







KOSTAT-UNFPA Summer Seminar on Population

Workshop 1. Demography in R

Day 3: Worked example of tidy processing

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29 July 2022

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1 Tidy data

1.1 Definition

Recall that tidy data follows a standard structure where each column is a variable, each row is an observation, and each cell is a value. Anything else is messy. Demographic data is often delivered in a tidy format. When it is not, then it can be reshaped into a tidy format.

1.2 Example of not-tidy data

The following layout (screenshot from Excel) is not tidy. This example data was manually extracted from here: https://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=demo_fasec&l ang=en, and the given format was concocted with EUROSTAT's online data widget.

	А	В	С	D	Е	F	G	Н	- 1	
1	Live births by mother's age and newborn's sex [demo_fasec]									
2				_						
3	Last update	28.06.21								
4	Extracted d	20.07.21								
5	Source of d	Eurostat								
6		,,,,,,,,,,								
7	SEX	Total								
8	UNIT	Number								
9										
10	AGE	GEO/TIME	2011	2012	2013	2014	2015	2016		
11	Total	Belgium	128,705	128,051	125,606	125,014	122,274	121,896		
12	Total	Czechia	108,673	108,576	106,751	109,860	110,764	112,663		
13	Total	Spain	470,553	453,348	424,440	426,076	418,432	408,734		
14	Total	Croatia	41,197	41,771	39,939	39,566	37,503	37,537		
15	15 years	Belgium	74	72	48	52	47	37		
16	15 years	Czechia	67	53	55	62	66	47		
17	15 years	Spain	414	379	391	375	390	341		
18	15 years	Croatia	35	36	42	27	24	18		
19	16 years	Belgium	171	161	183	180	130	119		
20	16 years	Czechia	224	225	204	230	177	223		
21	16 years	Spain	930	896	855	878	806	813		
22	16 years	Croatia	97	102	92	98	86	75		
23	17 years	Belgium	436	417	338	346	303	292		
24	17 years	Czechia	511	483	422	412	447	417		
25	17 years	Spain	1,784	1,677	1,581	1,489	1,493	1,394		
26	17 years	Croatia	228	191	172	182	182	158		
27	18 years	Belgium	822	753	606	621	559	523		
28	18 years	Czechia	818	952	801	771	752	744		
29	18 years	Spain	2,914	2,699	2,397	2,309	2,217	2,182		
30	18 years	Croatia	428	360	365	356	308	314		
31	19 years	Belgium	1,470	1,346	1,056	1,027	967	917		
32	19 years	Czechia	1,434				1,164	1,192		
33	19 years	Spain	4,160	3,866	3,529	3,365	3,220	3,113		

The main thing that makes it not-tidy are the years spread over columns. These should be stacked into to columns: TIME (per the original codes) and Births, which are the values in the cells. The fact that AGE is coded as an arithmetically unusable character string is something we'll want to recode, but it is orthogonal to the *tidiness* of the data. Finally, we will ensure that age-specific births sum up to the stated total births per year and country.

To follow along, create a folder in your project called Data. Then, go to the Data folder of the course repository on github: https://github.com/timriffe/KOSTAT_Workshop1/blob/master/Data/demo_fasec.xlsx and click Download. Move it to the data folder you just made. You can also do the same for a second file that we'll use later today: https://github.com/timriffe/KOSTAT_Workshop1/blob/master/Data/demo_pjan.xlsx

2 Tidy processing fertiltiy data

Today's entire session will be working with this smallish births dataset.

2.1 Reshape, Rescale, Recode it

We'll use the read_excel() function from the readxl package to get the data in. First let's look at the help file using ?read_excel. Visual inspection of the data shows us that we need to skip several rows, plus there's a note at the bottom of the sheet that we want to ignore. We specify

an explicit cell range using the argument range and giving spreadsheet coordinates range = "A10:H158" (determined based on visual inspection of the file).

```
library(tidyverse)
library(readxl)
# ?read_excel
Wide <- read_excel(</pre>
           path = "Data/demo fasec.xlsx",
           range = "A10:H158")
glimpse(Wide)
## Rows: 148
## Columns: 8
                <chr> "Total", "Total", "Total", "Total", "15 years", "15 years", "
## $ AGE
                <chr> "Belgium", "Czechia", "Spain", "Croatia", "Belgium", "Czech~
## $ `GEO/TIME`
                <dbl> 128705, 108673, 470553, 41197, 74, 67, 414, 35, 171, 224, 9~
## $ `2011`
                <dbl> 128051, 108576, 453348, 41771, 72, 53, 379, 36, 161, 225, 8~
## $ `2012`
## $ `2013`
                <dbl> 125606, 106751, 424440, 39939, 48, 55, 391, 42, 183, 204, 8~
                <dbl> 125014, 109860, 426076, 39566, 52, 62, 375, 27, 180, 230, 8~
## $ `2014`
## $ `2015`
                <dbl> 122274, 110764, 418432, 37503, 47, 66, 390, 24, 130, 177, 8~
## $ `2016`
                <dbl> 121896, 112663, 408734, 37537, 37, 47, 341, 18, 119, 223, 8~
```

2.1.1 pivot_longer() (pivot_wider())

These data are not tidy because TIME is spread over columns. Instead we should have a column called TIME, containing the years, and the cell values currently in the columns 2011 to 2016 should be in a column called births. The function pivot_longer() will do this for us. See <code>?pivot_longer</code> or the handout from the previous session.

A nice thing about pivot_longer is you can specify column ranges using names (see how we put back-ticks on years? That's so that 2011 doesn't get interpreted at the 2011th column!). Integer ranges will also do, as would listing out the columns by name or position. There are other tricks for intelligently picking out columns, see ?tidyr_tidy_select. From that help file, we see other things would have worked in our case, for example where(is.double), which is handy.

2.1.2 select() columns

Note, some of these columns are clean and ready to use (TIME and Births), but GEO/TIME is not a useful column name, and the values in AGE might not be useful in practice. Instead, maybe we want AGE to be an integer value so that we can use it to sort, or use it in a numeric way in a plot axis. Maybe also we don't want to have a Total value for AGE, but instead we want to ensure that the age-specific births add up to the total? This will give us a chance to showcase some more tidyverse tools:

Select, rearrange, and rename **columns** using **select()**. Notice that i) the left side of each = is the new column name, ii) the order you list the columns is the new order, iii) if you forget to list a column you lose it! iv) when in doubt, but back-ticks on the column name to ensure it won't be misinterpreted. **GEO/TIME** looks like math, so stick it in back-ticks...

```
Long <- select(
   .data = Long,
   Country = `GEO/TIME`,
   Age = AGE,
   Year = TIME,
   Births
)
head(Long)</pre>
```

```
## # A tibble: 6 x 4
##
     Country Age
                   Year
                          Births
##
     <chr>
             <chr> <chr>
                           <dbl>
## 1 Belgium Total 2011
                          128705
## 2 Belgium Total 2012
                          128051
## 3 Belgium Total 2013
                          125606
## 4 Belgium Total 2014
                          125014
## 5 Belgium Total 2015
                          122274
## 6 Belgium Total 2016
                          121896
```

2.1.3 pipes, %>%

Let's re-express the above using *pipes*. Pipes allow us to string together data operations into a single sequentially evaluated execution step. Thus far we have read in the data, reshaped it, and re-specified columns. All together, this becomes:

```
Long <-
  read_excel(
  path = "Data/demo_fasec.xlsx",
    range = "A10:H158") %>%

pivot_longer(
  cols = `2011`:`2016`,
  names_to = "Year",
  values_to = "Births") %>%

select(
  Country = `GEO/TIME`,
    Year,
    Age = AGE,
    Births)
```

This reads as "first read_excel(), then pivot_longer(), then select() columns, renaming some of them". Of course we should always take the time to then annotate the code, for posterity:

```
Long <-
  # first read in from Excel
  read excel(
    path = "Data/demo_fasec.xlsx",
    range = "A10:H158") %>%
  # stack years
  pivot_longer(
    cols = `2011`:`2016`,
    names_to = "Year",
    values_to = "Births") %>%
  # pick out the columns
  select(
     Country = `GEO/TIME`,
     Year,
     Age = AGE,
     Births)
```

Note: you can run this code by simply placing the cursor anywhere in the pipeline and pressing Ctrl + Enter. There is no need to select the whole statement before running, although this also works (you could in this case also click the green play arrow).

We will augment this pipeline step by step and then recompose it in its entirety at the end.

2.1.4 filter(), mutate(), group_by()

So what age classes do we have? unique() picks out just the unique values present in a vector.

```
# same thing using tidy:
Long %>%
 pull(Age) %>% # pull() extracts column as vector
unique()
```

```
## [1] "Total" "15 years" "16 years" "17 years" "18 years" "19 years"
## [7] "20 years" "21 years" "22 years" "23 years" "24 years" "25 years"
## [13] "26 years" "27 years" "28 years" "29 years" "30 years" "31 years"
## [19] "32 years" "33 years" "34 years" "35 years" "36 years" "37 years"
## [25] "38 years" "39 years" "40 years" "41 years" "42 years" "43 years"
## [31] "44 years" "45 years" "46 years" "47 years" "48 years" "49 years"
## [37] "Unknown"
```

The Age column should be changed to consist in just integers. But this raises another issue: what to do with the Total and Unknown ages? My preference is usually to redistribute unknowns proportional to the distribution of any *knowns*:

$$\widehat{Y_x} = Y_x + Y_{unknown} * \frac{Y_x}{\sum Y_x}$$

where the denominator excludes unknowns... This is just the same as rescaling the distributions of known ages to sum to the stated total

$$\widehat{Y_x} = Y * \frac{Y_x}{\sum Y_x}$$

(where x excludes unknowns)

Once we do one or the other of these operations, we'll end up with just ages 15 years through 49 years, and can convert using string operators. We can throw out either Total or Unknown using filter() to select rows. Calculations to redistribute can be done using the function mutate(). The basic structure of said operation would be something similar to:

```
# don't run this
Long %>%
# 1
mutate(TOT = Births[Age == "Total"]) %>%
# 2
filter(! Age %in% c("Total", "Unknown")) %>%
# 3
mutate(Births = Births / sum(Births) * TOT)
```

I'll first explain the basic logic, then why it won't *yet* work as expected. In step 1, we use mutate() to create a new column called TOT, which just repeats the respective value for each row of the data.

Now for the filter() statement.

2.1.5 Time out for logicals

Each value of TOT is intended to be the value of Births where Age is equal to "Total". Note == is a *logical* equals, meaning you're asking if values are equal. The result will be TRUE, FALSE, or NA if pertinent.

Example:

```
1:5 == 5
```

[1] FALSE FALSE FALSE FALSE TRUE

Other useful logical operators include != (inequality), <, >, <=, >=. Further logical functions include: is.na(), any(), all(). Each of these operators and functions is vectorized, meaning they can evaluate long vectors of expressions element-wise.

Here we want to use this logical vector to select values:

```
abcde <- c("a","b","c","d","e")
abcde[1:5 == 5]
```

[1] "e"

Namely, we get back the values where the logical vector evaluates to TRUE.

Given a column TOT, we can remove age classes equal to "Total" or "Unknown" with filter(). %in% is a logical operator for set membership.

```
c("a","d","k") %in% abcde
```

```
## [1] TRUE TRUE FALSE
```

Finally, mutate() can be used to do the rescale operation using our basic arithmetic.

2.1.6 group_by() (ungroup())

An issue that you may foresee at this point is that either of the above formulas is independent within each Country and Year. We can deal with this by declaring each combination of these two variables as an *independent* group using group_by(), and then removing groups when no longer

needed using ungroup(). That's just good housekeeping, but it keeps the pipeline rigorous: You can assume group declarations will persist until explicitly removed.

```
Long %>%

# add group metadata
group_by(Country, Year) %>%

# raise Total count to column for element-wise rescale
mutate(TOT = Births[Age == "Total"]) %>%

# throw out Total and Unknown ages
filter(! Age %in% c("Total", "Unknown")) %>%

# rescale proportions known to stated total
mutate(Births = Births / sum(Births) * TOT) %>%

# groups no longer needed, let's remove them:
ungroup()
```

Finally, we can clean up the Age column! Here I'll take the string substitution strategy, although other options would also work. gsub() looks for a pattern in the string "years" and replaces it. In this case, I replace with an empty string "", so "15 years" becomes "15", still a character string. We can then modify it in the same mutate() call: comma-separated statements in mutate() are evaluated in sequence, and they can be sequentially dependent!

```
## # A tibble: 840 x 5
                       Age Births
##
      Country Year
                                     TOT
##
      <chr>
              <chr> <int>
                            <dbl>
                                   <dbl>
##
   1 Belgium 2011
                       15
                             74.1 128705
    2 Belgium 2012
                             72.1 128051
##
                        15
    3 Belgium 2013
                        15
                             48.5 125606
##
   4 Belgium 2014
##
                        15
                             52.0 125014
##
    5 Belgium 2015
                        15
                             47.5 122274
    6 Belgium 2016
                             37.4 121896
##
                        15
##
    7 Czechia 2011
                        15
                             67.0 108673
##
    8 Czechia 2012
                        15
                             53.0 108576
                             55.0 106751
   9 Czechia 2013
                        15
## 10 Czechia 2014
                        15
                             62.0 109860
## # ... with 830 more rows
## # i Use `print(n = ...)` to see more rows
```

A helper function from the readr package parse_number() can do this for us in a single step:

```
Long2 %>%
  mutate(Age = parse_number(Age))
```

Note, you can also use pipes inside function calls, like mutate(), so the above could become:

```
replacement = "") %>%
as.integer())
```

```
# A tibble: 840 x 5
##
      Country Year
                                      TOT
                       Age Births
##
      <chr>
              <chr> <int>
                            <dbl>
                                   <dbl>
    1 Belgium 2011
                             74.1 128705
##
                        15
##
    2 Belgium 2012
                        15
                             72.1 128051
    3 Belgium 2013
##
                        15
                             48.5 125606
##
   4 Belgium 2014
                        15
                             52.0 125014
##
    5 Belgium 2015
                        15
                             47.5 122274
##
    6 Belgium 2016
                        15
                             37.4 121896
##
    7 Czechia 2011
                        15
                             67.0 108673
##
    8 Czechia 2012
                        15
                             53.0 108576
## 9 Czechia 2013
                             55.0 106751
                        15
## 10 Czechia 2014
                        15
                             62.0 109860
## # ... with 830 more rows
## # i Use `print(n = ...)` to see more rows
```

Depending on what you're doing, one or the other of these could be more *legible*. Human-legible code is more robust than illegible code, can we agree on this point?

2.1.7 Bringing it all together

There are times when it may make sense to keep steps separate, in separate data objects, but our first example is a case of wanting to keep all steps contained in a single pipeline. That's because the intermediate pieces are redundant and add no value. Combined into a single pipeline, we'd end up with something like this:

```
Births <-
  # first read in from Excel
  read_excel(
    path = "Data/demo fasec.xlsx",
    range = "A10:H158") %>%
  # stack years
  pivot_longer(
    cols = `2011`:`2016`,
    names_to = "Year",
    values_to = "Births") %>%
  # pick out the columns
  select(
     Country = `GEO/TIME`,
     Year,
     Age = AGE,
     Births) %>%
  # add group metadata
  group_by(Country, Year) %>%
  # raise Total count to column for element-wise rescale
  mutate(TOT = Births[Age == "Total"]) %>%
  # throw out Total and Unknown ages
  filter(! Age %in% c("Total", "Unknown")) %>%
  # rescale proportions known to stated total
  mutate(Births = Births / sum(Births) * TOT) %>%
```

```
## Rows: 840
## Columns: 4
## $ Country <chr> "Belgium", "Bel
```

This is a tidy pipeline. And tidy code, no matter who writes it, usually ends up looking something like this. To finish off the pipeline, I've sorted the rows. arrange(Country, Year, Age) sorts Age within Year within Country), and we delete the TOT column using subtraction inside select().

You see all those annotations between many of the pipe steps? That's not just for you, the reader. It's good practice to do that. Possibly because someone else might like to interpret your code, so why not make it easier, but also you should comment your code out of respect for future you, because future you won't remember what you were thinking when you wrote it.

2.2 Group to 5-year ages with summarize()

Aggregation typically implies a reduction in the number of rows in a data set. Let's see examples of grouping countries, grouping ages, and calculating marginal sums. Grouping ages or years often follows a similar logic. We will exploit the *modulo* operator, %%, which tells us the remainder after Euclidean division. Example:

```
a <- 1:10

a %% 2

## [1] 1 0 1 0 1 0 1 0 1 0

a %% 5

## [1] 1 2 3 4 0 1 2 3 4 0
```

That is the divisor (2 or 5) is subtracted away an integer number of times until what remains is smaller than the divisor. This is useful for redefining Age, see:

```
Age <- 0:20
Age - Age %% 5
```

```
## [1] 0 0 0 0 0 5 5 5 5 5 10 10 10 10 10 15 15 15 15 15 20
```

That is, subtracting Age modulo 5 from a vector of single ages tells you the lower bound of the five year age group that each single age lays within. We can then use this new age vector to group data, and finally we aggregate Births using summarize().

```
## # A tibble: 168 x 4
##
      Country Year
                      Age Births
##
      <chr>
              <int> <dbl> <dbl>
##
   1 Belgium
              2011
                       15 2977.
               2011
    2 Belgium
                       20 18239.
##
##
   3 Belgium
               2011
                       25 44498.
   4 Belgium
##
               2011
                       30 41905.
   5 Belgium 2011
##
                       35 17368.
    6 Belgium
              2011
                       40 3542.
##
##
   7 Belgium
              2011
                       45
                            175.
   8 Belgium
                       15 2751.
              2012
   9 Belgium
               2012
                       20 17631.
## 10 Belgium 2012
                       25 43746.
## # ... with 158 more rows
## # i Use `print(n = ...)` to see more rows
```

Births = sum(Births) might look strange. The left side is a single outgoing row, whereas the right side is a vector with five values. Our dataset of 840 rows is in this way reduced to 840/5 = 168 rows. This works out cleanly in our case because the age groups were cleanly divisible. Note the argument .groups = "drop" at the end of summarize(), this is just the same as adding %>% ungroup() at the end of the pipeline.

2.2.1 Marginal sums

The result of a summary statement could be just a single row, in this case a probably not-useful result.

```
##
      <chr>
             <int> <dbl>
##
   1 Belgium 2011 128705
##
   2 Belgium 2012 128051
##
   3 Belgium 2013 125606
##
   4 Belgium 2014 125014
   5 Belgium
##
              2015 122274
##
   6 Belgium
              2016 121896
##
   7 Croatia 2011 41197
   8 Croatia 2012 41771
##
   9 Croatia 2013 39939
##
## 10 Croatia 2014 39566
## # ... with 14 more rows
## # i Use `print(n = ...)` to see more rows
```

Likewise, we could group countries using case_when(). First we use case_when() then I'll explain how it works.

```
## # A tibble: 12 x 3
##
      Country_Group Year Births
##
      <chr>
                     <int> <dbl>
##
    1 A
                      2011 579226
##
    2 A
                      2012 561924
##
    3 A
                      2013 531191
    4 A
                      2014 535936
##
##
    5 A
                      2015 529196
##
    6 A
                      2016 521397
    7 B
                      2011 169902
##
##
    8 B
                      2012 169822
    9 B
##
                      2013 165545
## 10 B
                      2014 164580
## 11 B
                      2015 159777
## 12 B
                      2016 159433
```

2.2.2 case_when()

This helper function is a generalization of ifelse() or if_else(), as may be familiar from other programs such as Excel. case_when() is premised on you being able to delimit all cases given in your data exhaustively. Each case is comma separated and defined in formula notation, where ~ separates a left and a right side. On the left of ~ you define the case with a logical statement and on the right side you specify what to assign for that case. By the end of the case_when() statement all cases must be handled. Further, cases are handled in the order specified, so where pertinent it makes sense to list cases ordered from specific to general. If there is a most general case meaning something like everything else, then you can end case_when() with TRUE ~ 1 (or whatever value you want).

For example, just to demonstrate the concepts, say I have an algorithm where you start with an integer. If the integer is: 1. divisible by 6 then divide by 2 and add 1 2. divisible by 3 then multiply by 2 3. odd add 1 4. even add 2

This is a silly algorithm, I admit. Note only the first condition produces an *odd* result. Note, all integers are handled by conditions 1-4. Note that conditions 3 and 4 handle more cases than conditions 1 and 2. Note also that condition 1 is more specific than 2, because all numbers divisible by 6 are also divisible by 3, but not vice versa. Using <code>case_when()</code> and exercising our new modulo skills, an example would be:

```
a <- 1:17

case_when(a %% 6 == 0 ~ a / 2 + 1,

a %% 3 == 0 ~ a * 2,

a %% 2 == 1 ~ a + 1,

a %% 2 == 0 ~ a + 2)
```

```
## [1] 2 4 6 6 6 4 8 10 18 12 12 7 14 16 30 18 18
```

If we write the same cases but changing the order of the first two conditions, we see that condition (1) from the initial algorithm is never activated, because divisibility by 3 handles the case earlier.

```
case_when(a %% 3 == 0 ~ a * 2,
    a %% 6 == 0 ~ a / 2 + 1,
    a %% 2 == 1 ~ a + 1,
    a %% 2 == 0 ~ a + 2)
```

```
## [1] 2 4 6 6 6 12 8 10 18 12 12 24 14 16 30 18 18
```

2.2.3 Weighted means

Our main use of summarize() today will be for evaluating weighted means. More specifically, we'll calculate the mean age at childbearing.

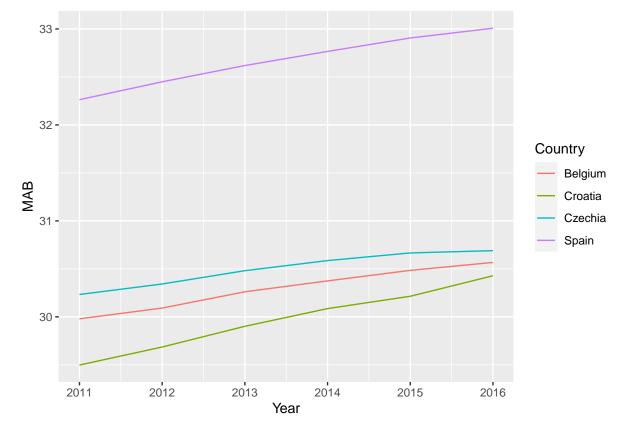
In general a weighted mean is defined as

$$\bar{x} = \frac{\sum x_i \cdot w_i}{\sum w_i}$$

For the mean age at childbearing, x is age (exact age at mid-interval we prefer), and w should be either birth counts or age-specific fertility. Since we don't have exposures (yet) to calculate fertility rates, we'll just use raw births by age as the weights.

While we're here, how about a plot teaser, even though we don't get serious about ggplot2 until next week:





Allow me to pose a question: All of these lines are increasing. These mean ages are based on observed births in each mother age group, which are a product of fertility rates and population size in each age group. How much of this trend do you suppose is due to changes in age-specific fertility rates versus changes in underlying population structure? To answer this question, we will need to obtain, harmonize, and merge population data to the birth counts data we've been working with. Let's get to it!

3 Process population data

Often we get data from different sources that needs to be merged (or joined) into a single merged dataset in order to carry out an analysis. In this case, I've pulled January 1st female population counts data from EUROSTAT, and to make things interesting it's formatted differently and has its own challenges. Our objective is to prepare this data to join it to the births data from above. This will allow us to repeat some concepts.

	A	В	С	D	Е	F	G	Н		J	K	L
1	Population of	n 1 January	by age and	sex [demo	pian1							
2			, Ly ago ama	oun [aumo_	ELSOI .							
3	Last update	05.07.21										
4	Extracted &	21.07.21										
5	Source of d	Eurostat										
6		~~~~										
7	SEX	Total										
8	UNIT	Number										
9												
10	TIME	GEO/AGE	Less than >	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 year
11	2012	Belgium	128,701	131,406	130,525	131,088	128,720	128,016	125,150	123,401		
12	2012	Bulgaria	66,214	69,093	72,786	70,163	67,761	66,918	65,285	64,906	62,602	6
13	2012	Czechia	108,753	119,389	121,285	122,912	118,351	108,766	102,953	98,187	94,357	9
14	2012	Denmark	59,306	64,168	63,842	66,358	65,529	66,506	65,597	65,693	65,667	6
15	2012	Germany in	658,332	679,236	669,579	689,691	689,748	677,338	687,268	705,953	706,644	72
16	2012	Estonia	14,728	15,945	15,765	15,940	15,588	-	14,097	13,625	-	
17		Ireland	73,041	71,754	72,015	71,481	69,861	66,438	64,950	65,078	63,891	
18	2012	Greece	105,955	110,528	115,191	112,981	108,392	110,138	106,916	105,638	105,963	10
19	2012	Spain	475,603	483,549	494,158	521,121	502,163	498,183	487,772	482,425	473,952	45
20	2012	France	789,537	808,857	803,643	809,817	808,910		809,663	805,752	803,913	81
21	2012	France (me	763,531	781,550	775,094	780,356	778,613	_	778,941	775,211	773,146	
22	2012	Croatia	40,948	43,350	44,149	43,130	41,278		42,435		-	
23		Italy	531,372	544,814	554,608		560,099		556,531	561,023		_
24	2012	Cyprus	9,737	10,112	10,058	9,757	9,217	9,415	8,960	9,060	8,868	_
25	2012	Latvia	18,573	19,333	21,175	,	22,253	21,073	20,283		-	_
26	2012	Lithuania	30,194	30,534	31,190	29,502	27,564	27,133				
27	2012	Luxembour		6,049	5,894	5,904	5,872	5,917	5,881	5,865	_	
28		Hungary	87,680	89,906	96,679	99,423	97,391	100,953	97,693			
29		Malta	4,161	3,948	4,068	4,101	3,842	3,830	3,860		_	_
30		Netherland?		184,741	185,690	185,962	182,690	185,808	187,772	-	_	_
31		Austria	77,562	80,080	77,993	79,552	78,512		80,685	,	,	_
32	2012	Poland	380,668	412,836	429,002	427,212	399,979		366,058	-	_	
33		Portugal	95,703	99,519	96,085	100,281	98,361	101,075		-	_	
34	2012	Romania	185,286	207,241	215,521	216,015	212,002	213,127	215,766			
35	2012	Slovenia	22,003	22,576	22,057	22,297	20,477	19,543	18,605	_	_	
36		Slovakia	60,598	57,712	59,722	57,077	54,439		54,527	54,020	_	_
37	2012	Finland	60,074	61,504	61,109	60,486	59,804	60,185	58,901	59,077	-	
38	2012	Sweden	112,114	117,242	114,285	112,444	111,284	110,630	106,635		_	
39	2012	Iceland	4,486	4,859	4,880	4,720	4,543	4,423	4,339	-		_
40	2012	Liechtenste		333	411	353	366		387	387		_
41	2012	Norway	60,466	62,521	63,713	62,810	61,257	61,671	60,120	-	-	
42	2012	Switzerland		81,663	80,196	79,522	78,288		77,450			
45	2012	United Kind	8∩8 517	707 560	797 737	788 447	770 3/11	758 633	735 8/10	71/ 606	607 812	S.S.

3.1 Read in population data

Source: https://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=demo_pjan&lang=en

```
## [1] 500 104
```

When we read this in, some rows are entirely NA values for population. It will be easier to filter() these out a after the population values are stacked in a single column.

3.2 Reformat for joining

To be able to join, we must be able to exactly match on each of our structural criteria: Country names, Year and Age categories.

3.2.1 Reshape to tidy

```
# check first and last age classes
# colnames(Pop)
```

```
Pop <-
               Pop %>%
               pivot longer(`Less than 1 year`:`Unknown`,
                                                                                                              names_to = "AGE",
                                                                                                               values_to = "Population") %>%
               rename(Country = `GEO/AGE`) %>%
               # is.na() gives TRUE if a value is NA, FALSE otherwise,
                #! negates this, so we keep all rows that are not NA
               filter(!is.na(Population))
glimpse(Pop)
 ## Rows: 39,423
 ## Columns: 4
 ## $ TIME
                                                                                                                       <chr> "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "2012", "20
 ## $ Country
                                                                                                                      <chr> "Belgium", 
                                                                                                                       <chr> "Less than 1 year", "1 year", "2 years", "3 years", "4 year~
## $ AGE
## $ Population <dbl> 62808, 64238, 63685, 63823, 62920, 62733, 61007, 60190, 588~
```

3.2.2 Recode Age classes

What age classes to we have?

```
Pop %>%

pull(AGE) %>%

unique()
```

```
##
     [1] "Less than 1 year"
                                  "1 year"
                                                           "2 years"
##
     [4] "3 years"
                                  "4 years"
                                                           "5 years"
##
     [7] "6 years"
                                  "7 years"
                                                           "8 years"
                                                           "11 years"
##
    [10] "9 years"
                                  "10 years"
##
    [13] "12 years"
                                  "13 years"
                                                           "14 years"
##
    [16] "15 years"
                                  "16 years"
                                                           "17 years"
    [19] "18 years"
                                  "19 years"
                                                           "20 years"
##
##
    [22] "21 years"
                                  "22 years"
                                                           "23 years"
                                  "25 years"
##
    [25] "24 years"
                                                           "26 years"
                                                           "29 years"
##
    [28] "27 years"
                                  "28 years"
##
    [31] "30 years"
                                  "31 years"
                                                           "32 years"
                                                           "35 years"
##
    [34] "33 years"
                                  "34 years"
##
    [37] "36 years"
                                  "37 years"
                                                           "38 years"
                                                           "41 years"
                                  "40 years"
##
    [40] "39 years"
##
    [43] "42 years"
                                  "43 years"
                                                           "44 years"
    [46] "45 years"
                                                           "47 years"
##
                                  "46 years"
##
    [49] "48 years"
                                  "49 years"
                                                           "50 years"
##
    [52] "51 years"
                                  "52 years"
                                                           "53 years"
                                  "55 years"
                                                           "56 years"
##
    [55] "54 years"
##
    [58] "57 years"
                                  "58 years"
                                                           "59 years"
                                  "61 years"
##
    [61] "60 years"
                                                           "62 years"
##
    [64] "63 years"
                                  "64 years"
                                                           "65 years"
##
    [67] "66 years"
                                  "67 years"
                                                           "68 years"
                                  "70 years"
                                                           "71 years"
##
    [70] "69 years"
##
    [73] "72 years"
                                  "73 years"
                                                           "74 years"
                                  "76 years"
                                                           "77 years"
##
    [76] "75 years"
```

```
##
    [79] "78 years"
                                  "79 years"
                                                            "80 years"
##
    [82] "81 years"
                                  "82 years"
                                                            "83 years"
##
    [85] "84 years"
                                  "85 years"
                                                            "86 years"
    [88] "87 years"
                                  "88 years"
                                                            "89 years"
##
    [91] "90 years"
                                  "91 years"
                                                            "92 years"
##
                                  "94 years"
    [94] "93 years"
                                                            "95 years"
##
##
    [97] "96 years"
                                  "97 years"
                                                            "98 years"
## [100] "99 years"
                                  "Open-ended age class" "Unknown"
```

Now what would be the easiest way to code this to integer? I'd say: we have special cases for ages 0, 1, the open age group (100) and unknown ages. Every other age follows an exact pattern. Therefore, I propose we treat this with <code>case_when()</code> handling all special cases first, then doing some sort of string operation to handle all other cases that follow the pattern "n years". This latter operation could either be a string operation that extracts digits, or a string substitution that deletes "years".

Check:

```
a <- c("10 years","11 years")
# standard
a %>% gsub(pattern = " years", replacement = "") %>% as.integer()

## [1] 10 11
# terse regular expression
a %>% gsub(pattern = "([0-9]+).*$", replacement = "\\1") %>% as.integer()

## [1] 10 11
# or a handy helper function from the readr package:
parse_number(a)
```

[1] 10 11

Any of these checks would work to handle "everything else" at the end of our case_when()

```
Pop <- # overwrite the same object here
Pop %>%
mutate(Age = case_when(
   AGE == "Less than 1 year" ~ 0,
   AGE == "Open-ended age class" ~ 100,
   AGE == "Unknown" ~ NA_real_,
   TRUE ~ parse_number(AGE)
),
Year = as.integer(TIME)) %>%
select(-TIME, -AGE)
```

3.2.3 Redistribute unknown ages

Here, rather than rescaling to the stated total as we did for Births, we take the other formula that applies the same principle, but framed in terms of redistributing counts with unknown age:

$$\hat{P}_x = P_x + \frac{P_x}{\sum P_x} * P_{Unkown}$$

where the denominator excludes P_{Unkown} . Once again, this operation is done inside mutate(). Note, we're using is.na() three different times as a logical selector! Here, if_else() is used

rather than case_when() because there is only one condition and it is faster to type out.

```
Pop <-
  Pop %>%
  # declare groups
  group_by(Country, Year) %>%
  # 1. move Unknown age up to column
  # 2. replace NAs w Os in the new UNK column
  mutate(UNK = Population[is.na(Age)],
         UNK = if_else(is.na(UNK), 0, UNK)) %>%
  # remove rows with Unknown age
  filter(!is.na(Age)) %>%
  # do the redistribution
  mutate(Population = Population + Population / sum(Population) * UNK) %>%
  # remove groups
  ungroup() %>%
  # remove column no longer needed
  select(-UNK)
```

3.3 Calculate exposures

Probably we'd rather join exposures to Births than January 1st population counts. One final calculation will allow us to introduce a join operation. The approximation we'd like to do is:

$$Exposure_x = \frac{P_x^{Jan1} + P_x^{Dec31}}{2}$$

In other words, just take the simple average of the population at the start and end of the year. We can approximate the end-of-year population using the following year's January 1st population. Our goal is to do this arithmetic like so mutate(Exposure = (P1 + P2) / 2), so the trick is to create a second Population column, consisting in the same Population column we already have, but back-dated one year.

We will do this in two ways, first using joins. To do this we create a copy of Pop, then deduct Year by one in that copy, then merge it back to the original Pop that we started with. In the process we'll also rename both versions of Population to P1 and P2 so that we don't get confused. The year-range for P2 will lose the most recent year, and it will also have one extra year on the lower end, due to the shift. When we join the objects together we want to do so only where we have overlapping combinations of Year (and Age and Country need to match too, but these will match exactly in our case).

3.3.1 joining datasets

There are different kinds of joining. Joins have a *left* and *right* side data object. Here are the basic ones, with some example data to make concepts clear:

A tibble: 3 x 3

```
## A B C
## <a href="mailto:chr">chr</a> <a href="mailto:chr">chr</a> <a href="mailto:chr">int></a>
## 1 a t 1
## 2 b u 2
## 3 c v 3

y
```

```
# A tibble: 3 x 3
##
     Α
            В
                       D
##
     <chr> <chr> <int>
                        3
## 1 a
            t
## 2 b
                        2
            u
## 3 d
                        1
            W
```

1. left_join() the left object is primary and the right object is secondary. (left side row count unchanged, but right side could grow or shrink)

```
left_join(x,y)
```

```
## Joining, by = c("A", "B")
## # A tibble: 3 x 4
                             D
##
            В
     <chr> <chr> <int> <int>
##
## 1 a
            t
                       1
                             3
## 2 b
                       2
                             2
            u
                       3
## 3 c
            V
                            NA
```

2. right_join() the right object is primary and the left object is secondary. (right side row count unchanged, but left side could grow or shrink)

```
right_join(x,y)
```

```
## Joining, by = c("A", "B")
## # A tibble: 3 x 4
                             D
##
            В
##
     <chr> <chr> <int> <int>
## 1 a
                             3
            t
                       1
                             2
## 2 b
                       2
            u
## 3 d
                     NA
                             1
```

3. inner_join() only keep combinations present in both the left and right. (row count can stay same or shrink)

```
inner_join(x,y)
```

```
## Joining, by = c("A", "B")
## # A tibble: 2 x 4
## A B C D
## <chr> <chr> <int> <int> ## 1 a t 1 3
## 2 b u 2 2
```

4. full_join() keep all combinations (row count can stay same or grow)

```
full_join(x,y)
```

```
## Joining, by = c("A", "B")
## # A tibble: 4 x 4
##
     Α
           В
                             D
##
     <chr> <chr> <int> <int>
## 1 a
           t
                      1
                             3
## 2 b
                      2
                             2
           u
## 3 c
                      3
           v
                            NA
## 4 d
                     NA
                             1
```

You see in each of these examples that we're politely told in the console which variables were used to determine structural combinations? In these examples, it made good default choices, but in general, we should specify which columns to consider, using the by argument:

```
left_join(x, y, by = c("A", "B"))
right_join(x, y, by = c("A", "B"))
inner_join(x, y, by = c("A", "B"))
full_join(x, y, by = c("A", "B"))
```

In our case, we want inner_join(by = c("Country", "Year", "Age)), make sense?

There are other kinds of joining too! Check out this Rstudio cheat sheet for data reshaping possibilities with dplyr: https://github.com/rstudio/cheatsheets/raw/master/data-transfor mation.pdf Here we're interested in the section called Combine Tables on page 2. These cheat sheets are pure gold when you're trying to think through something like this. Now, back to our beloved pipeline!

```
Exp <- Pop %>%

# jan 1 of this year = dec 31 of last year

mutate(Year = Year - 1) %>%

# back-dated, this becomes P2 of the previous year

rename(P2 = Population) %>%

# Join together where Year overlaps

inner_join(Pop, by = c("Country", "Year", "Age")) %>%

rename(P1 = Population) %>%

# Exposure calc averaging within-age

mutate(Exposure = (P1 + P2) / 2) %>%

# sort

arrange(Country, Year, Age)
```

3.3.2 Alternative way to compute exposure

We could also reframe the calculation of exposure as a moving average. Here the functions lead() and lag() are often helpful:

```
a <- 1:10
lead(a)
## [1] 2 3 4 5 6 7 8 9 10 NA
lag(a)
## [1] NA 1 2 3 4 5 6 7 8 9
```

```
# mimics our objective calculation:
(a + lead(a)) / 2
```

```
## [1] 1.5 2.5 3.5 4.5 5.5 6.5 7.5 8.5 9.5 NA
```

The trick is to sort the data as needed (time series within Country and Age), then declare discrete groups on Age and Country. The result is identical, and this approach is more parsimonius than the join approach demonstrated above.

```
Pop %>%
  arrange(Country, Age, Year) %>%
  group_by(Country, Age) %>%
  mutate(Exposure = (Population + lead(Population)) / 2) %>%
  ungroup() %>%
  filter(!is.na(Exposure)) %>%
  arrange(Country, Year, Age)
```

```
## # A tibble: 34,214 x 5
##
      Country Population
                            Age Year Exposure
##
      <chr>
                    <dbl> <dbl> <int>
                                          <dbl>
    1 Albania
                              0
                                 2012
                                         16674.
##
                    16436
##
    2 Albania
                              1
                                 2012
                                         16285
                    16479
##
   3 Albania
                              2
                                 2012
                                         16146.
                    16074
##
    4 Albania
                    16709
                              3
                                 2012
                                         16302
##
   5 Albania
                    16100
                              4
                                 2012
                                         16324
##
    6 Albania
                    16513
                              5
                                 2012
                                         16240
                              6
                                 2012
##
   7 Albania
                    18119
                                         17253
                              7
##
   8 Albania
                                 2012
                                         18529
                    19056
##
    9 Albania
                    19102
                              8
                                 2012
                                         19007
## 10 Albania
                              9
                    21091
                                 2012
                                         20036
## # ... with 34,204 more rows
## # i Use `print(n = ...)` to see more rows
```

3.4 Bring it all together

The above steps accentuate how designing a processing pipeline happens in stages, and sometimes needs to be mapped out in advance. It's inherently iterative, and often involves trial and error, especially when we are learning. When doing this sort of work, we always check the results as we go to ensure things are processing as expected. When complete, we can clean up everything into a parsimonious pipeline. This allows you (and others) to think through all the steps in a glance: because tidyverse verbs string together into sentences! We therefore now paste all the above Pop processing code into a minimal pipeline, duly annotated:

```
# recode age
 mutate(Age = case_when(
    Age == "Less than 1 year" ~ 0,
    Age == "Open-ended age class" ~ 100,
    Age == "Unknown" ~ NA_real_,
    TRUE ~ parse_number(Age)
 ),
 Year = as.integer(TIME)) %>%
  select(-TIME) %>%
  # Begin redistribution of pop with unknown age
 group_by(Country, Year) %>%
 mutate(UNK = Population[is.na(Age)],
         # Not each Country / Year has an Unknown age category
         UNK = ifelse(is.na(UNK), 0, UNK)) %>%
 filter(!is.na(Age)) %>%
  # The redistribution (only affects some subsets)
 mutate(Population = Population + Population / sum(Population, na.rm = TRUE) * UNK) %>%
 select(-UNK) %>%
 ungroup() %>%
  # set up moving avg version of exposure calcs
 arrange(Country, Age, Year) %>%
  # time-series within age and country
 group_by(Country, Age) %>%
 mutate(Exposure = (Population + lead(Population)) / 2) %>%
 ungroup() %>%
 filter(!is.na(Exposure)) %>%
 arrange(Country, Year, Age)
## Warning: 783 parsing failures.
## row col expected
## 101 -- a number Open-ended age class
## 102 -- a number Unknown
## 203 -- a number Open-ended age class
## 204 -- a number Unknown
## 305 -- a number Open-ended age class
```

See how this pipeline is in a single piece? This is because we avoided the self-join of the exposure calculation by using the lead() trick.

4 Work with merged data

... ... for more details.

4.1 Join Pop and Births

Note Pop has more Year (2012-2019), Age (0-100), and Country (41) values than does Births. However, Births has one year that Pop does not (2011). If we did left_join(Pop, Births) that would be clearly too much. If we did left_join(Births, Pop) then we'd be closer, but still have an extra year (2011) with no exposure available. Either of these (and by extension a full_join()) would still work, but would require extra filter() operations in order to get the data down to just the valid combinations of Country, Year, and Age. Hence, we use inner_join() again to create our new object, Dat.

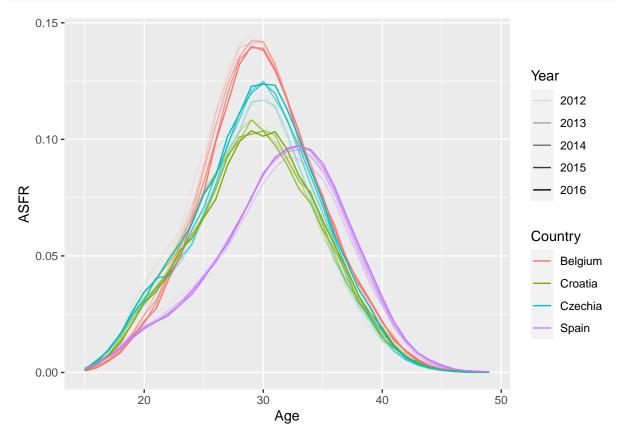
```
Dat <-
Births %>%
inner_join(Pop, by = c("Country", "Year", "Age"))
```

4.2 Calculate rates

Rate calculation is a straightforward mutate() statement. There is no need to apply groups, as age-specific fertility is done row-wise.

```
Dat <-
Dat %>%
mutate(ASFR = Births / Exposure)
```

Now a brief detour to examine the fertility curves and do a quick sanity check that TFR is as expected. An explanation of the ggplot code: everything inside aes() is a mapping of our data to coordinate or aesthetic properties. Since ggplots are composed of additive layers, we can keep adding layers using +. geom_line() is the geometric form that that mapping is translated to. Other geometric mappings are also possible. We'll breeze through several other low-key ggplot examples before more explicitly explaining things later in the workshop.



I can't visually integrate those curves, can you? So let's just do a quick check of TFR:

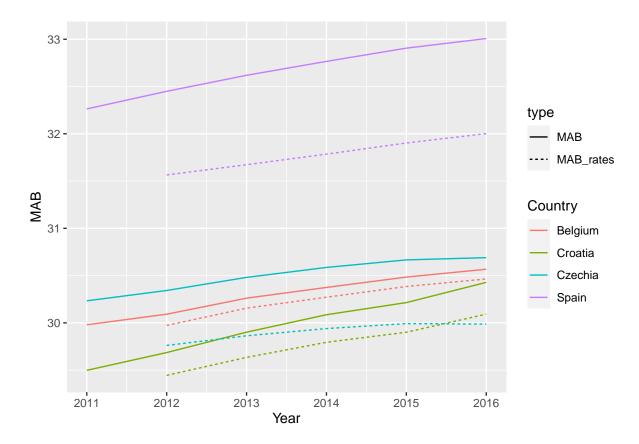
```
Dat %>%
  group_by(Country, Year) %>%
  summarize(TFR = sum(ASFR),
            .groups = "drop") %>%
  pivot_wider(names_from = Country, values_from = TFR)
## # A tibble: 5 x 5
##
      Year Belgium Croatia Czechia Spain
             <dbl>
##
     <int>
                     <dbl>
                             <dbl> <dbl>
## 1 2012
              1.80
                      1.51
                              1.45 1.32
## 2 2013
              1.76
                      1.46
                              1.46 1.27
## 3 2014
              1.74
                      1.46
                              1.53 1.32
## 4 2015
              1.70
                      1.40
                              1.57 1.33
## 5 2016
              1.68
                      1.42
                              1.63 1.34
```

Full disclosure: When setting up this exercise I at first downloaded Total population by Country, Year, and Age, and I literally didn't realize it until checking the TFRs. They were too small, so I re-downloaded denominators to be sure and that was the problem! Lesson: always do these side checks! If your script is cluttered with checks of this kind, then put them aside in a supplementary script.

4.3 Recalculate mean age at birth using rates

Now we can calculate the MAB using fertility rates rather than birth counts, which ought to reduce the effects of population structure.

```
Dat %>%
  # age midpoint
  mutate(Age = Age + .5) \%
  # independent groups
  group_by(Country, Year) %>%
  # weighted mean for MAB
  summarize(MAB_rates = sum(ASFR * Age) / sum(ASFR),
            .groups = "drop") %>%
  # join to previous estimate. We do full
  # because year range different, but we can plot everything
  full_join(MAB, by = c("Country", "Year")) %>%
  # stack so we can plot together easier, mapping MAB types to `linetype`
  pivot_longer(c(MAB_rates,MAB),
               names_to = "type",
               values_to = "MAB") %>%
  # remove NAs from asfr-weighted MAB (no 2011 info)
  filter(!is.na(MAB)) %>%
  ggplot(aes(x = Year,
             y = MAB,
             linetype = type,
             color = Country,
             group = interaction(Country, type))) +
  geom_line()
```



From this we see that trends are mostly the same, but not levels, and sometimes slopes are different. One could easily imagine a situation in which ASFR-weighted MAB gives a different trend than Birth-weighted MAB. One senses Czechia is close this case. Certainly levels can be quite different, and any discrepancy is due to departures from non-uniformity in population structure, which is an odd but precise way of putting it. In this case, if birth-weighted MAB is higher than ASFR-weighted MAB, it means that the population structure has more weight in higher ages.

5 Excercises

I will offer exercise solutions in a separate file, and walk through these in the following day's lecture in case there is no time in the current session.

- 1. plot time trends of TFR by country.
- 2. Use the time-series trick lead() and/or lag() to calculate a variable rt as:

$$r(t) = \frac{MAB(t+1) - MAB(t-1)}{2}$$

Here r(t) (in R call the new variable rt) approximates the average rate of change in the mean age at childbearing. We can use it to calculate an adjusted form of TFR, TFR adj like so:

$$TFR_{adj}(t) = \frac{TFR(t)}{1 - r(t)}$$

The basic idea is that if fertility is undergoing postponement, then the TFR of today is an underestimate (Bongaarts and Feeney (1998)). Without over-thinking the theory, can you

calculate a variable called TFR_adj, and compare it's (short) time trends with those of TFR in a plot?

3. Choose either rate-weighted MAB or birth-weighted MAB, but redo the calculations in terms of 5-year age groups. Assume that the average age within the interval is simply the midpoint (Age + 2.5). Does this change MAB very much?

References

Bongaarts, John, and Griffith Feeney. 1998. "On the Quantum and Tempo of Fertility." *Population and Development Review*, 271–91.