Discussion 11: Overview and Review!

EE 20 Spring 2014
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0.1 Fourier Series

First concept of the semester! Almost any periodic signal can be written as a linear combination of sinusoids
*lecture 4

$$x(t) = A_0 \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)$$

which can also be written as

$$A_0 + \sum_{k=1}^{\infty} \alpha_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} \beta_k \sin(k\omega_0 t)$$

The $\alpha_k s$ and the $\beta_k s$ make up the frequency representation of this signal and can be determined by

$$\alpha_k = \frac{2}{p} \int_0^p x(t) \cos(k\omega_0 t) dt$$

$$\beta_k = \frac{2}{p} \int_0^p x(t) \sin(k\omega_0 t) dt$$

where $\omega_0 = \frac{2\pi}{p}$

Similarly for the Fourier Series for a Discrete-time signal,
*lecture 5
Let $x(n)$ be a p-periodic discrete-time signal. Then

$$x(n) = A_0 + \sum_{k=1}^{K} A_k \cos(k\omega_0 n + \phi_k).$$

where

$$\omega_0 = \frac{2\pi}{p} \text{ and } K = \begin{cases} \frac{p}{2} & \text{if } p \text{ is even} \\ \frac{p-1}{2} & \text{if } p \text{ is odd} \end{cases}.$$

or

$$A_0 + \sum_{k=1}^{K} \alpha_k \cos(k\omega_0 n) + \sum_{k=1}^{K} \beta_k \sin(k\omega_0 n)$$

0.2 Complex Exponentials

Later, we talked about complex exponentials and how they are much simpler to deal with when it comes to Fourier series and time shifts and all of that fun stuff. Remember how $\cos(t)$ can be written as $\frac{e^{it}+e^{-it}}{2}$ and $\sin(t) = \frac{e^{it}-e^{-it}}{2}$. Therefore, for the Continuous Time Fourier Series, it can be rewritten in complex form

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}$$

and

$$X_k = \frac{1}{p}\int_{-p/2}^{p/2} x(t)e^{-ik\frac{2\pi}{p}t}dt$$

0.3 LTI System

Up to lecture 5, we only talked about signals and the different ways we can describe how to represent periodic ones, continuous-time and discrete-time. Now we moved on to system (mainly LTI system)! The two conditions for an LTI system is:

- linearity
  - homogeneity
  - additivity
- time invariance

You should remember how to prove each of these properties. If a system satisfies these two properties, it is LTI.
*lecture 6

Theorem 0.1 *For an LTI system, if the input signal is a sinusoid, the output signal will be a sinusoid of the same frequency, but may have its magnitude and/or phase changed.*

For an LTI system, What is unique about an LTI system is that it has a unique frequency response. First, the main result we want to show this lecture is that when you input $x(t) = e^{i\omega t}$, the output is $y(t) = H(\omega)e^{i\omega t}$, where $H(\omega)$ is the frequency response as shown in Figure **??**.

Figure 0.1: An LTI system with input $x(t) = e^{i\omega t}$ and output $y(t) = H(\omega)e^{i\omega t}$.

The frequency response is important because it tells you how much each frequency is attenuated or amplified.
If Figure **??** is true, if we send in a cosine into an LTI system, Figure **??** is also true.

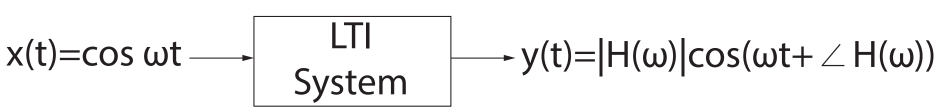


Figure 0.2: An LTI system with input $x(t) = \cos\omega t$ and output $y(t) = |H(\omega)|\cos(\omega t + \angle H(\omega))$.

Remember the different ways to calculate magnitude and phase of $H(\omega)$?

0.4 Convolution and Impulse Response

*lecture 9
Recall the delta function, $\delta(n)$. It is also known as the identity function because

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

Since any discrete-time signal $x(n)$ can be written as a *linear combination* of shifted deltas,

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \quad \Rightarrow \quad y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k).$$

This expression can be considered as defining an operation between two sequences $x(n)$ and $h(n)$ to get a third sequence $y(n)$; this operation is called the *convolution* operation '$*$':

$$y(n) = (h * x)(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k).$$

Basically, $h(n)$ is a time-domain representation of an LTI system. In order to get our output, we can convolve $x(n)$ with $h(n)$ to get $y(n)$.

0.5 Putting them together, Frequency and Impulse Response

*lecture 11
Because both $h(n)$ and $H(\omega)$ capture all the information about LTI systems, we can naturally obtain $H(\omega)$ from $h(n)$.

With this in mind, let H be a LTI system with frequency response $H(\omega)$ and impulse response $h(n)$. On the frequency side, if the input is $x(n) = e^{i\omega n}$, then the output is $y(n) = H(\omega)e^{i\omega n}$. At the same time on the time domain,

$$\begin{aligned} y(n) &= (h * x)(n) \\ &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=-\infty}^{\infty} h(k)e^{i\omega(n-k)} \\ &= \left(\sum_{k=-\infty}^{\infty} h(k)e^{-i\omega k}\right) e^{i\omega n}. \end{aligned}$$

Since k is merely a dummy variable for the summation, our result is a function only in ω,

$$H(\omega) = \sum_{k=-\infty}^{\infty} h(k)e^{-i\omega k},$$

where $h(n)$ is the impulse response of a LTI system. For a simple sanity check, note that there is no n term in the expression for $H(\omega)$; the frequency response of a system is completely independent of time.

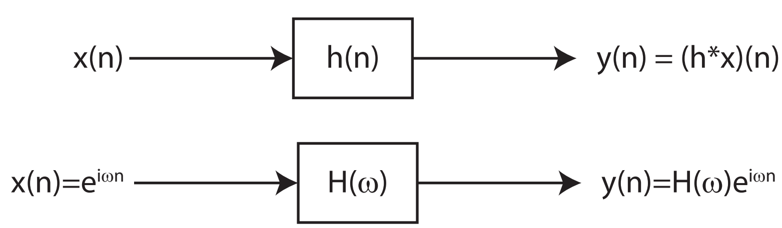
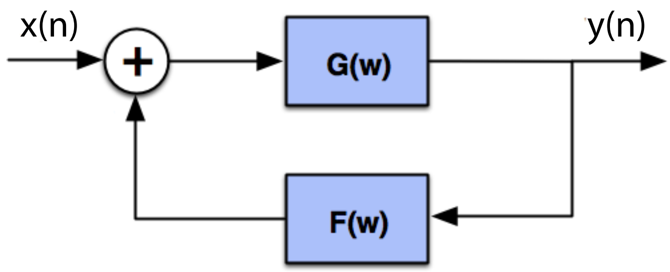


Figure 0.3

FREQUENCY RESPONSE IS THE FOURIER TRANSFORM OF IMPULSE RESPONSE!

0.6 Feedback

*Lecture 13
Consider the following feedback system



The overall frequency response was:

$$H(\omega) = \frac{G(\omega)}{1 - G(\omega)F(\omega)}$$

Why is feedback even useful? Remember how the use of feedback for our moving average LPF removed the side lobes of our frequency response. It allowed for a more graceful looking low-pass filter. In certain cases, it can make things well behaved and provide stability to control a system.
Also, it helps break down complicated systems. Try to turn a system on an exam into a feedback system and find the Fourier Transforms of each sub system to find the overall FT.

0.7 Fourier Transform

*Lecture 13
Up to this point, we were able to represent periodic signals in terms of its Fourier series. What about aperiodic signals? Similarly to how $H(\omega)$ related to $h(n)$, we can generalize it to any aperiodic signal by treating the period to be infinitely large.

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-i\omega n}.$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{i\omega n}d\omega.$$

The DTFT can be thought of as a generalized Fourier series. Similarly for continuous-time signals,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega$$

*Lecture 15
The CTFT can be written as,

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$$

Therefore, we have a way to represent the frequency-domain of a signal and make it easier to relate the input and output rather than by convolution.

$$Y(\omega) = X(\omega)H(\omega)$$

Review

Time domain properties	Finite/Periodic	Infinite Duration	
Continuous Time	FS	CTFT	Infinite Duration
Discrete Time	DFS/DFT	DTFT	Finite/Periodic
	Discrete Spectrum	Continuous Spectrum	Frequency domain properties

AND HAVE ALL FOURIER TRANSFORM PAIRS MEMORIZED IN YOUR HEAD

0.8 Sampling

Using all we have learned previously, we apply it to sampling. *More in-depth notes on sampling are on discussion 11 notes.

0.9 Spectrograms

A representation of frequencies as a function of time for a given signal. Things to understand and get out of the Shazam Lab

- Given a spectrogram, what would the signal look like
- How did we construct our own spectrogram
- How were spectrograms used for creating our own Shazam