3) $f(x_1, x_2) = x_1^3 - 3ax_1x_2 + x_2^3$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 - 3ax_2$$

$$\frac{\partial f}{\partial x_2} = -3ax_1 + 3x_2^2$$

$$\frac{\partial^2 f}{\partial^2 x_1} = 6x_1$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -3a$$

$$\frac{\partial^2 f}{\partial^2 x_2} = 6x_2$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = -3a$$

$$\nabla f = \begin{bmatrix} 3x_1^2 - 3ax_2 \\ -3ax_1 + 3x_2^2 \end{bmatrix}$$

$$H(f) = \begin{bmatrix} 6x_1 & -3a \\ -3a & 6x_2 \end{bmatrix}$$

$\Delta_1 = 6x_1$

$\Delta_2 = \det(H(f)) = (6x_1)(6x_2) - (-3a)(-3a)$

$36x_1x_2 - 9a^2$

for $(0,0)$ $\Delta_1 = 0$ $\Delta_2 = -9a^2$ -ve

$\Rightarrow$ indef.

for (a,a) $\Delta_1 = 6a$ $\Delta_2 = 27a^2$ (+ve)

+ve local minimizer $\leftarrow$ (+ve) (-ve) $\rightarrow$ indef.

① $3x_1^2 - 3ax_2 = 0$

② $-3ax_1 + 3x_2^2 = 0$

from 2: $3x_2^2 = 3ax_1 \Rightarrow x_1 = \frac{x_2^2}{2}$

replacein 1: $3\left(\frac{x_2^2}{a}\right)^2 - 3ax_2 = 0$

$3\left(\frac{x_2^4}{a^2}\right) - 3ax_2 = 0$

$3x_2^4 - 3ax_2 = 0$

$3x_2(x_2^3 - a^3) = 0$

$3x_2 = 0$	$x_2^3 - a^3 = 0$
$x_2 = 0$	$x_2 = a$
replacein 1 $\Rightarrow x_1 = 0$	$x_1 = a$

critical points @ $(0,0)$ and (a,a)

4) $f(x_1, x_2) = x_1^5 - x_1x_2^6$

$$\frac{\partial f}{\partial x_1} = 5x_1^5 - x_2^6$$

$$\frac{\partial f}{\partial x_2} = -6x_1x_2^5$$

$$\Rightarrow \nabla f(x) = \begin{bmatrix} 5x_1^5 - x_2^6 \\ -6x_1x_2^5 \end{bmatrix}$$

when set $= 0$
$x_1 = 0$
$x_2 = 0$

critical point @ $(0,0)$

$$\frac{\partial^2 f}{\partial^2 x_1} = 25x_1^4$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -6x_2^5$$

$$\frac{\partial^2 f}{\partial^2 x_2} = -30x_1x_2^4$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = -6x_2^5$$

$$\Rightarrow \nabla^2 f(x) = \begin{bmatrix} 25x_1^4 & -6x_2^5 \\ -6x_2^5 & -30x_1x_2^4 \end{bmatrix}$$

$= \det(\nabla^2 f(x)) = (25x^4)(-30x_1x_2^4) - (-6x_2^5)^2$

It can be +ve or -ve

undetemined

indefinite $\Rightarrow (0,0)$ is an inflection point

$$\nabla^2 f(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$