Evaluation by reduction

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

$\Longrightarrow$ `3 * (3 + 4)`

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

$\Longrightarrow$ `3 * (3 + 4)`

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

$\Longrightarrow$ `3 * (3 + 4)`

$\Longrightarrow$ `3 * 7`

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

$\Longrightarrow$ `3 * (3 + 4)`

$\Longrightarrow$ `3 * 7`

Evaluation by reduction

```
(1 + 2) * (3 + 4)
```

$\Longrightarrow$ `3 * (3 + 4)`

$\Longrightarrow$ `3 * 7`

$\Longrightarrow$ `21`

User-defined functions

```
square x = x * x
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$

```
2 * 2 + square 3 + square 4
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

$\Longrightarrow$ `4 + 9 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

$\Longrightarrow$ `4 + 9 + square 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

$\Longrightarrow$ `4 + 9 + square 4`

$\Longrightarrow$ `4 + 9 + 4 * 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
```

$\Longrightarrow$ `2 * 2 + square 3 + square 4`

$\Longrightarrow$ `4 + square 3 + square 4`

$\Longrightarrow$ `4 + 3 * 3 + square 4`

$\Longrightarrow$ `4 + 9 + square 4`

$\Longrightarrow$ `4 + 9 + 4 * 4`

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
⟹  2 * 2 + square 3 + square 4
⟹  4 + square 3 + square 4
⟹  4 + 3 * 3 + square 4
⟹  4 + 9 + square 4
⟹  4 + 9 + 4 * 4
⟹  4 + 9 + 16
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
⟹  2 * 2 + square 3 + square 4
⟹  4 + square 3 + square 4
⟹  4 + 3 * 3 + square 4
⟹  4 + 9 + square 4
⟹  4 + 9 + 4 * 4
⟹  4 + 9 + 16
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
⟹  2 * 2 + square 3 + square 4
⟹  4 + square 3 + square 4
⟹  4 + 3 * 3 + square 4
⟹  4 + 9 + square 4
⟹  4 + 9 + 4 * 4
⟹  4 + 9 + 16
⟹  4 + 25
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
⟹  2 * 2 + square 3 + square 4
⟹  4 + square 3 + square 4
⟹  4 + 3 * 3 + square 4
⟹  4 + 9 + square 4
⟹  4 + 9 + 4 * 4
⟹  4 + 9 + 16
⟹  4 + 25
```

User-defined functions

```
square x = x * x
```

```
square 2 + square 3 + square 4
⟹  2 * 2 + square 3 + square 4
⟹  4 + square 3 + square 4
⟹  4 + 3 * 3 + square 4
⟹  4 + 9 + square 4
⟹  4 + 9 + 4 * 4
⟹  4 + 9 + 16
⟹  4 + 25
⟹  29
```

Guards

```
max x y | x > y      = x
        | otherwise = y
```

Guards

```
max x y | x > y      = x
        | otherwise = y
```

```
max (max 5 3) 6
```

Guards

```
max x y | x > y      = x
        | otherwise = y
```

```
max (max 5 3) 6
```

Guards

```
max x y | x > y      = x
        | otherwise  = y
```

```
max (max 5 3) 6
```

$\Longrightarrow$ `max 5 6`

Guards

```
max x y | x > y      = x
        | otherwise  = y
```

```
max (max 5 3) 6
```

$\Longrightarrow$

```
max 5 6
```

Guards

```
max x y | x > y      = x
        | otherwise  = y
```

```
max (max 5 3) 6
```

$\Longrightarrow$ `max 5 6`

$\Longrightarrow$ `6`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

Pattern matching

```
product [1,2,3,4]
```

```
product [] = 1
product (n:ns)
        = n * product ns
```

Pattern matching

```
product [1,2,3,4]
```

```
product [] = 1
product (n:ns)
        = n * product ns
```

Pattern matching

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

```
product [] = 1
product (n:ns)
        = n * product ns
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

$\Longrightarrow$ `1 * 2 * 3 * 4 * product []`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

$\Longrightarrow$ `1 * 2 * 3 * 4 * product []`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

$\Longrightarrow$ `1 * 2 * 3 * 4 * product []`

$\Longrightarrow$ `1 * 2 * 3 * 4 * 1`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

$\Longrightarrow$ `1 * 2 * 3 * 4 * product []`

$\Longrightarrow$ `1 * 2 * 3 * 4 * 1`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
```

$\Longrightarrow$ `1 * product [2,3,4]`

$\Longrightarrow$ `1 * 2 * product [3,4]`

$\Longrightarrow$ `1 * 2 * 3 * product [4]`

$\Longrightarrow$ `1 * 2 * 3 * 4 * product []`

$\Longrightarrow$ `1 * 2 * 3 * 4 * 1`

$\Longrightarrow$ `1 * 2 * 3 * 4`

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
⟹  1 * 2 * 12
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
⟹  1 * 2 * 12
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
⟹  1 * 2 * 12
⟹  1 * 24
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
⟹  1 * 2 * 12
⟹  1 * 24
```

Pattern matching

```
product [] = 1
product (n:ns)
        = n * product ns
```

```
product [1,2,3,4]
⟹  1 * product [2,3,4]
⟹  1 * 2 * product [3,4]
⟹  1 * 2 * 3 * product [4]
⟹  1 * 2 * 3 * 4 * product []
⟹  1 * 2 * 3 * 4 * 1
⟹  1 * 2 * 3 * 4
⟹  1 * 2 * 12
⟹  1 * 24
⟹  24
```