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=====
% AUTHOR ..... [Lishan Huang]
% UPDATED .... [Jan 23]
% Task 3

lagrange interpolation formula

slagrange.m

%
=====
% AUTHOR ..... [Lishan Huang]
% UPDATED .... [Jan 23]
%
% Evaluate the Lagrange interpolation formula
%
% INPUT
% tau .... The vector of interpolation nodes (length n)
% rho .... The vector of values at the interpolation nodes (length n)
% x ..... A vector of values to evaluate the interpolating polynomial
%          at (length 1 to many (probably not n!))
%
% OUTPUT
% T :
%
=====
function F = lagrange(tau, rho, x)
%initialize F=0
t=tau;
p=rho;
F=0;
%create a for loop
    for k=1:length(t)
        %make L=1 before each loop where l means Lk in the function
        L=1;
        for i=1:length(t)
            if i~=k
                L=L.*(x-t(i))./(t(k)-t(i));
            end
        end
        F=F+p(k).*L;
    end
end

```

The first form of the barycentric formula

firstbaryeval.m

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%
=====
% AUTHOR ..... [Lishan Huang]
% UPDATED .... [Jan 23]
%
% Evaluate the first form of the barycentric formula
%
% INPUT
% tau .... The vector of interpolation nodes (length n)
% rho .... The vector of values at the interpolation nodes (length n)
% x ..... A vector of values to evaluate the interpolating polynomial
%          at (length 1 to many (probably not n!))
%
% OUTPUT
% T :
%
=====

function T = firstbaryeval(tau, rho, x)
%initialize t p T and a
    t=tau;
    p=rho;
    T=0;
    a=1;
    %calculate w(x) and store in a
    for k=1:length(t)
        a=a.*(x-t(k));
    end
    %create for loop to add up the function
    for k=1:length(t)
        b=1;
        %for loop for b(k)
        for j=1:length(t)
            if j~=k
                b=b.*((t(k)-t(j))).^-1;
            end
        end
        T=T+b.*p(k)./(x-t(k));
    end
    T=T.*a;
end

```

Script

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%The following is a code that using firstbaryeval function to
%interpolate
%complex numbers and plot the p(z) and the conjugate of z by real part
%and image part separately

```

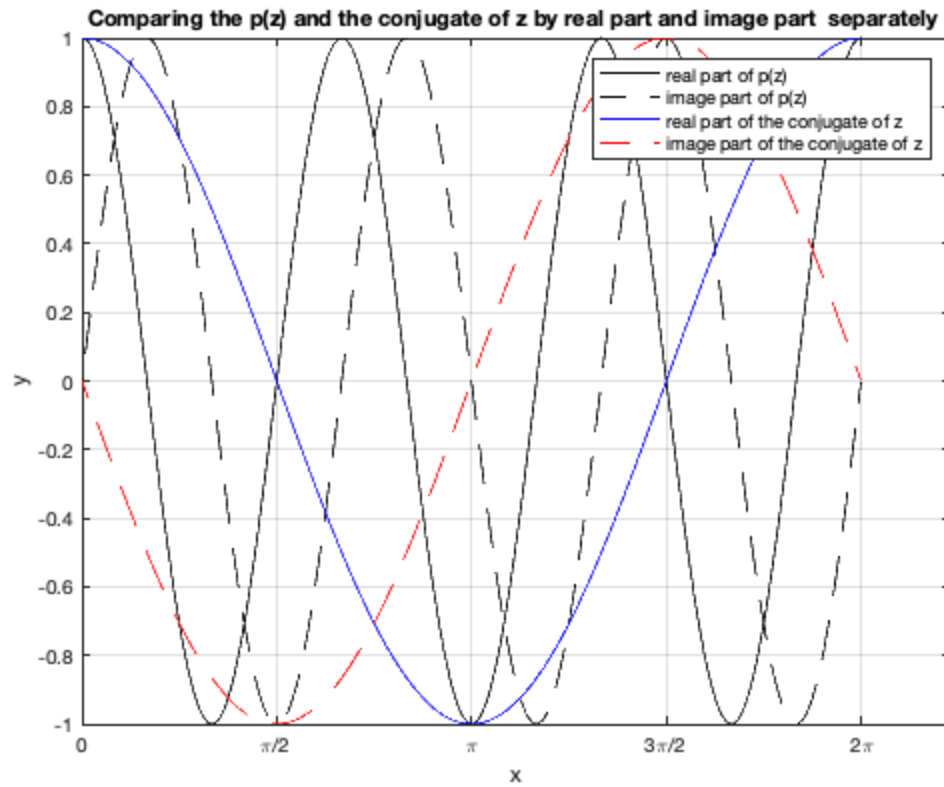
```

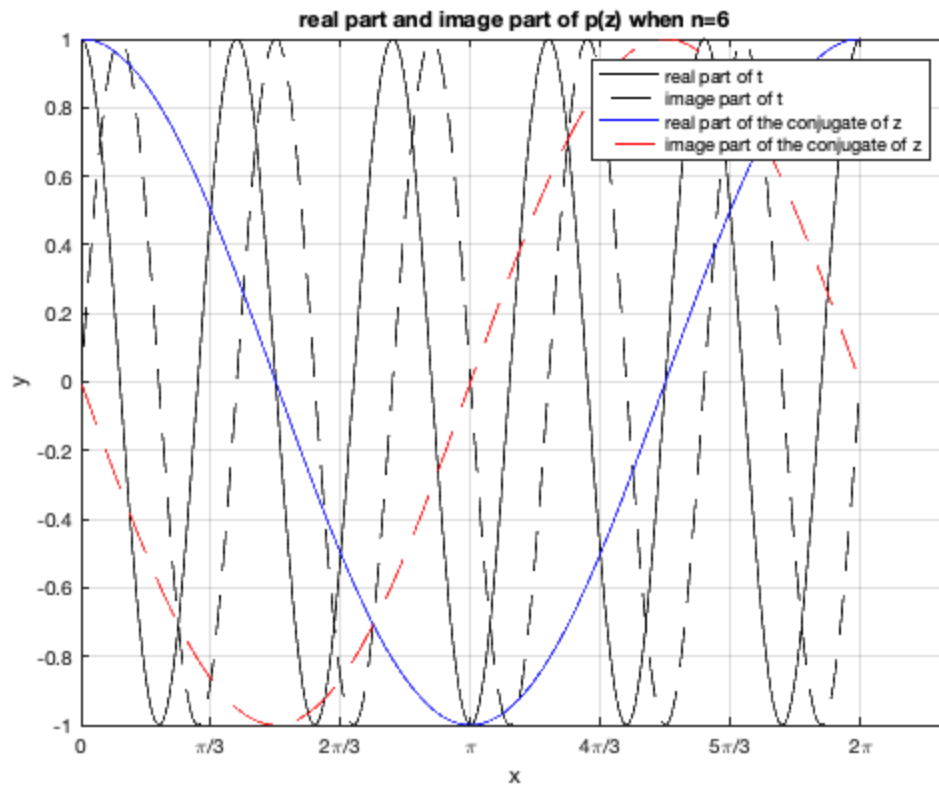
%create a vector with 2019 equal points between 0 and 2 pi
num1=linspace(0,2*pi,2019);
%create a vector with 5 equal points between 0 and 2 pi
num2=linspace(0,2*pi,5);
%define the function of z
z = @(x) cos(x)+1i*sin(x);
%define the function that inverse the image part of input
f = @(x) real(x) - 1i*imag(x);
%define tau and rho for firstbaryeval
tau=[1,1i,-1,-1i];
rho=[1,-1i,-1,1i];
%store the result of firstbaryeval in x
x=firstbaryeval(tau,rho,z(num1));
%plot the the real part of p(z) versus ? in solid black and the
    imaginary
%part of p(z) versus ? in dashed balck
plot(num1,real(x),'-k',num1,imag(x),'--k');
grid on
xticks([0 pi/2 pi 3*pi/2 2*pi])
xticklabels({'0','\pi/2','\pi','3\pi/2','2\pi'})
%keep on this graph plot other line
hold on;
%plot the real part and imaginary part of the conjugate of z
plot(num1,real(f(z(num1))), 'b',num1,imag(f(z(num1))), '--r')
%label axies
xlabel('x')
ylabel('y')
title('Comparing the p(z) and the conjugate of z by real part and
    image part separately ')
%add the legend to the graph
legend('real part of p(z)','image part of p(z)','real part of the
    conjugate of z','image part of the conjugate of z')
%create a vector with 6 equal points between 0 and 5 (n-1 where n=6)
a=linspace(0,5,6);
%define the function
tau=exp(2*pi*1i*(a)/6);
%define tau and rho
rho=(f(tau));
%store result of firstbaryeval in t
t=firstbaryeval(tau,rho,z(num1));
figure(2)
%plot the the real part of t and the imaginary part of t
plot(num1,real(t),'-k',num1,imag(t),'--k');
grid on
xticks([0 pi/3 2*pi/3 pi 4*pi/3 5*pi/3 2*pi])
xticklabels({'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'})
title('real part and image part of p(z) when n=6')
xlabel('x')
ylabel('y')

hold on;
%plot the real part and imaginary part of the conjugate of z
plot(num1,real(f(z(num1))), 'b',num1,imag(f(z(num1))), '--r')

```

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legend('real part of t','image part of t','real part of the conjugate  
of z','image part of the conjugate of z')
```





% *Text Answers*

% Do these curves agree with the polynomial interpolant at the points $\pi = 0$

%, $\pi/2$, π , and $3\pi/2$? Do they agree only at those points?

%

% Answer: Yes it agree with the polynomial interpolant at those points, and they agree at those points only, since intersection of corresponding point only occur in those points

%. What happens if you re-run your script with a higher number of points, say $n = 6$, and interpolate z

%at the points $z_k = \exp(2\pi i k/n)$ for $k = 0, 1, \dots, n - 1$?

%

% Answer: Since $e^{ix} = \cos(x) + i\sin(x)$, then $e^{(2\pi i k i/n)} = \cos(2\pi k/n) + i\sin(2\pi k/n)$ when $k=0,1,2,3,4,5$

and $n=6$, then when $k=1$: $e^{(2\pi i 0 i/6)} = \cos(0) + i\sin(0)$

$k=2$: $e^{(2\pi i 1 i/6)} = \cos(2\pi/6) + i\sin(2\pi/6) =$

$\cos(\pi/3) + i\sin(2\pi/6)$, it is obviously that this function is same as the previous $z = \cos(x) + i\sin(x)$ where $x = \text{linspace}(0, 2\pi, 6)$, and all of those interpolation points are intersect with $f(z)$, but the curves are not intersection in those point only. In addition, the period of the interpolation curve is changed and it equals to $5 = n-1$