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EQUATIONS FOR THE ANALYSIS OF THE ROSSITER-McLAUGHLIN EFFECT IN EXTRASOLAR PLANETARY TRANSITS

Alvaro Giménez
Research and Scientific Support Department, European Space Agency, ESTEC, Keplerlaan 1, 2200 AG Noordwijk ZH, The Netherlands;
agimenez@rssd.esa.int
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ABSTRACT

Analytical equations are presented for the computation of the Rossiter-McLaughlin effect in the radial velocity curves of stars hosting extrasolar planets and showing periodic transits. This is a well-known effect in the radial velocity curves of eclipsing binaries that results from the fractional visibility of the stars and the subsequent loss of symmetry for the integrated rotational contribution. In this paper, general and accurate equations are presented, allowing for the computation of the effect for spherical components. The nonorbital component of the radial velocity curve, during the transit of the planet, can thus be used to determine the rotational properties of the star, including any deviation of its rotational axis from the perpendicular to the orbital plane. For this purpose, the observational effects of stellar limb darkening, as well as the geometrical and orbital elements of the system, are taken into account. Results can be used immediately for the analysis of the spectroscopic observations during the transit of extrasolar planets and allow for the simultaneous solution of the light and radial velocity curves, therefore improving the precision of the derived elements by using two completely different sources of information in an integrated form. The effects of changes in various of the involved parameters are also discussed using the analytical properties of the equations.

Subject headings: binaries: eclipsing — planetary systems — techniques: radial velocities

1. INTRODUCTION

The study of extrasolar planets has been greatly enhanced during the last few years through the analysis of systems showing photometric transits. The light curve produced by the small dip in the total light of the star, due to the transit of the planet in front of it, can be analyzed to derive the geometrical parameters of the system, namely, the inclination of the orbit and the radii of both the star and the planet. Analytical equations for the study of these light curves have been recently presented by Giménez (2006, hereafter Paper I), using techniques earlier developed by Kopal (1979) for the analysis of eclipsing binaries.

The host star of an extrasolar planet also shows radial velocity variations due to its motion around the center of mass of the system. This has been the most common technique for discovering new planets, despite the small variations involved, and is the best way to derive their orbital parameters and masses. In the case of systems showing eclipses, accurate absolute dimensions (masses, radii, and mean densities) can be determined for the star and the planet by combining the spectroscopic and photometric information.

Radial velocities of the star, when observed during the transit, are nevertheless modified due to the fractional visibility of its surface. This has been a well-known effect in eclipsing binaries since very early in the last century, when it was pointed out by Schlesinger (1910), and established by Rossiter (1924) in the case of β Lyr, and McLaughlin (1924) for Algol. Since then, it has been extensively used to investigate the rotational properties of the component stars in close binaries. In particular, the Rossiter-McLaughlin (RM) effect allowed confirmation of both the expected stellar rotation in the direction of the orbital motion and an almost perpendicular position of the rotational axis with respect to the orbital plane. Important tests of the theory of tidal evolution in binary stars could thus be carried out.

For the analysis of the observational data, several theoretical developments of the RM effect, have been published. The first one by Petrie (1938), for spherical stars, was followed with a complete description of the transit behavior by Kopal (1942, 1959). In his developments, Kopal took into account the distortion of the component stars, as needed for close binary systems, and made use of the α functions representing the loss of light during the different phases of the eclipse, i.e., the corresponding surface integrals projected over the plane of the sky. A simplified approximation for spherical components, but taking into account the possibility of the rotational axis to be inclined with respect to the perpendicular to the orbital plane, was given by Hosokawa (1953), also making use of the α functions.

As soon as planets were found to transit in front of their host stars (Charbonneau et al. 2000), it was realized that sufficiently accurate measurements of radial velocities during the eclipses should also show evidence of the RM effect (Worek 2000). In fact, the prototypical transiting extrasolar planet in HD 209458 was found to be a clear case and, out of the now known transiting extrasolar planets, some other candidates are bright enough for this type of measurements to be performed, while many more cases are expected to be discovered in the coming years.

For the analysis of HD 209458, Queloz et al. (2000) used the radial velocity measurements by Mazeh et al. (2000), in fact the residuals from their orbital solution, establishing that the rotation of the star indeed takes place in the same direction of the orbital motion of the planet. More recent studies confirmed this result and allowed further insight into the co-alignment of the rotational axis with the pole of the orbit (Winn et al. 2005).

The equations adopted for the analysis of the observations were not all based on the previous developments by Hosokawa (1953) and Kopal (1959). The reasons were linked to the complicated nature of the expressions, based on the mathematical formulation of the α functions, and the need to use tabulated values to compute them, as given, e.g., by Kopal (1947). Queloz et al. (2000) therefore adopted a method based on the numerical integration of a large number of surface elements, and simple geometrical assumptions, to derive the expected effect. In a more recent study, Wittenmyer et al. (2005) adopted for the integration a synthetic model as described by Orosz & Hauschildt (2000). Synthetic radial velocity curves can of course be very accurate by taking into account all possible effects in the individual surface elements and can also allow for the simultaneous solution of the light and radial velocity variations, thus providing further constraints to the final solution. In the case of extrasolar planets, this is generally not necessary due to the almost spherical shape of the star undergoing eclipses, contrary to the case of the highly distorted components of close binaries for which these synthetic models were developed. On the other hand, Winn et al. (2005) made use of the analytical approximation provided by Ohta et al. (2005). Analytical equations are easier to integrate in a general study of the behavior of the system and provide better insight into the interpretation of the results. Unfortunately, the equations produced by Ohta et al. (2005) are also complicated to use and check, are only valid for a linear law of limb darkening (like those by Hosokawa 1953), and have different expressions for the partial and the annular part of the transit. In fact, their main contribution to the work of Hosokawa (1953) is to provide analytical expressions for the α functions as direct integrations of the fractional loss of light at any phase of the transit. This is somewhat equivalent to the development of Mandel & Agol (2002) for the computation of theoretical light curves.

In this paper, an improvement to the analytical approach is presented through a more direct integration of the involved equations, by means of which the RM effect can be easily calculated. Equations requiring little computing effort are obtained using the technique introduced by Kopal (1979) and already successfully applied to calculate the light curves of planetary transits in Paper I. The following equations are valid for any degree of limb darkening, inclination of the rotational axis of the star, and orbital eccentricity.

2. THE EQUATIONS

Let us from now on consider only the nonorbital part of the radial velocity. The orbital contribution, for nondistorted components, is a simple function of the orbital elements of the system and represents, in the case of extrasolar planets, more than 95% of the total radial velocity curve. Therefore, it is easy to correct the observations, during eclipse, for the orbital velocity contribution or to integrate them together with the nonorbital terms.

At any phase of the orbit, θ, the correction to the orbital radial velocity of the star, δV, will be given by

$$\delta V = \frac{\int_S V' J \cos\gamma \, d\sigma}{\int_S J \cos\gamma \, d\sigma}, \qquad (1)$$

where both integrals, in the numerator and the denominator, are extended over the visible fraction area S of the eclipsed star; J stands for the brightness distribution, over the apparent disk of the star, of surface element $d\sigma$; γ is the so-called angle of foreshortening, i.e., the angle between the surface normal and the line of sight; and V' represents the radial component of the stellar rotational velocity at any surface element.

By means of equation (1), the non-orbital contribution to the radial velocity of a star undergoing eclipses can be easily calculated using the α functions representing the fractional visibility of the star. It can be immediately seen that the denominator of equation (1) is identical to the fractional luminosity of the star and, since in the case of extrasolar planets the relative luminosity of the planet with respect to the star is negligible, we can write

$$\int_S J \cos\gamma \, d\sigma = 1 - \alpha^0, \qquad (2)$$

where α^0 stands for the zero-order function of the type α^m given by Kopal (1959). For the numerator, some more elaborate expressions involving the rotational axis of the star are needed. A simple solution is possible when the position of the rotational axis of the star remains constant during the transit of the planet. In the most probable case, with the axis of rotation parallel to that of the orbital angular momentum, this will always be true. In the general case, inclined rotation of the star with respect to the orbital motion, periodical oscillations of the rotational axis will lead to changes in the projected position. Fortunately, the short duration of planetary transits when compared to the orbital period, as well as the negligible expected stellar distortion, allow for the assumption of the position of the rotational axis to be constant. This assumption was always adopted by previous authors considering inclined rotation, but not by Kopal, who only derived equations for co-aligned stellar and orbital rotation angles, consistent with the application of the equations in close binary systems where precession and nutation phenomena could not be ignored.

With these assumptions, the radial component of the rotational velocity, V', can be formulated in terms of the position of the rotational axis and easily integrated in the numerator of equation (1). For this purpose, it is convenient to define the position of the rotational axis by means of two angles: the inclination with respect to the line of sight I, which will in general be different from the inclination of the orbit i, and the angle between the position of its projection on the plane of the sky with that of the pole of the orbit, β. These definitions are equivalent to those adopted by Hosokawa (1953) and showed in his Figure 1. In the case of co-aligned rotation, we will obviously have $I = i$ and $\beta = 0$. Using the definition of the α functions, the numerator in equation (1) can now be rewritten as

$$\int_S V' J \cos\gamma \, d\sigma = \frac{V^*}{\delta} \alpha_n^1, \qquad (3)$$

where the rotational parameter V^* is given by

$$V^* = V \sin I (\sin\beta \cos i \cos\theta - \cos\beta \sin\theta), \qquad (4)$$

and V is the equatorial rotational velocity of the star, while δ denotes the projected relative separation of the centers of the planet and the star. The problem of finding an explicit formulation for δV is thus reduced to having expressions for the α^m functions of order 0 and 1. This was done in Paper I for $m = 0$, and it is also possible for $m = 1$. It should also be noted that, for a central transit, the α^1 function goes to zero much faster than δ, so equation (3) is numerically stable.

In order to proceed, we obviously need a good knowledge of the brightness distribution over the surface of the star. Like in Paper I, a general law of limb darkening can be adopted,

$$J(\mu) = J(1)\Big[1 - \sum_{n=1}^{N} u_n (1 - \mu^n)\Big], \qquad (5)$$

where $\mu = \cos\gamma$, $J(1)$ is the intensity of radiation emerging normally to the surface, and u_n are the limb-darkening coefficients. With this formulation, an arbitrary degree of accuracy can be achieved for a sufficiently large value of N without any loss of generality. Note that, as it was mentioned in Paper I, in the case of $N = 2$ the quadratic limb-darkening coefficients are not defined as

$$J(\mu) = J(1)[1 - u_a(1 - \mu) - u_b(1 - \mu)^2], \qquad (6)$$

which is the generally adopted second-order law of limb darkening. Instead, $u_1 = u_a + 2u_b$ and $u_2 = -u_b$.

Using equation (5) for $J(\mu)$, it was shown by Kopal (1979), that associated α_n^m functions can be defined as

$$\alpha^m = \sum_{n=0}^{N} C_n \alpha_n^m, \tag{7}$$

where the coefficients C_n are given by

$$C_0 = \frac{1 - \sum_{n=1}^{N} u_n}{1 - \sum_{n=1}^{N} (n u_n / n + 2)}, \tag{8}$$

for $n = 0$, and

$$C_n = \frac{u_n}{1 - \sum_{n=1}^{N} (n u_n / n + 2)}, \tag{9}$$

for $n > 0$. In this way, the geometrical elements can be decoupled from the radiation parameters, facilitating the analysis.

With these definitions, equation (1) can finally be written as

$$\delta V = \frac{V^*}{\delta} \frac{\sum_{n=0}^{N} C_n \alpha_n^1}{1 - \sum_{n=0}^{N} C_n \alpha_n^0}, \tag{10}$$

where V^* is given by equation (4) and it has been realistically assumed that the β and I angles do not oscillate during the short duration of the transit. The associated α_n^0 functions are identical to those described, and explicitly evaluated, in Paper I, while the associated α_n^1 functions of index 1 represent the fractional loss in radial velocity during eclipses. This result is the generalization of equation (5.11) from Kopal (1990, chap. III), for any position of the rotational axis and is equally valid for any degree of limb darkening.

It is now possible to evaluate the α_n^1 functions, complementing our knowledge of those with index $m = 0$ used for the light variations of nondistorted components. The same nomenclature can be used, i.e., the relative radii of the star r_s and the planet r_p, their ratio $k = r_p/r_s$, the relative separation of their centers δ, and the inclination of the orbit i. Moreover, we use the method introduced by Kopal (1979), who reformulated the problem of evaluating the fractional loss of light as a cross-correlation of two apertures: one representing the star undergoing eclipse, and the other the eclipsing disk (in our case, the transiting planet). With this approach, it is found that the α_n^1 functions can be expressed using a Hankel transform and subsequently integrated, with

$$\begin{aligned}\alpha_n^1(b, c) = {} & \frac{cb^2(1-b)\Gamma(\nu)(1-c^2)^{\nu+1}}{\Gamma(\nu+2)\Gamma(\nu+2)} \sum_{j=0}^{\infty} (-1)^j (2j + \nu + 3) \\ & \times \frac{\Gamma(\nu + j + 3)}{\Gamma(j+1)} \left[G_j(\nu+3, \nu+2; 1-b)\right]^2 \\ & \times G_j(\nu+3, 2; c^2)\end{aligned} \tag{11}$$

in terms of Jacobi polynomials of the type $G_n(p, q; x)$, where the parameters $b \equiv r_p/(r_s + r_p) = 1/(1+k)$ and $c \equiv \delta/(r_s + r_p) = b\delta/r_p$ have been introduced, together with the abbreviation $\nu \equiv n/2 + 1$. Although written in a different form, equation (11) is equivalent to equation (3.11) in Kopal (1990, chap. III), as given for the analysis of nondistorted eclipsing binaries. The degree of the Jacobi polynomials have, however, been made the same to facilitate their computation using known recursive expressions and Γ functions.

The G notation for Jacobi polynomials has been adopted for consistency with the expressions provided by Kopal, who used it because of the Gaussian hypergeometric series involved in the integration of the α functions. They are related to the usual $P_n^{(\alpha,\beta)}(x)$ Jacobi polynomials by

$$G_n(p, q; x) = \frac{\Gamma(n+1)\Gamma(q)}{\Gamma(n+q)} P_n^{(q-1, p-q)}(1 - 2x). \tag{12}$$

In the case of circular orbits, we obviously have, for any orbital phase θ, that the relative separation between the center of the star and the planet is

$$\delta^2 = 1 - \cos^2\theta \sin^2 i, \tag{13}$$

while in the case of eccentric orbits, the same separation is given by

$$\delta^2 = \left[\frac{(1-e^2)}{1 - e\sin(\theta - \omega)}\right]^2 (1 - \cos^2\theta \sin^2 i), \tag{14}$$

where e represents the orbital eccentricity and ω is the position of the periastron, measured from the line of the nodes.

Equation (11) is the most general expression for the associated α_n^1 functions in algebraic form, valid for any degree n of limb darkening, eccentricity, and type of eclipse, be it total, annular, a transit, or an occultation. The right-handside of the equation is a convergent series of terms and the precision that can be achieved, under the adopted assumptions, is more than enough for the needs of the observational data. This precision is, of course, obtained as a function of the degree of the series of terms in the Jacobi polynomials, but no practical problem in evaluating α_n^1 for any value of j, even well above 1000, has been found, so no limitation is encountered for the geometrical evaluation of the RM effect. The application of equation (11) in the analysis of radial velocity curves is not only simple but also very fast. The computer code needed for the evaluation of the total effect at any orbital phase, for a given set of parameters, requires a small number of lines of FORTRAN making use of available mathematical libraries. Subroutines for such computations can be provided by the author on request and can be easily integrated with those reported in Paper I, thus allowing for the simultaneous solution of radial velocity and light variations.

3. COMPARISON WITH PREVIOUS FORMULATIONS

The main result shown in § 2 is that the RM effect can be expressed mathematically in a simple form for any degree of limb darkening, position of the rotational axis of the star, orbital eccentricity, and type of transit, be it annular or partial. The only assumptions are that the components can be treated as spherical in shape and the star shows no significant precessional effects. Let us now have a look to some simplified versions of the equations and compare them with earlier results.

Assuming that the rotational axis of the star is co-aligned with the perpendicular to the orbital plane, then $\beta = 0$, $I = i$, and equation (10) takes the form

$$\delta V = -V \sin i \frac{\sin\theta}{\delta} \frac{\sum_{n=0}^{N} C_n \alpha_n^1}{1 - \sum_{n=0}^{N} C_n \alpha_n^0}, \tag{15}$$

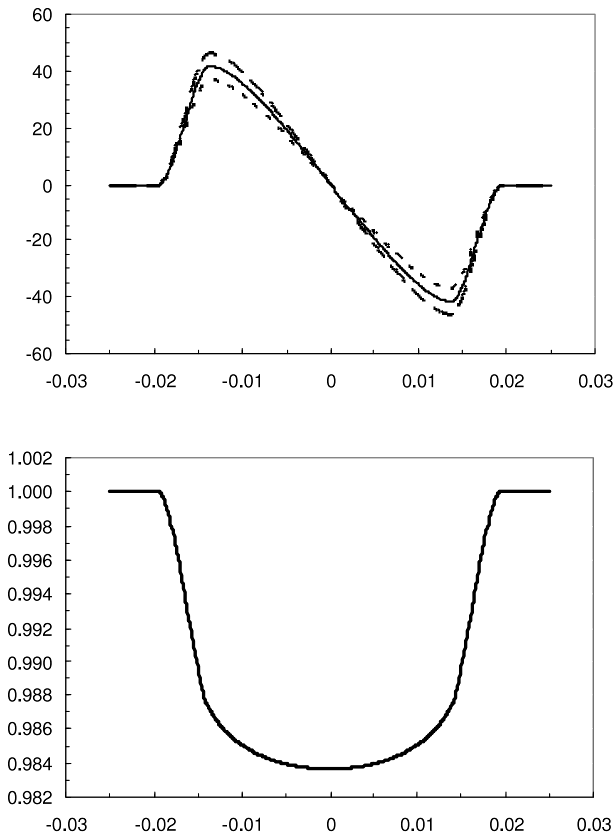

FIG. 1.—*Top:* Simulation of the rotational correction to the radial velocity during a transit, with parameters similar to those of HD 209458, as δV, in m s^{-1}, vs. orbital phase. Changes in the equatorial rotational velocity of the star are indicated with solid, dashed, and dotted curves for $V = 4.5$, 5, and 4 km s^{-1}, respectively. *Bottom:* The light curve for the same adopted geometrical parameters is shown as normalized luminosity vs. orbital phase.

which, with different nomenclature, is the same expression given by Kopal (1990, chap. III) as equation (5.11). On the other hand, for a linear limb-darkening law, but inclined rotational axis, equation (10) can be written as

$$\delta V = \frac{V^*}{\delta} \frac{(1-u)\alpha_0^1 + \alpha_1^1}{(1-u)(1-\alpha_0^0) + u(2/3 - \alpha_1^0)}, \qquad (16)$$

in excellent agreement with equation (4) of Hosokawa (1953). Hosokawa (1953) unfortunately did not provide the corresponding expression for nonlinear limb darkening or, for that matter, any closed expression for the α functions, which were only available in tabulated form.

Explicit equations for the computation of the α functions were provided recently by Ohta et al. (2005), but only for linear limb darkening and with a complicated formulation, including different expressions for total or partial phases of the transit. It is not simple to directly compare equations (10) and (11) with the results by Ohta et al. (2005), but, for the case of no limb darkening, during the total phases of the transit, $\alpha_0^0 = k^2$ and $\alpha_0^1 = k^2\delta/r_p$. Using these values, equation (10) becomes

$$\delta V = \frac{V^*}{r_p} \frac{k^2}{1-k^2}, \qquad (17)$$

which is identical to equation (25) of Ohta et al. (2005), where the position of the planet is given in units of the semimajor axis of the orbit.

4. OBSERVATIONAL PROPERTIES

The main characteristics of the RM effect, from an observational point of view, can now be reviewed with the help of the obtained analytical expressions. The most immediate observational consequence of equation (10) is that, if the rotation of the star takes place in the same direction as the orbital motion of the planet, the maximum value of δV will occur before the central phase of the transit, at $\theta = 0$, and the minimum after it, during the egress phases. Figure 1 shows a simulation of the RM effect for a planetary transit similar to that observed in HD 209458, i.e., with a stellar rotation of 4.5 km s^{-1}, a relative radius of the star $r_s = 0.12$, a ratio of radii $k = 0.12$, and an orbital inclination $i = 86\overset{\circ}{.}7$. A second-order law of limb darkening was adopted with the values $u_a = u_b = 0.32$, as described by equation (6), which are also close to those of the star in HD 209458. The RM effect is represented for a co-aligned rotation with the perpendicular to the orbital plane and in the same direction of the orbital motion, together with the corresponding effect of increasing or decreasing, respectively, the rotational velocity by 0.5 km s^{-1}. The amplitude of the curve changes with the value of the equatorial rotational velocity and the effect becomes zero at phase $\theta = 0$ while the areas above and below the line for $\delta V = 0$ are equal.

Of course, changes in the inclination angle I will affect δV like the rotational velocity, since the actual observable parameter is $V \sin I$, as given by equation (4). In Figure 2, the effect of changing the β angle is shown. Here β affects the symmetry of the curve by differentiating the areas above and below $\delta V = 0$, as well as the position of the zero point before or after the central phase of the transit, depending on the sign of β. Extreme cases of inclination are shown in Figure 3, where the RM effect is displayed when the rotational axis coincides with the line of the nodes and the transit takes place over the receding or the advancing hemisphere of the star. The shape of these curves is quite different and such a behavior has not yet been observed. Coplanarity as a result of tidal evolution is not to be expected and, if confirmed, should be attributed to the formation mechanism.

Of course, when the rotational axis lies on the line of sight, $I = 0$, there is no projected rotational velocity, and the RM

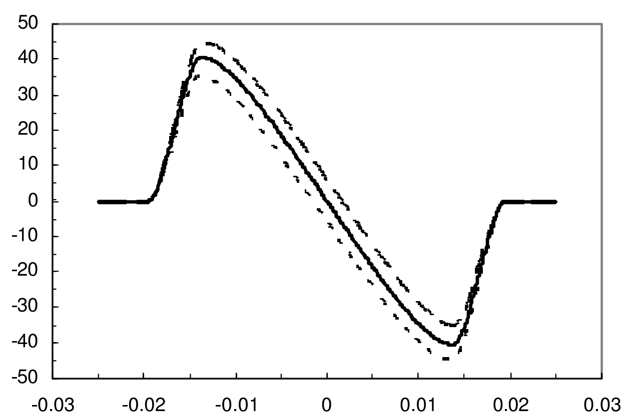

FIG. 2.—Effect of the inclination of the rotational axis of the star for a fixed value of $I = i - 10°$ in the δV vs. orbital phase plane. The solid, dashed, and dotted lines represent the position of the angle β with 0°, 10°, and −10°, respectively.

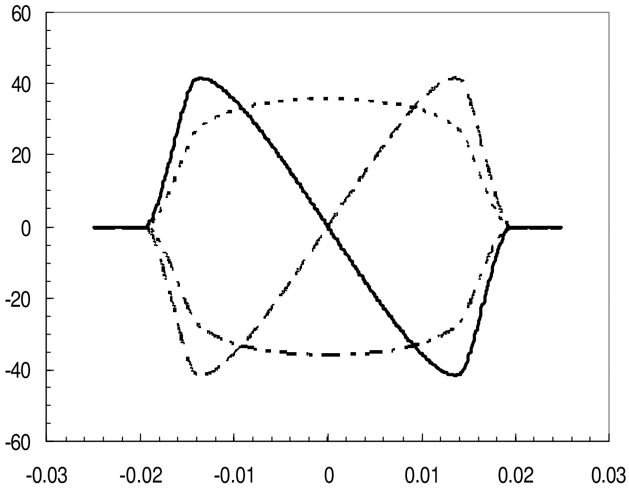

FIG. 3.—Simulated RM correction to radial velocities for different positions of the stellar rotational axis. The solid line shows the behavior for co-aligned rotation in the same direction as the orbital motion, while the dashed curve shows the case of retrograde rotation. With dotted and dot-dashed lines, the extreme cases of the rotational axis on the plane of the orbit are shown for a transit over the receding or advancing hemispheres, respectively.

effect disappears for any phase. In addition, the dashed line of Figure 3 shows the RM effect when the stellar rotation takes place in the direction opposite to the orbital motion. The difference in shape from the reference continuous line is obvious. Actual data for HD 209458, as shown, e.g., by Queloz et al. (2000), clearly indicate that in this case rotation indeed takes place in the direction of the orbital motion.

The effect of adopting no limb darkening, instead of the reference second-order law, is indicated in Figure 4. Limb darkening decreases the amplitude of the RM effect and modulates the variations with phase but otherwise shows a straight inclined line during the total phases of the transit. Values of linear or nonlinear limb darkening show no significant differences when kept within realistic values, i.e., $u \approx u_a + u_b$. This was not the case in the analysis of the photometric light curves, and a second-order law

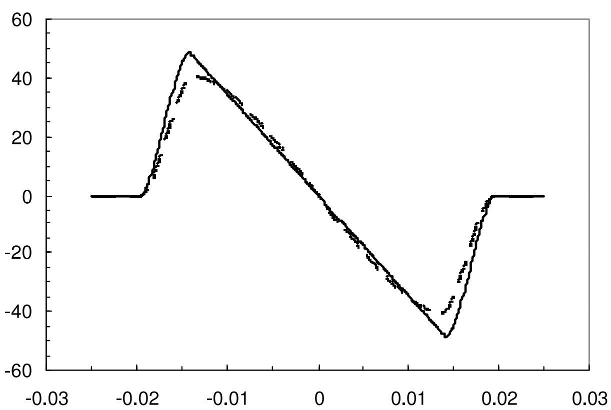

FIG. 4.—Simulated rotational correction to the radial velocity during a transit with parameters similar to those of HD 209458. The solid line shows the behavior for no limb darkening compared to a nonlinear limb darkening with coefficients $u_a = u_b = 0.32$ represented by the dashed line. This is in fact indistinguishable from the case of linear limb darkening with $u = 0.64$.

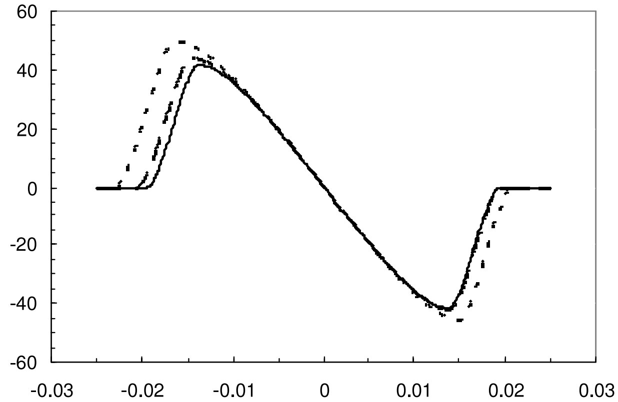

FIG. 5.—Changes in the RM effect, represented in the δV(m s^{-1}) vs. θ plane, due to different values of the eccentricity: 0 (*solid line*), 0.15 (*dashed line*), and 0.30 (*dotted line*). In each case, the position of the periastron is $\omega = 0$.

was considered necessary only to keep internal consistency in the analysis of the data.

For any phase outside of transit, $\delta > r_s + r_p$ and the α functions α_n^0 and α_n^1 are always identical to 0, leading, of course, to $\delta V = 0$. For $\delta < r_s - r_p$ the transit will be annular, and for values $r_s + r_p > \delta > r_s - r_p$ the transit is partial. The orbital phase, where the RM effect starts (or finishes), is given by the definition of δ to be

$$\theta_1 = \arccos\sqrt{\frac{1-(r_s+r_p)^2}{\sin^2 i}} \qquad (18)$$

for circular orbits. In case of eccentric orbits, equation (14) indicates that a nonsymmetric start and end of the RM effect should be observed, except for a position of the periastron $\omega = \pi/2$ or $3\pi/4$, when the line of the apses coincides with that of conjunctions.

Figure 5 shows the effect of the orbital eccentricity for a position of the periastron $\omega = 0$ and parameters otherwise similar to those of HD 209458. The asymmetry in the position of the first and last contact points is clearly displayed, together with

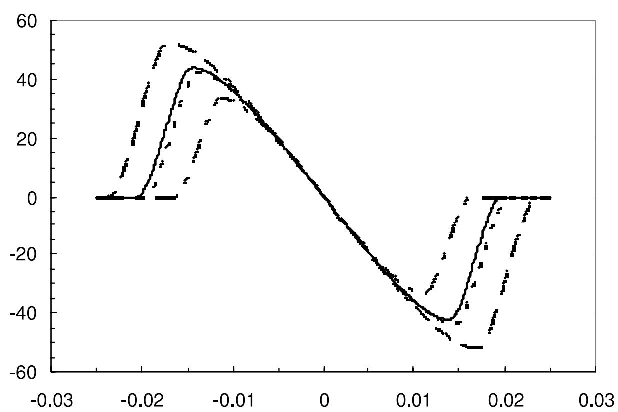

FIG. 6.—Same as Fig. 5 for changes in the position of the periastron. For an eccentricity of 0.15, ω is given the values 0 (*solid line*), $\pi/2$ (*dashed line*), π (*dotted line*), and $3\pi/4$ (*dot-dashed line*).

the corresponding displacement of the maximum and minimum values. Adopting $e = 0.15$, the effect of changing the position of the periastron is indicated in Figure 6. The small asymmetry seen at $\omega = 0$ is reversed for $\omega = \pi$ and disappears for $\pi/2$ and $3\pi/4$, where the RM effect then increases or decreases, respectively, the amplitude or the position of the minimum value of δV as a consequence of the change in the apparent relative radii.

5. CONCLUSIONS

Analytical equations for the study of the rotational properties of stars hosting transiting extrasolar planets are given. The algorithms can be easily integrated with similar equations for the computation of light curves and can thus analyze both the radial velocities and the photometric data with a simultaneous solution. A comparison of the RM effect with the light curve corresponding to the same parameters is given in Figure 1.

The main condition for the validity of the equations presented in this paper is that both the star and the planet can be treated as circular disks moving in front of each other. This is indeed a realistic approximation, since the mutual distortions caused by rotation and tides can be neglected for most extrasolar planetary systems (Giménez 2006). Furthermore, the possible inclined position of the rotational axis of the star is assumed to be constant during the transit phases, which is an even more realistic assumption for the conditions prevailing in extrasolar planetary systems.

The analytical equations presented in this paper can be used to estimate the Rossiter-McLaughlin effect in any new possible observational candidate. The maximum value of the Rossiter effect is expected at the internal tangency phase that corresponds to $\delta = r_s - r_p$. This is the exact solution for no limb-darkened atmospheres and, in the case of linear or nonlinear limb darkening, the corresponding phase of maximum δV is only just inside the internal tangency phase depending on the actual coefficients. A good estimation of the corresponding orbital phase is given by,

$$\theta_2 = \arccos\sqrt{\frac{1 - (r_s - r_p)^2}{\sin^2 i}}. \tag{19}$$

Concerning the predicted maximum value of δV of the RM effect at θ_2, equation (17) can be used, as it is valid for no limb darkening. Furthermore, the equations presented in this paper can be of great help in the analysis of differential effects in the radial velocity curve, which is used for probing the atmospheres of transiting planets, by following the method described by Snellen (2004).

Equations (10) and (11) are valid for any type of eclipse and any degree of limb darkening, as well as for orbital eccentricity, and accurate enough for all possible applications from a geometrical point of view. Nevertheless, if the observed radial velocities do not represent accurately the geometric situation, appropriate empirical corrections should be applied, as discussed by Winn et al. (2005). This could be the case when the method used to derive the radial velocity is based on fits of observed and template spectra, together with wavelength-dependent instrumental response parameters. The equations presented in this paper can also be used to compute the corresponding convolution of profiles.

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