Week 2 Discrete

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Combinations of Sets

Let S be a set of n elements, A *combination* of set S is an unordered selection of r elements in S. The result is a subset, A, of S. The number of r-subset or r-combination of S with n elements is denoted by $\binom{n}{r}$ and the formula is:

$$\tbinom{n}{r} = \tfrac{n!}{r!(n-r)!}$$

For example, if $S = \{a, b, c, d\}$, there are 3-subsets of S:

$$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$$

Obvious rules for combinations:

$$\tbinom{n}{r} = 0 \text{ if } r > n,$$
$$\tbinom{0}{r} = 0 \text{ if } r > 0$$
And,
$$\tbinom{n}{0} = 1,\ \tbinom{n}{1} = n,\ \tbinom{n}{n} = 1$$

Properties of Combinations

$$\tbinom{n-1}{k} + \tbinom{n-1}{k-1} = \tbinom{n}{k}$$

Combinations of Mulit-Sets

Multisets are sets that hold multiple copies of the same element. For example, the multiset $\{a, a, b, c, c, c\}$, also written as $\{2a, b, 3c\}$

r-permutations of a multiset S is an ordered arrangment of r elements of S. While there are many formulas for the amount of permutations of a multiset, one existst that gives the number of full permutations of S:

$$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}$$