The atmospheric properties of FGK stars using wavelet analysis

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Introduction

Wide angle photometric surveys have been extremely successful in finding exoplanets around bright nearby stars in conjunction space based observatories. An example of such ground-based surveys are WASP (2006PASP..118.1407P), HATnet (2004PASP..116..266B), KELT (2007PASP..119..923P) along with space based missions CoRoT (1995ASPC....76..426C) and K2 (2014PASP..126..398H). Constraining the mass and radius of the an exoplanet requires assumptions the host star: Teff, (Fe/H) and Log g. These can then be combined with the best fitting orbital solutions to interpolate evolutionary models and determine the mass, radius of both components and the age of the system (2015A&A...575A..36M).

Completing the orbital solution requires quality transit photometry and radial velocity measurements. It is possible to use the co-add spectra from radial velocity measurements to determine Teff, (Fe/H) and Log g for the host star. These spectra provide quality radial velocity measurements but the co-added spectrum can have low signal-to-noise (SNR) introducing high-order noise unique to each star. It is typical to observe low-order systematics due to poor blaze function corrections and merged echelle orders. Atmospheric parameters for exoplanet hosts discovered by WASP are determined by-hand with experienced spectroscopists who are cautious to avoid the influence of these effects on the determined atmospheric parameters. Such matters are explored in Jofre et al., 2017 (2017A&A...601A..38J).

We have developed a method to determine the atmospheric parameters of single-lined FGK stars from low-quality echelle spectra. Our method uses wavelets to filter out noise and low order systematics. We find our method to be self-consistent and in agreement with other spectroscopic routines.

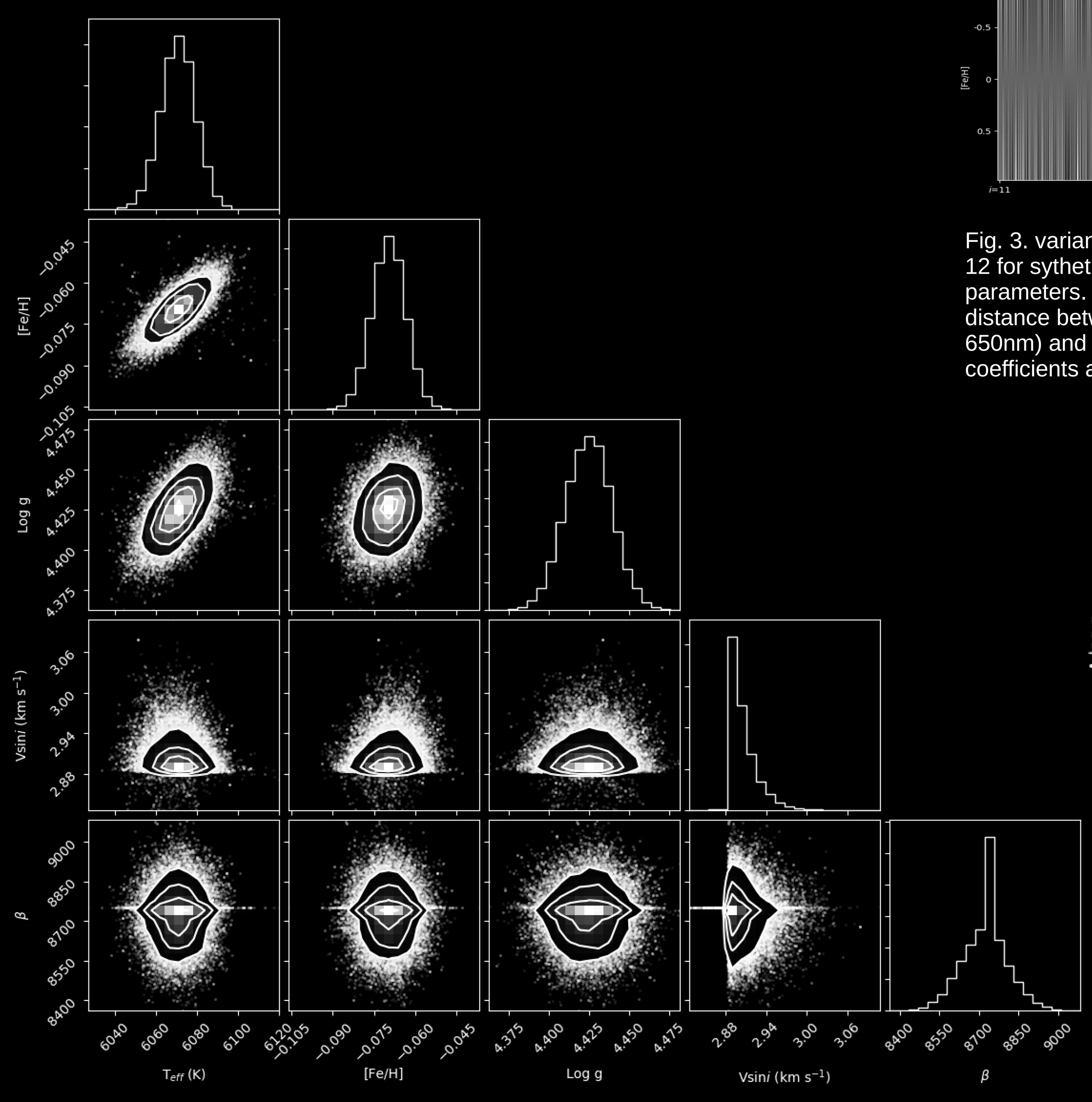


Fig. 3. Example corner plot for WASP-20.

Wavelet decomposition

Wavelet decomposition shares similarities with Fourier decomposition. A Fourier decomposition finds similarities of a data set to the sinusoidal and cosinusoidal basis functions at different frequencies and is routinely used search for repeating signals such as light-curve transits and modulation (1977Ap&SS..51..439K, 2017ApJ...834...59G). A wavelet decomposition has the advantage of being able to capture temporal information alongside frequency information and uses a different set of basis functions called wavelets. One of the most popular sets of wavelets are the Daubechies kind (Fig. 1), named after Ingrid Daubechies who pioneered the field.

From the mother wavelet , daughter wavelets can be generated to capture temporal and frequency information. With careful selection, we can generate a set of daughter wavelets which minimise the overlap between convolutions across different scales leading to the discrete wavelet transform (DWT). The DWT for a spectrum, f(lambda), is given by

$$WT_{f(\lambda)}(i,k) = \frac{1}{\sqrt{2^i}} \int f(\lambda)\psi\left(\frac{\lambda - k2^i b_0}{2^i}\right) d\lambda = \langle f(\lambda), \psi_{i,k}(\lambda)\rangle$$

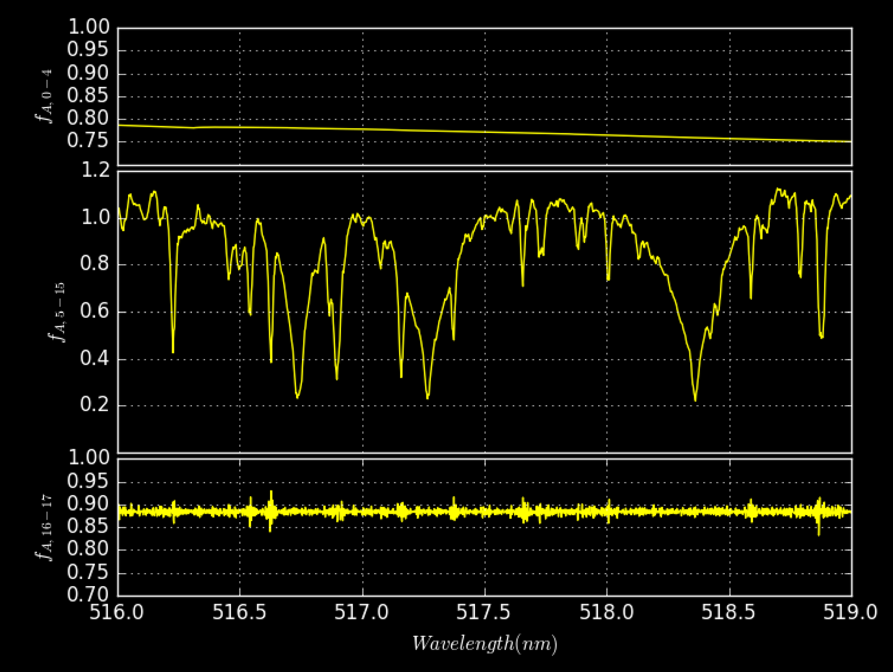


Fig. 2. Wavelet reconstruction around the Mg triplets using filtered coefficients for a spectrum between 450-650 nm with 2^{17} values. Plotted is the reconstruction using coefficients with i=0-3 (top), coefficients with i=4-14 (middle) and coefficients i=15-17 (bottom).

where phi is the daughter wavelet, k and b_0 are chose to minimise the overlap between subsequent convolutions and $Wt_{i,k}$ represents a wavelet coefficient at a particular scale (i) and part of the spectrum (k). A DWT is performed for all values of i and k, and we group the coefficients into bands with constant values of i. One particular requirement for a DWT is that the number of values in the spectrum is a dyadic number (e.g. 2^x; we chose a value of x=17 for this work), a DWT involves x number of bands (values of I) which hold information covering scales from twice the sample rate to the whole spectrum. Each scale, i, covers twice the wavelength (pixel) range as the previous. This opens the door for to select coefficient which correspond to line opacities and exclude scales corresponding to noise low-order systematic trends.

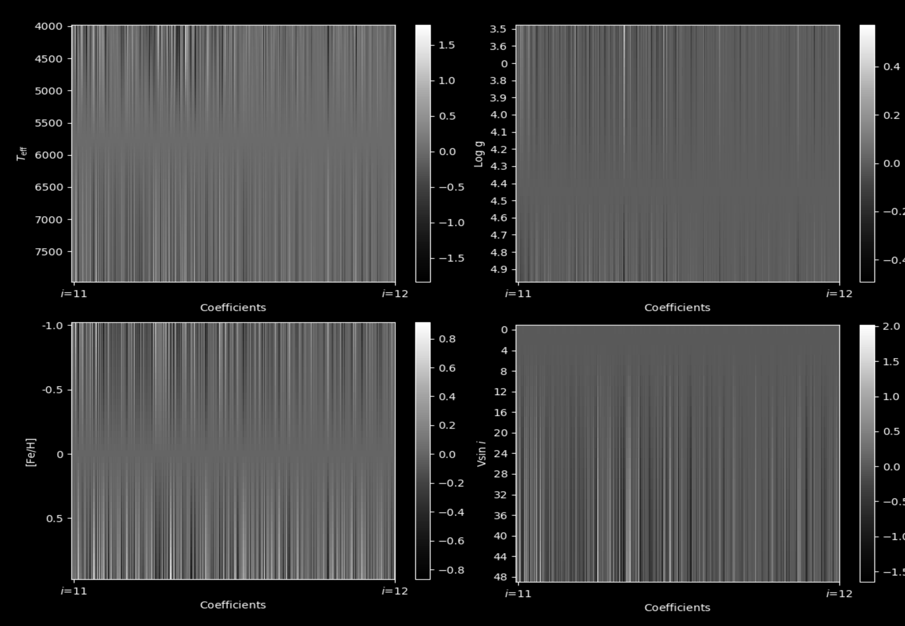


Fig. 3. variance of wavelet coefficients between values of I = 11-12 for synthetic spectra (450-650 nm with 2^{17} values) of solar parameters. In each plot, one parameter was changed. The distance between i=11-12 is a pseudo scale for wavelegth (450-650nm) and it is possible to identify which lines wavelet coefficients are sensitive to.

We demonstrate the ability of wavelets to capture and remove noise and low-order systematic for the spectrum of WASP-20 in Fig. 2. This is analogoues to Fourier filtering with the exception of differing basis functions. In Fig. 3 we demonstrate that wavelet coefficients are sensitive to atmospheric parameters by performing a DWT on synthetic spectra (450-650 nm with 2^{17} values) with solar values. We see a clear evolution of wavelet coefficients opening the door to compare coefficients from spectra to those from a synthetic grid.

Bayesian Fitting

We use the Markov chain Monte Carlo method to determine the posterior probability distribution (PPD) for T_{eff}, (Fe/H), Log g and Vsini given an observed spectrum. Our method is a global chi-squared minimising routine which compares subsets of wavelet coefficients (i = 4-14 assuming 2^{17} values between 450-650 nm) to those from a pre-synthesised grid of spectra. Our grid was synthesised with the radiative transfer code SPECTRUM (1994AJ....107..742G) using MARCS model atmospheres (2008A%26A...486..951G) and version 5 of the GES atomic linelist provided within iSpec (2014A%26A...569A.111B) with solar abundances from Asplund, 2009 (2009ARA%26A..47..481A). Values of macroturbulence and microturbuloence are estimates using calibrations from Doyle, 2015 (2015PhDT........16D).

We compare the subset of wavelet coefficients from an interpolated model (WT_m) to those from those from the data (WT_d) in the following Baysian framework:

1. The probability of observing a spectrum for a given model is given by

$$\mathbf{p(m|d)} \propto \mathcal{L}\mathbf{(d|m)p(m)}$$

2. The vector model of parameters is given by

$$\mathbf{m} = (\mathrm{T_{eff}}, [\mathrm{Fe/H}], \log g, \mathrm{Vsini})$$

3. We use the liklihood function

$$\mathcal{L}(\mathbf{d}|\mathbf{m}) = \exp(-\chi^2/2)$$

where

$$\chi^2 = \frac{(WT_\mathbf{d} - WT_\mathbf{m})^2}{\beta\sigma^2_{MC}}$$

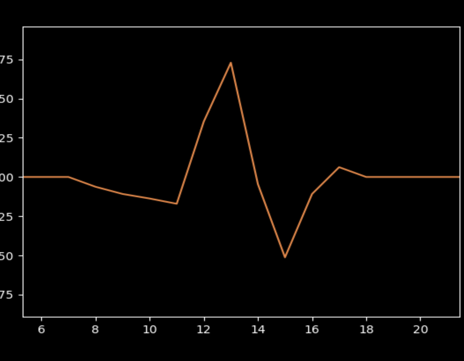


Fig. 1. Daubechies k=4 wavelet used in this work.

Where σ is the standard deviation of wavelet coefficients determined from flux uncertaintiy in a Monte Carlo fashion and β is a "jitter" term to account for additional noise, incomplete atomic data, deviations from solar metalicity scaling, lines formed under non-LTE and other unaccounted errors.

We sample parameter smace using the python package *emcee* (2013PASP..125..306F). *Emcee* uses affine-invariant ensemble sampling with the parallel stretch move algorithm (2010CAMCS...5...65G). We run a single walker for 5000 draws to find the best least-squares solution and step size before initialising 100 walkers in the *emcee* framework and generating 1000 draws. An example corner plot is shown in Fig. 3. The model with the highest loglikliehood is accepted. Uncertainties are estimated from the calibration sample.

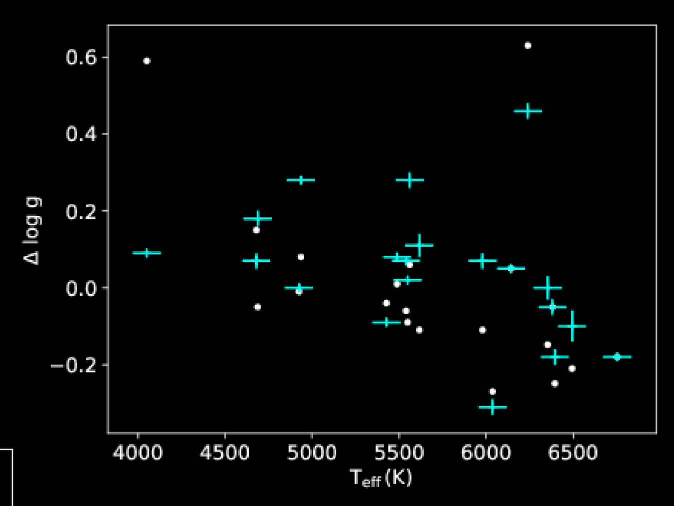
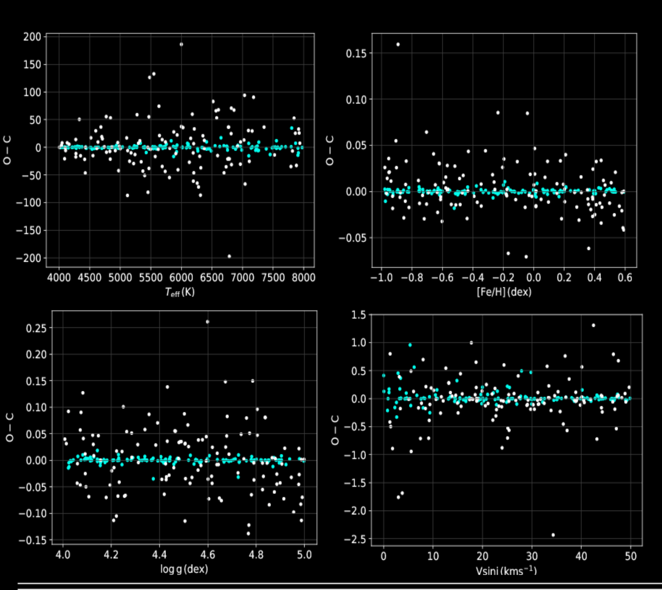


Fig. 6. Difference in logg (spectroscopic – transit photometry) correlated against temperature. Plotted are the values from wavelet analysis (white) against those quoted by Doyle et al. 2014 (cyan).

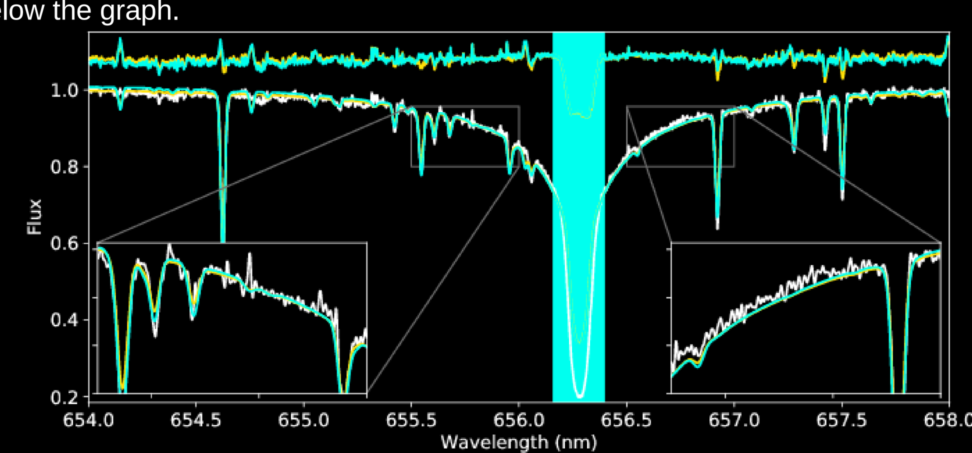
Self consistency



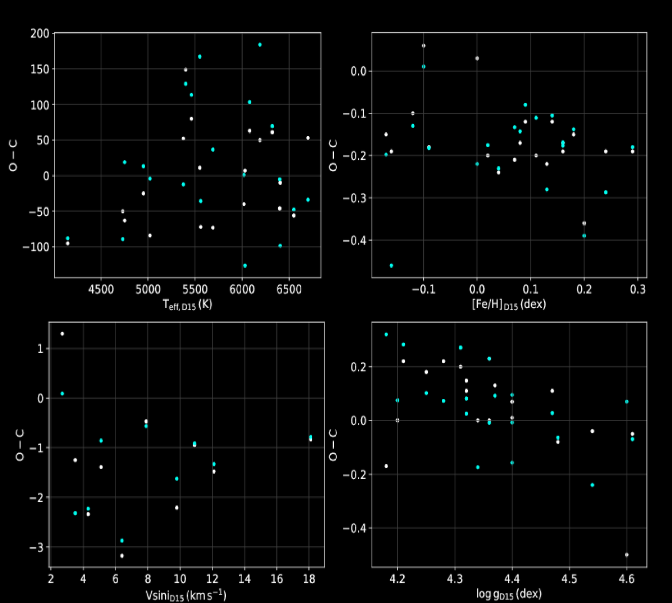
	Prior on log g?	σ	μ
$T_{\rm eff}$ (K)	no	46.0	-3.2
	yes	3.1	0.2
[Fe/H] (dex)	no	0.040	-0.003
	yes	0.020	-0.001
$V \sin i$ (kms^{-1})	no	0.47	0.05
	yes	0.17	-0.06
$\log g$ (dex)	no	0.060	-0.002
	yes	0.020	0.001

Fig. 4. Self consistency tests of the wavelet analysis method. We plot the determined atmospheric parameters for 216 spectra with no priors on Log g (top plots – white) and with priors on Log g (cyan). We tabulate the results of this test below the graph.

We assess the quality of our interpolation and the wavelet analysis procedure by interpolating 512 spectra from our grid of models. We analysed 216 spectra with the aforementioned method and 216 spectra with priors on log g from interpolation with uncertainty expected from transit photometry (σ ≈ 0.02 dex; 2013A%26A...558A.106M). In Fig. 4 we see that including a prior on Log g significantly reduces the scatter between interpolated and determined atmospheric parameters. In reality, we must be cautious using such a prior as significant deviations between spectroscopic and transit log g have been seen (2013A%26A...558A.106M, 2014A%26A...572A..95M, 2017MNRAS.469.4850D). We also find this trend in our calibration sample (Fig. 6)



Calibration sample



	Prior on log g?	σ	μ
$T_{\rm eff}$ (K)	no	85.00	31.00
	yes	86.00	14.00
[Fe/H] (dex)	no	0.06	– 0.15
	yes	0.10	– 0.18
$V \sin i$ (kms^{-1})	no	1.35	– 0.79
	yes	0.62	– 1.33
$\log g$ (dex)	no	0.13	0.08
	yes	0.14	0.05

Notes. Values of σ and μ for $V \sin i$ excluded stars where macroturbulence, ξ_t, was set to 0 km s^{-1}.

Fig. 5. Results of the calibration sample used to benchmark wavelet analysis against a "by-hand" approach for 20 exoplanet host stars. Each star was measured twice, once with no priors on Log g (white) and once with priors on Log g from transit photometry in the relevant discovery papers. The results of this test are tabulated alongside the graph.

We calibrate the wavelet method for 20 exoplanet-host stars discovered by WASP which were diligently analysed by-hand by Doyle et al. 2013 (2013MNRAS.428.3164D). Doyle used high quality spectra from the HARPS spectrograph (2001Msngr.105....1Q) to determine atmospheric parameters. To test the robustness of the wavelet method, we use lower-quality, co-added spectra from the CORALIE spectrograph used to provide spectroscopic orbits for each system. Individual spectra was reduced using the CORALIE standard data reduction pipeline (1996A&AS..119..373B) and corrected into the laboratory reference frame before co-adding and normalising using iSpec routines. Each spectrum was re-sampled to have 2^{17} values between 450-650 nm and analysed with the wavelet method, keeping coefficients i=4-14. Each spectrum was then remeasured using log g priors from (2013A&A...558A.106M) or the relerelevant discovery paper.

The results are summarised and tabulated in Fig. 5. Our method determines Teff to within 220 K of those found by Doyle. Our mesumeasurements of (Fe/H) are systematically offset by 0.18 dex. This could be due to numerous reasons highlighted in Jofre et al. 2016 (e.g. continuum placement, line lists, hyper-fine structure, microturbulence, -element abundance, atmospheric models, blends and radiative transfer codes; 2016arXiv161205013). To rule out wavelets as the cause, we re-measure a few stars usying different wavelets (Daubechies k=20, Harr k=2 & bspline k=103) and found no difference in results. We also ensured that the reconstructed spectra using wavelet coefficients i=4-14 did not smear out line depths. In this work, we find no benefit to using priors on Log g from transit photometry, which actually increases O-C in Fig. 5. We also find a discrepancy between spectroscopic and photometric Log g correlated with temperature (Fig. 6) suggesting photometric priors may not be useful for non solar-like stars. The internal precision of the wavelet method is small and so we use the largest uncertainties quoted in the table in Fig. 5 as our error estimates for parameters determined with the wavelet method. We also find that a minuminimum SNR of 40 for the spectrum is required before accuracies become unusable (Fig. 7).

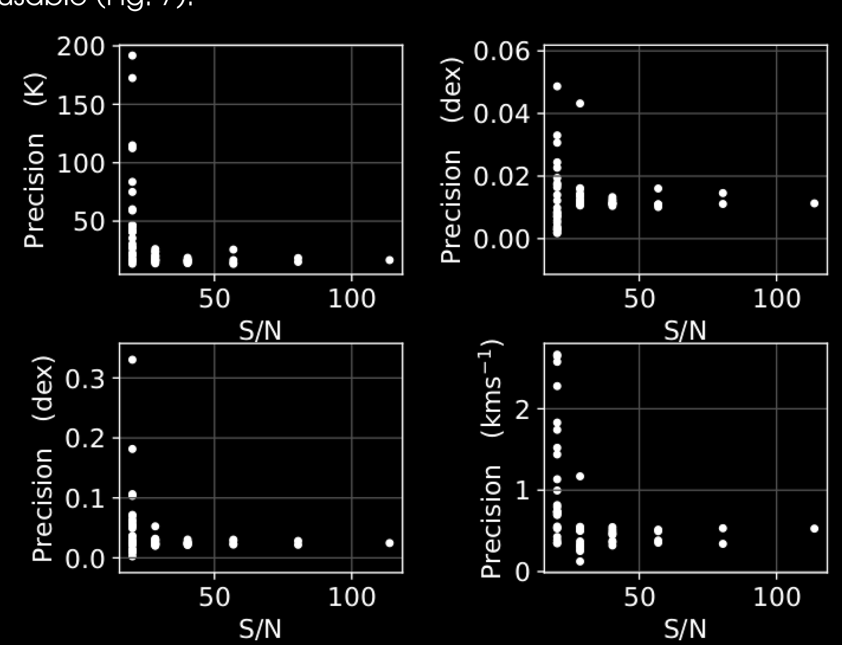


Fig. 7. The internal precision of the wavelet method as a function of spectrum SNR. When SNR < 40, the precision blows up to unusable values rendering the method obsolete.

Future prospects

The wavelet method has already been used to measure the atmospheric parameters of EBLM J0555-57 (2017A&A...604L...6V), an F-type star with a Saturn-mass companion located just above the hydrogen-burning mass limit. We intend to use the wavelet method to estimate the atmospheric of similar FG-M binaries (EBLMs) as part of the EBLM project (2013A&A...549A..18T, 2014A&A...572A..50G, 2017A&A...604L...6V, 2017arXiv170705211), which requires accurate atmospheric parameters to interpolate evolutionary models and estimate the mass and radius of both components. As part of this project, we have obtained SALT HRS spectra which will be analysed in a similar fashion and is the subject of a future paper. A further development of this method would be to utilise data graphed in Fig. 3 to create a more sophisticated weighting scheme. The current Monte Carlo approach does a good job at margenalising the most variant wavelet coefficients from uncertain flux measurements. However, in Fig. 3 we see that only certain wavelet coefficients contribute Log g information. One approach would be to weight coefficients corresponding to sensitive lines (Mg 1 b or Na 1 D for Log g) in a way which would not skew other atmospheric parameters due to degeneracies (Fig. 3).

As more spectrographs come online, we require a way to quickly characterise spectra. Numerous methods are routinely used in the community and are often combined into "nodes" to provide reliable atmospheric parameters of stars from large spectroscopic surveys (2014A&A...570A.122S). The methods generally fall into two categories: measurements of equivalent width and synthetic spectrum fitting. At its heart, the wavelet method is a global spectral fitting routine and is subject to the same uncertainties from atomic line data , atmospheric models, etc. Our method is not perfect, but does an excellent job at quickly the atmospheric parameters for medium-to-low quality spectra.

We have implemented the wavelet method in a python module called waveletspec. Source code is available from:

https://github.com/samgill844/waveletspec

Or

http://www.astro.keele.ac.uk/~sgill/