#2 $f(x) = 2x_1^2 + x_1 x_2 + x_2^2 + x_2 x_3 + x_3^2 - 6x_1 - 7x_2 - 8x_3 + 9$

$f(x) = 2x_1^2 + x_2^2 + x_3^2 + x_1 x_2 + x_2 x_3 - 6x_1 - 7x_2 - 8x_3 + 9$

a) $\nabla f(x) = \begin{pmatrix} 4x_1 + x_2 - 6 \\ 2x_2 + x_1 + x_3 - 7 \\ 2x_3 + x_2 - 8 \end{pmatrix}$ $Hf(x) = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

b) at $(0,0,0)$

$\nabla f(x) = \begin{pmatrix} -6 \\ -7 \\ -8 \end{pmatrix}$ $Hf(x) = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

c) $f(x) = f(x^*) + f'(x^*)(x - x^*) + \dfrac{f''(z)}{2!}(x - x^*)^2$

$x^* = (0,0,0)$

$f(x_1, x_2, x_3) = f(0,0,0) + \dfrac{df(0,0,0)}{dx_1}(x_1 - 0) + \dfrac{df(0,0,0)}{dx_2}(x_2 - 0) + \dfrac{df(0,0,0)}{dx_3}(x_3 - 0)$

$+ \dfrac{1}{2!} \left[\dfrac{df(0,0,0)}{dx_1^2}(x_1 - 0)^2 + \dfrac{df(0,0,0)}{dx_2^2}(x_2 - 0)^2 + \dfrac{df(0,0,0)}{dx_3^2}(x_3 - 0)^2 \right.$

$\left. + 2\dfrac{df(0,0,0)}{dx_1 dx_2}(x_1 - 0)(x_2 - 0) + 2\dfrac{df(0,0,0)}{dx_1 dx_3}(x_1 - 0)(x_3 - 0) + 2\dfrac{df(0,0,0)}{dx_2 dx_3}(x_2 - 0)(x_3 - 0) \right]$

$f(x_1, x_2, x_3) = 9 - 6x_1 - 7x_2 - 8x_3 + \dfrac{1}{2!}\left[4x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1 x_2 + 0 + 2x_2 x_3 \right]$

$f(x_1, x_2, x_3) = 9 - 6x_1 - 7x_2 - 8x_3 + 2x_1^2 + x_2^2 + x_3^2 + x_1 x_2 + x_2 x_3$

$f(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + x_3^2 + x_1 x_2 + x_2 x_3 - 6x_1 - 7x_2 - 8x_3 + 9$