#4 a) $f(x,y,z) = e^{x^2+y^2+z^2} - x^4 - y^6 - z^6$

$$\nabla f(x,y,z) = \begin{pmatrix} 2x\, e^{x^2+y^2+z^2} - 4x^3 \\ 2y e^{x^2+y^2+z^2} - 6y^5 \\ 2z\, e^{x^2+y^2+z^2} - 6z^5 \end{pmatrix}$$

~~let $\alpha = x^2+y^2+z^2$~~

$$H = \begin{pmatrix} (4x^2+2)e^{x^2+y^2+z^2} - 12x^2 & 4xy e^{x^2+y^2+z^2} & 4xz\, e^{x^2+y^2+z^2} \\ 4xy\, e^{x^2+y^2+z^2} & (4y^2+2)e^{x^2+y^2+z^2} - 30y^4 & 4yz\, e^{x^2+y^2+z^2} \\ 4xz\; e^{x^2+y^2+z^2} & 4yz\; e^{x^2+y^2+z^2} & (4z^2+2)e^{x^2+y^2+z^2} - 30z^4 \end{pmatrix}$$

$\nabla f(x,y,z) = 0$ if $x=0,\ y=0,\ z=0$

H @ critical point $= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow$ eigenvalues

$\lambda_1 = 2 \quad \lambda_1 = 2 \quad \lambda_3 = 2$

$(0,0,0)$ is a local minimizer of f

b) $f(x,y) = x^3 + \cancel{E}^{3y} - 3xe^y$

$$\nabla f(x,y) = \begin{pmatrix} 3x^2 - 3e^y \\ 3e^{3y} - 3e^y - 3xe^y \end{pmatrix} = \begin{pmatrix} 3x^2 - 3e^y \\ 3e^{3y} - 3xe^y \end{pmatrix}$$

① $3x^2 - 3e^y = 0$

② $3e^{3y} - 3xe^y = 0$

from ② $x = e^{2y}$

replace in ① $3(e^{2y})^2 - 3e^y = 0 \Rightarrow e^y(e^{3y} - 1) = 0$

$e^y \neq 0$

$e^{3y} = 1$ for equ to be 0

$y = 0 \Rightarrow x = 1$

Critical point @ $(1,0)$

$$H = \begin{pmatrix} 6x & -3e^y \\ -3e^y & 9e^{3y} - 3e^y - 3xe^y \end{pmatrix}$$

$$= \begin{pmatrix} 6x & -3e^y \\ -3e^y & 9e^{3y} - 3xe^y \end{pmatrix}$$

@ $(1,0)$ $H = \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix}$

positive definit

$\Rightarrow (1,0)$ is a local minimizer of f.