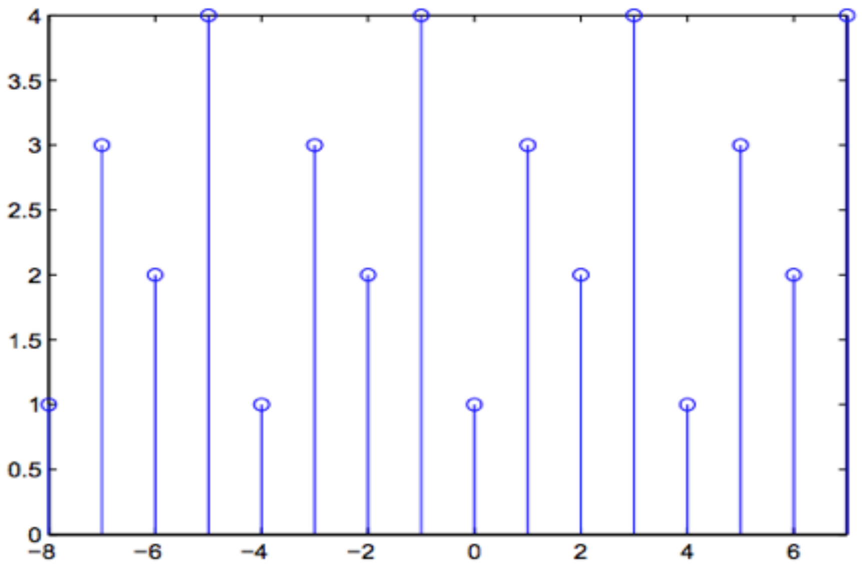
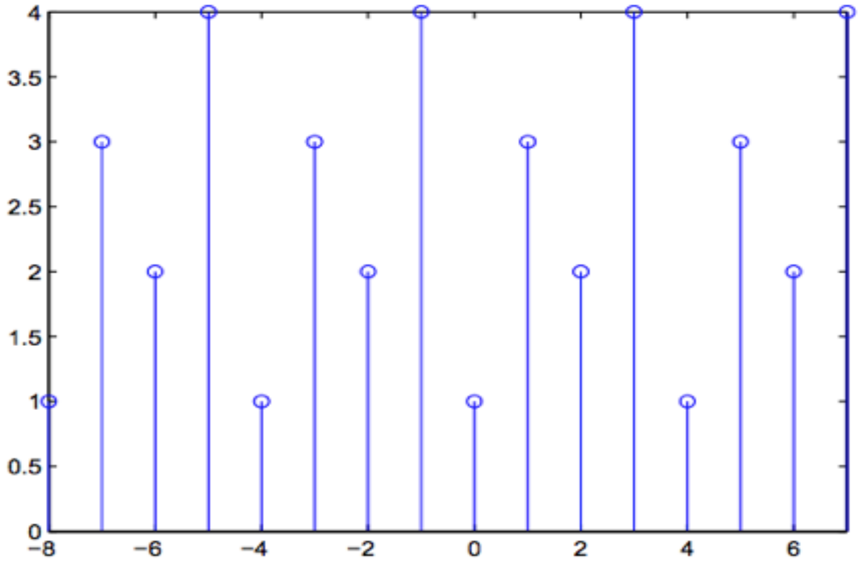
Discrete-Time Fourier Series

Discrete-Time Signal



What is the period of this signal?

Discrete-Time Signal



What is the period of this signal? 4! It repeats every 4 samples

Discrete-Time Fourier Series

$$x(n) = A_0 + \sum_{k=1}^{K} \alpha_k \cos(k\omega_0 n) + \sum_{k=1}^{K} \beta_k \sin(k\omega_0 n)$$

Given that x(n) is the previous signal,
what is the size of the DTFS in matrix form?
Compute it. What is ω_0?

Discrete-Time Fourier Series

$$x(n) = A_0 + \sum_{k=1}^{K} \alpha_k \cos(k\omega_0 n) + \sum_{k=1}^{K} \beta_k \sin(k\omega_0 n)$$

Period (p) = 4 which means K = p/2 = 2, so there are 2K + 1 = 5 coefficients, A0, α1, α2, β1, β2

ω_0 = 2π/4 = π/2

Matrix Form

Notice that β2 is not included because all of the β2 is multiplied by sin(2ω_0n) = sin(πn) = 0.

$$\begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{pmatrix} = \begin{pmatrix} 1 & \cos(\omega_0 * 0) & \sin(\omega_0 * 0) & \cos(2\omega_0 * 0) \\ 1 & \cos(\omega_0 * 1) & \sin(\omega_0 * 1) & \cos(2\omega_0 * 1) \\ 1 & \cos(\omega_0 * 2) & \sin(\omega_0 * 2) & \cos(2\omega_0 * 2) \\ 1 & \cos(\omega_0 * 3) & \sin(\omega_0 * 3) & \cos(2\omega_0 * 3) \end{pmatrix} \begin{pmatrix} A_0 \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{pmatrix}$$

Matrix is given by

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & \cos(\pi/2) & \sin(\pi/2) & \cos(\pi) \\ 1 & \cos(\pi) & \sin(\pi) & \cos(2\pi) \\ 1 & \cos(3\pi/2) & \sin(3\pi/2) & \cos(3\pi) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 \end{pmatrix}$$

CT to DT

Why discrete-time?

When we process signals with computers (when we enter MATLAB land), signals must be represented in discrete-time.

We digitize the signal by **sampling**, which will be covered in greater depth later in the semester.

CT to DT

$x(t) = \sin(500\pi t)$

Sampling rate is 1000 samples/second.

What is the resulting discrete-time signal?

CT to DT

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Sampling rate is 1000 samples/second.

What is the resulting discrete-time signal?

CT frequency = 250 cycles/second

DT frequency = 250cycles/second / (1000 samples/second)

= ¼ cycles/sample

CT to DT

$x(t) = \sin(500\pi t)$

Sampling rate is 1000 samples/second.

What is the resulting discrete-time signal?

CT frequency = 250 cycles/second

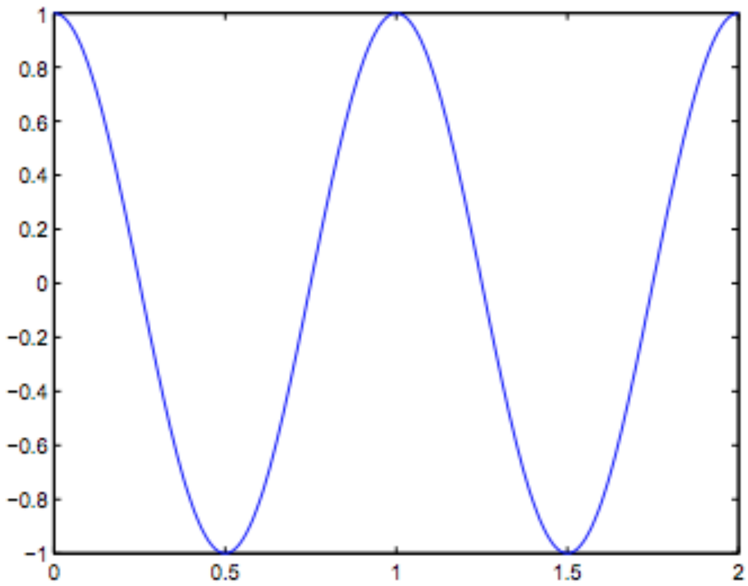
DT frequency = 250cycles/second / (1000 samples/second)

= ¼ cycles/sample **fd = fc/fs!**

Sampling

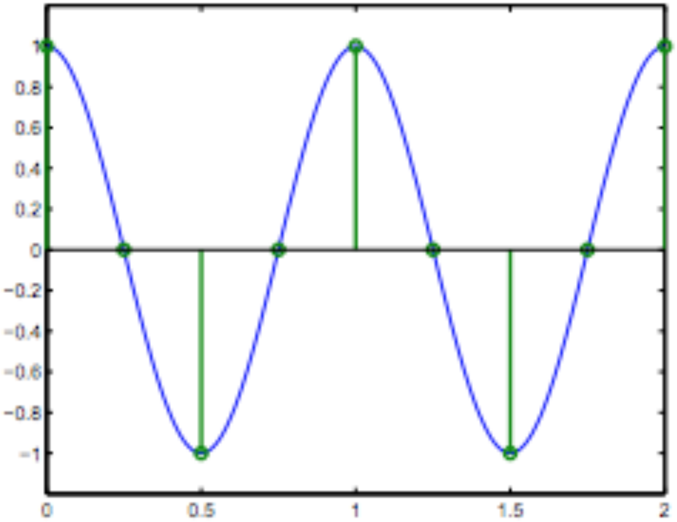
$x(t) = \cos(2\pi t)$

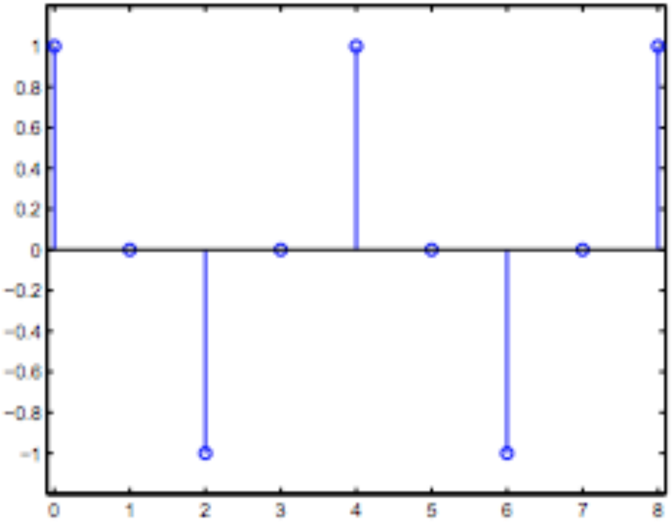
Original signal:



Sampling

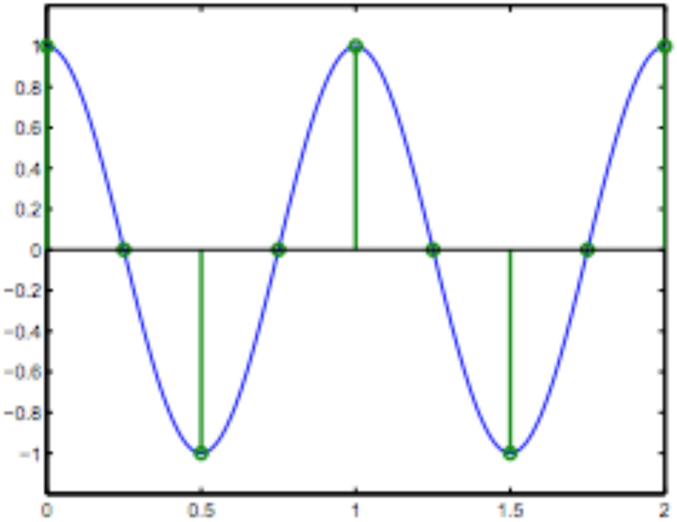
Sampling rate = 4 samples/second

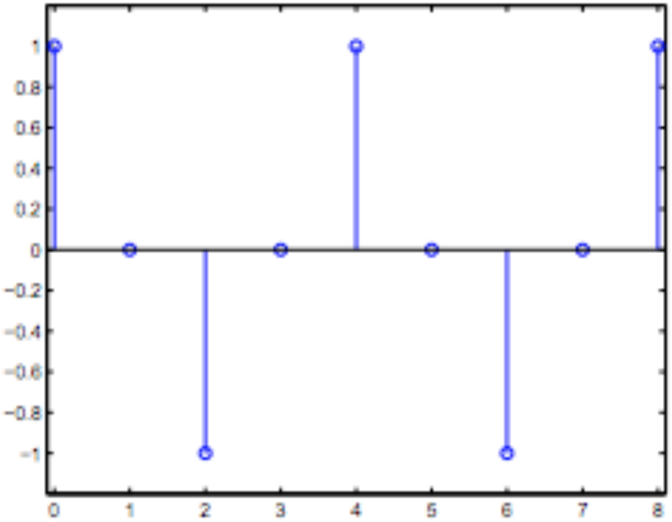




Sampling

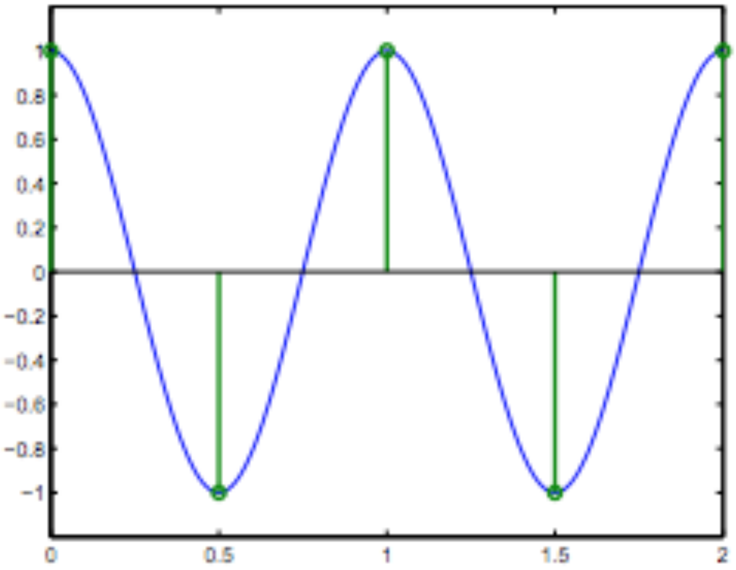
Sampling rate = 4 samples/second

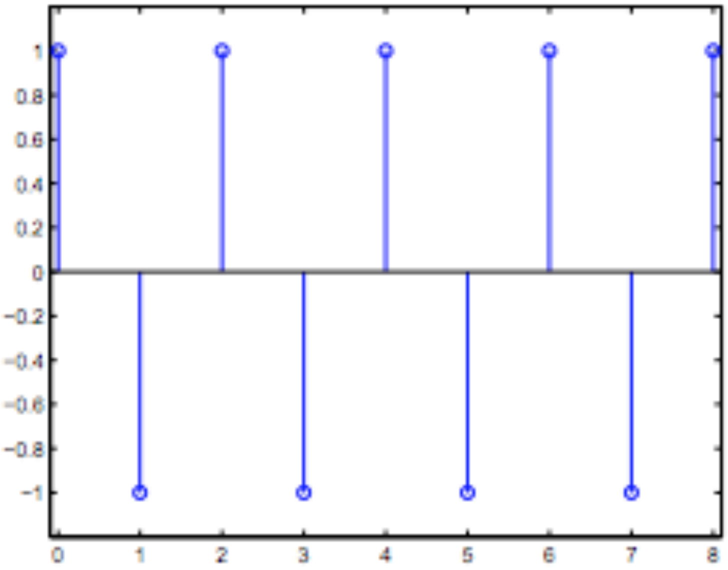




Sampling

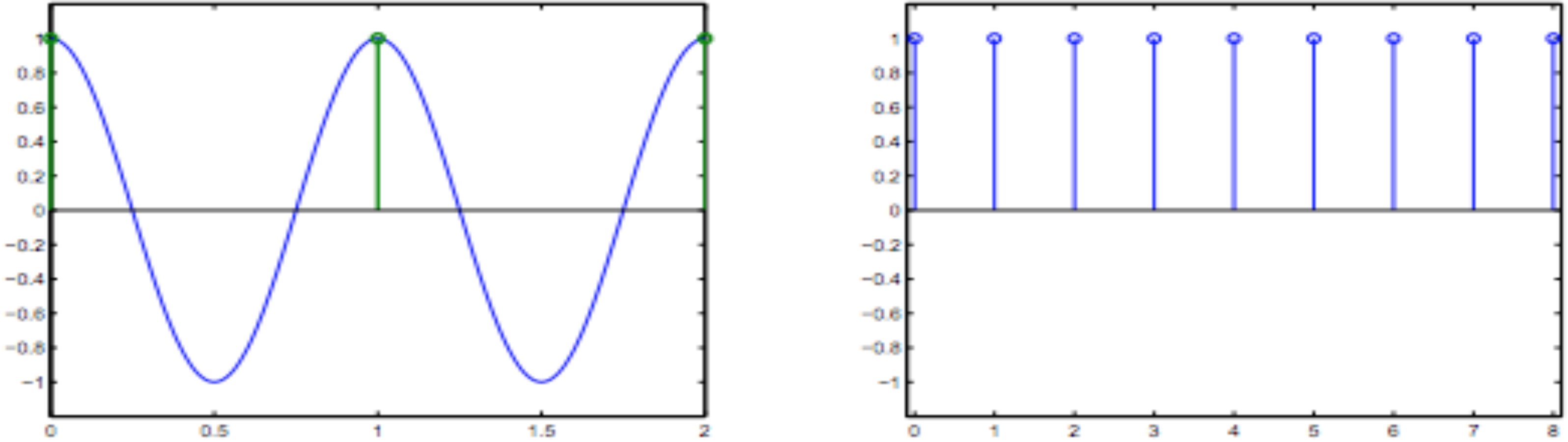
Sampling rate = 2 samples/second





Sampling

Sampling rate = 1 samples/second



This doesn’t look like the original signal! Lost information. Example of **aliasing**…but we’ll get to that later

Discrete-Time Period

In continuous time, we know that the period is related by

$p = 1/f$ where f is the CT frequency

How about in the discrete time?

Discrete-Time Period

$x(t) = \cos(2\pi t)$

fs = 0.5 samples/second

fd = 1/0.5 = 2 samples/second

Is p = ½? Nope. p is still 1.

For discrete-time signals, if **fd = a/b** where a and b are relatively prime, **p = b**