#1 $f(x,y,z) = \begin{pmatrix} x^2+yz \\ \sin(xyz)+z \end{pmatrix}$

a) Jacobian of f at $(x,y,z)=(-1,0,1)$

let $f(x,y,z) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} x^2+yz \\ \sin(xyz)+z \end{pmatrix}$

$f_1(x,y,z) = x^2+yz$

$f_2(x,y,z) = \sin(xyz)+z$

$Jf = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \end{pmatrix}$

$Jf = \begin{pmatrix} 2x & z & y \\ yz\cos(xyz) & xz\cos(xyz) & xy\cos(xyz)+1 \end{pmatrix}$

$\frac{\partial f_1}{\partial x} = 2x$

$\frac{\partial f_1}{\partial y} = z$

$\frac{\partial f_1}{\partial z} = y$

$\frac{\partial f_2}{\partial x} = yz\cos(xyz)$

$\frac{\partial f_2}{\partial y} = xz\cos(xyz)$

$\frac{\partial f_2}{\partial z} = xy\cos(xyz)+1$

Jf at $(-1,0,1) = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

$Jf = df$

b) directional derivative of f at $(-1,0,1)$ is $df \cdot di$ where di is a directional vector $\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$

$\begin{pmatrix} -2 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} -2dx+dy \\ -dy+dz \end{pmatrix}$

directional derivative is 0 for $di = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$