#7 $f(x_1, x_2) = 100(x_2 - x_1)^2 + (1-x_1)^2$

$$\nabla f^T \cdot d < 0 \quad \text{where} \quad d = \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$

$$\nabla f(x_1, x_2) = \begin{pmatrix} 2 \cdot 100(x_2 - x_1^2)(-2x_1) + 2(1-x_1)(-1) \\ 2 \cdot 100(x_2 - x_1^2) \end{pmatrix}$$

$$\nabla f(x_1, x_2) \text{ at } (0,0) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$\nabla f(x_1, x_2)^T = \begin{pmatrix} -2 & 0 \end{pmatrix}$$

$$\nabla f(x_1, x_2)^T \cdot \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix} = -2dx_1 + 0 \, dx_2 < 0$$

$$\Rightarrow -2dx_1 < 0$$

for this to be always true:

dx_1 should be stricktly +ve

$$\Rightarrow d = \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix} \text{ where } dx_1 \text{ is a +ve real number and } dx_2 \text{ is a Real number}$$

$$dx_1 \in \mathbb{R}^+ \, , \, dx_2 \in \mathbb{R}$$