EE20N Fall 2013 Discussion Questions

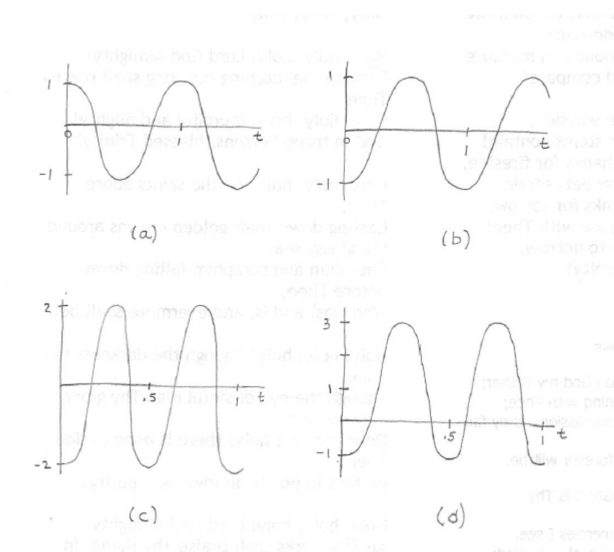
1. Plot the following signals

   (a) $y(t) = cos(2\pi t)$

   (b) $y(t) = cos(2\pi t + \pi)$

   (c) $y(t) = -2cos(4\pi t)$

   (d) $y(t) = 1 + 2cos(4\pi t - \pi)$

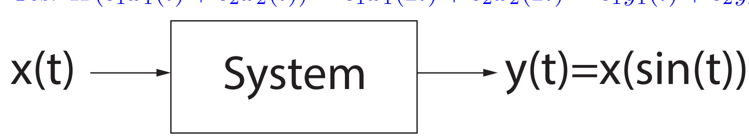


2. Determine whether the following systems are linear

   (a)

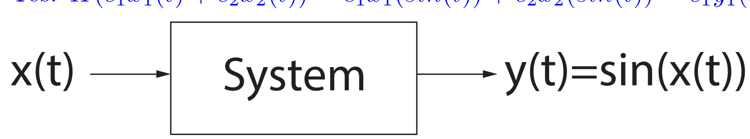
   Yes. $H(c_1x_1(t) + c_2x_2(t)) = c_1x_1(2t) + c_2x_2(2t) = c_1y_1(t) + c_2y_2(t)$

   (b)



   Yes. $H(c_1x_1(t) + c_2x_2(t)) = c_1x_1(sin(t)) + c_2x_2(sin(t)) = c_1y_1(t) + c_2y_2(t)$

   (c)



   No.
   $H(c_1x_1(t) + c_2x_2(t)) = sin(c_1x_1(t) + c_2x_2(t))$
   $c_1y_1(t) + c_2y_2(t) = c_1 sin(x_1(t)) + c_2 sin(x_2(t))$

3. Given $z_1 = e^{i\frac{2}{3}\pi}$ and $z_2 = \frac{1}{2} - \frac{1}{2}i$,

   (a) Plot on the complex plane z_1, z_2, z_1^*, and z_2^*
   $z_1 = e^{i\frac{2\pi}{3}} = cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3}) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$
   $z_2 = \frac{1}{2} - \frac{1}{2}i = \sqrt{(\frac{1}{2})^2 + (\frac{-1}{2})^2} e^{i(tan^{-1}(\frac{-1/2}{1/2}))} = \frac{\sqrt{2}}{2}e^{-i\frac{\pi}{4}}$

(b) Determine $z_1 + z_2$ in polar form as well as Cartesian form. Is polar form easier or Cartesian form easier?
$z_1 + z_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} + \frac{1}{2} - i\frac{1}{2} = 0 + i\frac{\sqrt{3}-1}{2} = \frac{\sqrt{3}-1}{2}e^{i\frac{\pi}{2}}$

(c) Determine $z_1 z_2$ in polar form as well as Cartesian form. Is polar form easier or Cartesian form easier?
$z_1 z_2 = e^{i\frac{2\pi}{3}}\frac{\sqrt{2}}{2}e^{-i\frac{\pi}{4}} = \frac{\sqrt{2}}{2}e^{i\frac{5\pi}{12}} = \frac{\sqrt{2}}{2}cos(\frac{5}{12}\pi) + i\frac{\sqrt{2}}{2}sin(\frac{5}{12}\pi)$

(d) Find the 4th roots of 1.
We want to solve $x^4 = 1$. Since x is complex, we have $x = ae^{i\phi}$ for some real numbers a and ϕ. Then $(ae^{i\phi})^4 = 1 \to a^4 e^{i4\phi} = 1 \cdot e^{i2k\pi}$ for some integer k. This means that $a = 1$, and $4\phi = 2k\pi$. Thus we have $\phi = \frac{k\pi}{2}$. So the 4th roots of 1 are $\{1, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}\}$.

(e) Find the 6th roots of $1 + 2i$.
Same thought process as above. The magnitude is $a = (\sqrt{5})^{1/6} = 5^{1/12}$, and the phase is $(\arctan(2/1) + 2\pi k)/6$ for $k = \{0, 1, 2, 3, 4, 5\}$.