#3

$$X_{k+1} = \frac{1}{2}\left(X_k + \frac{a}{X_k}\right) \implies a > 0$$

a)

$$\lim_{k \to \infty} X_k = \ell$$

$$\ell = \frac{1}{2}\left(\ell + \frac{a}{\ell}\right)$$

$$\ell = \frac{\ell^2 + a}{2}$$

$$\ell^2 - a = 0$$

$$\ell = \sqrt{a} \implies \lim_{k \to \infty} X_k = \sqrt{a}$$

b) NR method:
$$X_{k+1} = X_k - \frac{f(X_k)}{f'(X_k)}$$

$$\implies X_{k+1} = X_k - \frac{f(X_k)}{f'(X_k)} = \frac{X_k^2 + a}{2X_k}$$

$$X_{k+1} = \frac{X_k f'(X_k) - f(X_k)}{f'(X_k)} = \frac{X_k^2 + a}{2X_k}$$

$$f'(X_k) = 2X_k$$

$$X_k(2X_k) - f(X_k) = X_k^2 + a$$

$$2X_k^2 - a - X_k^2 = f(X_k)$$

$$f(X_k) = X_k^2 - a$$

$$f(X) = x^2 - a$$