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Comp 250 Assignment 2

Q2

(a)

$$t(n) = t(n/2) + \log_2 n \qquad t(1) = 0$$

$$t(n/2) = t(n/4) + \log_2(n/2)$$

$$\Rightarrow \quad t(n) = t(n/4) + \log_2(n/2) + \log_2 n$$

$$t(n/4) = t(n/8) + \log_2(n/4) + \log_2(n/2)$$

$$\Rightarrow \quad t(n) = t(n/8) + \log_2(n/4) + \log_2(n/2) + \log_2(n)$$

$$\therefore \quad t(n) = t(n/2^m) + \log_2(n/(2^{m-1})) + \log_2(n/(2^{m-2})) + \ldots + \log n$$

$n = 2^m$,

$$\therefore \quad t(2^m) = t(2^m/2^m) + \log_2(2^m/2^{m-1}) + \log_2(2^m/2^{m-2}) + \ldots + \log_2 2^m$$

$$= t(1) + \log_2 2 + \log 2^2 + \log_2 2^3 + \ldots + \log 2^m$$

$$= t(1) + (1 + 2 + 3 + \ldots + m)$$

$$= t(1) + \frac{m(m+1)}{2} = \frac{m(m+1)}{2}$$

Since $m = \log_2 n$:

$$t(n) = \frac{(\log_2 n)((\log_2 n) + 1)}{2}$$

$$= \frac{(\log_2 n)^2}{2} + \frac{\log_2 n}{2}$$

$\therefore$ $\boxed{t(n) \text{ is } O((\log_2 n)^2)}$ Answer.

(b) Prove $\frac{(\log_2 n)^2}{2} + \frac{(\log_2 n)}{2}$ with mathematical induction.

(Perform induction on variable m:

$m = \log_2 n$

Base Case: $(m = 0)$

$$t(2^0) = 0$$

$$t(1) = 0 \quad \text{Base case Proved.}$$

$$t(2^k) = \frac{k^2}{2} + \frac{k}{2} \longrightarrow \text{induction hypothesis}$$

P.T.O