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ORIGINAL ARTICLE

Cylindrically symmetric cosmological model of the universe in modified gravity

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Abstract In this paper, we have constructed the cosmological models of the universe in a cylindrically symmetric space time in two classes of $f(R,T)$ gravity (Harko et al. in Phys. Rev. D 84:024020, 2011). We have discussed two cases: one in the linear form and the other in the quadratic form of R. The matter is considered to be in the form of perfect fluid. It is observed that in the first case, the pressure and energy density remain the same, which reduces to a Zeldovich fluid. In the second case we have studied the quadratic function of $f(R,T)$ gravity in the form $f(R) = \lambda(R + R^2)$ and $f(T) = \lambda T$. In the second case the pressure is in the negative domain and the energy density is in the positive domain, which confirms that the equation of state parameter is negative. The physical properties of the constructed models are studied.

Keywords Modified gravity · Perfect fluid · Space-time · Accelerated expansion

1 Introduction

Recent cosmological observations on the accelerated expansion of the universe gives a new dimension in the study of cosmology. A lot of observations (Riess et al. 1998; Perlmutter et al. 1999; Seljak et al. 2005; Eisenstein et al. 2005; Spergel et al. 2007; Komatsu et al. 2009) have already confirmed the late time cosmic dynamics. It is also believed that the transition from a decelerated phase to an accelerated phase has been occurred at a transition redshift $z_{da} \sim 1$ (Farooq and Ratra 2013; Capozziello and Luongo 2014). The reason of late time dynamics is still unknown. In order to study this, two different approaches can be used: first, the cosmic speed up is explained through the inclusion of an exotic dark energy in the matter field in General Relativity. The dark energy, usually represented by a cosmological constant, corresponds to an isotropic fluid with almost constant energy density and negative pressure. Many alternative models have also been proposed in recent times: canonical scalar fields like quintessence (Ratra and Peebles 1988; Sahni and Starobinsky 2000), phantom fields (Caldwell 2002), k-essence (Picon et al. 2000, 2001), tachyons (Sen 2002), quintom (Feng et al. 2005; Guo et al. 2005); parametrised dark energy candidates such as ghost dark energy (Urban and Zhitnitsky 2009, 2010; Ohta 2011), holographic dark energy (Li 2004), Ricci dark energy (Cao et al. 2009) and agegraphic dark energy (Cai 2007; Wei and Cai 2008) or a consideration of a unified dark fluid (Ananda and Bruni 2006; Xu et al. 2012; Tripathy et al. 2015). Dark energy is required to explain the cosmic speed up and dark matter is required to explain the emergence of Large Scale Structure in the universe. According to the Planck data (Ade et al. 2014a, 2014b, 2014c), the dark energy leads the cosmic mass-energy budget with a lion share having $\Omega_\Lambda = 0.691 \pm 0.006$ followed by dark matter having $\Omega_{dm} = 0.259 \pm 0.005$. However, the lack of a comprehensive understanding of the nature and behavior of these components either in particle form or of scalar fields has triggered to look for alternative theories of gravity (Martino et al. 2015). Further, the need of two unknown components to explain physical phenomena can be interpreted as a break down of the very theory at astrophysical scales (Martino et al. 2015; Capozziello and Laurentis 2011). Bamba et al. (2012) have introduced a general class of generalized holographic dark energy, where they have also explained that

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ghost dark energy in a particular case of an in-homogeneous EoS fluids.

Second the modification of the geometrical part of the gravitational interaction by including higher order curvature invariant in the Einstein–Hilbert action. Among various ways to modify the gravitation theory, $f(R)$ theory is a straightforward approach, where the Ricci curvature scalar R in the action is replaced by more general function of R (Capozziello et al. 2003; Nojiri and Odintsov 2003, 2011; Carroll et al. 2004; Nojiri et al. 2006; Sotiriou and Liberati 2007; Sotiriou and Faraoni 2010; Felice and Tsujikawa 2010; Capozziello and Laurentis 2011). $f(R)$ gravity models have been widely used with emphatic results and success in stellar formation and evolution (Astashenok et al. 2015; Laurentis and Martino 2013), structure formation and evolution of the universe (Abdelwahab et al. 2012; Carloni et al. 2009; Nojiri and Odintsov 2014; Bamba and Odintsov 2015), cluster of galaxies (Schmidt et al. 2009; Ferraro et al. 2011; Terukina and Yamamoto 2012), gravitational waves and massive gravitation (Clifton and Barrow 2010; Laurentis and Capozziello 2011).

Recently, Harko et al. (2011) proposed a modified theory of gravity dubbed as $f(R,T)$ gravity. This theory is a generalization of $f(R)$ theory where the gravitational Lagrangian is taken as an arbitrary function of the Ricci scalar R and the trace T of the stress energy tensor. The gravitational field equations are obtained in metric formalism, which follow from the covariant divergence of the stress energy tensor. The dependence of trace T may be introduced by exotic imperfect fluids or quantum effects (conformal anomaly). The covariant divergence of the stress energy tensor is zero and as a result the motion of test particles is non-geodesic. An extra acceleration is always present due to the coupling between matter and geometry.

Recently many authors have investigated different issues in cosmology with late time acceleration in $f(R,T)$ gravity. Jamil et al. (2012) reconstructed some cosmological model in $f(R,T)$ gravity and it was concluded that the dust fluid reproduced ΛCDM, phantom non-phantom era and the phantom cosmology. It is worthy to note that the first law of black hole thermodynamics is violated for $f(R,T)$ gravity. Reddy et al. (2013) have investigated a spatially homogeneous and anisotropic Bianchi type-V cosmological model in $f(R,T)$ theory. Mishra and Sahoo (2014) have investigated Bianchi type VI_h cosmological model in this theory. The model obtained by them in consistence with the present day accelerated expansion of the universe, when $h=1$. Shamir (2015) have obtained the exact solutions of Bianchi type V metric in modified gravity. Recently Mishra et al. (2015) have constructed non-static cosmological model in $f(R,T)$ gravity and dealt with the higher order of the trace of energy momentum tensor. Most recently Mishra et al. (2016) have studied the dynamics of an anisotropic universe in $f(R,T)$ gravity using a re-scaled functional.

In this paper, we are intending to investigate the $f(R,T)$ gravity in a cylindrically symmetric space-time. The matter field is considered to be in the form of perfect fluid. The arrangement of the paper is as follows: Sect. 1 describes introduction and the basic formalism of $f(R,T)$ gravity. In Sect. 2, we have set up the field equations and derived its exact solutions. In Sect. 3, the physical properties of constructed models are studied and concluding remarks are given in Sect. 4. At the end a list of references has been added which are refereed for the study.

The field equations of $f(R,T)$ gravity are derived from the Hilbert Einstein type variational principle. The action for the modified $f(R,T)$ gravity is (Harko et al. 2011)

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \tag{1}$$

Here $f(R,T)$ is an arbitrary function of Ricci scalar R, T is the trace of stress energy tensor T_{ij}, L_m is the matter Lagrangian density. We can define the energy momentum tensor as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2}$$

The matter Lagrangian is assumed to be dependent only on the metric tensor g_{ij} and not on the derivatives. So the stress energy tensor of the matter is defined as

$$T_{ij}=g_{ij}L_m-\frac{\delta L_m}{\delta g^{ij}} \tag{3}$$

where the trace $T=g^{ij}T_{ij}$

The $f(R,T)$ field equations are obtained by varying the action S with respect to the metric tensor g_{ij} as

$$\begin{aligned} &f(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\Box-\nabla_i\nabla_j)f_R(R,T)\\ &\quad=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\theta_{ij} \end{aligned} \tag{4}$$

where,

$$\theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{\alpha\beta}} \tag{5}$$

and

$$f_R(R,T)=\frac{\partial f(R,T)}{\partial R};\qquad f_T(R,T)=\frac{\partial f(R,T)}{\partial T} \tag{6}$$

$$\Box\equiv\nabla^i\nabla_j \tag{7}$$

and ∇_i denotes covariant derivative.

On contraction, Eq. (4) can be obtained as,

$$\begin{aligned} &f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)\\ &\quad=8\pi T-f_T(R,T)(T+\theta) \end{aligned} \tag{8}$$

Here $\theta = \theta_i^i$ From Eq. (6), a relation can be derived between Ricci scalar R and the trace T of the energy momentum tensor.

From (5), the variation of the stress energy is obtained. Using matter Lagrangian L_m, the stress energy tensor of matter is given by

$$T_{ij} = (\rho + p)u_{ij} - p g_{ij} \tag{9}$$

where $u^i = (0, 0, 0, 1)$ is the four velocity in comoving coordinates where $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$. ρ is the energy density and p is the pressure density of the fluid. L_m can be considered as $-p$ as there is no unique definition of Lagrangian matter.

From (5), the variation of stress energy of perfect fluid is obtained and is given by

$$\theta_{ij} = -2T_{ij} - p g_{ij} \tag{10}$$

Since the field equations depend on the tensor θ_{ij} also, in the case of $f(R, T)$ gravity depending on nature of matter source, several theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are obtained. There are three classes of these models given by Harko et al. (2011)

$$\begin{aligned} f(R, T) &= R + 2f(T) \\ &= f_1(R) + f_2(T) \\ &= f_1(R) + f_2(R) f_3(T) \end{aligned} \tag{11}$$

From Eq. (4), we can obtain the gravitational field equations of $f(R, T)$ gravity as

$$\begin{aligned} R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} \\ + \left[2pf'(T) + f(T)\right] g_{ij} \end{aligned} \tag{12}$$

where, prime denotes differentiation with respect to argument.

2 Field equations and its solutions

We have considered the cylindrically symmetric space time in the form

$$ds^2 = E^2 dt^2 - G^2\left(dx^2 + dy^2\right) - E^2 dz^2 \tag{13}$$

along with the convention $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = t$. E and G are the functions of cosmic time t. The matter tensor for the perfect fluid θ_{ij} is obtained is

$$\theta_{ij} = -2T_{ij} - p g_{ij} = (\rho, -p, -p, -p) \tag{14}$$

2.1 Case-I

In this case, we have considered the first class (Harko et al. 2011) in the form $f(R) = R$; $f(T) = \lambda T$ such that $f(R, T) = R + 2\lambda T$, where λ is a constant. Hence, the field equations for $f(R, T)$ gravity can be obtained as:

$$\frac{1}{E^2}\left[\frac{G_{44}}{G} + \frac{E_{44}}{E} - \frac{E_4^2}{E^2}\right] = (8\pi + 3\lambda)p - \lambda\rho \tag{15}$$

$$\frac{1}{E^2}\left[\frac{G_4^2}{G^2} - \frac{2G_4E_4}{GE} + \frac{2G_{44}}{G}\right] = (8\pi + 3\lambda)p - \lambda\rho \tag{16}$$

$$\frac{1}{E^2}\left[-\frac{G_4^2}{G^2} - \frac{2G_4E_4}{GE}\right] = (8\pi + 3\lambda)\rho - \lambda p \tag{17}$$

The suffix 4 after a field variable denotes ordinary differentiation with respect to t. Since the number of unknowns is more than the number of equations, we need an extra condition to solve the field equations. Therefore, we consider a relationship between the field variables in the form $E = G^m$, where m is a scalar. Now, from Eqs. (15)–(17), we get,

$$E = G^m = (k_1 t + c_1)^{\frac{m}{2}} \tag{18}$$

where k_1, c_1 are constants. Subsequently the space-time (12) can be obtained as

$$\begin{aligned} ds^2 = (k_1 t + c_1)^m dt^2 - (k_1 t + c_1)\left(dx^2 + dy^2\right) \\ - (k_1 t + c_1)^m dz^2 \end{aligned} \tag{19}$$

The pressure and the energy density for case-I can be obtained as,

$$p = \rho = \frac{(-2m - 1)k_1^2}{4(k_1 t + c_1)^{m+2}(8\pi + 2\lambda)} \tag{20}$$

Since the energy density cannot be negative, either the constant m is restricted as $m < -\frac{1}{2}$ or the value of the cosmological constant λ need to be constrained.

In this model, from Fig. 1 we observe that $p \geq 0$ and $\rho \geq 0$. The variation of pressure and density has been shown with cosmic time depending upon the values of the parameter λ (gravitational constant). It is also observed that the pressure and density remain non negative for a particular range of values of λ, i.e., $\rho, p \geq 0$ for $\lambda \in (-\infty, -12.58]$. The large values of ρ and p in the beginning suggest that pressure and density dominate the early universe but for sufficiently large time, they became negligible. We also found that the energy density and pressure are gradually decreasing functions of time. They both attains positive constant value and latter vanishes at $t \to \infty$. But if λ doesn't belong to the preferred range, then strong negative pressure may be found, which shows that the universe is dominated by dark energy (DE) at late time causing the accelerated expansion of the

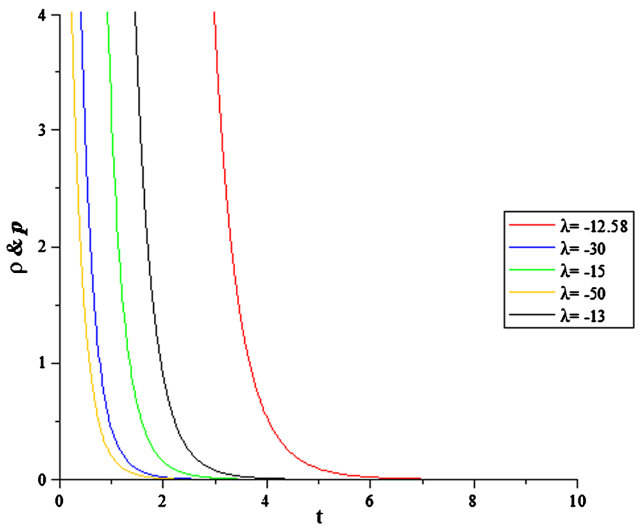

Fig. 1 Variation of pressure and density with cosmic time for $f(R,T)$ model 1 for four representative values of λ (gravitational constant)

universe. These observations show similar behavior with the FLRW universe at infinite time. We have also seen that, if we consider $f(R)=\lambda R$, for $\lambda>1$, the model behaves similar as in $f(R)=R$. So, the first two classes of Harko et al. (2011) is showing similar behavior in this space time.

2.2 Case II

In the previous case, we have observed that the model behaves similar in different consideration of the function in the linear form. In this case, we have considered the quadratic form of the function, where $f(R)=\lambda(R+R^2)$ and $f(T)=\lambda T$. With this assumption, the set of field equations for $f(R,T)$ gravity can be formed as

$$\frac{1}{E^2}\left[\frac{G_{44}}{G}+\frac{E_{44}}{E}-\frac{E_4^2}{E^2}\right] + \frac{1}{E^4}\left[\left(\frac{4G_{44}}{G}+\frac{2G_4^2}{G^2}+\frac{2E_{44}}{E}-\frac{2E_4^2}{E^2}\right) \times\left(\frac{E_{44}}{E}-\frac{E_4^2}{E^2}-\frac{G_4^2}{G^2}\right)\right]=\alpha p-\Lambda \tag{21}$$

$$\frac{1}{E^2}\left[\frac{G_4^2}{G^2}-\frac{2E_4G_4}{EG}+\frac{2G_{44}}{G}\right] + \frac{1}{E^4}\left[\left(\frac{4G_{44}}{G}+\frac{2G_4^2}{G^2}+\frac{2E_{44}}{E}-\frac{2E_4^2}{E^2}\right) \times\left(\frac{2G_{44}}{G}+\frac{G_4^2}{G^2}+\frac{E_4^2}{E^2}-\frac{E_{44}}{E}-\frac{4E_4G_4}{EG}\right)\right] =\alpha p-\Lambda \tag{22}$$

$$\frac{1}{E^2}\left[-\frac{G_4^2}{G^2}-\frac{2G_4E_4}{GE}\right] + \frac{1}{E^4}\left[\left(\frac{4G_{44}}{G}+\frac{2G_4^2}{G^2}+\frac{2E_{44}}{E}-\frac{2E_4^2}{E^2}\right) \times\left(-\frac{E_{44}}{E}+\frac{E_4^2}{E^2}-\frac{G_4^2}{G^2}-\frac{2E_4G_4}{EG}\right)\right]=\alpha\rho+\Lambda \tag{23}$$

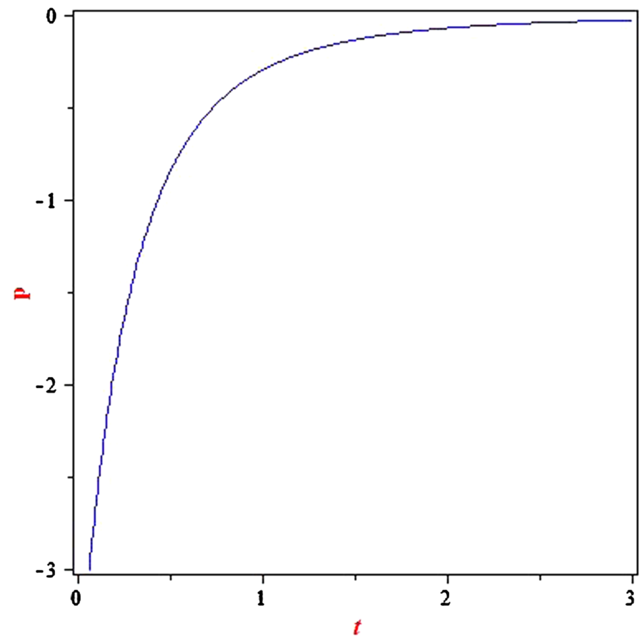

Fig. 2 Pressure vs. cosmic time

The above set of field equations are highly non-linear with four unknowns with three equations. In order to have a deterministic solution, we have considered a relation between the metric potential in the form of $E=G^m$, where m is a constant. On solving, we obtained

$$E=G^m=\left[\frac{2(1+m)}{3}(k_2t+c_2)\right]^{\frac{2m(1+m)}{3}} \tag{24}$$

where k_2, c_2 are integrating constants. Subsequently, we obtain the pressure and energy density for this model and observed that unlike in the previous case, in this case the pressure and energy density are not same. This indicates that the model is not static and the equation state parameter found to be negative, which further indicates the accelerated expansion of the universe. In the following Figs. 2 and 3, we have represented the pressure and energy density curve for a representing value of $m=0.732$. One candidate is the dark energy and the other is through the cosmological constant. Since in this case the ratio of pressure and energy density are negative, it is observed that the pressure is in the negative domain and in increase in time the energy density decreases but in the positive domain for the representing value of the constant $m=0.732$. The details are shown in Figs. 2 and 3 below.

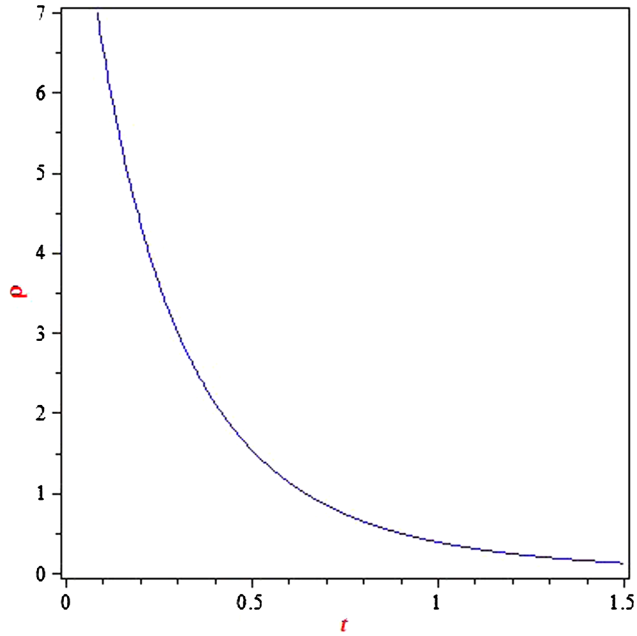

Fig. 3 Energy density vs. cosmic time

3 Physical properties of the model

In this section, we will discuss the physical parameters of the model constructed in Sect. 2. For the first case: the volume and average scale factors for the model can be obtained respectively as, $V=\sqrt{(-g)}=(k_1t+c_1)^{m+1}$ and $R=V^{\frac{1}{3}}=(k_1t+c_1)^{\frac{m+1}{3}}$. It has been observed that the spatial volume V is zero at $t=-\frac{c_1}{k_1}$ and increases with time. At this epoch, the expansion scalar is infinite which shows the universe starts evolving with a constant amount of volume ($V=c_1^{m+1}$) at $t=0$, with big bang scenario. As $t\to\infty$, the volume becomes infinite. Also, we find that the metric potential or the spatial scale factors are alike and are zero at $t=-\frac{c_1}{k_1}$. Hence, each of these models has a point-type singularity. At this epoch, all physical and kinematic parameters diverge. Both the volume and average scale factor for the models increases with increase in time and vanish at $t=-\frac{c_1}{k_1}$. It is found that at $t\to 0$, $R\to c_1^{\frac{m+1}{3}}$ and at infinite time the scalar curvature vanishes.

The Hubble parameter and the scalar expansion for the models can be obtained as, $H=\frac{k_1}{3}(\frac{m+1}{k_1t+c_1})$ and $\Theta=3H=k_1(\frac{m+1}{k_1t+c_1})$. The graph of Hubble's parameter with respect to cosmic time has been constructed as in Fig. 4 for the represented values of the constants. Figure 4 shows when time increases the Hubble parameter decreases and vice-versa. It may be noted that for $m=-1$, the Hubble parameter vanishes and when the metric potential vanishes, it remains undefined. As $t\to\infty$, the Hubble parameter and expansion scalar attain a constant value.

The deceleration parameter q which is a measure of the cosmic accelerated expansion of the universe and the behavior of the model is governed by the sign of the parameter. In this case, the deceleration parameter is found to be $q=-(\frac{m-2}{m+1})\frac{1}{(k_1t+c_1)^{\frac{m+1}{3}}}$. It is observed here from Fig. 5 that for $m>2$, the deceleration parameter is negative and it decreases in the increase in time.

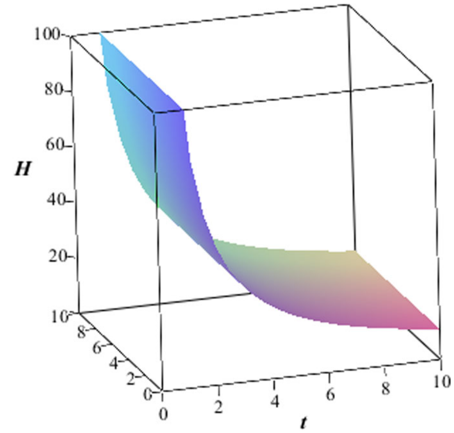

Fig. 4 Evolution of Hubble's parameter (H) w.r.t. cosmic time (t) for $k_1=0.2$, $m=10$ and $c_1=0.5$

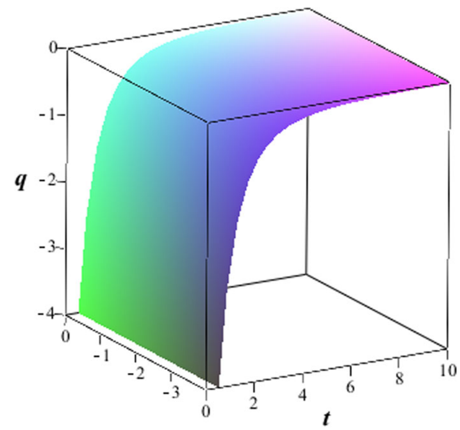

Fig. 5 Variation of Deceleration parameter (q) with cosmic time (t) for $m=10$, $k_1=0.25$ and $c_1=0.5$

In order to understand the geometrical nature of the universe, the state finder diagnostic has been calculated as $r=\frac{R_{444}}{RH^3}=\frac{(m-2)(m-5)}{(m+1)^2}$ and $s=\frac{r-1}{3(q-\frac{1}{2})}$. It has been found that for $m=1$, the state finder pair having value $[1,0]$, which behaves like the ΛCDM model. For other value of m, r is still remain a constant; however the value of s depends on the time t.

In the second case, we obtain the average scale factor and volume as $V=R^3=[\frac{2}{3(1+m)}\cdot(k_2t+c_2)]^{\frac{2(m+1)^2}{3}}$. This indicates that when $t\to 0$, $V\to$ constant and the volume becomes infinite in infinite future. The average scale factor is also behaving similar. Both the volume and average scale factor for the models increases with increase in time and vanish at $t=-\frac{c_2}{k_2}$. The metric potential vanishes at $t=-\frac{c_2}{k_2}$,

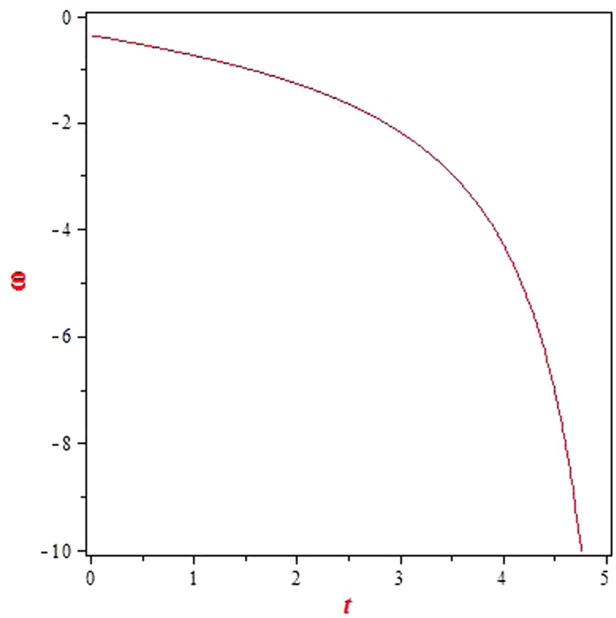

Fig. 6 EoS parameter vs. cosmic time

which is again similar as in the previous case. Hence, each of these models has a point-type singularity.

The scalar expansion and Hubble parameter can be obtained as $\Theta = 3H = \frac{8k_2}{9}(k_2 t + c_2)$. So, in increase in time, the Hubble parameter increases and both scalar expansion and Hubble parameter vanishes at $t = -\frac{c_2}{k_2}$; for $k_2 \neq 0$. The deceleration parameter is found to be $-(\frac{2m^2+4m-7}{2m^2+4m+2})$, which is independent of the cosmic time. However, for an large value of m, the deceleration parameter approaches to the value -1.

For the same representing value of $m = 0.732$, the equation of state parameter ($\omega = \frac{p}{\rho}$) is found to be negative. The model initially starts from the quintessence region and with increase in time it enters in to the phantom region. The graphical representation is as in Fig. 6.

To assesses the geometric features of the model, the state finder diagnostic pair to be analyzed. The pair can be obtained as $r = \frac{R_{444}}{RH^3}$ and $s = \frac{r-1}{3(q-\frac{1}{2})}$. It is found that for the same representing value of $m = 0.732$, the state finder diagnostic pair $[r, s] = [1, 0]$.

4 Concluding remarks

In this paper, we have presented a cylindrically symmetric cosmological model in $f(R,T)$ gravity in two forms such as $f(R,T) = R + 2\lambda T$; $f(R,T) = \lambda(R + R^2) + \lambda T$. In the first model, the pressure and energy density are found to be same, so the Zeldovich fluid model is directly obtained from the field equations. It has been observed that the metric potential in both this case vanishes at the constant time $t = -\frac{c_1}{k_1}$ for case I and $t = -\frac{c_2}{k_2}$ for case II. Also, the energy density is found to be initially constant and vanishes at infinite future.

In the second model, we assume the quadratic form of Ricci scalar. Here, the behavior of the pressure and energy density are different in the sense that they are not same. The pressure is in the negative domain; however the energy density is in the positive side. Further, the Equation of State parameter is found to be negative, which is also indicating the accelerated expansion of the universe. This singular model of the universe possesses a point singularity when $t \equiv t_s = -\frac{c_2}{k_2}$. The volume scale factor and metric potentials vanishes at the singular point. The cosmological parameters H, Θ, q, r, s are all infinity at the point of singularity. All these observations suggest that the universe starts its expansion with certain volume, may be endowed with strong negative pressure from $t = t_s$ and it will continue to expand. Moreover, these observations resembles with FLRW universe at infinite time.

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