

#4 Theorem

$$f(b) - f(a) = (b - a)f'(c)$$

$$f: [a, b] \rightarrow \mathbb{R}$$

~~a < b~~

$$\text{for } f: [a, b] \rightarrow \mathbb{R}^2$$

$$f: [0, 1] \rightarrow \mathbb{R}^2$$

$$f(x) = (f_1(x), f_2(x)) = (x(1-x), x^2(1-x))$$

$$f_1: [0, 1] \rightarrow \mathbb{R} \quad f_1(x) = x(1-x) = x - x^2 \quad \text{continuous \& differentiable}$$

$$f_2: [0, 1] \rightarrow \mathbb{R} \quad f_2(x) = x^2(1-x) = x^2 - x^3 \quad \text{continuous \& differentiable}$$

$$f = (f_1, f_2) \Rightarrow \begin{array}{l} \text{continuous } [0, 1] \\ \text{differentiable } (0, 1) \end{array}$$

$$f(1) - f(0) = (1(1-1), 1^2(1-1)) - (0(1-0), 0^2(1-0)) = (0, 0) - (0, 0) = (0, 0)$$

$$f(x) = (x(1-x), x^2(1-x)) = (x - x^2, x^2 - x^3)$$

$$f'(x) = (1 - 2x, 2x - 3x^2)$$

\Rightarrow yes the theorem remains true for $f: [a, b] \rightarrow \mathbb{R}^2$