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Comp 250 Assignment 2

Q2

(a)

$$t(n) = t(n/2) + \log_2 n \qquad t(1)=0$$

$$t(n/2) = t(n/4) + \log_2(n/2)$$

$$\Rightarrow t(n) = t(n/4) + \log_2(n/2) + \log_2 n$$

$$t(n/4) = t(n/8) + \log_2(n/4) + \log_2(n/2)$$

$$\Rightarrow t(n) = t(n/8) + \log_2(n/4) + \log_2(n/2) + \log_2(n)$$

$$\therefore t(n) = t(n/2^m) + \log_2(n/(2^{m-1})) + \log_2(n/(2^{m-2})) + \dots + \log n$$

$n = 2^m$,

$$\therefore t(2^m) = t(2^m/2^m) + \log_2(2^m/2^{m-1}) + \log_2(2^m/2^{m-2}) + \dots + \log_2 2^m$$

$$= t(1) + \log_2 2 + \log_2 2^2 + \log_2 2^3 + \dots + \log_2 2^m$$

$$= t(1) + (1+2+3+\dots+m)$$

$$= t(1) + \frac{m(m+1)}{2} = \frac{m(m+1)}{2}$$

Since $m = \log_2 n$:

$$t(n) = \frac{(\log_2 n)((\log_2 n)+1)}{2}$$

$$= \frac{(\log_2 n)^2}{2} + \frac{\log_2 n}{2}$$

$\therefore$ $\boxed{t(n) \text{ is } O((\log_2 n)^2)}$ Answer.

(b) Prove $\frac{(\log_2 n)^2}{2} + \frac{(\log_2 n)}{2}$ with mathematical induction.

(Perform induction on variable m:

$m = \log_2 n$

Base Case: $(m=0)$

$t(2^0) = 0$

$t(1) = 0$ Base case Proved.

$$t(2^k) = \frac{k^2}{2} + \frac{k}{2} \longrightarrow \text{induction hypothesis}$$

P.T.O

Induction Step:

$$t(2^{k+1}) = t(2^k \times 2)$$
$$= t\left(\frac{2^k \times 2}{2}\right) + (k+1)$$
$$= t(2^k) + k+1$$
$$= \frac{k^2}{2} + \frac{k}{2} + k+1$$
$$= \frac{k^2+k+2k+2}{2}$$
$$t(2^{k+1}) = \frac{(k^2+2k+1)}{2} + \frac{k+1}{2}$$
$$\boxed{t(2^{k+1}) = \frac{(k+1)^2}{2} + \frac{k+1}{2}}$$ Proven.

(d) $t(n) = \sqrt{n^2+100n} - n$

$$t(n) = \sqrt{n^2+100n} - n \times \frac{\sqrt{n^2+100n}+n}{\sqrt{n^2+100n}+n}$$
$$= \frac{\cancel{n^2}+100n-\cancel{n^2}}{\sqrt{n^2+100n}+n}$$
$$= \frac{100n}{\sqrt{n^2+100n}+n}$$
$$= \frac{100n}{n\sqrt{1+100/n}+n}$$
$$= \frac{100}{(\sqrt{1+100/n})+1} \leqslant 50 \quad \text{for all } n$$
$$= 2 \leqslant \sqrt{1+100/n}+1 \quad \text{for all } n$$
$$= 1 \leqslant \sqrt{1+100/n} \quad \text{(square both sides)}$$
$$= 1 \leqslant 1+100/n \quad \text{for all } n$$
$$= 0 \leqslant 100/n \quad \text{for all } n.$$
$$= 100 \geqslant 0 \quad \text{which is true}$$
$$\therefore \quad t(n) = O(1)$$

(c) $(n_1 \times n_2)(n_1 \times n_2)(n_1 \times n_2)\ldots$ (Since both n_1 and n_2 have N_1 and N_2 digits).

If we divide this into groups of 2 we will get: $n/2$.

Since grade school multiplication is $O(\log_{10}(n)^2)$ this will be $\frac{n}{2} \log(n)^2$

Repeat steps:

$$t(n) = \frac{n}{2} \log(n)^2 + \frac{n}{4} \log n^4 + \ldots + \frac{n}{2^k} \log n^{2k}$$

until $2^k = n$, $(k = \log_2 n)$

$$t(n) = \frac{n}{2^k} \log n^{2k} + \frac{n}{2^{k-1}} \log n^{2k-1} + \ldots + \frac{n}{2^0} \log n^{2^0}$$

$$= \sum_{i=1}^{k} \left((2^k \log n)(n/2^k) \right)$$

$\therefore O(n) = nK \log_{10}(n)$, where $k = \log_2 n$