2)

a) $f(x) = x + \sin(x)$

* $f'(x) = 1 + \cos(x)$

$f'(x) = 0$ at critical points

$1 + \cos(x) = 0$; $\cos(x) = -1$ for $x = \pi + 2\pi n = (2n+1)\pi$

* $f''(x) = -\sin(x)$

@ $(2n+1)\pi$: $f''((2n+1)\pi) = -\sin((2n+1)\pi)$
$= -\sin(2n\pi + \pi)$
$= -\sin(\pi) = 0$

* $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

$\Longrightarrow$ local minimum @ $x = (2n+1)\pi$

global min & max does not exist

b) $f(x) = (2x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - 1)^2$

$\frac{\partial f}{\partial x_1} = 4(2x_1 - x_2) = 8x_1 - 4x_2$

$\frac{\partial f}{\partial x_2} = \cancel{-4(2x_1 - x_2) + 2(x_2 - x_3)}$
$-4x_1 + 4x_2 - 2x_3$

$\frac{\partial f}{\partial x_3} = \cancel{-2(x_2 - x_3) + 2(x_3 - 1)}$
$-2x_2 + 4x_3 - 2$

$\Rightarrow H(x) = \begin{bmatrix} 8 & -4 & 0 \\ -4 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}$

$\Delta_1 = 8 > 0$

$\Delta_2 = \det\begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix}$
$= 8(4) - (-4)(-4)$
> 0

$\Delta_3 = 8\begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix} - (-4)\begin{vmatrix} -4 & -2 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} -4 & 4 \\ 0 & -2 \end{vmatrix}$
$= 8[(4)(4) - (-2)(-2)] + 4[(-4)(4)] + 0 > 0$

since $\Delta_1 > 0$, $\Delta_2 > 0$, $\Delta_3 > 0 \Rightarrow f''(x) > 0$ we have a global minimum

$f'(x) = 0$ to find critical points

$$\left.\begin{array}{l} 8x_1 - 4x_2 = 0 \\ -4x_1 + 4x_2 - 2x_3 = 0 \\ -2x_2 + 4x_3 = 2 \end{array}\right\} \begin{array}{l} 3 \text{ equ.} \\ 3 \text{ unk.} \end{array}$$

$x_1 = \frac{1}{2}$
$x_2 = 1$
$x_3 = 1$

global min at $(\frac{1}{2}, 1, 1)$