

#1

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$3 \times 2$

$$A^T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$2 \times 3$

$$a) \quad A A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 13 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 5 & 8 \\ 8 & 14 \end{bmatrix}$$

$X \Rightarrow$  eigenvectors of  $A A^T$

$Y \Rightarrow$  eigenvectors of  $A^T A$

$$A^+ = Y \Sigma^+ X^T = \begin{bmatrix} -\frac{8}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \end{bmatrix} = \begin{bmatrix} -8 & -2 & 4 \\ 5 & 2 & -1 \end{bmatrix}$$

$$\Rightarrow A^+ A = \begin{bmatrix} -\frac{4}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \text{Rank}(A) = 2$$

$$b) \quad R(A) = \text{span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$P_{R(A)} = A A^+ = \frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$$

$$c) \quad A^+ A = \begin{bmatrix} 5 & 8 \\ 8 & 14 \end{bmatrix}$$