1) $f(x+y) = f(x) + f(y)$; $f(ax) = af(x)$

a) $f(x) = |x|$

* $|x+y| \neq |x| + |y|$

$|ax| \stackrel{?}{=} a|x|$

* $|a||x| \neq a|x|$

not linear

b) $f(x) = x^2 + 2x + 2$

$f(x+y) = (x+y)^2 + 2(x+y) + 2$

$x^2 + y^2 + 2xy + 2x + 2y + 2$

$f(x) + f(y) = x^2 + y^2 + 2x + 2y + 4$

* $f(x+y) \neq f(x) + f(y)$

$af(x) = ax^2 + 2ax + 2a$

$f(ax) = a^2x^2 + 2ax + 2$

* $af(x) \neq f(ax)$

not Linear

c) $f(x) = \frac{1}{x}$

* $\frac{1}{x+y} \neq \frac{1}{x} + \frac{1}{y}$

* $\frac{a}{x} \neq \frac{1}{ax}$

not linear

d) $f(x,y) = \sin(xe^y)$

* $\sin((x_1+x_2)e^{(y_1+y_2)}) \neq \sin(x_1e^{y_1}) + \sin(x_2e^{y_2})$

* $ab\sin(xe^y) \neq \sin(axe^{by})$

not linear

e) $f(x,y) = \cos(x)y + x^2y$

$f(x_1,y_1) = \cos(x_1)y_1 + x_1^2y_1$

$f(x_2,y_2) = \cos(x_2)y_2 + x_2^2y_2$

$f(x_1,y_1) + f(x_2,y_2) = \cos(x_1)y_1 + x_1^2y_1 + \cos(x_2)y_2 + x_2^2y_2$

$= y_1(\cos(x_1) + x_1^2) + y_2(\cos(x_2) + x_2^2)$

$f(x_1+x_2, y_1+y_2) = \cos(x_1+x_2)(y_1+y_2) + (x_1+x_2)^2(y_1+y_2)$

$= (y_1+y_2)\left[\cos(x_1+x_2) + (x_1+x_2)^2\right]$

$= (y_1+y_2)\left[\cos(x_1)\cos(x_2) - \sin(x_1)\sin(x_2) + x_1^2 + x_2^2 + 2x_1x_2\right]$

$abf(x,y) = ab\cos(x)y + abx^2y$

$f(ax,by) = b\cos(ax)y + ba^2x^2y$

not linear