Problem 4.1

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February 21, 2019

What is Semidefinite Programming?

In semidefinite programming,one minimizes a linear function subject to the constraint that an affine combination of symmetric matrices is positive semidefinite.
Such a constraint is non linear and non smooth, but convex, so semidefinite programs are convex optimization problems.
They have a particular structure that makes their solution computationally tractable by interior-point methods.

Question 4.1

$$\min_{x} f(\mathbf{x}) = x_{11} + x_{12}$$

With constraints

$$g_1(\mathbf{x}) = x_{11} + x_{22} = 1$$
$$g_2(\mathbf{x}) = \mathbf{X} \succeq 0$$

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{pmatrix}$$

Solution

cvxopt solver is used to find the solution.Reformulate the given problem as,

$$\min_{x} \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \\ x_{22} \end{pmatrix}$$

$$\text{s.t } \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \\ x_{22} \end{pmatrix} = 1$$

$$x_{11} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} + x_{12} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_{22} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \preceq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

General Format

Minimize Cx

Subject to Ax $=$ b

and Gx $\leq h$

Code

```
from cvxopt import matrix
from cvxopt import solvers
c = matrix([1.,1.,0.])
G = [ matrix([[-1., 0., 0., 0.],[ 0., -1., -1., 0.],[0., 0., 0., -1.]]) ]
Aval = matrix([1.,0.,1.],(1,3))
bval = matrix([1.])
h = [ matrix([[0., 0.], [0., 0.]]) ]
sol = solvers.sdp(c, Gs=G, hs=h,A=Aval, b=bval)
print(sol['x'])
print(sol['x'][0]+sol['x'][1])
print('found at' ,sol['x'][0] ,'and' ,sol['x'][1])
```

Solution

```
     pcost       dcost       gap    pres   dres   k/t
 0:  5.0000e-01  5.0000e-01  2e+00  3e-16  2e+00  1e+00
 1:  2.9243e-02  2.7087e-02  2e-01  5e-16  2e-01  1e-01
 2: -2.0474e-01 -2.0483e-01  4e-03  2e-16  3e-03  2e-03
 3: -2.0708e-01 -2.0708e-01  4e-05  4e-16  3e-05  2e-05
 4: -2.0711e-01 -2.0711e-01  4e-07  8e-17  3e-07  2e-07
 5: -2.0711e-01 -2.0711e-01  4e-09  3e-16  3e-09  2e-09
Optimal solution found.
[ 1.46e-01]
[-3.54e-01]
[ 8.54e-01]

-0.207106778823
('found at', 0.14644661058847713, 'and', -0.35355338941152276)


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(program exited with code: 0)
Press return to continue
```

And last

Thank You