

#2

$$\min \|Ax - b\|^2$$

$$\|Ax - b\|^2 = (Ax - b)^T (Ax - b)$$

$$(Ax - b)^T (Ax - b) = (x^T A^T - b^T) (Ax - b)$$

$$(x^T A^T A x - b^T A x - x^T A^T b + b^T b)$$

$$\text{gradient} = 0$$

$$\nabla (x^T A^T A x - b^T A x - x^T A^T b + b^T b) = 0$$

$$2A^T A x - A^T b - A^T b = 0$$

$$2A^T A x = 2A^T b$$

$$A^T A x = A^T b \quad \text{normal equations:}$$

(*)

$$\begin{bmatrix} I_m & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} r \\ x \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

$$r = b - Ax$$

\Rightarrow components of x (**) satisfy the normal equations. (*)