#1

$$A = \begin{bmatrix} 2 & 7 \\ -1 & -6 \end{bmatrix}$$

$$\det(\lambda I - A) = 0$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 7 \\ -1 & -6 \end{bmatrix}\right) = 0$$

$$\det\begin{bmatrix} \lambda-2 & -7 \\ 1 & \lambda+6 \end{bmatrix} = 0 \implies (\lambda-2)(\lambda+6) - (1)(-7) = 0$$

$$\lambda^2 + 6\lambda - 2\lambda - 12 + 7 = 0$$

$$\lambda^2 + 4\lambda - 5 = 0$$

$$\lambda = 1 \quad , \quad \lambda = -5$$

for $\lambda = 1$

$$(1I - A)X = 0$$

$$\left(1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 7 \\ -1 & -6 \end{bmatrix}\right)\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -7 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} -1 & -7 & | & 0 \\ 1 & 7 & | & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 7 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$X_1 + 7X_2 = 0$$

$$X_1 = -7X_2$$

$$X_2 = X_2 \implies V = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$$

for $\lambda = -5$

$$(-5I - A)X = 0$$

$$\left(-5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 7 \\ -1 & -6 \end{bmatrix}\right)\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -7 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} -7 & -7 & | & 0 \\ 1 & 1 & | & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$X_1 + X_2 = 0$$

$$X_1 = -X_2 \quad V = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$X_2 = X_2$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\det(\lambda I - B) = 0$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \right) = 0$$

$$\begin{vmatrix} \lambda-1 & -1 & 0 \\ 0 & \lambda-1 & -2 \\ 0 & 0 & \lambda-3 \end{vmatrix} = (\lambda-1)[(\lambda-1)(\lambda-3)+2] - (-1)\begin{bmatrix} 0 & 0 \\ 0 & \lambda-3 \end{bmatrix}^{\to 0} = (\lambda-1)(\lambda^2-4\lambda+5)$$
$$= 0$$

$$\lambda = 1 \qquad \lambda = 3$$

for $\lambda = 1$

$$(1I - B)x = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{rcases} x_2 = 0 \\ x_3 = 0 \\ x_1 = x_1 \end{rcases} \Rightarrow v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

for $\lambda = 3$

$$(3I - B)x = 0$$

$$3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$RREF: \begin{bmatrix} 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_1 - \frac{1}{2}x_3 &= 0 \\ x_2 - x_3 &= 0 \end{aligned} \Rightarrow \begin{aligned} x_2 &= x_3 \\ x_1 &= \frac{1}{2}x_3 \end{aligned} \quad \text{let } x_3 = 1 \quad v = \begin{pmatrix} \frac{1}{2} \\ 1 \\ 1 \end{pmatrix}$$