

#6  
a)  $f(x,y) = (x^2+4)^2 + y^2$

$$\frac{\partial f}{\partial x} = 4x(x^2+4) \Rightarrow \nabla f(x,y) = \begin{pmatrix} 4x(x^2+4) \\ 2y \end{pmatrix}$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\nabla f(x,y) = 0 \Rightarrow \nabla f(x,y) = 0 \text{ @ } (0,0), (2,0), (-2,0)$$

$$4x(x^2+4) = 0$$

stationary points

$$4x = 0 \quad x = 0$$

$$x^2+4 = 0 \quad x = \pm 2$$

$$2y = 0 \quad y = 0$$

$$H(x,y) = \begin{bmatrix} 12x^2-16 & 0 \\ 0 & 2 \end{bmatrix}$$

$$12x^2-16 = 0 \Rightarrow x = \frac{2}{\sqrt{3}}, \quad x = -\frac{2}{\sqrt{3}}$$

$$12x^2-16 > 0 \quad x > \frac{2\sqrt{3}}{3} \quad x < -\frac{2\sqrt{3}}{3}$$

$$H(x,y) > 0 \text{ when } x > \frac{2\sqrt{3}}{3} \quad x < -\frac{2\sqrt{3}}{3}$$

evaluate  $H(x,y)$  @ critical points

@ (0,0)  $H(0,0) = \begin{bmatrix} -16 & 0 \\ 0 & 2 \end{bmatrix}$  non definite (0,0) is a stationary point

@ (2,0)  $H(2,0) = \begin{bmatrix} 32 & 0 \\ 0 & 2 \end{bmatrix} > 0 \Rightarrow$  necessary conditions are satisfied

@ (-2,0)  $H(-2,0) = \begin{bmatrix} 32 & 0 \\ 0 & 2 \end{bmatrix} > 0 \Rightarrow$  necessary conditions are satisfied

(2,0), (-2,0) are local minima

$$b) f(x, y) = \frac{1}{2}x^2 + x \cos(y)$$

$$\frac{\partial f}{\partial x} = x + \cos(y)$$

$$\frac{\partial f}{\partial y} = -x \sin(y)$$

$$\nabla f(x, y) = \begin{pmatrix} x + \cos(y) \\ -x \sin(y) \end{pmatrix}$$

$$H(x, y) = \begin{bmatrix} 1 & -\sin(y) \\ -\sin(y) & -x \cos(y) \end{bmatrix}$$

$$\nabla f(x, y) = 0$$

$$\textcircled{1} x + \cos(y) = 0$$

$$\textcircled{2} -x \sin(y) = 0$$

eq 2 = 0 when  $x = 0 \Rightarrow$  from eq 1  $y = n\pi$  for  $n \neq 0$

$\frac{\pi}{2}, \frac{3\pi}{2}, \dots$

When  $y = n\pi \Rightarrow$  from eq 1  $x = \pm 1$

$$\begin{matrix} (0, n\frac{\pi}{2}) & (1, n\pi) & (-1, n\pi) \\ \downarrow & \downarrow & \downarrow \\ n \neq 0 & \text{odd} & \text{even} \\ & \pi, 3\pi, \dots & 0, 2\pi, 4\pi, \dots \end{matrix}$$

$$H(0, n\frac{\pi}{2}) = \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 0 \end{bmatrix} \Rightarrow \text{undetermined}$$

$$H(1, n\pi)_{\text{odd}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{local minimum}$$

$$H(-1, n\pi)_{\text{even}} = \begin{bmatrix} 1 & 0 \\ 0 & +1 \end{bmatrix} \Rightarrow \text{local minimum}$$



$$c) f(x, y) = \sin(x) + \sin(y) + \sin(x+y)$$

$$\nabla f(x, y) = \begin{pmatrix} \cos(x) + \cos(x+y) \\ \cos(y) + \cos(x+y) \end{pmatrix}$$

$$H(x, y) = \begin{bmatrix} -\sin(x) - \sin(x+y) & -\sin(x+y) \\ -\sin(x+y) & -\sin(y) - \sin(x+y) \end{bmatrix}$$

Critical points.

$$\begin{aligned} \nabla f(x, y) = 0 \quad \cos(x) + \cos(x+y) = 0 \quad \cos(x+y) = -\cos(x) \\ \cos(y) + \cos(x+y) = 0 \quad \cos(x+y) = -\cos(y) \end{aligned}$$

$$\cos(x+y) = -\cos(x) = -\cos(y)$$

$$\Rightarrow x = y \text{ en } @ (\pi, \pi)$$

$$H(\pi, \pi) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ Undetermined}$$