

Simplex Solver

January 24, 2024

Problem

Given the following linear system and objective function, find the optimal solution.

$$\begin{aligned} & \max 2x_1 + x_2 \\ & \begin{cases} x_1 + 3x_2 \leq 4 \\ x_1 + x_2 \leq 2 \\ 2x_1 \leq 3 \end{cases} \end{aligned}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} x_1 + 3x_2 + s_1 = 4 \\ x_1 + x_2 + s_2 = 2 \\ 2x_1 + s_3 = 3 \end{cases}$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 1 & 3 & 1 & 0 & 0 & 4 \\ 1 & 1 & 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 & 1 & 3 \\ \hline -2 & -1 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_1 and the departing variable is s_3 .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 0 & 3 & 1 & 0 & -1/2 & 5/2 \\ 0 & 1 & 0 & 1 & -1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 1/2 & 3/2 \\ \hline 0 & -1 & 0 & 0 & 1 & 3 \end{array} \right] \begin{matrix} s_1 \\ s_2 \\ x_1 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_2 .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ 0 & 0 & 1 & -3 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 1/2 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 & 7/2 \end{array} \right] \begin{array}{l} s_1 \\ x_2 \\ x_1 \\ \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 1, s_2 = 0, s_3 = 0, x_1 = \frac{3}{2}, x_2 = \frac{1}{2}, z = \frac{7}{2}$$