Discussion 2 Notes

Linearity: easiest to show when proving additivity and homogeneity at the same time

i.e: $x(t) \rightarrow \boxed{\text{System}} \rightarrow y(t) = x(2t)$

1. $y_1(t) = x_1(2t)$ * important to show input/output relationships first

   $y_2(t) = x_2(2t)$

2. Let $\hat{x}(t) = \alpha x_1(t) + \beta x_2(t)$; $\alpha, \beta \in \mathbb{R}$

3. Expected output, $\hat{y}(t) = \hat{x}(2t)$

   $= \alpha x_1(2t) + \beta x_2(2t)$

Does $\hat{y}(t) = \alpha y_1(t) + \beta y_2(t)$? If so, then system is linear

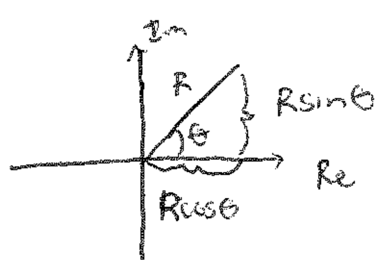
This just basically means that putting 2 scaled inputs into the system results in the same output as if you put the 2 inputs individually

Trig Identity

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$. easier to add sinusoids this way.

Complex Numbers

$Re^{i\theta} = R\cos\theta + iR\sin\theta$



Complex Roots

$y = x^n = Re^{i\theta}$

$y^{1/n} = R^{1/n} e^{i\left(\frac{\theta + 2\pi k}{N}\right)}$ for $k = 0, 1, \ldots, n-1$

i.e $y = x^4 = 1 = e^{i0}$

root : $\{1, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}\}$