#6 $\begin{pmatrix} x^4 & x^3 & x^2 \\ x^3 & x^2 & x \\ x^2 & x & 1 \end{pmatrix}$ the matrix has a +ve diagonal

Quadratic form

$x^4\alpha_1^2 + x^2\alpha_2^2 + 1\alpha_3^2 + 2x^3\alpha_1\alpha_2 + 2x^2\alpha_1\alpha_3 + 2x\alpha_2\alpha_3$

Quadratic form is +ve

$\Rightarrow$ +ve semidefinite

Proof : determinant test det1: $x^4 > 0$ det2: $x^4x^2 - x^3x^3 = x^6 - x^6 = 0$ det3: 0

$$x^4\begin{vmatrix} x^2 & x \\ x & 1 \end{vmatrix} - x^3\begin{vmatrix} x^3 & x \\ x^2 & 1 \end{vmatrix} + x^2\begin{vmatrix} x^3 & x^2 \\ x^2 & x \end{vmatrix} = x^4(x^2 - x^2) - x^3(x^3 - x^3) + x^2(x^4 - x^4) = 0$$

$\Rightarrow$ +ve semidefinite and not +ve definite