Induction Step:

$$t(2^{k+1}) = t(2^k \times 2)$$
$$= t\left(\frac{2^k \times 2}{2}\right) + (k+1)$$
$$= t(2^k) + k+1$$
$$= \frac{k^2}{2} + \frac{k}{2} + k+1$$
$$= \frac{k^2+k+2k+2}{2}$$
$$t(2^{k+1}) = \frac{(k^2+2k+1)}{2} + \frac{k+1}{2}$$
$$\boxed{t(2^{k+1}) = \frac{(k+1)^2}{2} + \frac{k+1}{2}}$$ Proven.

(d) $t(n) = \sqrt{n^2+100n} - n$

$$t(n) = \sqrt{n^2+100n} - n \times \frac{\sqrt{n^2+100n}+n}{\sqrt{n^2+100n}+n}$$
$$= \frac{\cancel{n^2}+100n-\cancel{n^2}}{\sqrt{n^2+100n}+n}$$
$$= \frac{100n}{\sqrt{n^2+100n}+n}$$
$$= \frac{100n}{n\sqrt{1+100/n}+n}$$
$$= \frac{100}{(\sqrt{1+100/n})+1} \leq 50 \quad \text{for all } n$$
$$= 2 \leq \sqrt{1+100/n} + 1 \quad \text{for all } n$$
$$= 1 \leq \sqrt{1+100/n} \quad \text{(square both sides)}$$
$$= 1 \leq 1+100/n \quad \text{for all } n$$
$$= 0 \leq 100/n \quad \text{for all } n.$$
$$= 100 \geq 0 \quad \text{which is true}$$
$$\therefore \quad t(n) = O(1)$$