Structure and Randomness in Iannis Xenakis' *Analogique A*

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Introduction

Randomness, or the quality of being unpredictable, is a central concept in Western thinking and, as such, is an often-explored topic for composers of Western art music. Two schools of thought exist for reconciling randomness with a rational understanding of the world. One is to reject the existence of randomness entirely. The argument for fate, or determinism, claims that everything is predetermined and, given sufficient information or powers of prophesy, could be predicted. The second answer is chance, that individual events can't be predicted, but their likelihood can.[1] Games of chance, that rely on this assumption, have also existed since antiquity, because they manage to be both fair and unpredictable.[2] Games with two dice, for example, decrease that unpredictability, by making some outcomes more likely than others (figure 1). The result of the dice is unpredictable enough that you can bet on it, but predictable in the long term so some results are riskier bets than others.

An early example of randomness in music is Mozart's *Musikalisches Würfelspiel* or "Musical Dice Game" which allows anyone without musical background to compose a waltz using only a pair of dice (and Mozart's list of waltz phrases that fit together nicely).[3] In that piece,

[1]Eagle, Antony. *Stanford Encyclopedia of Philosophy*, Winter 2016 ed., s.v. "Chance versus Randomness." Accessed online at https://plato.stanford.edu/archives/win2016/entries/chance-randomness/.

[2]Glimne, Dan. *Encyclopedia Britannica*, 15th ed., s.v. "Dice." 2006. Accessed online at https://www.britannica.com/topic/dice

[3]Mozart, Wolfgang Amadeus. *Musikalisches Würfelspiel* K516f. Bonn: N. Simrock, n.d.(1793).

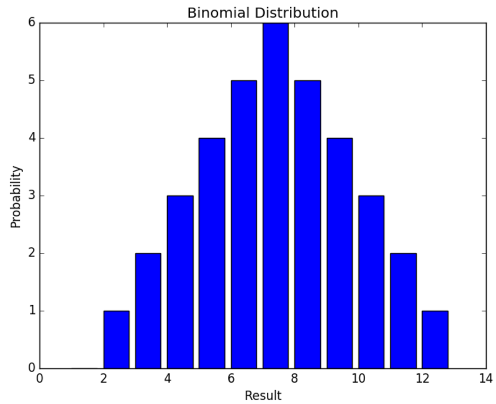


Figure 1: The distribution of probabilities for the sum of two dice rolls.

the micro-level of the composition, the notes and rhythms, was fixed by Mozart, but the order of the measures was left to chance. Randomness became more of a topic of interest for composers as it became better understood with the development of statistics in the late 19th and early 20th centuries. A notable composition from this period is French Dada artist Marcel Duchamp's 1913 work "Erratum Musical" or "Musical Misprint" which sets the text of a French dictionary definition of the verb "to impress" with pitches chosen at random.[4] Duchamp's work randomizes the micro-level, unlike Mozart's, and does not otherwise structure the piece.

John Cage, one of the best-known American composers of the 20th century avant-garde, was greatly influenced by Duchamp. He used chance music as a rejection of the trend towards increased structural and procedural complexity in music pursued by serialists like Pierre Boulez and Milton Babbitt and wrote what he called aleatoric music, where some elements of the composition were left to either the performer's improvisation or the sound of the performance space itself. The famous *4'33"*, composed in 1952, serves as the most provocative example of such a work, where the performer does not make any sound at all, and asks that the audience listen to the ambient sounds of the performance space and the audience (coughing, rustling of papers, etc.) for four minutes and thirty-three seconds.

[4] Chen, Ya-Ling. "Erratum Musical, 1913." *Tout-fait: The Marcel Duchamp Studies Online Journal.* `http://www.toutfait.com/issues/issue_1/Music/erratum.html`

Iannis Xenakis was a 20th century Greek avant-garde music composer, a contemporary of Cage, who had a lot to say regarding randomness in music. He rejected tonal music, serial music and aleatoric music because they did not correctly represent his view of the world as predictably chaotic. He disliked tonal music because it imposes arbitrary hierarchies of tonal functions which create highly deterministic linear processes (melodies) and simultaneous events (chords).[5] He also rejected serialism because it destroys itself through complexity – the processes that were used to create the music were just as deterministic and restrictive as tonality, but not audible, which, according to Xenakis, made them self-defeating.[6] Finally, he rejected aleatoric music because it is not really random, it just passes the task of composition onto some other entity — either the performer, who is conditioned by their background, or the space, which is conditioned on a whole host of other factors. His solution is to calculate chance formally and with precision, to avoid influencing the results, which appears to be the marriage of the serialist goal of avoiding unconscious bias (towards, for instance, a single tonal center) when composing with chance music. [7] Xenakis claims,

> Chance is a rare thing and a snare. It can be constructed up to a certain point with great difficulty, by means of complex reasoning which is summarized in mathematical formulae; it can be constructed a little, but never improvised or intellectually imitated...to play with sound like dice — what a truly simplistic activity! But once one has emerged from this primary field of chance worthless to a musician, the calculation of the aleatory, that is to say stochastics, guarantees first that in a region of precise definition slips will not be made, and then furnishes a powerful method of reasoning and enrichment of sonic processes.[8]

The word "stochastic" bears further definition. It comes from the Greek root *stochos*, which means target or goal, and initially entered English as an adjective to describe a process that relies on a collection of random variables. Informally, a stochastic process is unpredictable at the micro-level, but follows predictable patterns at the macro-level. A good example is a game in which a person flips a coin, then takes a step forward if the coin is heads, and takes a step backwards if the coin is tails, then repeats this process a large number of times.

[5] Xenakis, Iannis. Formalized Music; Thought and Mathematics in Composition, p. 5. Bloomington, IN: Indiana University Press, 1971.

[6] Ibid. p. 5-8

[7] Ibid. p. 38

[8] Ibid. pp. 38-39

While you can never predict what the person's next step will be, no matter how many steps are taken, if you were to bet on where they end up at the end of the process, it would be smart to bet on somewhere near where they started, since about half of the steps will go forward and half will go backwards.

In this paper, we will delve into this concept of stochastic music by examining one of Xenakis' first experiments in stochastic technique, *Analogique A* for strings, composed between 1958 and 1959. The piece is an application of what Xenakis calls Markovian Stochastic Music, which is a process for composition he spells out at length in his book *Formalized Music*. Xenakis had two goals for this piece, for it to demonstrate that stochastic music could create new forms and new second-order sonorities. Since listeners could not perceive either of those two phenomena, Xenakis considered *Analogique A et B* to be a failure. The piece becomes rather ironic in that context, since Xenakis centered his famous critique of serialism around the argument that serial music had too much structure that was imperceptible to the listener, then composed and published a work that fell victim to exactly that critique.

So should the piece be regarded as a failure? Agostino DiScipio offers an argument to the contrary.

> ...the problematic aspects of *Analogique A et B* ultimately represent the very element giving this music a strongly peculiar, almost unique character, also quite palpable when listening to it. Some of the problems Xenakis raised in composing this work, and that he apparently left without satisfying solutions, resulted into choices and decisions that remained (and probably had to remain) non-formalized. This music is not the audible trace of a thoroughly formalistic approach, but the result of a clash between different domains of rationality. It entailed not only a substantial body of theoretical premises, but also important manual, nonformalized adjustments and arbitrary choices. It represents less an unsatisfactory experiment than a work expressive of a lively and intricate dialectic between formalization and intuition. Interesting is not simply the opposition formalization vs. intuition, machine vs. human, but the dialectic between such terms in the actual process of knowledge that we call composing. The two elements are inextricably intertwined, so interlaced that they can hardly be separated in actual experience (neither the composer's, nor the listener's).[9]

[9]Di Scipio, Agostino. Formalization and Intuition in Analogique A et B Definitive Proceedings of the International Symposium Iannis Xenakis Athens, 2005, http://iannis-xenakis.org/Articles/Di%20Scipio.pdf.

Regardless of how the piece is critically evaluated, though, it is clear that there is some sort of inconsistency between Xenakis' goals for the piece, the systems he used when composing and the piece itself. What do other theorists have to say about *Analogique A et B* and other works by Xenakis? Do they find similar inconsistencies?

DiScipio focuses his analysis of *Analogique A et B* on the formalized decisions that Xenakis makes when composing, namely his use of repeated samples from a protocol to demonstrate his stochastic process, how he presented those samples in the final piece, which protocols he used and how he arrived at them.[10] His interest is not in understanding *Analogique A* itself in a vacuum, but in understanding its ideological context and how cybernetics and information theory impacted the composition. He labels some aspects of the piece's composition as formalized, meaning they were discussed in *Formalized Music* and others as intuitive, meaning they weren't. Kerry Hagan conducts on *Analogique B* what he calls "Genetic Analysis" or analysis by generating the piece from scratch, only according to Xenakis' stated methods, leaving the rest to chance, then compares the generated work to the original.[11] He notes that there is a sense of intentionality in the original which does not manifest in the generated work, but does not give any specific examples of what makes him say that. This analysis leads him to conclude that Xenakis did not use randomness to determine much of the micro level of his composition and merely made decisions that could have reasonably occurred at random according to his taste.

Some theorists have analyzed similar pieces by Xenakis and found inconsistencies like this as well. Robert Wannamaker analyzes *Herma*, one of Xenakis' later piano works, through a combination of PC set theory, mathematical set theory (as used by Xenakis in constructing the piece) and James Tenney's language of temporal gestalt formation.[12] These techniques attempt to capture the piece both from the side of its imperceptible underlying structure and, using Tenney's ideas, how Wannamaker perceives it when listening. He goes on to

[10] Ibid.

[11] Hagan, Kerry. 2005. "Genetic Analysis of Analogique B." Paper presented at Electroacoustic Music Studies Network, Montreal, October 2005. http://www.ems-network.org/IMG/EMS2005-Hagan.pdf.

[12] Wannamaker, Robert. 2001. "Structure and Perception in *Herma* by Iannis Xenakis." Music Theory Online 7 (9). http://www.mtosmt.org/issues/mto.01.7.3/mto.01.7.3.wannamaker.html

highlight the inconsistencies between the set theoretical structure of the piece and the perceptible structures, and speculates as to why those differences exist.

Other theorists have analyzed Xenakis' music using unconventional techniques, similar to Hagan. Thor Kell recreates *Mists*, one of Xenakis' later works, using a computer program and analyzes the differences between the generated piece and the original in a few key passages.[13] He notes that the passages in the original are a bit too perfect, that if they were generated at random, they were either the result of incredible luck or additional parameters that Xenakis does not disclose, and suggests that they were composed by hand to appear random.

Not all analysis of Xenakis focuses on his formalization, though. Ronald Squibbs, whose dissertation is the foremost analytical authority on Xenakis, approaches seven of his works in a deliberate, formal manner.[14] Squibbs breaks down each work into segments of interest, then discusses the characteristic similarities and differences between segments, in terms of pitch sets, rhythms and dynamics. Of particular interest to Squibbs are random walks, arborescences and applications of Xenakis' sieve theory, which are frequent in the pieces he examines. Keller and Fenneyhough create a mathematical model of *ST/10-1*, another of Xenakis' pieces, intending to document an undocumented step in the composition of the piece: how Xenakis quantized rhythmic events to the 4:5:6 polyrhythmic structure evident in the score.[15] They find that their approach creates similar musical event distributions to the ones in the original, which suggests that their model was similar to the approach Xenakis used.

In the following sections, we will explore the product and process of *Analogique A* and try to determine the extent to which Xenakis' composition of the piece reflects the structure he claims it has in *Formalized Music*. As we will see, a great deal of information about the piece was omitted from that compositional methodology, but what remains is fairly accurate to

[13]Kell, Thor. Mistify: Dynamically Recreating Xenakis' Mists Proceedings of the 9th Conference on Interdisciplinary Musicology CIM14. Berlin, Germany 2014. http://tide-pool.ca/papers/Thor%20Kell%20-%20CIM%202014.pdf.

[14]Squibbs, Ronald J., An Analytical Approach to the Music of Xenakis PhD diss., Yale University, 1996.

[15]Keller, Damian, Fenneyhough, Brian. Analysis by Modeling: Xenakis's ST/10-1 080262 Journal of New Music Research, 33:2, 161-171. http://dx.doi.org/10.1080/0929821042000310630.

the piece, with the notable exception of the expected densities of sonic events, which do not seem to describe the piece. The unusual densities, uneven distribution of pitches and non-formalized components of the work, like rhythm and playing technique make it seem likely that the micro-level of the piece was composed intuitively, without the use of stochastic techniques, to fit a stochastic form.

We will start by going through the discussion of the piece presented in *Formalized Music* and build up Xenakis' formalization of Markovian Stochastic Music. Once we have the process behind the piece, we will use it, along with a couple of computational analysis methods, to critically examine the piece itself and see exactly what the disconnects are between the process and product.

Process

In order to understand *Analogique A*, we must understand the method that Iannis Xenakis used to create it. Xenakis, fortunately, devoted two chapters to *Analogique A et B* is his book, *Formalized Music*.[16] In this section, we will start from Xenakis' general theory of music and build up enough background to understand the specific methods used in this piece. We'll approach this writing not only as an explanation of a process and discuss what it says, but also as a text for analysis itself and discuss what it doesn't say. This sort of inquiry will hopefully grant us insight into what the process of the piece actually was, so we can better compare it with its product.

For context, *Analogique A*, and its sister composition, *Analogique B* are, as the title suggests, analogues. Both pieces were composed in 1958-9, while Xenakis was working for the architect Le Corbusier, and were constructed using the same precompositional methodology, except *A* was composed for nine strings while *B* was composed for electronics. While *Analogique B* became historically noteworthy as the first instance of granular synthesis [17],

[16] Xenakis, Iannis. Formalized Music; Thought and Mathematics in Composition. Bloomington, IN: Indiana University Press, 1971.

[17] Granular synthesis is a method of constructing complex electronic sounds from short, simple sonic "grains", which inspired a later generation of composers

Xenakis considered the whole composition *Analogique A et B* a failure, according to DiScipio, because he left many of the problems it raised without satisfying answers.[18] As such, he did not repeat the technique, though *Syrmos*, a composition from later in 1959, replicates some of its ideas about equilibrium and perturbation.

Grains and Screens

Xenakis starts his discussion with the concept of an atomic sonic particle, which he calls a "grain." A grain has properties, including pitch, intensity (amplitude), duration and timbre, and we can discuss the set of grains that occur within a given moment in time (of length Δt) by projecting the grains onto the Cartesian plane (Figure 2), where the x-axis is pitch and the y-axis is intensity. Xenakis calls the resultant projection a screen. All sonic events, Xenakis claims, can be represented by sets of sonic grains. He compares this concept of a sound to "a multicolored firework in which each point of light appears and instantaneously disappears against a black sky".[19]

Just as the physical theory of kinematics can describe all motion, but is impractical to use for real physical systems, all sound can be tediously described as ordered sets of screens. Xenakis claims to solve this problem just as physicists do, by discussing sonic grains statistically. He divides a screen into cells based on regions of intensity and pitch and measures the density of each cell. Since these screens are sets, they exhibit all of the properties of other mathematical sets: we can subtract them, take their union, intersection and complement, measure their ranges, averages, variances and entropy in various dimensions (e.g. pitch, intensity, duration) and define linear transformations on them.

The theoretical basis of this interpretation is surprisingly similar to physical science for describing something as abstract as music. Scholars have speculated that Xenakis saw himself

[18]Di Scipio, Agostino. Formalization and Intuition in Analogique A et B Definitive Proceedings of the International Symposium Iannis Xenakis Athens, 2005, http://iannis-xenakis.org/Articles/Di%20Scipio.pdf.

[19]Ibid., p. 43.

[4]Kollias, Phivos-Angelos. 2011. "Iannis Xenakis and Systems Thinking" Proceedings of the Xenakis International Symposium, London, 2011.

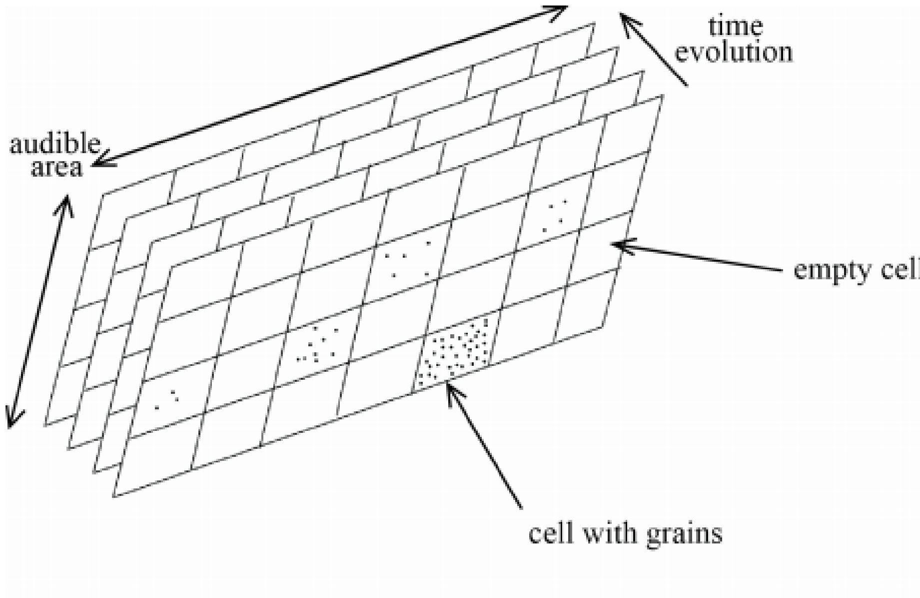


Figure 2: Representation of Xenakis' screens (Kollias'[4] reproduction of the diagram from p. 51 of Formalized Music). Here, the vertical dimension is intensity (quiet-loud) and the horizontal is pitch (low-high).

establishing a theoretical foundation for music, just as nineteenth century mathematicians searched for the foundations of mathematics, and felt the need to create a mathematically well-formed system, similar to a scientific theory. Squibbs argues that Xenakis stands apart from other composers interested in indeterminacy, such as Cage, Wolff, Brown, Stockhausen and Boulez, since he focuses so heavily on scientific models of indeterminacy and probability theory.[5] Xenakis clearly prioritizes the creation of a formal and consistent theory of music, but states that his approach deals with perceptions of sound rather than physical phenomena. However, Makis Solomos notes that Xenakis tends to prefer metaphors from the natural sciences to metaphors from pure mathematics.[6] DiScipio points out that Xenakis often evokes the names of important figures in the history of science, especially in the fields of thermodynamics and information theory and how both fields intersect with probability and randomness.[7] He argues that Xenakis was familiar with the ideas of cybernetics

[5]Squibbs, Ronald J., "An Analytical Approach to the Music of Xenakis" PhD diss., Yale University, 1996.
[6]Solomos, Makis "Xenakis' Thought through his Writings", Journal of New Music Research, 33(2), 2004, p.125–136.
[7]Di Scipio, Agostino. "Formalization and Intuition in Analogique A et B" Definitive Proceedings of the "International Symposium Iannis Xenakis" Athens, 2005

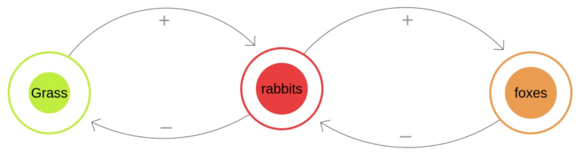


Figure 3: Example system. The amount of grass positively affects the number of rabbits, and the number of rabbits positively affects the number of foxes, but the number of foxes negatively affects the number of rabbits, and the number of rabbits negatively affects the amount of grass. This system will reach a stable point, since an increase in any one quantity will cause a corresponding decrease in that quantity.

and general systems theory, which heavily influenced his approach. Phivos-Angelos Kollias continues this line of inquiry and argues that cybernetics is essential to the understanding of Xenakis' music.[8]

The comparison between Xenakis' formalization of music and cybernetics bears explaining. Cybernetics is the study of processes as systems, or sets of interacting elements. It claims that rather than processes existing in strict cause-effect chains, every element of a system has some effect on other elements of the system, either positive feedback (one causes the other) or negative feedback (one inhibits the other), and systems will either reach stable points, where elements are in equilibrium (like the system in figure 3), or form positive or negative feedback loops until they collapse. That equilibrium is what scientists call dynamic equilibrium – things are constantly changing at the micro level (using the example in figure 3, rabbits are always eating grass and wolves are always eating rabbits), but not changing at the macro level. As we'll see, *Analogique A et B* is Xenakis' attempt to convey the concept of falling into equilibrium through music.

Transformations on Screens

After discussing the basics of grains and screens, Xenakis delves deeper into the concept of a transformation.[9] Since transformations on screens are the same as those in linear algebra,

[8] Kollias, Phivos-Angelos. 2011. "Iannis Xenakis and Systems Thinking" Proceedings of the Xenakis International Symposium, London, 2011.

[9] By transformation, I mean the mathematical concept, which is another name for a function. It is often easier to think of these transformations as occurring in time, where the earlier screen transforms into the later screen, though Xenakis does not use them in this way. All of the following claims about transformations

they can be expressed as the product with a matrix where the value of the matrix in row n and column m is 1 if the transformation maps screen m onto screen n, or kinematic diagrams where each screen is represented by a letter and arrows denote which screens transform into other screens (see figure 4 for an example).

T: ↓	A	B	C	D
	D	C	D	B

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

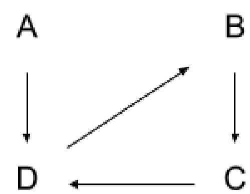


Figure 4: The same transformation, represented as a table, matrix and kinematic diagram.

Transformations can be either deterministic, where a given screen always transforms into the same screen when the transformation is applied, or stochastic, where a given screen could transform into one of several different screens, with fixed probabilities. If the transformation is stochastic, it can still be represented by matrix multiplication as long as the values in the matrix are transition probabilities instead of ones and zeros. Consider a length n vector where n is the number of screens. If that vector has a 1 in the position of the start screen and 0s elsewhere, multiplication of that vector with the transition probability matrix will yield a vector of probabilities over what the next screen will be. Further, repeated multiplication of that probability vector with the transition matrix will yield the probabilities that the screen will end up in that screen after that many transformations (see figure 5).

Like all functions that map a set onto itself, these transformations on screens are iterable, meaning they can be repeatedly applied to their output (e.g. iterating a function f five

on screens are true of transformations on sets in general, but since this paper has no need to be so abstract, I will talk in terms of transformations on screens.

$$\begin{bmatrix}0\\0\\1\\0\end{bmatrix} * \begin{bmatrix}0 & .2 & .2 & 0\\.5 & 0 & .3 & 1\\0 & .8 & 0 & 0\\.5 & 0 & .5 & 0\end{bmatrix} = \begin{bmatrix}.2\\.3\\0\\.5\end{bmatrix}$$

$$\begin{bmatrix}.2\\.3\\0\\.5\end{bmatrix} * \begin{bmatrix}0 & .2 & .2 & 0\\.5 & 0 & .3 & 1\\0 & .8 & 0 & 0\\.5 & 0 & .5 & 0\end{bmatrix} = \begin{bmatrix}.06\\.6\\.24\\.1\end{bmatrix}$$

Figure 5: Example of using multiplication to calculate the probabilities that after two applications of the given transformation, screen three will be each of four different screens.

times would be calculating $f(f(f(f(f(x)))))$. Some functions, when iterated on, reach stable points (i.e. $f(x) = x$), which here means that application of a transformation to a vector of screen probabilities does not change the probabilities. For example, if we only are discussing two screens, A and B, and our transition matrix is,

$$\begin{bmatrix}.2 & .8\\.8 & .2\end{bmatrix}$$

If we start with a screen A and repeatedly apply this transformation, the probabilities that we will be in each screen are as follows:[10]

Stage	P(A)	P(B)
0	1	0
1	.2	.8
2	.68	.32
3	.39	.61
4	.57	.43
5	.46	.54
6	.52	.48
7	.49	.51
8	.5	.5
9	.5	.5

Here, the stable point occurs when there is a 50% chance of being in screen A and 50% chance of being in screen B. With a little algebra, the stationary point of any transformation

[10] Students of probability theory will recognize this as a Markov process, though it is a very different approach to Markov processes than other composers (e.g. Lejaren Hiller) have used before Xenakis.

can be easily calculated.[11] Drawing the connection back to cybernetics, this stable point is a point of dynamic equilibrium. More applications of the transformation will cause a screen to potentially transform from A into B and from B into A, but the probability of being in either screen won't change.

Xenakis finally sets forward the concept of entropy on a transformation, which measures how disordered or unpredictable the transformation tends to be. Importantly, neither zero entropy (when there is a 100% chance that all of the screens will have one value) nor maximum entropy (when everything happens with equal probability) is particularly interesting in music. Since iterating on a function with a stable point tends towards the point of maximum entropy (the stable point), the most musically interesting procedure is to create a "perturbation" of minimum entropy (screens are all the same) and allow it to naturally trend towards maximum entropy (screens are equally distributed).

Pre-compositional method of *Analogique A et B*

Now that we have built up enough theory, we can discuss the pre-compositional method for *Analogique A et B*. Xenakis decides that pitch, intensity and density (the three dimensions along which he measures the differences between screens) should each be assigned two contrasting configurations (see figure 6), where each of four clouds of grains is placed at a different position along that dimension. For each dimension, then, Xenakis decides upon two transition probability matrices (see figure 7) and chooses which one to use at a given timestep based on the state in a separate dimension.

This coupling together of dimensions violates the Markov property, which describes stochastic processes where the probability distribution over future states depends only on the current state. Without the Markov property, we can't calculate probability distributions over possible screens using matrix multiplication, so Xenakis expands the definition of state to include all three dimensions. Taking each possible combination of these three partitions

[11] What I call a stationary point is closely related to an eigenvector, and techniques for finding it are discussed in any introductory linear algebra book.

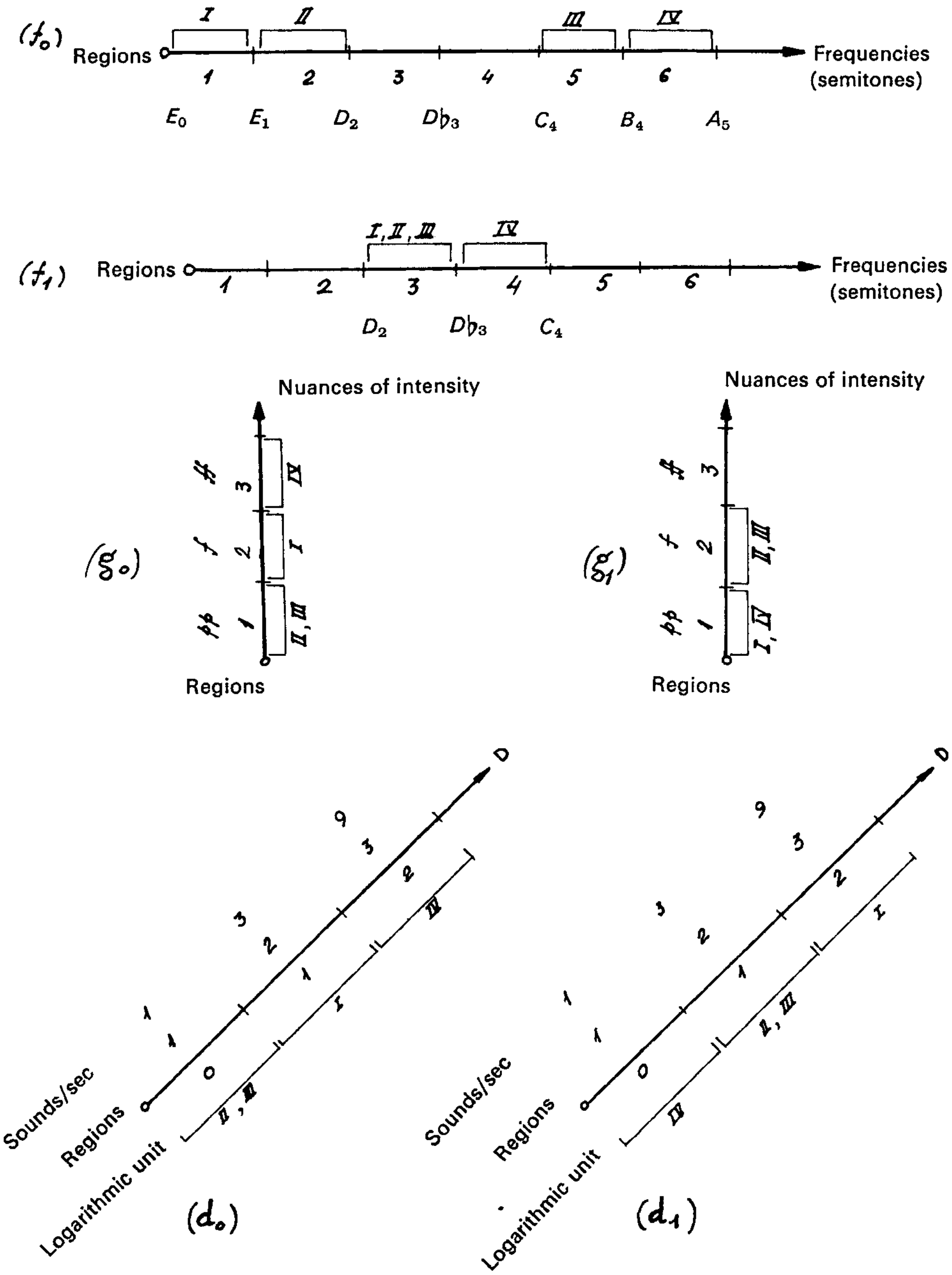


Figure 6: partitions of pitch, intensity and density space into two configurations. Roman numerals correspond to locations of specific clouds.

4. Each of these three variables will present a protocol which may be summarized by two matrices of transition probabilities (MTP).

(ρ)

$\downarrow$	X	Y
X	0.2	0.8
Y	0.8	0.2

(σ)

$\downarrow$	X	Y
X	0.85	0.4
Y	0.15	0.6

The letters (ρ) and (σ) constitute the parameters of the (MTP).

MTPF (of frequencies)

(α)

$\downarrow$	f_0	f_1
f_0	0.2	0.8
f_1	0.8	0.2

(β)

$\downarrow$	f_0	f_1
f_0	0.85	0.4
f_1	0.15	0.6

MTPG (of intensities)

(γ)

$\downarrow$	g_0	g_1
g_0	0.2	0.8
g_1	0.8	0.2

(ε)

$\downarrow$	g_0	g_1
g_0	0.85	0.4
g_1	0.15	0.6

MTPD (of densities)

(λ)

$\downarrow$	d_0	d_1
d_0	0.2	0.8
d_1	0.8	0.2

(μ)

$\downarrow$	d_0	d_1
d_0	0.85	0.4
d_1	0.15	0.6

(e_0)

$\downarrow$											
f_0	f_1	d_0	d_1	g_0	g_1	g_0	g_1	f_0	f_1	d_0	d_1
λ	μ	α	β	λ	μ	β	α	γ	ε	γ	ε

Figure 7: Transition matrices and coupling between dimensions, which is read "if currently in a screen with frequencies f_0, use transition matrix λ for density."

MTPZ

↓	A $(f_0g_0d_0)$	B $(f_0g_0d_1)$	C $(f_0g_1d_0)$	D $(f_0g_1d_1)$	E $(f_1g_0d_0)$	F $(f_1g_0d_1)$	G $(f_1g_1d_0)$	H $(f_1g_1d_1)$
$A(f_0g_0d_0)$	0.021	0.357	0.084	0.189	0.165	0.204	0.408	0.096
$B(f_0g_0d_1)$	0.084	0.089	0.076	0.126	0.150	0.136	0.072	0.144
$C(f_0g_1d_0)$	0.084	0.323	0.021	0.126	0.150	0.036	0.272	0.144
$D(f_0g_1d_1)$	0.336	0.081	0.019	0.084	0.135	0.024	0.048	0.216
$E(f_1g_0d_0)$	0.019	0.063	0.336	0.171	0.110	0.306	0.102	0.064
$F(f_1g_0d_1)$	0.076	0.016	0.304	0.114	0.100	0.204	0.018	0.096
$G(f_1g_1d_0)$	0.076	0.057	0.084	0.114	0.100	0.054	0.068	0.096
$H(f_1g_1d_1)$	0.304	0.014	0.076	0.076	0.090	0.036	0.012	0.144

Figure 8: Markov transition probability matrix Z (MTPZ), which was used to create *Analogique A et B*.

yields $2^3 = 8$ screens, which Xenakis labels A through H. These states yield a new eight by eight transition probability matrix, which Xenakis calls MTPZ (figure 8).

Sequences of screens created by repeated application of transformation are notably an outside-time structure nested within an inside-time structure. In Xenakis' own words, the structures of sonic events exist in four stages, ordered from completely outside-time, to completely inside-time:

> Let there be three events, a, b, c emitted successively.
>
> First stage: Three events are distinguished, and that is all.
>
> Second stage: A "temporal succession" is distinguished, i.e. a correspondence between events and moments. There results from this
>
> $$a \text{ before } b \neq b \text{ before } a \text{ (non-commutativity).}$$
>
> Third stage: Three sonic events are distinguished which divide time into two sections within the events. These two sections may be compared and then expressed in multiples of a unit. Time becomes metric and the sections constitute generic elements of set T. They thus enjoy commutativity....
>
> Fourth stage: Three sonic events are distinguished; the time intervals are distinguished; and independence between the sonic events and the time intervals is recognized. An algebra outside-time is thus admitted for sonic events, and a secondary temporal algebra exists for temporal intervals; the two algebras are otherwise identical. [12]

[12] Xenakis, Iannis. "Symbolic Music." In Formalized Music; Thought and Mathematics in Composition, p. 160–161. Bloomington, IN: Indiana University Press, 1971.

The structure of sonic events within a single screen is only defined with respect to pitch, duration and intensity, which puts it in the first stage (it is important to note that while duration occurs along the same axis as time, it does not position different sonic events relative to each other in time). The structure of a sequence of screens, on the other hand, is inside-time, but is only as far as the second stage in this list, since a sequence of screens says nothing about the relative distances between sonic events.

So how does Xenakis get from concepts of screens and transformations to music? The key is a kinematic diagram:[20]

$$E \to P_A^0 \to P'_A \to E \to P'_C \to P_C^0 \to P_B^0 \to P'_B \to E \to P'_A$$

Each of these symbols is a protocol for composition taken from a probability distribution over possible screens that emerges from either a perturbation or a return to equilibrium. By protocol, Xenakis means procedure or algorithm that consists of repeatedly sampling from a probability distribution over different screens. Specifically,

- E is equilibrium, the stable point of transformation Z (figure 8), the vector

$$[.17, .13, .13, .11, .14, .12, .1, .1]$$

- P_A^0 is perturbation towards screen A, the vector

$$[1, 0, 0, 0, 0, 0, 0, 0]$$

- P'_A is the probability vector after one application of transformation Z on P_A^0,

$$[.021, .084, .084, .336, .019, .076, .076, .304]$$

[20] Xenakis, Iannis. "Markovian Stochastic Music: Applications." In Formalized Music; Thought and Mathematics in Composition, p. 96. Bloomington, IN: Indiana University Press, 1971.

- P_B^0 and P_C^0 are like P_A^0 except for screens B and C, respectively.
- P_B' and P_C' are like P_A' except one application of transformation Z on P_B^0 and P_C^0, respectively.

Any composer of stochastic music must come up with some procedure for crossing the boundary between abstract probability space and fixed music. This "protocol" system is just one of the solutions that Xenakis came up with over the course of his career. For an example of another, two of his solo piano works, *Mists* and *Evryali* make use of a slightly different technique. Instead of putting a number of samples from a probability distribution in series, he places them in parallel, branching into polyphony at every stochastic decision point (he calls these figurations arborescences and Squibbs discusses them at length).[13]

The use of these protocols, rather than just taking a single screen, applying the transformation to get a new screen, then apply the transformation again to get a new screen, etc. seems odd. It reveals that the object of Xenakis' interest is not the effect of a transformations on screens, but the effect of transformations on *populations* of screens. The result is a compositional goal that seems more like an outside-time sonic sculpture, with lots of screens existing in parallel, projected into a piece of music that can necessarily only examine one screen at a time.

In order to make this sonic sculpture into a linear composition, Xenakis sets the Δt (time length) of one screen to be 1.11 seconds, which he notates as a half note, then explores each of these ten protocols with 30 sequential screens, sampled from the probability distribution of the protocol. Since $30 * \Delta t = 15$ measures, and there are ten protocols to explore, that's 150 measures, the length of *Analogique A*.

While the focus for analysis in this paper is on A, it only reveals half of the work *Analogique A et B*. The two analogues, A and B are for strings and electronics, respectively. They use the same Markovian stochastic technique, the same kinematic diagram (shown on the previous page) and the same transition function. B adopts a different partition of frequency (not

[13] Squibbs, Ronald J., "An Analytical Approach to the Music of Xenakis" PhD diss., Yale University, 1996.

pitch, he is synthesizing these with sine waves), intensity and density that is significantly more precise and elaborate, since it does not need to rely on acoustic instruments or live musicians.

Product

While examining the precompositional method Iannis Xenakis used to create *Analogique A* is helpful for understanding it, analyzing the techniques used to compose a piece is not analyzing that piece. In this section, we will go through some high-level observations about a recording of *Analogique A et B* and its score. We will then discuss three computational tools, two of which produced useful analytical results and one that did not, which leads us to conclusions about the relationship between product and process as well.

High-Level Observations

Upon listening to a recording[21] of *Analogique A et B*, several features of the composition stand out. First, the parameterization of dynamics and density are readily audible – individual musicians within the ensemble have different parts and may be playing in different registers, at different dynamics. The density of each instrumental voice changes quite abruptly. A bassist may play four quick notes in a row, then remain silent for the next measure, which gives the piece a jerky, unpredictable nature.

These characteristics are to be expected. The piece is investigating sonic grains, and the best way to approximate that using an acoustic string instrument is with short notes. The abrupt changes in density, intensity and register are direct consequence of the juxtaposition of different screens. If anything, this texture is proof of concept that granular synthesis can create rapid, discontinuous shifts in the fabric of a sound. If Xenakis' metaphor of the complex sound as "a multicolored firework in which each point of light appears and

[21] The recording I use is, to my knowledge, the only widely available recording of the piece. It is available on the album *Iannissimo!* recorded by the Charles Zacharie Bornstein STX ensemble for Xenakis' 75th birthday at St. Peter's church in NYC.

Protocol	E	A0	A'	E	C'	C0	B0	B'	E	A'
A Start	0:00	0:54	1:28	2:10	2:55	3:30	4:06	5:05	5:45	6:52
A End	0:32	1:25	2:00	2:53	3:30	3:58	4:44	5:33	6:20	7:30
B Start	0:32	1:13	1:30	2:03	3:30	???	4:44	5:33	6:18	???
B End	0:54	1:26	1:35	2:10	???	4:06	5:05	5:45	???	6:52
Overlap?	No	Yes	Yes	No	Yes, with C0	Yes	No	No	A little	No

Table 1: Start and end times for each protocol (??? indicates that two protocols are played continuously and are difficult to differentiate). The tape was played by Paul Miller, who performs under the name DJ Spooky (the track says "Featuring DJ Spooky on electronics") based on time cues by Joseph Pehrson, which indicates that the overlaps are endemic to the composition, despite not being specified in the score.

instantaneously disappears against a black sky" holds true, then he has succeeded in splicing together short segments of film from the video recordings of several different fireworks.

The piece is certainly unpredictable and discontinuous, but being unpredictable does not make it stochastic. The relationships between the different screens are difficult to perceive and the large-scale shifts between high and low entropy with respect to the transformation are entirely inaccessible to the listener. It is difficult to tell that the composition is highly structured stochastic music and not just free atonality.

Another notable feature of this recording is the relationship between *Analogique A* for strings and *Analogique B* for electronics. After each protocol of A is played by live musicians, the corresponding protocol of B is played from electromagnetic tape on a speaker system. Recall the same sequence of protocols and the same transition function is used to construct both pieces, so the "E" protocol of A is played, then the "E" protocol of B is played, etc. This alternation is not consistent: sometimes the sections overlap, sometimes the protocol from B is contained entirely within the protocol from A, sometimes two sections from either A or B are played consecutively, see Table 1.

The inconsistent alternation gives the piece a dialectic feel, that it is a conversation between acoustic and electronic sounds. Just as the sequences of screens in each protocol feel like an atemporal sonic sculpture that is projected into a temporal piece of music, *Analogique A* and *Analogique B* feel like distinct sonic sculptures that are meant to be observed simultaneously and separately, like two exhibits in the same gallery, and Xenakis' best effort to convey that feeling once they are projected into time is to alternate irregularly between them, as if to simulate a viewer's attention shifting from one exhibit to the other, then back again, seeing both at the same time, then focusing on one again, and so on.

Turning now to the score, we can see the exact instrumentation, for three violins, three cellos and three basses, and the reason for the difficult-to-identify rhythm. Within each instrument, the first player has notes quantized to an eighth note grid, the second player has notes quantized to a quarter note triplet grid and the third player has notes quantized to an eighth note quintuplet grid. The score used is a copy of an original notated in Xenakis' handwriting. The fact that they are ordered 4 against 3 against 5 instead of 3 against 4 against 5 in this version, putting the slowest voice in the middle, makes the score difficult to follow. It places the emphasis in the visual representation of the music less on the intuition of the polyrhythm (which is what you would see in the music of, say, Nancarrow, who tries very hard to make the polymetric subdivisions feel natural) and more on the sonic grains being scattered through time, only quantized to a grid at all so that humans can count them.

There are nine fermati notated in the score, one after each 15 measure protocol, and the playing technique of each instrument changes after each fermata. These techniques include *sourd* (muted), *pizzicato* (plucked), *sul ponticello* (on the bridge) and *frappe col legno* (slap with the wood of the bow), their occurrences are displayed in table 2. Notably, playing techniques correlate clearly with protocol: in equilibrium, all instruments are playing *arco*, *sans sourd* and normal position. In P'_A, the only other protocol that appears twice, violins play pizzicato, cellos are muted and basses are muted and playing *sul pont* both times. The perturbations, P^0_A, P^0_B and P^0_C share muted violins, but no other characteristics. P'_B is the strangest, since all instruments are playing *frappe col legno*, a kind of homogeneity not seen in any of the other perturbation protocols.

These observations reveal some of the holes in the discussion of the piece in *Formalized Music*. Xenakis does not even mention that *Analogique A* and *Analogique B* are meant to be performed together, let alone discuss his choice of rhythmic organization or string

Measure	Protocol	Violin	Cello	Bass
0	E			
15	P_A^0	sourd		pizz
31	P'_A	pizz	sourd	sourd, sul pont
46	E	arco	sans sourd	sans sourd, pos norm
64	P'_C	pizz		frappe col legno
79	P_C^0	sourd	1 and 3 sourd	2 and 3 pizz
94	P_B^0		sans sourd	sourd, sul pont
109	P_B^0	sans sourd, frappe col legno	frappe col legno	sans sourd, frappe col legno
124	E	arco	arco	arco
140	P'_A	pizz	sourd	sourd sul pont

Table 2: Playing techniques marked in the score.

⇓

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
0	0	0	0	1	1	0	0	2	0	1	0

⇓

[0,0,0,0,0.38,0.38,0,0,0.76,0,0.38,0]

Figure 9: Segment of music, its frequency table and its corresponding bag-of-words vector.

techniques. Table 3 contains a list of topics he does and does not discuss. While it might be tempting to conclude that all of the included components are formalized and all of the omitted components are intuitive (like DiScipio does), that conclusion seems premature. Xenakis didn't write *Formalized Music* as an instruction manual, he wrote it to discuss his ideas about composition, and does not need to include every step of his method. Rather than focus on the omissions, we will now look at the elements of the composition that he does discuss, namely the protocols and screen construction, and see whether he represented them accurately. We'll investigate these claims quantitatively.

What Xenakis Discusses	What Xenakis Omits
• his goal: stochastic processes creating new sounds and new forms • the contents of each dimension at each screen • the transitions from each combination of regions in a single dimension to the next • how the mechanisms progress towards stable states • how the state of a mechanism can be made into a protocol • which protocols he examines	• how he arrives at the transition probabilities • how he arrives at the protocols he wants to explore • how he gets from ranges of pitches, intensities and densities to lists of notes • how he positions those in time • where the 3-4-5 rhythm comes from, and also where duration comes from • why he gives different timbral instructions to different instruments through the piece • why he spreads out protocols over time, one screen at a time, you lose track of the transition in the mix

Table 3: List of notable inclusions in and exclusions from chapters 2-3 of *Formalized Music*

Bag-of-Words Model

The bag-of-words model is a simplifying data representation technique, common in natural language processing, that assumes that a set of objects for study that exist in a specific order do not depend on that order or even their absolute quantities to present their meaning, but do depend on their relative frequencies, or which occur more often and by what amount. With this assumption in place, a given collection of objects can be represented by a normalized vector in n-dimensional space, where n is the number of possible objects. These vectors exhibit all of the properties of vectors, namely two "bags" of words with the same relative frequencies are equal and the angle between two non-equal vectors is an indication of their similarity. The analogy that gives rise to the name is the canonical example from statistics of the bag of marbles, except each marble is replaced with a strip of paper with a word written on it, so the only thing that affects what you draw from the bag is the relative frequencies of the words.

It is notable that this method of analysis is not limited to pitch: we can also use it to study dynamics (where we look at the relative number of *forte* vs. *piano* notes), note durations, instrument, or even combine two parameters, like measuring the amount of time a given pitch is sounded. For working with this piece, though, pitch and dynamics are the attributes we care about the most, since they are two of the three attributes Xenakis formalizes.

While this model is not usually used on music data since it ignores time, it is useful for us because Xenakis also ignores time for most of his compositional process. Recall the four stages of inside-time structures (discussed in the previous section). While the position of sonic events in time is important in music, and disregarding it is not insignificant, Xenakis' stochastic approach doesn't introduce temporal organization on its sonic objects until the end, converting pitch collections into a bag-of-words undoes that last step rather nicely, allowing us to correctly determine the similarity of passages based on their screen, rather than their melodies or other structures in time.

After inputting the first three fifths of the score into a machine-readable format (musicXML files, created using MuseScore[22]), we were able to use music21, a library in the python programming language, to visualize different subsets of the score as data.[23] For example, consider figure 10, a pitch vs. time graph of a four bar passage (beats 16-32, which are measures 4-8 and screens 8-16, placing them within the first "E" protocol of the piece). Suddenly, the "clouds" that exist in each screen become visible. This sample contains four measures, so it contains eight screens. The first two draw from the f_0 frequency distribution, the next from f_1, the next from f_0, the next two from f_1, and so on. Durations do not seem to be sorted out well, which makes sense since note duration is not at all formalized in Xenakis' method. While this particular graph doesn't show note intensity, the density dimension can be seen in a few spots, for instance, the first and second screens use the same pitch set, but the second has much lower density in the higher pitch regions.

Another lens we can take is to consider pitches outside of time, independent of screen or protocol. Figure 11 shows the frequency (number of times per piece) of each pitch and pitch-class. The pitch diagram shows the dominance of f_0, which was all of the low and

[22] An open-source music notation editor

[23] Cuthbert, Michael Scott and Christopher Ariza. "music21: A Toolkit for Computer-Aided Musicology and Symbolic Music Data." in J. Stephen Downie and Remco C. Veltkamp (Eds.). 11th International Society for Music Information Retrieval Conference (ISMIR 2010), August 9-13, 2010, Utrecht, Netherlands. pp. 637-642.

high notes, over f_1, the middle notes. Most of the piece consists of screens A,B,C and D, which all use the f_0 pitchset. Xenakis also seems to avoid the borders between the two pitch ranges, focusing on the extremes of f_0 and the interior of f_1, which indicates that he was prioritizing the differences between these pitchsets and probably not choosing pitches at random. We can also see that there is no dominant pitch class collection, so the piece is not significantly biased towards a single tonal key, or even a less traditional chord or sieve. There is, though, a slight preference towards five notes common to the C whole tone scale (pitches 0,2,4,6 and 8), which indicates that Xenakis did not deliberately even out his use of pitch classes, like the twelve-tone method would.

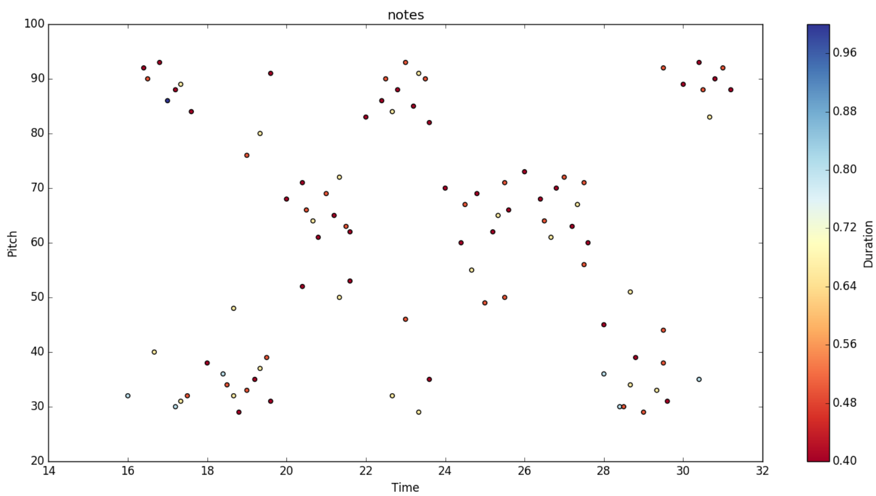


Figure 10: Pitch (midi number, where 28 is E2 and 94 is A#7) vs. time (measured in beats) for measure 4-8 of *Analogique A*, with color indicating note length, from red (an eighth note quintuplet) to blue (a quarter note).

Self-Similarity Matrices

After introducing the bag-of-words model onto our piece, a helpful technique that becomes available to us is self-similarity matrices (SSMs), introduced by Foote to help visualize musical form.[24] An SSM is a matrix where the value, usually represented by color, at position (a, b) is the similarity of point a in the data with point b, so the diagonal (where

[24] Foote, Jonathan. “Visualizing Music and Audio Using self-similarity.” in Proceedings of the ACM Multimedia 1999, Orlando, FL, pp. 77-80

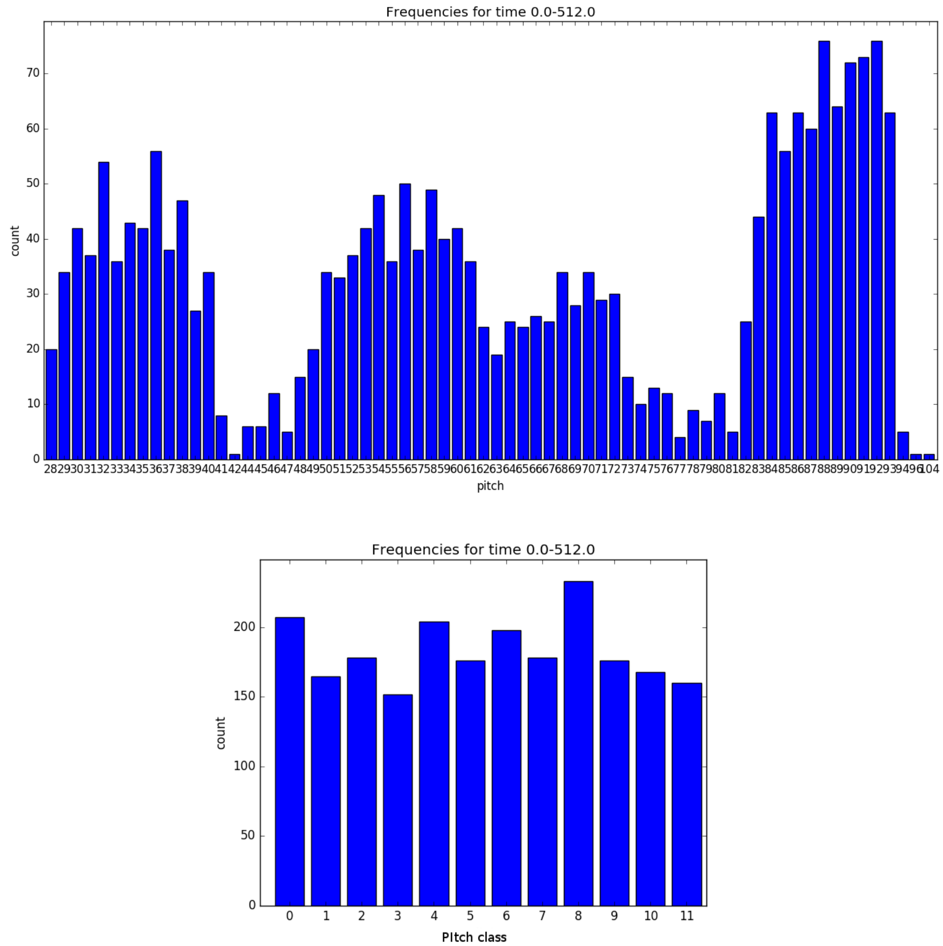


Figure 11: Pitch and pitch-class distribution for the first 90 measures of *Analogique A*.

$a = b$) is always perfectly similar, has a value of 1, and two data points that have nothing in common have a value of 0. While Foote uses this approach on raw audio, Martorell and Gomez apply it to PC set theory analysis.[25] They extract PC sets from a variety of twentieth century works on several levels of granularity, then use different metrics of set equivalence to create SSMs and expose the structure of the pieces.

Instead of using PC-sets, which don't make sense for use with music that rejects octave equivalence, we will use the pitch sets that we get from the bag-of-words model. Since bags of words are vectors, we can calculate self-similarity on a scale from 0 to 1 using the vector

[25] Martorell, Agustin, Gomez, Emilia. "Systematic multi-scale set-class analysis", in Proceedings of the 15th International Society for Music Information Retrieval Conference, 2014.

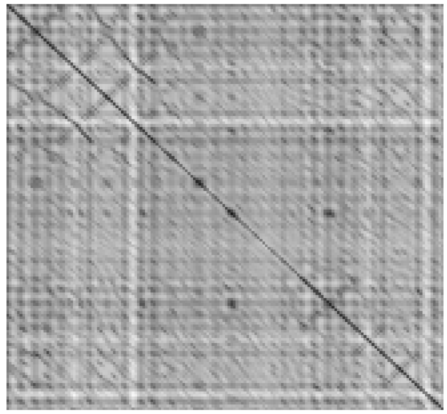


Figure 12: self-similarity matrix of Webern's Op. 27/1, from Martorell and Gomez

dot product: all of our segments of music are represented as normalized vectors, their dot product will be 1 if they are identical and 0 if they are perpendicular, which means they have very few notes or intensities in common. A measure of music is perfectly similar to itself, so there are 1's running up the diagonal, where the x coordinate equals the y coordinate. Also, the similarity between measure a and measure b is the same as the similarity between measure b and measure a, so the matrix is symmetrical across the diagonal.

As an example of this technique, consider Figure 13, the SSM for a Mozart string quartet. We can use the self-similarity of pitches, compared measure-by-measure, to detect repeated material, and infer form. The parallel red lines right before $(32, 32)$ indicates a four measure passage that repeats, which turns out to be parallel period in the second theme. The blue line around the $x = 33$ vertical is a measure with a diminished seventh chord that never repeats (a different diminished seventh chord appears in the recapitulation, in fact, it also causes a blue line, around $x = 92$), The reds and yellows around $(72, 0)$ indicate similarity between the start of the piece and measure 72, which turns out to be the start of the recapitulation (Figure 14 shows the score in these locations). The lack of similarity immediately after that (i.e. $(80, 8)$) indicates that the transition section from the exposition is not repeated note-for-note, a reminder that this particular approach is only looking at pitch (not pitch-class) sets and not relative pitch or rhythm, which may be more appropriate if our ultimate goal was analyzing Mozart. The reds and yellows around $(110, 64)$ indicate that the end of the

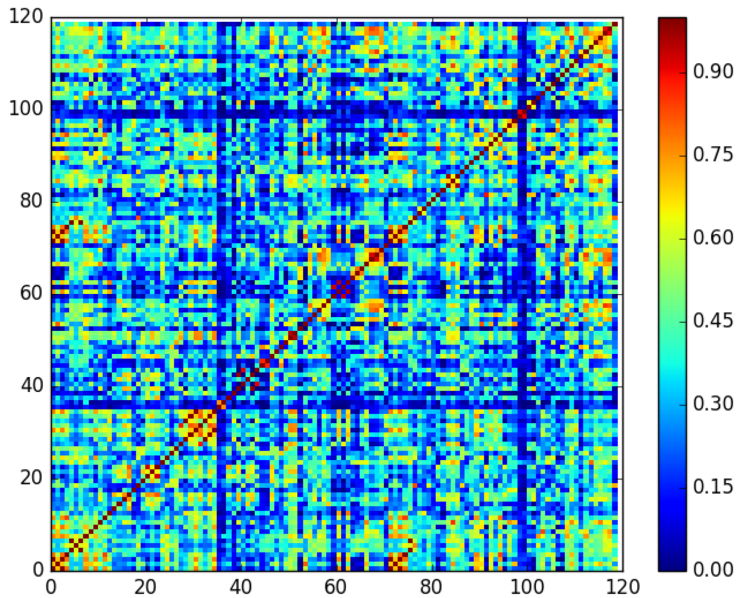


Figure 13: SSM for pitches in Mozart string quartet K155, first movement.

piece borrows from the end of the development.

While this approach seems redundant for a Mozart string quartet, since the form and underlying logic of the piece are accessible to any listener familiar with classical-era sonatas, it serves as an example of the sorts of things this methodology prioritizes, namely direct quotation or rearrangements of the same pitch sets, and ignores, like transpositions of the same melody, and will contrast with the very different-looking SSM of the Xenakis.

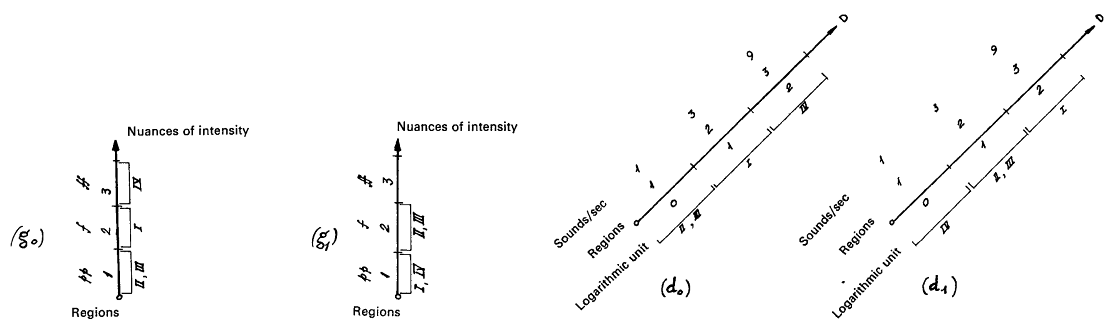
We create three self-similarity matrices from *Analogique A*, one for each dimension that Xenakis specifies, shown in Figure 15. The pitch matrix makes the protocols analyzed (the first six of the ten) very visible. It looks rather different from the Mozart SSM. Namely, in general the matrix is more blue, meaning it is less self-similar, but the greenish/yellow sections (the second and last protocol) at $[30, 60]$[26] and $[160, 190]$ are more self-similar. In fact, those self-similar sections are the perturbations, P_A^0 and P_C^0, which are entirely homogeneous and use the same pitchset, which makes them similar. Each protocol isn't

[26]The notation I use for ranges of numbers is conventional in mathematics to refer to continuous ranges, $[a, b]$ means from a to b, inclusive. These can be used to specify squares, where $([a, b], [c, d])$ means within the square with lower left corner at point (a, c) and upper right corner at (b, d).

Figure 14: Score excerpts from Mozart K155 First movement, m. 32-40, 1-4, 72-76

perfectly self-similar, though, which indicates that the pitches are differently distributed within each half-measure. The bands of color in the $([30, 60], [0, 30])$ region, for example, show which screens have that f_0 pitchset and which don't.

The intensity and density SSMs tell different stories. The $[30, 60]$ and $[160, 190]$ ranges are almost perfectly self-similar, which is bizarre given screen A and screen C should use different intensities. Closer examination of the data supports this point: in protocol P_A^0, almost all of the notes are *pianissimo* (between 80% and 100% of the notes in each screen), even though in screen A, which uses intensity g_0 and density d_0 (Figure 6, partially reproduced below), *pianissimo* notes should be at a density of about 2 per screen (14%) and *fortissimo* notes should be around 9 (64%). The density SSM is almost perfectly homogeneous, except for a couple of blue lines, which indicate empty measures (which have fermati to maintain the right overlaps with the electronic sounds), and the last protocol, which has surprisingly inconsistent density.



Reproduction from Figure 6.

We can reach an important conclusion from this data through comparison with the charts from *Formalized Music* displayed in figure 6. Unexpected intensities could always be due to an error in the score (if a *pianissimo* note was supposed to be followed with several *forte* notes, but that *forte* was not written, all of those notes would be interpreted as *pianissimo*), but assuming the score is correct, unusual intensity distributions raise a lot of questions. The last protocol P_B^0 should have a constant density, since it uses only one screen, but is not particularly self-similar.[27] Since Xenakis' product is not consistent with what it is

[27] At first, this made me doubt my math and search for a mistake, but there is no mistake. For context, since I can't use the vector dot product on scalar density values to calculate their similarity, I use a/b where

supposed to be given his process, either he used some other procedure, different from the one he described, to construct the piece or followed no procedure at all.

Failed Clustering Attempts

One early goal of this project was to use K-means clustering, a technique to separate data into categories based on similarity, to label screens, then analyze the real populations of screens versus the ideal probability distributions in *Formalized Music* for each protocol. Clustering didn't work nearly as well as SSMs, though, because of the inherent coupling of dimensions: two screens using different intensity distributions and the same density distribution might be more similar than two screens using the same intensity distribution and different density distributions.

Attempting to solve this problem, we first split the piece based on density, then tried clustering the low-density and high-density segments separately, but none of the screens had densities as high as Xenakis' description of the density dimension would have me believe (the maximum screen density was 15.77 sounds/sec the minimum was 1.8 sounds/sec and the average was 10.9 sounds/sec, when d_0 should have 14 sounds/sec and d_1 should have 16 sounds/sec).

There are three potential explanations for this inconsistency. First, Xenakis' conception of sound density could be based on the distances between sonic events, and not the number of sonic events in a screen (e.g. a couple notes in quick succession surrounded by silence are high sonic density, even though most of the screen is empty). This interpretation seems unlikely, since Xenakis was a trained engineer and would know the standard physical definition of density. Second, Xenakis' definition of density could be entirely based on relative density. This interpretation is more likely, given Formalized Music, since Xenakis discusses density in terms of perception, but still seems unusual given the strictly quantitative sonic densities of *Analogique B*. Third, Xenakis could have composed the piece intuitively, labeling density both explicitly and incorrectly for some unknown reason. While we can never reject the

a is the smaller value between the two screens.

hypothesis that the methods in Formalized Music are fabricated and unrelated to Xenakis' actual composing, embracing that skepticism seems like jumping to conclusions.

Summary

We used several different lenses, computational and non-computational, for analysis at different levels of granularity (examining the form of the whole piece, the individual protocols and the individual screens), with the ultimate goal of showing the lack of consistency between the process described in *Formalized Music* and the product of the piece.

Examining the piece on a large scale reveals a great deal of compositional features Xenakis didn't discuss, such as the dialectic between electronics and acoustics and the changes of string playing technique in sync with changes in protocol. Digitizing the score allowed us to mostly confirm Xenakis' claims about the micro-structure of the piece, but see a slight bias towards one whole tone scale, which supports the claim that the micro-structure was composed without stochastic techniques. Self-similarity matrices revealed a strange and inconsistent approach to density that is directly at odds with *Formalized Music* and made clustering impossible.

These analytical results make it seem incredibly likely that the notes and rhythms of the piece were composed entirely from intuition and do not demonstrate any elements of stochastic composition. That means that Hagan's observation that his recreation of the piece sounded less intentional than the original was entirely founded and DiScipio's thesis that the piece presents a unique dialectic between formalization and intuition holds more conclusively than he claims. As such, the piece exemplifies Xenakis' ideas at only the ordering of the screens, and nowhere else, making the stochastic-ness of the piece difficult to perceive; which is likely central to why Xenakis considered the piece a failure.

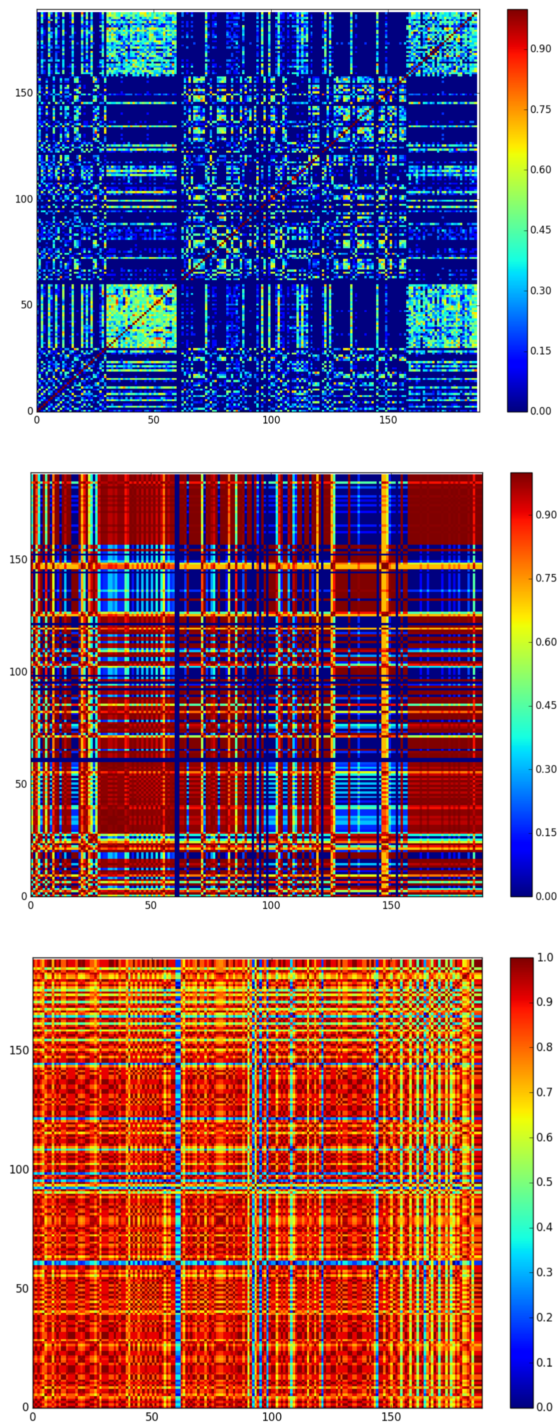


Figure 15: Self-similarity matrices for pitch, intensity and density, respectively, across the first 180 screens (90 measures) of *Analogique A*.

Conclusion

In this paper, we investigated the process and product of one of Iannis Xenakis' first pieces of stochastic music and found its formalization in Xenakis' writing incomplete and inaccurate. These issues support the conclusion that every component of the composition except for the ordering of the screens was composed intuitively in order to appear random.

Analogique A et B is a wonderful study into how formalization in music can have unexpected consequences. Xenakis invented a complex conceptual framework for music and composed using it in an attempt to investigate stochastic forms and second-order sonority. The resulting piece, though, did not successfully demonstrate either of those concepts (by Xenakis' standards of success) because they were inaudible. Instead, Xenakis created an exploration of dialectics in composition.

This interpretation resonates much better with my understanding of the piece. *Analogique A et B* contains tightly-woven dialectics on every level: the acoustic sounds of *A* versus the electronics of *B*, the low entropy of the perturbations versus the high entropy of equilibrium and the structured randomness of stochastic music versus the unstructured deliberateness of intuitive composition. For me at least, that makes the piece an extremely compelling work of art, and examining the holes and inaccuracies in its formalization only strengthens that narrative.

Bibliography

Chen, Ya-Ling. "Erratum Musical, 1913." *Tout-fait: The Marcel Duchamp Studies Online Journal*. `http://www.toutfait.com/issues/issue_1/Music/erratum.html`

Cuthbert, Michael Scott and Christopher Ariza. "music21: A Toolkit for Computer-Aided Musicology and Symbolic Music Data." in J. Stephen Downie and Remco C. Veltkamp (Eds.). 11th International Society for Music Information Retrieval Conference (ISMIR 2010), August 9-13, 2010, Utrecht, Netherlands. pp. 637-642.

Di Scipio, Agostino. Formalization and Intuition in Analogique A et B Definitive Proceedings of the "International Symposium Iannis Xenakis" Athens, 2005, http://iannis-xenakis.org/Articles/Di%20Scipio.pdf.

Foote, Jonathan. "Visualizing Music and Audio Using self-similarity." in Proceedings of the ACM Multimedia 1999, Orlando, FL, pp. 77-80

Glimne, Dan. *Encyclopedia Britannica*, 15th ed., s.v. "Dice." 2006. Accessed online at https://www.britannica.com/topic/dice

Hagan, Kerry. 2005. "Genetic Analysis of Analogique B." Paper presented at Electroacoustic Music Studies Network, Montreal, October 2005. http://www.ems-network.org/IMG/EMS2005-Hagan.pdf

Kell, Thor. Mistify: Dynamically Recreating Xenakis' Mists Proceedings of the 9th Conference on Interdisciplinary Musicology CIM14. Berlin, Germany 2014. http://tide-pool.ca/papers/Thor%20Kell%
%20CIM%202014.pdf.

Keller, Damian, Fenneyhough, Brian. Analysis by Modeling: Xenakis's ST/10-1 080262 Journal of New Music Research, 33:2, 161-171. http://dx.doi.org/10.1080/0929821042000310630.

Kollias, Phivos-Angelos. 2011. "Iannis Xenakis and Systems Thinking" Proceedings of the Xenakis International Symposium, London, 2011.

Martorell, Agustin, Gomez, Emilia. Systematic multi-scale set-class analysis, in Proceedings of the 15th International Society for Music Information Retrieval Conference, 2014.

Mozart, Wolfgang Amadeus. *Musikalisches Würfelspiel* K516f. Bonn: N. Simrock, n.d.(1793).

Solomos, Makis "Xenakis' Thought through his Writings", Journal of New Music Research, 33(2), 2004, p.125–136.

Squibbs, Ronald J., An Analytical Approach to the Music of Xenakis PhD diss., Yale University, 1996.

Wannamaker, Robert. 2001. "Structure and Perception in *Herma* by Iannis Xenakis." Music Theory Online 7 (9). http://www.mtosmt.org/issues/mto.01.7.3/mto.01.7.3.wannamaker.html

Xenakis, Iannis "Analogique A." Paris: E.F.M. Technisonor, 1967.

—— Formalized Music; Thought and Mathematics in Composition, p. 5. Bloomington, IN: Indiana University Press, 1971.

—— *Iannissimo!*. Charles Zacharie Bornstein STX ensemble. Vandenburg VAN 0003. 1997, CD.