

#7  $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \vdots \\ \vdots & & & \vdots \\ a_{m1} & \dots & \dots & a_{mn} \end{bmatrix}$

$$f(y) = g(Ay)$$

Chain Rule:  $f: \mathbb{R}^k \rightarrow \mathbb{R}^m$  and  $g: \mathbb{R}^m \rightarrow \mathbb{R}^n$

let  $h: \mathbb{R}^k \rightarrow \mathbb{R}^n$  be defined by  $h(x) = g(f(x))$ .

$$\Rightarrow \nabla h(x) = \nabla f(x) \nabla g(f(x)) \quad \forall x \in \mathbb{R}^k$$

let  $K(y) = Ay$

$$\Rightarrow f(y) = g(K(y))$$

$$\nabla f(y) = \nabla K(y) \nabla g(K(y))$$

$$\nabla f(y) = \nabla(g(Ay)) = A^T \nabla g(Ay)$$

$$\nabla^2 f(y) = \nabla^2(g(Ay)) = A^T \nabla^2 g(Ay) \cdot A$$