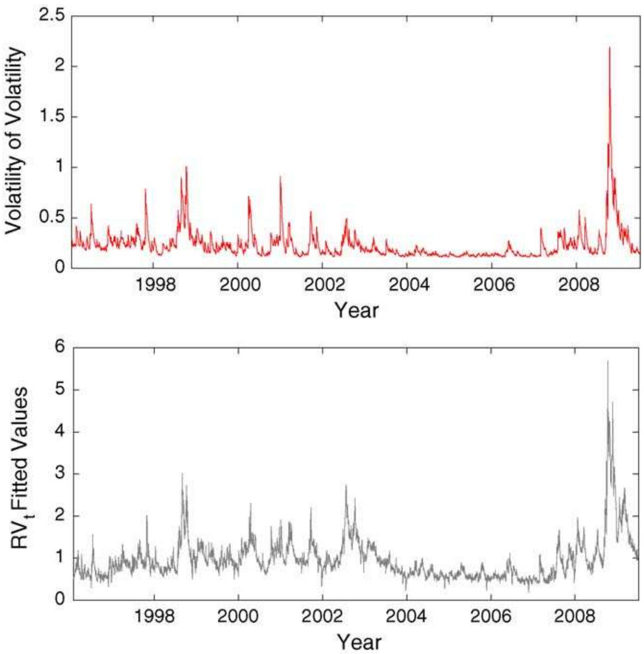

Fig. 4. Volatility of realized volatility series (top) and realized volatility fitted values (bottom) for a HAR-GARCH realized volatility model with leverage effects.

The general specification for our model in autoregressive fractionally integrated and heterogenous autoregressive versions are:

$$\begin{aligned}
&\text{DARV-HAR model}:\\
&r_t = \mu_t + RV_t\varepsilon_t,\\
&RV_t = \phi_0 + \phi_1 RV_{t-1} + \phi_2 RV_{5,t-1} + \phi_3 RV_{22,t-1}\\
&\qquad + \lambda_1 I(r_{t-1} < 0)r_{t-1} + \lambda_2 I(r_{5,t-1} < 0)r_{5,t-1}\\
&\qquad + \lambda_3 I(r_{22,t-1} < 0)r_{22,t-1} + h_t\eta_t,\\
&h_t^2 = \theta_0 + \theta_1 VL_t^2,
\end{aligned} \tag{3}$$

where r_t is the log return at day t, μ_t is the conditional mean for the returns, RV_t is the realized volatility, ε_t is i.d. $N(0,1)$, ψ_t shifts the unconditional mean of realized volatility, d denotes the fractional differencing parameter, $\Phi(L)$ is a polynomial with roots outside the unit cirvle, L the lag operator, I is the indicator function, $r_{j,t-1}$ is a notation for the cumulated returns $\sum_{i=t-j}^{t-1} r_{t-i}$, $RV_{j,t-1} = \sum_{i=t-j}^{t-1} RV_{t-i}$, h_t is the volatility of the realized volatility, η_t is i.i.d. with $E(\eta_t)=0$ and $E(\eta_t^2)=1$, ε_t and η_t are allowed to be dependent and $VL_t = E(RV_t|\mathcal{F}_{t-1})$.

Following the evidence of fractional integration in realized volatility, ARFIMA models are the standard in this literature. Fractionally integrated models have been estimated for example in Refs. [3,4,7,12,20,27], among others. Nevertheless, the estimation of ARFIMA models in this context has encountered a few shortcomings. Although $I(d)$ processes are a seemingly reasonable approximation for the data generating process of volatility series, there is no underlying theory to formally support this specification. Instead, the results of Refs. [13,16] challenge fractional integration as the correct specification for realized volatility series by showing that long memory properties can be engendered by structural breaks or regime switching. Scharth and Medeiros [22] discuss how estimates of the fractional differencing parameter are subject to excessive variation over time.

Given the lack of stronger support for a strict interpretation of fractional integration evidence and the higher computational burden in estimating and forecasting this class of models, some researchers have chosen to apply simpler time series models which are consistent with high persistence in the relevant horizons (like the HAR model of the last section), even though they do not rigorously exhibit long memory (hence being labeled 'quasi-long memory' models) Refs. [9,22] highlight the impact of leverage effects for the dynamics of realized volatility. The latter argues for the existence of regime switching behavior in volatility, with large falls (rises) in prices being associated with persistent regimes of high (low) variance in stock returns. The authors show that the incorporation of cumulated daily returns as a explanatory variable brings modeling advantages by capturing this effect. While Scharth and Medeiros [22] consider multiple regimes in a nonlinear model, we focus on a simpler linear relationship to account for the large correlation between past cumulated returns and realized volatility.

To account for the non-gaussianity in the error terms we follow [11] and assume that the i.i.d. innovations η_t follow the standardized normal inverse Gaussian (with we denote by *NIG**), which is flexible enough to allow for excessive kurtosis and skewness and reproduce a number of symmetric and asymmetric distributions. A more complex approach would rely on the generalized hyperbolic distribution, which encompasses the NIG distribution and requires the estimation of an extra parameter. On the other hand typical distributions with support on the interval $(0,\infty)$, which would be a desirable feature for our case, were strongly rejected by preliminary diagnostic tests.

Finally, to model the asymmetry in the conditional return distribution we let η_t and ε_t be dependent and model this relation via a bivariate Clayton copula. The copula approach is a straightforward way to account for non-linearities in this dependence relation and has the important advantage of not requiring the joint estimation of the return and volatility equations in our model. Let $U = \Phi(\varepsilon_t)$ and $V = 1 - \Upsilon(\nu_t)$, where $\Phi(\cdot)$ and $\Upsilon(\cdot)$ are the corresponding normal and *NIG** cdfs for ε_t and η_t, respectively. The joint CDF or copula of U and V is given by:

$$C_\kappa \equiv P(U \leq u, V \leq u) = (u^{-\kappa} + u^{-\kappa} - 1)^{-1/\kappa} \tag{4}$$

In this specification, returns and volatility are negatively correlated and display lower tail dependence (days of very low returns and very high volatility are linked, where the strength of this association is given by the parameter κ). While this approach can be extended with the use of more sophisticated copula functions or by allowing for time variation in the dependence parameter, the static Clayton copula is a simple and parsimonious way to adequately account for this volatility feedback effect.

To reduce bias on our estimators and avoid distortions of the error distribution, we control the mean of the dependent variable for day of the week and holiday effects using dummies. Refs. [20,22] show that volatility sometimes tend to be lower on Mondays and Fridays, while substantially less volatility is observed around certain holidays.

3.1. Estimation and density forecasting

We estimate the the dually asymmetric realized volatility model by maximum likelihood. The fact that the conditional volatility of volatility h_t depends on the conditional mean of the realized volatility brings no issues for the estimation. The log-likelihood function is given by:

$$\begin{aligned}\ell(\hat{d}, \hat{\phi}, \hat{\psi}, \hat{\lambda}, \hat{\theta}, \hat{\alpha}, \hat{\beta}; \quad RV_{1\ldots T}, X_{1\ldots T}) &= T\log(\hat{\alpha}) - T\log(\pi) + \sum_{t=1}^{T}\log[K_1(\hat{\alpha}\hat{\delta}(1+\hat{y}_t^2)^{1/2})] \\ &\quad - 0.5\sum_{t=1}^{T}\log(1+\hat{y}_t^2) + T\hat{\delta}(\hat{\alpha}^2-\hat{\beta}^2)^{1/2} + \hat{\delta}\hat{\beta}\sum_{t=1}^{T}\hat{y}_t - 0.5\sum_{t=1}^{T}\log(\hat{h}_t)\end{aligned} \tag{5}$$

where X collects the additional explanatory variables, α and β are the tail heaviness and asymmetry parameters of the standardized NIG distribution. $\gamma = (\hat{\alpha}^2 - \hat{\beta}^2)^{1/2}$ and $\hat{y}_t = (\hat{\eta}_t/\hat{h}_t - \hat{\omega})/\hat{\delta}$, where ω and δ are the location and scale parameters associated with the standardized NIG distributed with parameters α and β.

The copula specification for the joint distribution of return and volatility innovations allows us to estimate the copula by maximum likelihood in a separate stage once we have obtained estimates for η_t and ε_t from the marginal models. For simplicity we estimate the mean of returns μ_t by the sample mean (since the daily expected return is very small μ_t is immaterial for our analysis), so that $\widehat{\varepsilon_t} = (r_t - \widehat{\mu})/RV_t$.

Unfortunately an analytical solution for the return density implied by our flexible normal variance–mean mixture hypothesis (realized volatility is distributed normal inverse gaussian and returns given volatility are normally distributed) is not available. Except for a few cases such as one day ahead point forecasts for realized volatility, many quantities of interest based on our model have to be obtained by simulation. We consider the following Monte Carlo method which can be easily implemented and made accurate with realistic computational power. Conditional on information up to day t we implement the following general procedure for simulating joint paths for returns and volatility (where $\sim$ is used to denote a simulated quantity):

1. In the first step the functional form of the model is used for the evaluation of forecasts $\widehat{RV}_{t+1}$ and $\widehat{h}_{t+1}$ conditional on past realized volatility observations, returns, and other variables.
2. Using the estimated copula we randomly generate S pairs of return $(\tilde{\varepsilon}_{t+1,j}, \quad j=1,\ldots,S)$ and volatility $(\tilde{\eta}_{t+1,j}, \quad j=1,\ldots,S)$ shocks with the according marginal distributions. Antithetic variables are used to balance the return innovations for location and scale.
3. We obtain S simulated volatilities through $\widetilde{RV}_{t+1,j} = \widehat{RV}_{t+1} + h_t\tilde{\eta}_{t+1,j}$, $j=1,\ldots,S$. Each of these volatilities generate a returns $\tilde{r}_{t+1,j} = \hat{\mu}_t + \widetilde{RV}_{t+1,j}\tilde{\varepsilon}_{t+1,j}$.
4. This procedure can be iterated in the natural way to generate multiple paths for returns and realized volatility.

3.2. Pricing European options with realized volatility

To price European options on the S&P 500 index using the framework discussed above, we simulate returns and volatility under the risk neutral distribution. As it is well known, the existence of a risk neutral dynamics follows from absence of arbitrage and mild regularity conditions. To keep the pricing framework tractable, we follow the approach of Stentoft [25] and assume that investors require no premium for being exposed to realized volatility risk. In this case the risk neutral RV dynamics are the same as the physical dynamics. The risk neutral system is:

$$
\begin{aligned}
r_t &= \mu_t^* + RV_t\varepsilon_t^*,\\
RV_t &= \phi_0 + \phi_1 RV_{t-1} + \phi_2 RV_{5,t-1} + \phi_3 RV_{22,t-1}\\
&\quad + \lambda_1 I(r_{t-1} < 0) r_{t-1} + \lambda_2 I(r_{5,t-1} < 0) r_{5,t-1}\\
&\quad + \lambda_3 I(r_{22,t-1} < 0) r_{22,t-1} + h_t\eta_t,\\
h_t^2 &= \theta_0 + \theta_1 VL_t^2 + \theta_2\eta_{t-1}^2,
\end{aligned} \tag{6}
$$

where ε_t^* is distributed $N(0,1)$. μ_t^* is such that $E^Q(\exp[\mu_t + RV_t\varepsilon_t^*]) = \exp rf_t$, where rf_t is the daily risk-free rate (assumed constant during the life of the option) and $E^Q(\cdot)$ is the expectation under the risk neutral measure.

4. Empirical illustration

In this section we perform a brief empirical analysis of our option pricing model. For conciseness we focus on put options with 9–60 calendar days to expiration. Defining moneyness by $M = S_t/X$, where S_t is the underlying index price at the time when the option is observed and X is the strike price, we divide the options into the following groups: at-the-money ($0.98 < M < 1.02$), out-the money ($1.02 < M < 1.05$), in-the-money ($0.95 < M < 0.98$), deep out-of-the-money ($1.05 < M < 1.1$) and the deep in-the-money ($0.9 < M < 0.95$). We consider some alternative models: (i) the Black and Scholes price (where the volatility is given by the mean of realized volatility over the last month), (ii) GARCH/NIG prices (for the GARCH, EGARCH, GJR-GARCH and NGARCH models). See (for example) Stentoft [24] for the theoretical background and details, (iii) two variations of our realized model: in both cases the specification is exactly as in the main model, except that we consider a GARCH(1,1) model for the volatility of realized volatility (h_t in the notation of the last sections) and first alternative does not allow for lagged leverage effects.

The results are summarized in Table 1 below, where we focus on the mean absolute pricing error metric. As expected, both GARCH and RV prices are substantial improvements over the Black and Scholes prices. Among the GARCH models, the EGARCH stands out as having a much superior performance than the others, followed by the GJR, NGARCH and GARCH specifications. Perhaps surprisingly, the RV-HAR-GARCH models are inferior to the EGARCH model. Nevertheless, the dually asymetric model presented in this paper substantially reduces pricing errors at all moneyness categories when compared to the best GARCH model. Table 2 illuminates the reason for better performance of the model: contrary to all the other specifications it does not underprice in general these put options (and in particular the out-of-the-money ones, a well known deficiency of the Black Scholes and other models). The reason why the model achieves the results is the conjunction of leverage effects and the specification for the volatility

Table 1
Mean absolute pricing errors.

	ATM	OTM	ITM	DOTM	DITM
Black and Scholes	4.26	4.89	3.13	5.11	2.33
GARCH/NIG	3.69	4.07	2.86	4.16	2.04
EGARCH/NIG	2.68	2.59	2.31	2.36	1.89
GJR-GARCH/NIG	3.23	3.09	2.77	2.65	2.04
NGARCH/NIG	3.61	3.75	3.06	3.92	2.36
RV-HAR-GARCH	3.31	4.13	2.30	4.92	1.60
RV-HAR/AE-GARCH	2.90	3.24	2.32	3.58	1.78
DARV-HAR	1.87	1.77	1.61	1.71	1.31

Table 2
Mean pricing errors.

	ATM	OTM	ITM	DOTM	DITM
Black and Scholes	−2.27	−3.63	−0.45	−4.13	0.42
GARCH/NIG	−1.40	−2.39	0.01	−2.80	0.51
EGARCH/NIG	−1.80	−1.78	−1.62	−1.74	−1.67
GJR-GARCH/NIG	−2.06	−2.01	−1.58	−1.64	−1.41
NGARCH/NIG	−3.08	−3.35	−2.45	−3.61	−1.96
RV-HAR-GARCH	−2.69	−3.81	−1.59	−4.74	−1.00
RV-HAR/AE-GARCH	−2.29	−2.85	−1.75	−3.33	−1.35
DARV-HAR	0.25	0.23	0.04	0.30	−0.28

of realized volatility: increases in the volatility may breed even more volatility through a higher volatility of volatility – the model generates much fatter tails than the alternatives.

5. Conclusion

As exemplified by the long memory property case, the volatility literature grows in the middle of a theoretical gap. There currently exists no fully compelling theoretically parametric model of asset returns and the theoretical underpinnings of the patterns observed in volatility processes are not fully understood. Nevertheless, the importance of empirical findings of the volatility literature for option pricing and other applications should not be understated. In this paper we have analyzed an options pricing framework based on a new realized volatility model that captures all the relevant empirical regularities of the realized volatility series of the S&P 500 index, the dually asymmetric realized volatility (DARV) model of Allen et al. [1]. Our results, though not extensive enough to be conclusive, indicate that the presence of persistent leverage effects and time varying volatility risk are essential for understanding common option pricing anomalies.

In particular, our model emphasizes the fact that realized volatility series are themselves very volatile. Relevantly, this volatility risk is highly positively related to the level of volatility. Even though the conditional variance of returns is very persistent and can be in general well estimated, it is important to recognize that large errors arise in predicting the (*ex post*) realized volatility. Due to the high nonlinearity of expected option payoffs on future volatility, this risk becomes a major issue for pricing these derivatives. However, we have not here considered the volatility risk premium. Accounting for this risk premium in the present discrete time framework is currently an important technical challenge in the volatility literature.

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