The Macroeconomic Effects of Fiscal Adjusments in The UK

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1 Introduction

- motivate
- research gap / contribution
- summarise/ outline structure

This paper proceeds as follows, Section 2 provides a review of the related literature, focusing on the costs of high indebtedness and evidence on the effects of fiscal consolidation. We also provide a synthesis of the methods used to estimate fiscal multipliers, highlighting the use of Structural Vector Autoregressions (SVARs). This provides us with sufficient context and foundations to proceed with section 3 where we introduce the data and reduced form model. Here the Vector Error Correction Model (VECM) representation is considered as a generalisation of the VAR model which can yield efficiency gains when properly specified. We considered the merits of this representation and perform pre-testing of the specification. Following this, in Section 4 we introduce the identifying assumptions to allow us to recover the structural shocks. With our model now defined, Section 5 presents the results of this paper, focusing on the Impulse Response Function (IRF) and Forecast Error Variance Decomposition (FEVD). Overall, the IRFs of interest are statistically and practically significant; in Section 6 we consider robustness checks such as parameter stability and a lower lag lenght to see whether the results are sensitive to the specification. Finding that the result of low fiscal multipliers in the UK are robust to the specification, Section 7 proceeds with a discussion of policy implications. In Section 8 we then constextualise the results of this study and scope for future research, so that the results can be appropriately appreciated. Section 9 concludes.

2 Literature Review

2.1 Costs of high indebtedness

Warmedinger, Checherita-Westphal, and De Cos (2015) emphasise the importance of public debt sustainability for ensuring macroeconomic stability. Recent economic crises have been met with government intervention, leading to further strains on public finances. For instance, Sutherland, Hoeller, and Merola (2012) draw attention to the fiscal challenges facing countries following the Global Financial Crisis (GFC). They also note that gross government debt has exceeded 100% of GDP for the OECD as an aggregate. These concerns have been exacerbated following the Covid pandemic where governments implemented fiscal measures to mitigate the economic costs of the pandemic (IMF, 2023).

Makin and Layton (2021) highlight that governments must employ fiscal responsibility to protect their economies from the risks that high indebtedness exposes them to. There are several mechanisms through which these risks may be presented. High indebtedness reduces the ability of countries to effectively respond counter cyclically to economic shocks. Additionally, when the interest rate exceeds the growth rate, the economy is unable to sustain the cost of growth. While Blanchard (2019) suggests that the low interest environments relative to growth seen recently have been persistent historically (which would suggest that debt could be self-stabilising due to economic growth outpacing the cost of debt), he also argues that there would still be a welfare cost in such a scenario. This is because Blanchard argues that capital accumulation reduces in response to high indebtedness. This aligns with the analysis of Alesina and Ardagna (1998) that "Wealth rises when future tax burdens decline": economic agents anticipate greater tax burdens due to the excessive levels of public indebtedness, and consequently confidence worsens leading to lower consumption and investment.

Kumar and Woo (2015) provide further evidence of the cost of public indebtedness, finding that greater indebtedness is associated with lower economic growth. They find noticeable non-linearities in this result, with the most severe effect occurring when public indebtedness exceeds 90% of GDP. Ilzetzki, Mendoza, and Vegh (2013) suggest that shocks to government speding can have strong negative effects on output at debt levels of as little as 60% of GDP. Recalling that public indebtedness has exceeded 100% in the OECD overall (Sutherland, Hoeller, and Merola ,2012), these results will be of particular interest for advanced economies. This highlights the importance of fiscal consolidation to ensure the long-term resilience of the economy. The IMF (2024) argue that economies should rebuild their fiscal buffers to reduce debt vulnerabilities, proposing that fiscal adjustments would need to be in the region of 4% of GDP. Sutherland, Hoeller, and Merola (2012) have argued for more aggressive adjustments, exceeding 5% of GDP in order to bring debt down to 50% of GDP. This figure would require the UK to halve its current debt levels (ONS, 2025). Thus, achieving this objective would require significant fiscal adjustments. **The next subsection synthesises the literature on fiscal consolidation and its effects.**

2.2 Fiscal Consolidaton

While the importance of fiscal consolidation has been highlighted, it is crucial that these measures are not at the expense of the broader economy. By investigating forecast errors for a sample of European countries, Blanchard and Leigh (2013) find that larger anticipated fiscal consolidation was associated with lower growth. This result was interpreted as due to the fiscal multipliers being greater than anticipated by forecasters. Consequently, fiscal tightening would have further dampened demand, meaning that improvements to government finances through fiscal consolidation would be offset by reduced growth. Gechert, Horn, and Paetz (2017) adopt a similar methodology, finding that austerity measures in the Euro Area deepened the GFC, with large persistence in their results. This suggests that, even in the long run, poorly implemented fiscal adjustments could be counterproductive. Fatas and Summers (2018) extend this research, investigating the long-term effects that fiscal adjustments have had on GDP. Their analysis suggests that fiscal consolidations have failed to lower the debt-to-GDP ratio due to a hysteresis effect of contractionary fiscal policy. This research underscores the need to effectively quantify fiscal multipliers, allowing policymakers to better understand the potential trade-offs between various economic objectives.

Despite this, the literature has struggled to gain consensus on the size, and even the sign of fiscal multipliers (Caldara and Kamps, 2008). Similarly, Ramey (2016) notes that there has been conflicting evidence in the literature as to which fiscal policy has the greater multiplier, noting that the lack of precision of estimates have made it difficult to make meaningful comparisons. The next subsection focuses on the methodologies used to estimate fiscal multipliers. Before proceeding, we discuss further evidence in the literature of heterogeneity in fiscal multipliers.

Canzoneri et al (2016) suggest that fiscal multipliers can vary throughout the business cycle, with greater effects being observed during recessions. Auerbach and Goronichenko (2012) also provide evidence of the greater effectiveness of fiscal policy during recessions. Furthermore, they note that there can be heterogeneity of multipliers within specific fiscal instruments, noting that military spending had the greatest multipliers. Adding to the literature on state dependent multipliers, Ghassibe and Zanetti (2022) note that the cause of a recession determines the extent of fiscal multipliers. While they report government spending to have larger multipliers during demand-driven recessions, they note that these can be ineffective when the recession is driven by supply side factors. On the other hand, they note that policies such as reductions in income taxes can be effective during supply-driven downturns, as they can help to boost aggregate supply.

Ilzetzki, Mendoza, and Vegh (2013) suggest that the heterogeneity in the estimates reported in the literature can be attributed to differences in structural characteristics of the economy considered. Key characteristics that the authors emphasise include: the level of development, public indebtedness, and openness to trade.

Alesina, Favero, and Giavazzi (2015) compare multipliers due to spending and tax adjustments. They find that tax-based adjustments have more severe effects than those based on adjustments to expenditure. They attribute this result to business confidence. They argue that sentiment recovers quickly in response to government spending cuts, however increasing revenue does not address the root of the problem and thus confidence struggles to recover. Similarly, they attribute confidence to their finding that adjustments as less effective when subject to reversals: this creates noise when future adjustments are made, agents are unable to ascertain the reliability of announcements, and consequently their effectiveness decreases. This reinforces the importance of research to better understand the fiscal multiplier for different policy instruments, particularly as this may vary across countries and over time.

2.3 Synthesis of Methodology

Capek and Cuasera (2020) simulated 20 million fiscal multipliers, highlighting how methodological choices contribute to the heterogeneity in estimates of fiscal multipliers prevalent in the literature. Consequently, they advocate for explicitly outlining modelling choices and assumptions. Similarly, Gechert (2017) provides a synthesis of the methodologies used to estimate fiscal multipliers, highlighting competing definitions for the fiscal multiplier and possible issues in its estimation. Among these issues, Gechert (2017) highlights potential omitted variable bias in the VAR model. When such variables are excluded from the VAR model, Structural VAR (SVAR) tools such as IRFs may lose their interpretation due to contamination with the unobeserved effect.

DSGE

Structural Vector Autoregressions (SVARs) have been prominent in the literature to estimate fiscal multipliers. Various approaches to identification have been used, with XXX (YYYY) noting that after accounting for the empirical specification, the competing identifying approaches have little effect on the estimated multipliers. Blanchard and Perotti (2002) pioneered this strand of the research, leveraging methodologies previously popularised by the monetary economics. To identify their SVAR, Blanchard and Perotti leverage instituional information. They provide a definition for the fiscal variables and highlight that government expenditure is predetermined within a quarter. Recursive measures to identfication have been employed by Fatas and Mihov (YYYY) and Fernandex (2008). Fernandez argues that Uhlig and Mountford (200Y) apply restrictions on the signs of the impulse response functions. Caldara and Kamps (2008) reviews the literature on SVAR identification. Caldara and Kamps (2017) introduce a new approach for identification.

Auerbach and Gorodnichenko (2012) explore state-dependent fiscal multipliers using regime-switching models. They find evidence of substantial differences in multipliers between recessionary and expansionary periods.

Add DSGE lit for context

3 Econometric Methodology

Despite the literature highlighting possible importance of state dependence, we focus on a linear VAR model. Reason is that little research has been performed on mulitpliers for the UK, scope is to ...

This research employs a structural vector autoregressive (SVAR) approach to estimating fiscal multipliers. In this section we outline the empirical methodol-

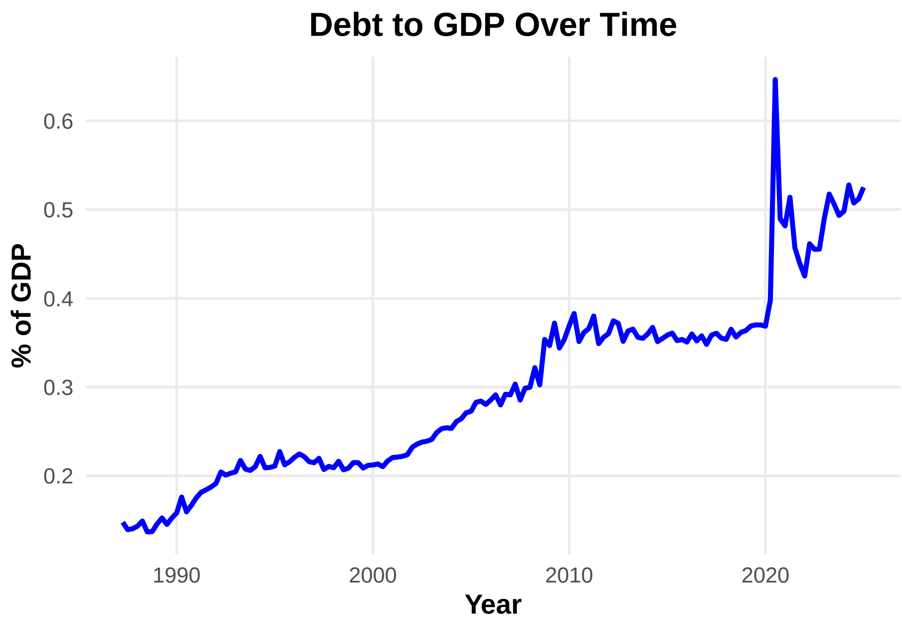


Figure 1: Debt to GDP Over Time

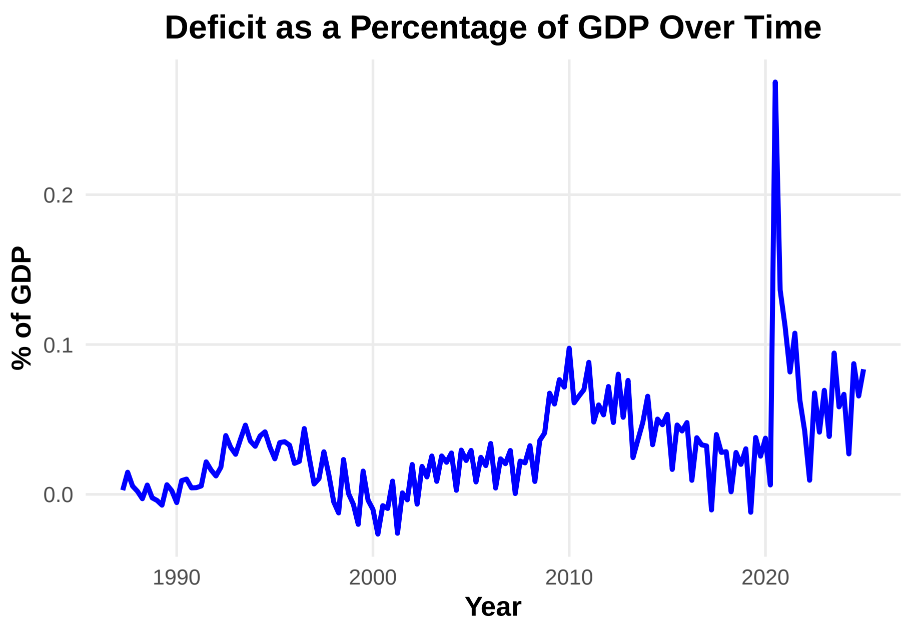


Figure 2: Government Deficit Over Time (as a percentage of GDP)

ogy, as well as the data. We use quarterly data from 1987:1 to 2023:3, modelling this using a six-dimensional VAR at with a lag length of 4. This lag order follows Blanchard and Perroti (2002), and suggests that past values of the endogenous variables continue to have an effect for up to a year. While statistical procedures were not used to determine this lag order, it has an econometric rationale: Kilian and Lutkepohl (2016) note that due to poor finite sample properties of lag selection techniques, it may be appropriate to impose a fixed lag length. Particularly when concerned with impulse response analysis, the risks of underestimating the lag length exceed those of using a larger order which may better reflect the dynamics of the data at the cost of inefficient estimates. Given these considerations, and the degrees of freedom restrictions imposed by the sample size, the lag order of 4 was determined.

Regarding the endogenous variables included in the VAR model, Blanchard and Perotti (2002) investigate the effects of fiscal shocks using a three-dimensional VAR model consisting of GDP, government expenditure, and government revenue. While such a model could be used to estimate the effects of fiscal shocks, Gechert (2017) highlights the potential issues of omitted variable bias. Therefore, we augment the model to include also a short term interest rate, the GDP deflator rate, and the exchange rate index. These variables are included to account for the effects of monetary policy, price levels, and interactions with the rest of the world respectively. Consequently, the impulse response functions reported later are better interpreted as the response of GDP to the fiscal variables, ceteris paribus.

Consistent with Fernández (2006), all variables are log-transformed prior to estimation, except for the short-term interest rate, which enters the model in levels. This will faciliate an intuitive interpretation of results. Furthermore, the fiscal variables and GDP are used in real terms. Following Capek and Cuaresma (2020), who highlight that data for estimating fiscal multipliers is typically seasonally adjusted, we applied the X-13 ARIMA-SEATS method to our variables. Gross Domestic Product (GDP) was an exception, as it was sourced after seasonal adjustment. This process reduces noise in the data, leading to more meaningful results. The fiscal variables are defined at the general level, including both local and central government finances (ONS, 2024) Additionally, the fiscal variables are defined following the European System of Accounts (ESA, 2010). In particular, government expenditure represents the outflows associated with government activities, including consumption, investment, and transfers. The inflows to the government, government revenue, consists of receipts net of transfer and interest payments. These definitions follow those used elsewhere in the literature, minimising the effect of methodological differences on this paper's results (Gechert, 2017).

The fiscal variables are sourced in current prices, therefore we process these into real values by dividing by the deflator rate...

- *Need to describe the data features and definitions*

The data series across the sample period are plotted in the data appendix. A key feature of these series is the apparent presence of a deterministic linear trend driving the series upwards (with the exception of the exchange rate and mean short run rate). This suggests that the VAR model may need treatment in order to impose stationarity. ...

The fiscal variables are obtained from the ONS and are the sum of local, central, and general government.

- *Cf Blanchard and Perotti (2002).*
- debt sustainability (inflation/ interest rate changing real cost of debt despite no change in deficit ceteris paribus)

3.0.1 Model

3.1 VECM

The reduced-form VAR model can be written as:

$$X_t = \mu + A_1 X_{t-1} + \cdots + A_p X_{t-p} + \epsilon_t \tag{1}$$

where:

$$X_t = \begin{pmatrix} G_t \\ R_t \\ GDP_t \\ ERI_t \\ \tau_t \\ P_t \end{pmatrix}$$

Here the vector X_t defines the endogenous variables, as previously mentioned, we have assumed a lag order (p) of 4. The term μ in equation 1 captures the deterministic component of the model. The evidence from the time series plots of the data suggest this to be a linear trend.

Kilian and Lutkepohl (2016) show that the VAR model can be reparameterised as a Vector Error Correction Model (VECM) by subtracting the lagged variables and rearranging terms:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t \tag{2}$$

When correctly specified, this representation can yield efficiency gains: including the error correction term may allow the model to capture important dynamics in the long run relationships between variables (Martin, Hurn, and Harris;

2013). We therefore perform tests for cointegration to determine the appropriate specification before estimating the model.

Given the small sample sizes associated with macroeconomic time series, various Monte Carlo simulations have been performed to assess the finite sample properties of cointegration tests. Hubrich, Lutkepohl, and Saikkonen (2001) find that the Likelihood Ratio type tests are among the best performing, with other classes of tests having worse power or size. Therefore we focus on this class of cointegration tests. Madadala and Kim (1998) provide a review of studies investigating Johansen's cointegration tests, highlighting that: sample sizes upwards of 100 may be needed to appeal to asymptotic results, tests can be misleading when too few variables are included, and insufficient lag lengths can lead to size distortions. Fortunately, these considerations do not appear to be a concern for this research. As previously discussed, due to concerns with underestimating the lag order when evaluating IRFs, we assume a lag order of 4, which we believe to be at least equal to the true lag parameter. [1] Additionally, we have added additional control variables to the VAR model to mitigate omitted variable bias (Gechert, 2017) which should alleviate the concern raised by Madadala and Kim. Nevertheless, Madadala and Kim highlight further complications of the test. We therefore consider this in detail before testing for cointegration.

Lutkepohl, Saikkonen, and Trenkler (2001) find that in small samples, while trace tests had size distortions, they had power advantages relative to the maximum eigenvalue test. This suggests that there may be merit in considering both tests. Cheung and Lai (1993), using a Monte Carlo simulation, find that there is finite sample bias in Johansen's cointegration tests, with both the maximimum eigenvalue and trace tests finding cointegration at a rate greater than implied by asymptotic theory. They therefore apply a scaling factor to the asymptotic critical values given by

$$SF = \frac{T}{T - kp} \tag{3}$$

where T is the number of observations, k the number of variables in the system, and p the lag order. Gonzalo and Lee (1998) provide further evidence of bias in Johansen's cointegration tests, identifying cases where the test detects spurious cointegration. In light of this, we ensure the asymptotic results reported as standard are scaled to account for the finite sample bias in the test.

Aside from the type of cointegration test, Hubrich, Lutkepohl, and Saikkonen (1998) highlight that improperly specified deterministic terms invalidate cointegration testing, leading to reduced power. Given that we have suggested the model to follow a deterministic linear trend, we ensure this is captured in our

[1] While not relied on for determining the lag order, lag order selection procedures including the Akaike Information Criteria suggest that a lag as low as 2 may be appropriate for the levels VAR. Due to the considerations mentioned, we decide to use a fixed lag order of 4. The sensitivity of the final results to the lag order is considered as a robustness check.

test. Martin, Hurn, and Harris (2013) highlight that a constant deterministic component in cointegration implies a linear trend in the VAR. We therefore include a constant term in cointegration when reporting the following Johansen's tests for cointegration.

Watson (2000): specification of deterministic components affects the power of tests

We begin by performing Johansen's maximum eigenvalue test for cointegration, which compares the null hypothesis of rank r, to the alternative of rank r+1. Accounting for the finite sample bias in the test, we find insufficient evidence to reject the null hypothesis of no cointegrating relations at the 5% significance level. Consequently, the VECM reduces to a VAR in the differences of variables (Kilian and Lutkepohl, 2016). Per the previous review of cointegration tests, we also perform Johansen's trace test. This test has as the alternative hypothesis that the rank is greater than assumed under the null hypothesis. Here we reject the null hypothesis of no cointegration (both under the asymptotic case and after applying Cheung and Lai's (1993) finite sample correction). Moving to the trace test's next null hypothesis (a single cointegrating rank), we fail to reject this null once applying the finite sample scaling factor.

This shows some discrepancy in the specification suggested by the two tests. Kilian and Lutkepohl (2016) highlight the asymmetric consequences of imposing a unit root. When the underlying data generating process possesses a unit root, the reduced form model can benefit from increased efficiency in estimation by imposing a unit root. Failing to impose a unit root in such a case would only reduce the precision of the Least Squares estimates. In contrast, when the underlying process does not follow a unit root, incorrectly imposing one would result in over-differencing - yielding an inconsistent estimator. While this argument would apply to both cases (cointegrating rank 0 or 1), as previously mentioned, the case of no cointegration reduces to a VAR in differences which is more robust to misspecification than the VECM. Thus, given the ambiguity in the results of the tests for cointegration, and recalling that the Johansen's tests has been noted to detect spurious cointegration, we choose the VAR in differences representation of the model. Given the differenceing, the lag order reduces to 3.

For robustness we also perform Phillips-Ouliaris' (1990) residual based test for cointegration. Using this test we again fail to reject the null hypothesis of no cointegration, suggesting that a VECM is not an appropriate representation for our data.

Add code results to appendix?

NB on unit root tests: failing to reject H0 does not mean we accept H0!!!! **(Lutkepohl)**

4 Identification

Thus far we have only defined the reduced form VAR. The residuals in this model are not meaningful as they are linearly dependent, instead, the interest of this research lies in the structural model. The structural shocks are mutually uncorrelated and have clear interpretations (Kilian and Lutkepohl, 2016). These shocks will allow us to construct FEVDs and IRFs to report the findings of this paper.

The structural model can be written as

$$\beta_0 X_t = \mu + \beta_1 X_{t-1} + \cdots + \beta_p X_{t-p} + u_t \tag{4}$$

Here u_t denotes the structural errors. We also observe that equation 1 for the reduced form model can be rewritten as:

$$X_t = \mu + B_0^{-1} B_1 X_{t-1} + \cdots + B_0^{-1} B_p X_{t-p} + \epsilon_t \tag{1a}$$

The term B_0^{-1} is referred to as the structural impact multiplier matrix. Comparing equations 1a and 4, it is clear that knowledge of the structural impact multiplier matrix or its inverse would allow us to move between the structural and reduced form representations, thus recovering the structural shocks. More explicitly, we can write the structural shocks as functions of the reduced form residuals:

$$\epsilon_t = B u_t \tag{5}$$

As outlined in the literature review, there have been numerous approaches to identifying the structural parameters. This study uses recursive sign restrictions, leveraging the Cholesky decomposition. To recover the structural shocks using this approach, we assume B to be a lower triangular matrix and that the structural shocks have unitary variance [2]. This corresponds to a causal ordering of the transmission of shocks: the first variable is considered the most exogenous, affecting all others. In contrast, the variable ordered last is considered the most endogenous, being contemporaneously effected by shocks to all other variables, whilst having no instantaneous effect itself on the other variables. Having defined this recursive ordering, given that the matrix B is assumed to be lower triangular, we can estimate it using the Cholesky decomposition of the covariance matrix of the reduced-form residuals (Σ_ϵ):

$$\Sigma_\epsilon = E[\epsilon_t \epsilon_t'] = BB' \tag{6}$$

[2] The assumption of unitary variance for the structural shocks is not restrictive as it can be imposed by rescaling the variables. This assumption, however means that the reduced form covariance matrix reduces to equation 6.

Therefore we have that $B = Chol(\Sigma_\epsilon)$

This paper uses the following recursive ordering:

$$(G, R, GDP, ERI, T, P) \tag{7}$$

With the variables corresponding to government expenditure, the short term interest rate, GDP, the Exchange Rate Index, net taxes, and the GDP deflator respectively.

The matrix B has the form:

$$B = \begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & 0 & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & 0 \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{65} \end{pmatrix} \tag{8}$$

Thus, the structural shocks u_t can be recovered as:

$$u_t = B^{-1}\epsilon_t \tag{5a}$$

Killian and Lutkepohl (2017) highlight that identification of the structural parameters is not a purely statistical concern, the restrictions must also be economically meaningful for the resulting structural parameters to be identified. Therefore we proceed with an exposition of the economic assumptions implicit in our identifying restrictions imposed by the matrix B. The following is comparable to Fernandez's (2006) identifying assumptions.

Blanchard and Perotti (2002) argue that the use of quarterly data allows government spending to be treated as predetermined with respect to the rest of the variables within the quarter. This is motivated by implementation lags for changes to government spending and consequently this is ordered first. Given physical constraints, the interest rate is assumed not to react contemporaneously to price, net taxes, output, or the exchange rate. Thus the short term rate is considered the next most exogenous variable. However monetary policy shocks are assumed to affect output, net taxes, prices, and the exchange rate contemporaneously. Fernandez (2006) justifies this assumption by noting that interest movements are anticipated and thus they can be transmitted to real variables relatively quickly. Shocks to the exchange rate are assumed to affect net taxes contemporaneously as households adjust to changes in the cost of imports. Investment plans take time to adjust, as does consumption due to internal habit which leads to highly persistent levels of consumption. Consequently, we do not expect shocks to net taxes to affect activity. However, shocks to economic activity is expected to affect contemporaneous tax receipts to the government as households respond to prevailing economic conditions. Despite the aforementioned internal habit, this behaviour is expected as households attempt to smooth fluctuations in their consumption path, consequently they may adjust their behaviours towards consumption and saving, which is expected to have tax implications. Due to price stickiness, prices do not react contemporaneously to shocks to GDP.

5 Results

Given that we have performed cointegration tests, it would suggest that the appropriate model has been chosen, and thus we would expect the stability of the VAR model. This is supported by inspecting the eigenvalues of the VAR model, the greatest of which is 0.77 suggesting that we do not have issues of explosive roots. We therefore proceed by presenting the results of the SVAR analysis, focusing on Forecast Error Variance Decompositions (FEVDs) and Impulse Response Functions (IRFs)

5.1 IRFs

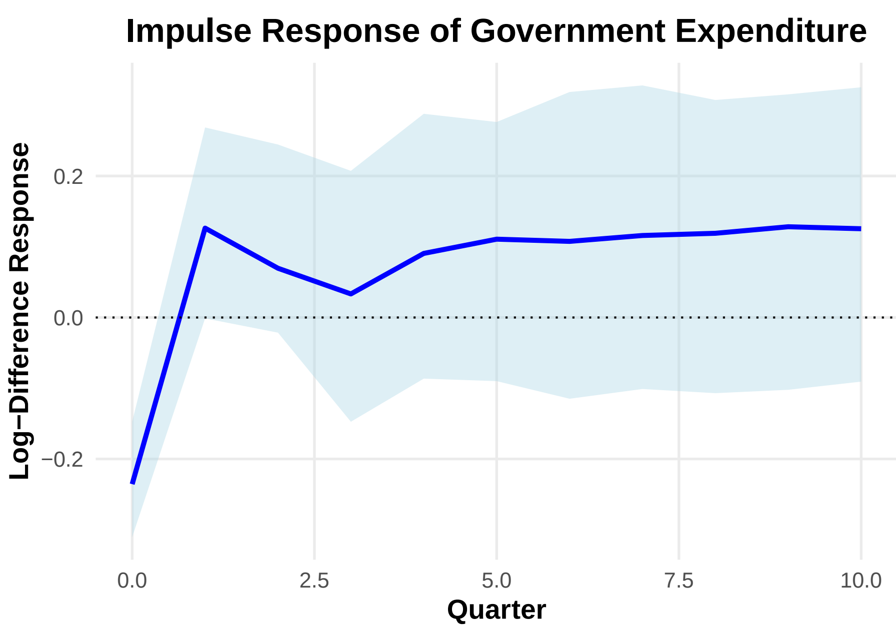


Figure 3: Impulse Response Function of GDP Following a Shock to Expenditure

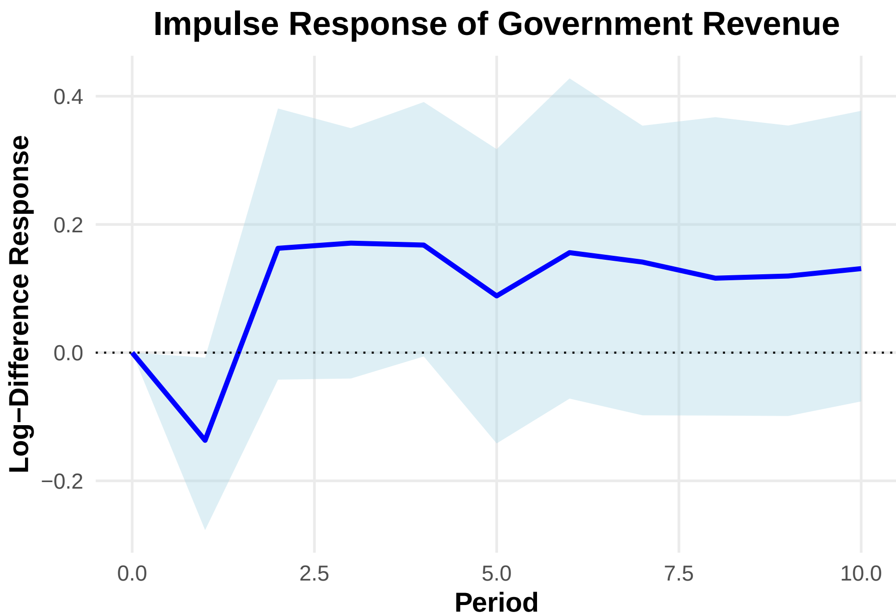


Figure 4: Impulse Response Function of GDP Following a Shock to Revenue

We report the cumulative structural impulse response functions up to a horizon of 10 quarters including bands for the 68% confidence intervals. Kilian and Lutkepohl (2016) highlight that given the short sample sizes VAR models are typically estimated on, this level is more appropriate than the typical 5% significance level. Furthermore, given the low sample size we do not appeal to asymptotic results, instead we report a bootstrap confidence interval. Using a 68% confidence interval therefore requires fewer iterations to accurate estimate this interval. The IRF function from the VARS library in R implements Efron's (1992) percentile interval. It should be highlighted that unlike standard confidence intervals constructed using standard errors, Efron's percentile interval are often asymmetric about the point estimate (Efron, 1981).

Here we focus on the response of GDP to the fiscal variables, the appendix includes comprehensive details of the IRFs for each of the variables. Figure 1 summarises the results for a shock to government expenditure **(growth?)**. Given the model specification, the results can be interpreted in terms of percentage growth. We find a point estimate of -0.24, suggesting that a standard deviation shock to the percentage growth of expenditure is associated with a reduction in GDP growth of **(-22% of a standard deviation?)**. The confidence interval for this IRF is - (-0.29 , -0.08). As 0 is not contained in this interval, it suggests that there is a statistically significant crowding out effect of government expenditure. Nevertheless, tracing the IRF out over time, by the next quarter the cumulative response is positive, stabilising at a value of 0.13 by the 10th quarter. However, the percentile intervals for these later IRFs contain 0 and thus these results are not statistically significant.

Considering now the response of GDP to a shock to government revenues, we find no effect in the quarter of the shock. There is a temporary negative effect in the following quarter, however this is reversed as we report a positive cumulative IRF by the second quarter. Similar to the response to government expenditure, following the 2nd quarter no further effects are detected. Again, our percentile intervals include 0, and thus the multiplier for government revenue is not statistically significant. Interestingly, the reported point estimates for government spending and revenue have similar magnitudes, despite much of the literature reporting differences.

5.2 FEVDs

Given that we have used a VAR in differences, with a logarithmic functional form for the variables initially in levels, we can interpret the FEVD as illustrating how each shock contributes to volatility in the growth in each variable. Considering GDP, we find that 95% of variability in GDP growth is explained by itself a quarter after a shock. By 10 quarters, this percentage reduces to 68%. While a reduction, this still highlights a notable degree of persistence in GDP. In fact, the fiscal variables explain only 20% of the Mean Squared Prediction Error in GDP growth. This supports the IRF results, which finds negligible impact of fiscal policy on GDP.

cf IRFs, large persistence in GDP growth consistent with the low multipliers observed, after 10 periods, the fiscal variables combimned explain only c 20% of variability in GDP ...

5.3 Counterfactual

6 Robustness

6.1 Chow test for Structural Breaks

Lutkepohl and Kilian (2016) highlight paramater instability as a major concern for SVAR analysis. Following the Global Financial Crisis (GFC) there were major changes to government finances (source?), which may have lead to changes in the structural parameters. This can also be seen in the plots in the data appendix, where the trends appear to change in 2008. We thus consider as a test for robustness whether the parameters have remained stable across the period, using 2008 as a potential breakpoint.

Using an asymptotic likelihood based Chow test we find evidence to support this alternative hypothesis of parameter instability. Nevertheless, Lutkepohl and Candelon (2000) highlight that when the sample size is low relative to the nuber of parameters, the Chow test may have distorted size. Given that we are considering a 6-dimensional VAR in this analysis, with a sample period of less than 40 yeas at a quarterly frequency, this issue is likely present in this analysis. Therefore we also consider a bootstrap version of the Chow statistic the authors show to have more desirable finite sample properties.

Following Lutkepohl and Candelon (2000), we implement our recursive residual-based boostrap for the Chow test as follows:

1. Estimate the restricted VAR model (pooling both periods around the GFC)
2. Obtain the demeaned residuals from the restricted model.
3. Generate bootstrap residuals by randomly drawing from the centered residuals with replacement.
4. For each iteration, estimate the restricted and unrestricted models (where we allow the parameters to vary across the samples).
5. Compute the Chow statistic (Here we use the same likelihood based Chow test as with our asymptotic testing).

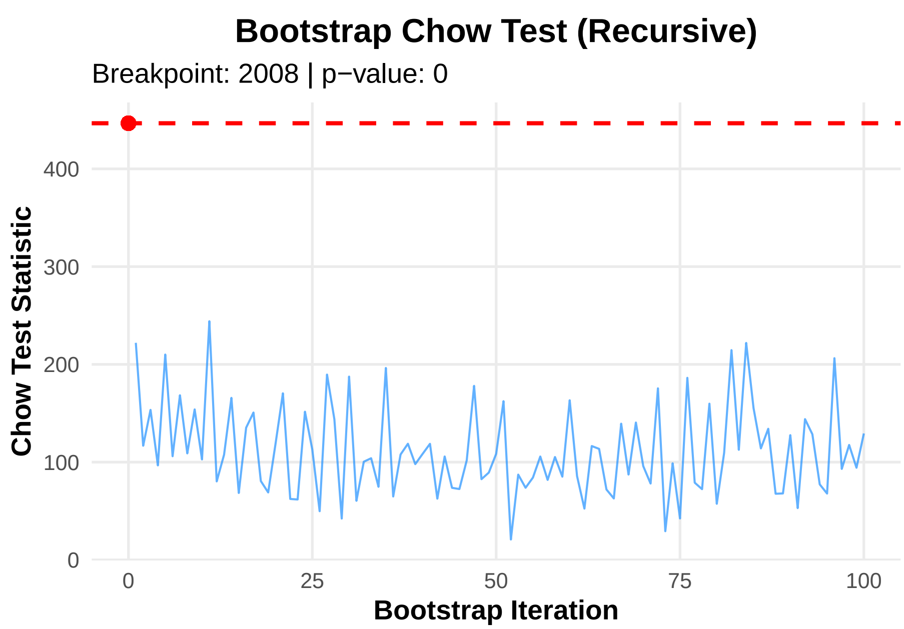


Figure 5: Bootstrap Chow Test Results

Through this process we can derive the empirical distribution of the bootstrap time series. *Chow statistic under the null hypothesis of parameter stability.* We then compute the pseudo p-value as the percentage of times the bootstrap statistic exceeds the test statistic. Using 100 replications, we find a p-value of 0, this can be further seen in figure 3 where the test statistic exceeds even the greatest boostrap statistic by a factor of 2. This supports the previous asymptotic test of parameter instability. We therefore estimate the SVAR on the post-GFC sample to determine whether there are differences in the estimated fiscal multipliers in the most recent period.

6.1.1 Post GFC Shocks

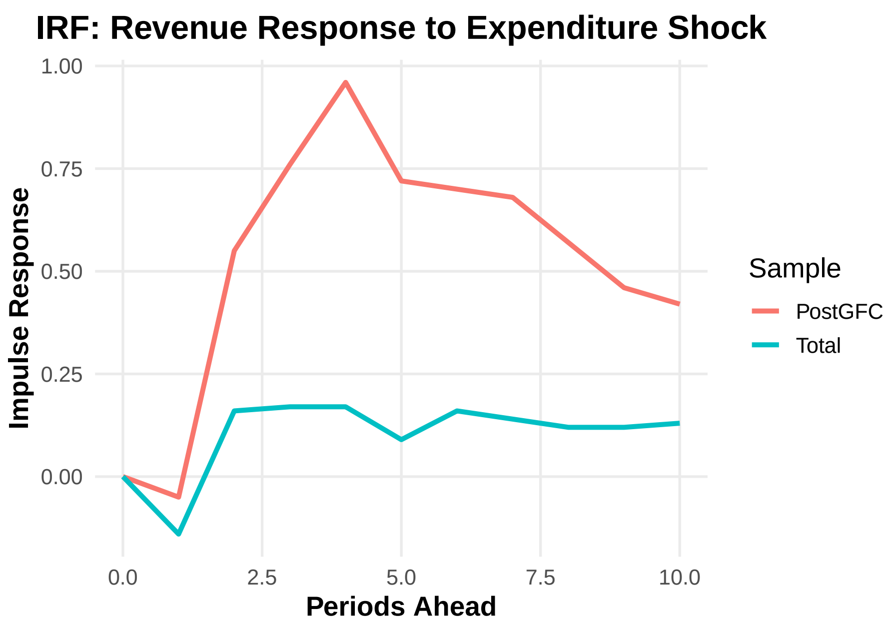


Figure 6: GDP Responses to Government Revenue Compared

Figures 3 and 4 compare the IRFs under the restricted model and unrestricted model (post GFC). Only the point estimates are reported, given the reduced sample size while keeping the number of model parameters fixed, we expect greater uncertainty in the IRF estimates, reflected in the confidence intervals. Consequently, we do not expect any of the previously insignificant results to change. [3]

[3] Along with the IRFs for all of the variables, the confidence intervals for the post GFC specification can be seen in the appendix. It should be noted that the confidence intervals under this specification show noticeable asymmetry. Efron (1981) highlights that this is expected in small samples. This thus supports the use of the restricted model in order to obtain

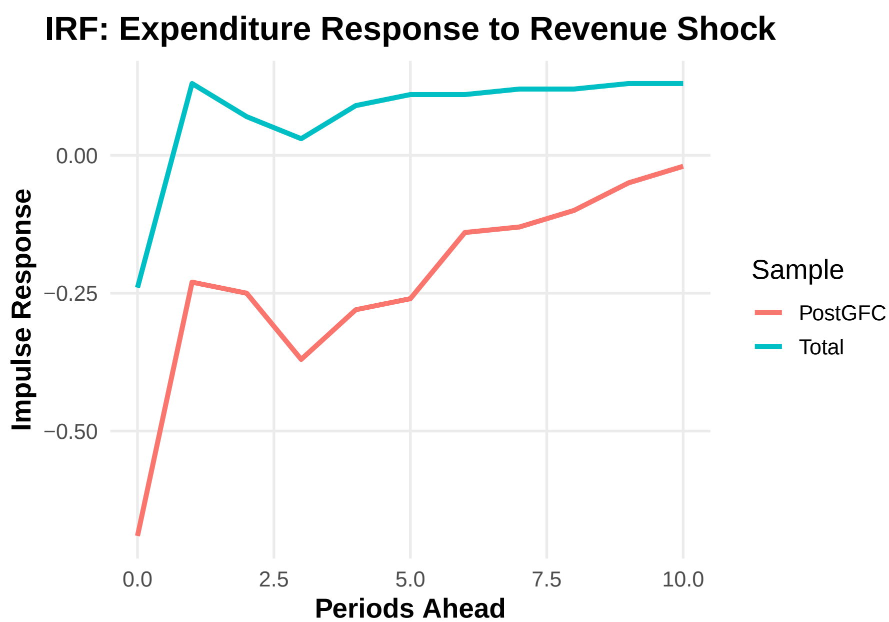


Figure 7: GDP Responses to Government Compared

We see little difference in IRFs between the restricted and unrestricted models. While for revenue there is a noticeable discrepancy between the models after a year, with a factor of over 3, by 10 quarters this gap has closed. Additionally, the instantaneous response is essentially the same.

For expenditure, the post GFC IRF follows a similar path to the total period, despite overall reporting lower point estimates by approximately 0.5. Nevertheless, the results for the restricted model are contained within the confidence intervals for both expenditure and revenue. Thus this does not negate this paper's results.

6.2 Lag length

When defining the empirical specification, we highlighted that the lag order was chosen a priori, and that information criteria suggested a lower order lag was compatible with the data. Therefore, here we estimate the model on a single lag (consistent with 2 lags in the levels VAR). Given the evidence of parameter instability, this is reported for both the full sample and the post GFC data. Figures 5 and 6 report the IRFs for government expenditure and revenue respectively across all of the specifications used for robustness testing.

more meaningful estimates which we can test for significance.

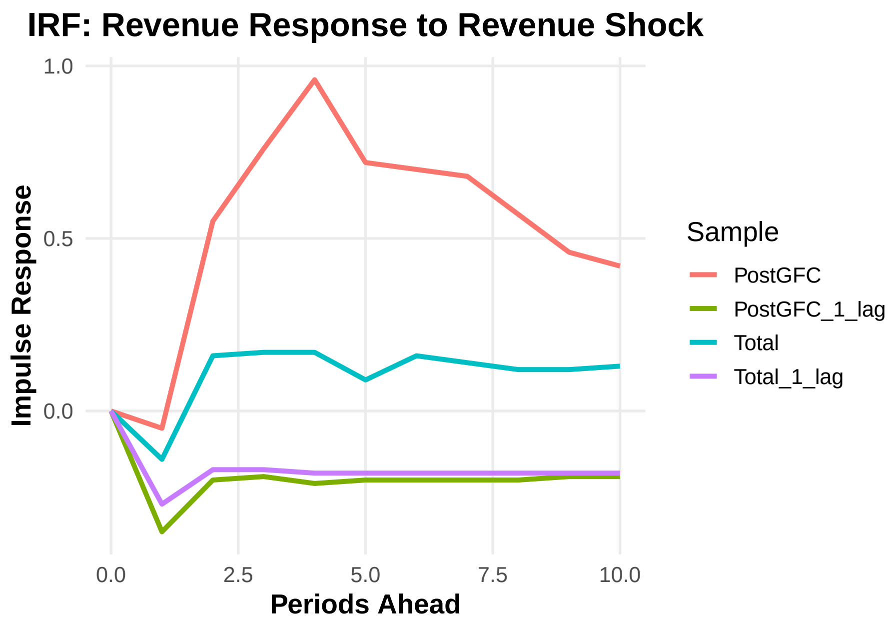


Figure 8: GDP Responses to Government Revenue Compared 2

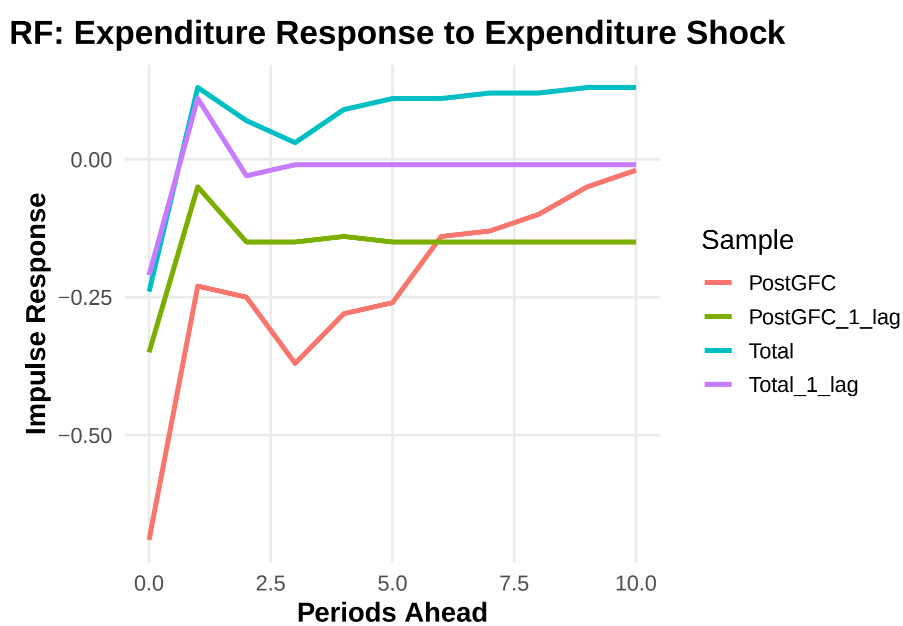


Figure 9: GDP Responses to Government Compared 2

consider reduced lag length (improved efficiency/ parsimony)

7 Discussion/ Policy Implications

This analysis has found fiscal multipliers that are practically and statistically insignificant

suggests that there is scope to reduce the government deficit through more aggressive fiscal consolidation measures and thus mitigate against the growing costs of the large deficit.

cf with research and theory

Lucas critique/ extrapolating.

7.1 Limitations

- potential heterogeneity within expedniture/ revenue
- debt sustainability (effects of policy on inflation and interest rate, changing real cost)

8 Conclusion

Summary of literature, this research's results, contributions, and future implications.

9 Bibliography

Alesina, A. and Ardagna, S., 1998. Tales of fiscal adjustment. Economic policy, 13(27), pp.488-545.

Cheung, Y.W. and Lai, K.S., 1993. FINITE-SAMPLE SIZES OF JOHANSEN'S LIKELIHOOD RATIC) TESTS FOR. Oxford Bulletin of Economics and statistics, 55(3), p.1025.

Efron, B., 1981. Nonparametric standard errors and confidence intervals. canadian Journal of Statistics, 9(2), pp.139-158.

Efron, B., 1992. Bootstrap methods: another look at the jackknife. In Breakthroughs in statistics: Methodology and distribution (pp. 569-593). New York, NY: Springer New York.

Efron, B. and Tibshirani, R., 1986. Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy. Statistical science, pp.54-75.

Lüutkepohl and Candelon (2000) https://www.econstor.eu/bitstream/10419/62173/1/723877181.pdf

Lüutkepohl, H., Saikkonen, P. and Trenkler, C., 2001. Maximum eigenvalue versus trace tests for the cointegrating rank of a VAR process. The Econometrics Journal, 4(2), pp.287-310.

Maddala, G.S. and Kim, I.M., 1998. Unit roots, cointegration, and structural change.

Martin, V., Hurn, S. and Harris, D., 2013. Econometric modelling with time series: specification, estimation and testing. Cambridge University Press.

UK government debt and deficit: December 2023 (2024) UK government debt and deficit - Office for National Statistics. Available at: https://www.ons.gov.uk/economy/governmentpublicsectorandtaxes/publicspending/bulletins/ukgovernmentdebtanddeficitforeurostatmaast/december2023 (Accessed: 11 August 2025).

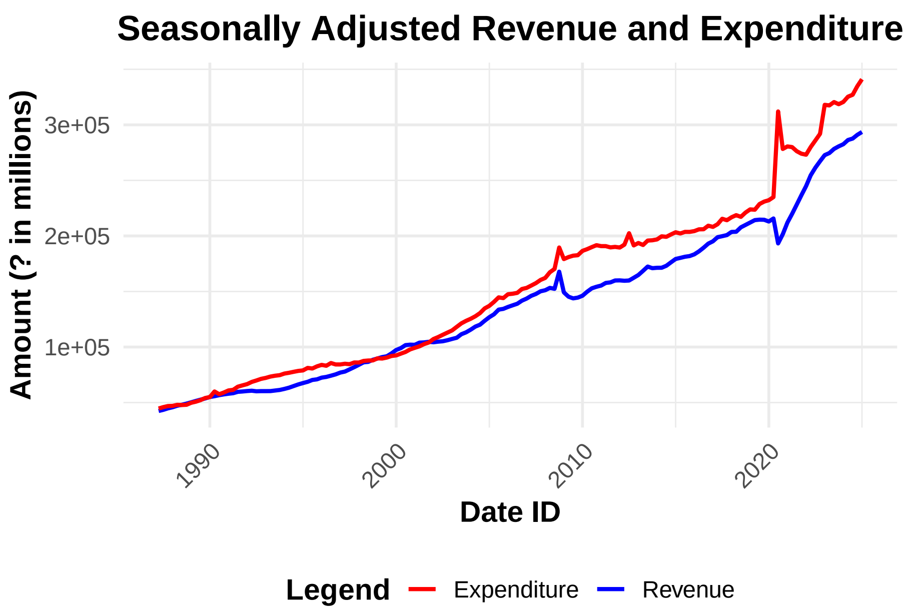
10 Data Appendix

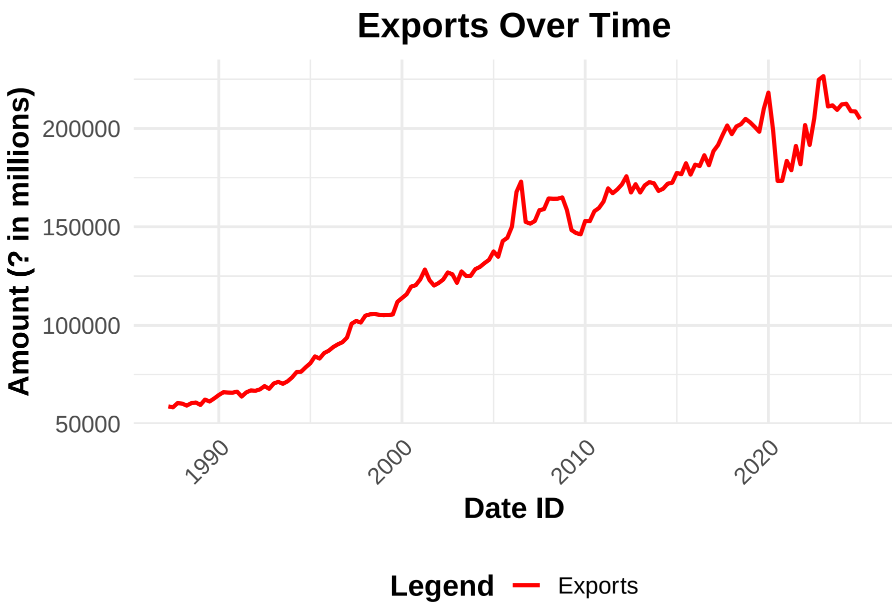
10.1 Data Sources

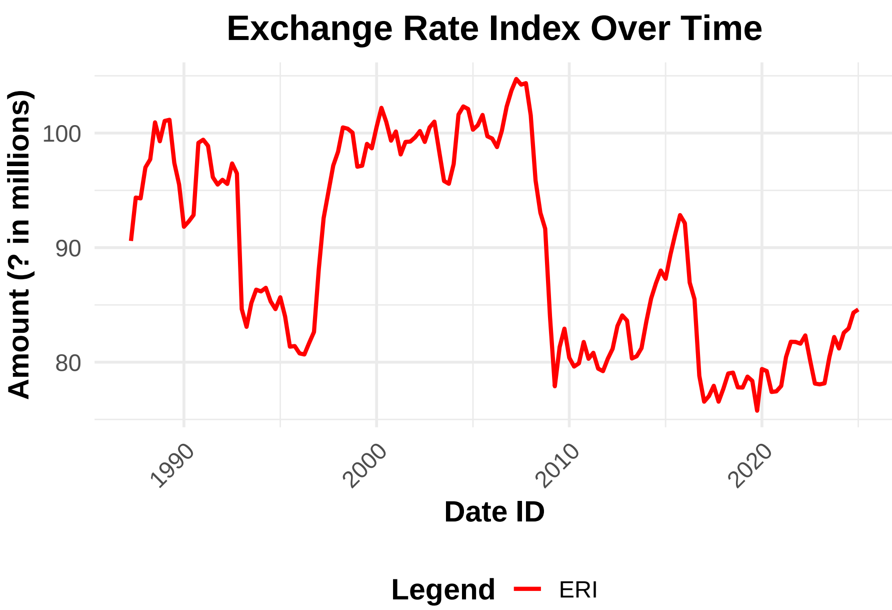
- Fiscal Variables (not seasonally adjusted):
  https://www.ons.gov.uk/economy/governmentpublicsectorandtaxes/publicspending/datasets/esatable25quarterlynonfinancialaccountsofgeneralgovernment
- UK Exports (seasonally Adjusted, %):
  https://fred.stlouisfed.org/series/XTEXVA01GBQ188S
- UK Exports (seasonally Adjusted, £millions):
  https://fred.stlouisfed.org/series/NXRSAXDCGBQ
- LFS (Pop aged 16-64):
  https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/employmentandemployeetypes/timeseries/lf2o/lms
- GDP (SA) and defaltor rate:
  https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/quarterlynationalaccounts/latest#data-on-gdp-quarterly-national-accounts
- interest rate (SR. This is dates of changes to the policy rate. Have interpolated to get quarterly data):
  https://www.bankofengland.co.uk/monetary-policy/the-interest-rate-bank-rate

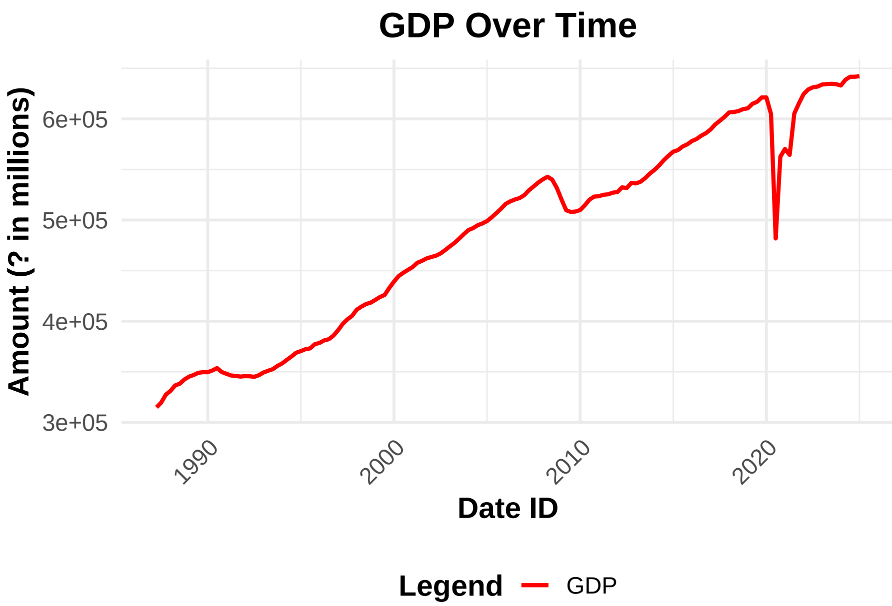
- 3 month interest rate:
  https://fred.stlouisfed.org/series/IR3TIB01GBM156N

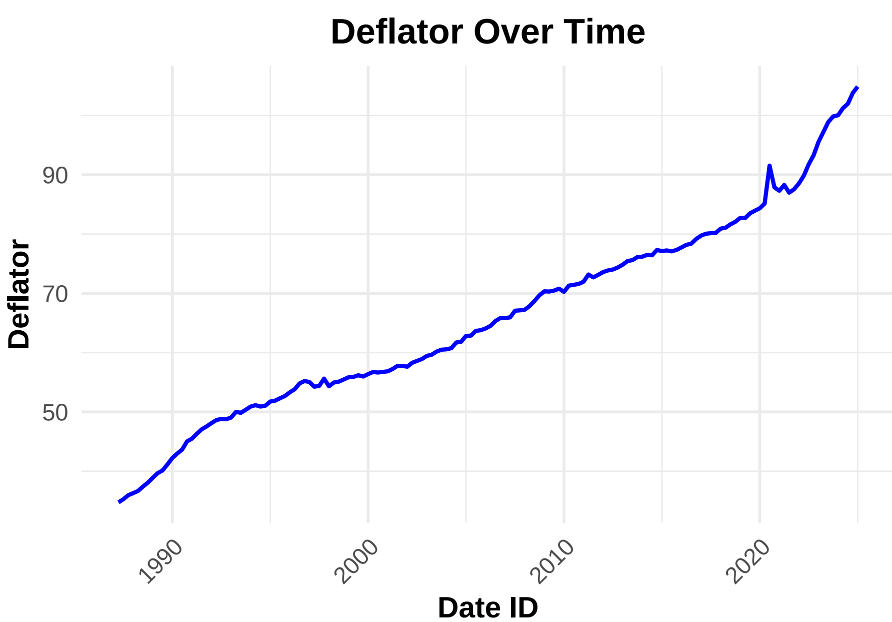
10.2 Data Plots

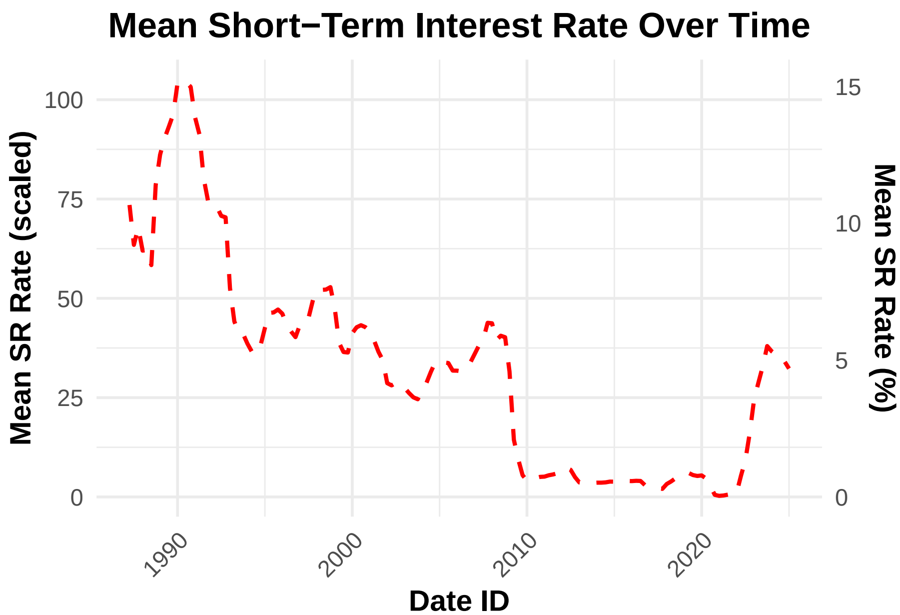



Exports Over Time
Amount (? in millions)
200000
150000
100000
50000
1990
2000
2010
2020
Date ID
Legend
Exports


Exchange Rate Index Over Time
Amount (? in millions)
100
90
80
1990
2000
2010
2020
Date ID
Legend
ERI


GDP Over Time
Amount (? in millions)
6e+05
5e+05
4e+05
3e+05
1990
2000
2010
2020
Date ID
Legend
GDP


Deflator Over Time
Deflator
90
70
50
1990
2000
2010
2020
Date ID



11 Technical Appendix

11.1 VECM Code/ Results

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: maximal eigenvalue statistic (lambda max) , with linear trend
##
## Eigenvalues (lambda):
## [1] 0.217684885 0.147708935 0.129491909 0.099310793 0.049570272 0.003934635
##
## Values of teststatistic and critical values of test:
##
##           test 10pct  5pct  1pct
## r <= 5 |  0.58  6.50  8.18 11.65
## r <= 4 |  7.52 12.91 14.90 19.19
## r <= 3 | 15.48 18.90 21.07 25.75
## r <= 2 | 20.52 24.78 27.14 32.14
## r <= 1 | 23.65 30.84 33.32 38.78
## r = 0  | 36.33 36.25 39.43 44.59
```

```
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##                    log_expenditure.l4 mean_SR_Rate.l4  log_GDP.l4  log_ERI.l4
## log_expenditure.l4          1.0000000       1.0000000   1.0000000  1.00000000
## mean_SR_Rate.l4             0.2535931      -0.4900211   0.1424397  0.01579774
## log_GDP.l4                 -5.0243568     -38.8551346  33.4819175  1.62964250
## log_ERI.l4                 -6.8708786      -3.2610158  -6.8979045  0.08817142
## log_revenue.l4              7.3539147      24.8544304 -16.7540388 -2.01870716
## log_deflator.l4           -12.3241618     -27.5232600   6.5705770  0.90530315
##                    log_revenue.l4 log_deflator.l4
## log_expenditure.l4    1.000000000        1.000000
## mean_SR_Rate.l4       0.008693287       -0.367051
## log_GDP.l4            0.957845679       94.066463
## log_ERI.l4            0.402492149      -28.218390
## log_revenue.l4       -0.703754374      -59.239380
## log_deflator.l4      -1.357161793        8.870339
##
## Weights W:
## (This is the loading matrix)
##
##                   log_expenditure.l4 mean_SR_Rate.l4   log_GDP.l4   log_ERI.l4
## log_expenditure.d       -0.001939026   -0.0072192210  0.004251129 -0.079396721
## mean_SR_Rate.d          -0.092190836    0.1187102324  0.068775084 -0.974163924
## log_GDP.d               -0.001520989    0.0046481301 -0.009448068  0.042986902
## log_ERI.d                0.001507855    0.0029834205  0.001991425 -0.001656115
## log_revenue.d           -0.003901146    0.0017497996 -0.004155118  0.054226296
## log_deflator.d           0.005865819   -0.0009867915  0.002538979 -0.014646030
##                   log_revenue.l4 log_deflator.l4
## log_expenditure.d    0.005921121    1.258999e-04
## mean_SR_Rate.d       0.185626803    2.375608e-03
## log_GDP.d           -0.007439052   -1.102677e-06
## log_ERI.d           -0.061740204    9.288839e-05
## log_revenue.d        0.005718839    1.197667e-04
## log_deflator.d       0.007320707    3.261393e-05

##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: trace statistic , with linear trend
##
## Eigenvalues (lambda):
## [1] 0.217684885 0.147708935 0.129491909 0.099310793 0.049570272 0.003934635
```

```
##
## Values of teststatistic and critical values of test:
##
##           test 10pct  5pct   1pct
## r <= 5 |   0.58  6.50  8.18  11.65
## r <= 4 |   8.11 15.66 17.95  23.52
## r <= 3 |  23.59 28.71 31.52  37.22
## r <= 2 |  44.11 45.23 48.28  55.43
## r <= 1 |  67.77 66.49 70.60  78.87
## r = 0  | 104.10 85.18 90.39 104.20
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##                   log_expenditure.l4 mean_SR_Rate.l4  log_GDP.l4  log_ERI.l4
## log_expenditure.l4          1.0000000       1.0000000   1.0000000  1.00000000
## mean_SR_Rate.l4             0.2535931      -0.4900211   0.1424397  0.01579774
## log_GDP.l4                 -5.0243568     -38.8551346  33.4819175  1.62964250
## log_ERI.l4                 -6.8708786      -3.2610158  -6.8979045  0.08817142
## log_revenue.l4              7.3539147      24.8544304 -16.7540388 -2.01870716
## log_deflator.l4           -12.3241618     -27.5232600   6.5705770  0.90530315
##                   log_revenue.l4 log_deflator.l4
## log_expenditure.l4    1.000000000        1.000000
## mean_SR_Rate.l4       0.008693287       -0.367051
## log_GDP.l4            0.957845679       94.066463
## log_ERI.l4            0.402492149      -28.218390
## log_revenue.l4       -0.703754374      -59.239380
## log_deflator.l4      -1.357161793        8.870339
##
## Weights W:
## (This is the loading matrix)
##
##                  log_expenditure.l4 mean_SR_Rate.l4   log_GDP.l4   log_ERI.l4
## log_expenditure.d       -0.001939026   -0.0072192210  0.004251129 -0.079396721
## mean_SR_Rate.d          -0.092190836    0.1187102324  0.068775084 -0.974163924
## log_GDP.d               -0.001520989    0.0046481301 -0.009448068  0.042986902
## log_ERI.d                0.001507855    0.0029834205  0.001991425 -0.001656115
## log_revenue.d           -0.003901146    0.0017497996 -0.004155118  0.054226296
## log_deflator.d           0.005865819   -0.0009867915  0.002538979 -0.014646030
##                  log_revenue.l4 log_deflator.l4
## log_expenditure.d    0.005921121    1.258999e-04
## mean_SR_Rate.d       0.185626803    2.375608e-03
## log_GDP.d           -0.007439052   -1.102677e-06
## log_ERI.d           -0.061740204    9.288839e-05
## log_revenue.d        0.005718839    1.197667e-04
## log_deflator.d       0.007320707    3.261393e-05
```

```
##
## Coefficient matrix of lagged endogenous variables:
##
## A1:
##                 log_expenditure.l1 mean_SR_Rate.l1  log_GDP.l1  log_ERI.l1
## log_expenditure         0.41704150     0.008671635 -0.26304605 -0.23876012
## mean_SR_Rate           -2.49936033     1.370564254  1.59494334  3.54915247
## log_GDP                 0.32753775     0.002606754  1.05396372  0.11643232
## log_ERI                -0.10598489     0.002211027 -0.01272697  1.34594063
## log_revenue             0.07191423     0.007031786  0.40807678  0.07815003
## log_deflator           -0.04249691     0.002395032 -0.01889660 -0.12600635
##                 log_revenue.l1 log_deflator.l1
## log_expenditure      0.1364673      0.48997805
## mean_SR_Rate         3.0532294     14.89607794
## log_GDP             -0.1758971     -0.11151071
## log_ERI             -0.1578925     -0.04591769
## log_revenue          0.6712035      0.28623263
## log_deflator         0.1160997      0.83179033
##
##
## A2:
##                 log_expenditure.l2 mean_SR_Rate.l2 log_GDP.l2 log_ERI.l2
## log_expenditure         0.58113859    -0.011212939  0.5223282  0.4112738
## mean_SR_Rate            3.10269034    -0.270926073  4.9380190 -6.0657799
## log_GDP                -0.20994646    -0.006199741 -0.1239854 -0.2571705
## log_ERI                 0.19479104    -0.007809499  0.5150612 -0.4471687
## log_revenue            -0.01595271    -0.012897691 -0.1490417 -0.1555298
## log_deflator            0.08650219    -0.001111717  0.1785929  0.1603943
##                 log_revenue.l2 log_deflator.l2
## log_expenditure     -0.4424538      0.03930104
## mean_SR_Rate        -6.7879440     -7.30512033
## log_GDP              0.3767903     -0.03590739
## log_ERI             -0.1691644      0.47275639
## log_revenue          0.4008437      0.19302001
## log_deflator        -0.1930533      0.16872860
##
##
## A3:
##                 log_expenditure.l3 mean_SR_Rate.l3    log_GDP.l3  log_ERI.l3
## log_expenditure         0.18115221    0.0016592633   0.136131247 -0.28153586
## mean_SR_Rate           -4.72447571   -0.2310662263 -13.460650256  5.55384541
## log_GDP                -0.16526338   -0.0009326237  -0.009489033  0.38594896
## log_ERI                -0.18716264    0.0064196467  -0.699640374  0.17327906
## log_revenue            -0.10660055    0.0028215412  -0.352118618  0.36787725
## log_deflator           -0.02682298    0.0003299150  -0.086603424 -0.09100948
##                 log_revenue.l3 log_deflator.l3
```

```
## log_expenditure     0.11808158     -0.23014873
## mean_SR_Rate        9.52110039    -13.36851855
## log_GDP            -0.05366651     -0.02284539
## log_ERI             0.56069867     -0.62381040
## log_revenue        -0.02830796     -0.51590732
## log_deflator        0.07725310      0.07468500
##
##
## A4:
##                 log_expenditure.l4 mean_SR_Rate.l4  log_GDP.l4  log_ERI.l4
## log_expenditure        -0.18127133    0.0003903169 -0.38567103  0.12234499
## mean_SR_Rate            4.02895487    0.1080490827  7.39088758 -2.40378596
## log_GDP                 0.04615111    0.0041398986  0.08715269 -0.23476025
## log_ERI                 0.09986434   -0.0004387929  0.18973017 -0.08241132
## log_revenue             0.04673789    0.0020550605  0.11268434 -0.26369319
## log_deflator           -0.01131648   -0.0001256994 -0.10256485  0.01631823
##                 log_revenue.l4 log_deflator.l4
## log_expenditure     0.17364545     -0.27523349
## mean_SR_Rate       -6.46434935      6.91373571
## log_GDP            -0.15841187      0.18900840
## log_ERI            -0.22255310      0.17838864
## log_revenue        -0.07242798      0.08473303
## log_deflator        0.04283722     -0.14749523
##
##
## Coefficient matrix of deterministic regressor(s).
##
##                     constant
## log_expenditure  0.006759702
## mean_SR_Rate    -0.378107357
## log_GDP         -0.003706264
## log_ERI          0.005405675
## log_revenue     -0.004180014
## log_deflator     0.022188051
```

```
## [1] "varest"
```

```
## [1] "vec2var"
```

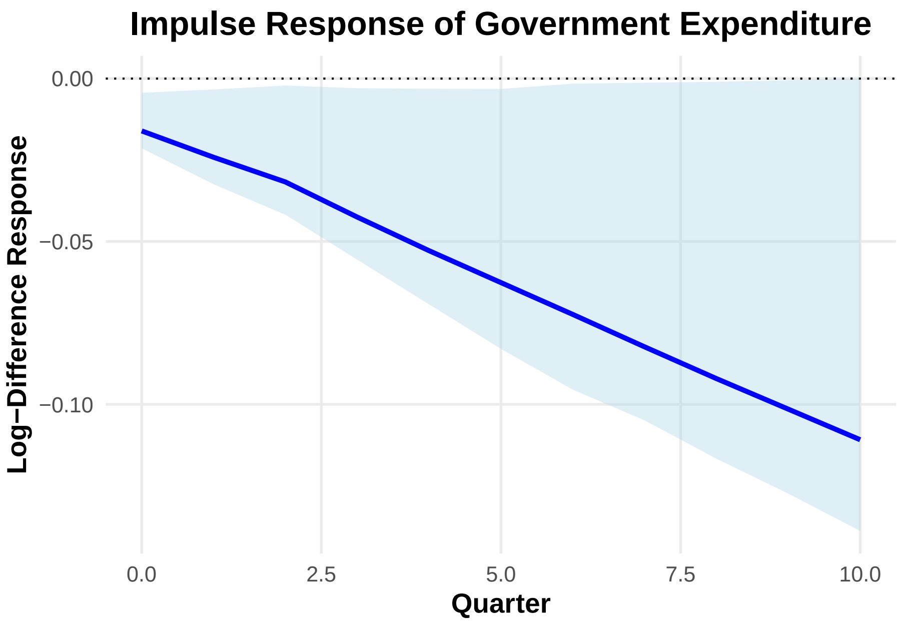


Figure 10: Impulse Response Function of GDP Following a Shock to Expenditure

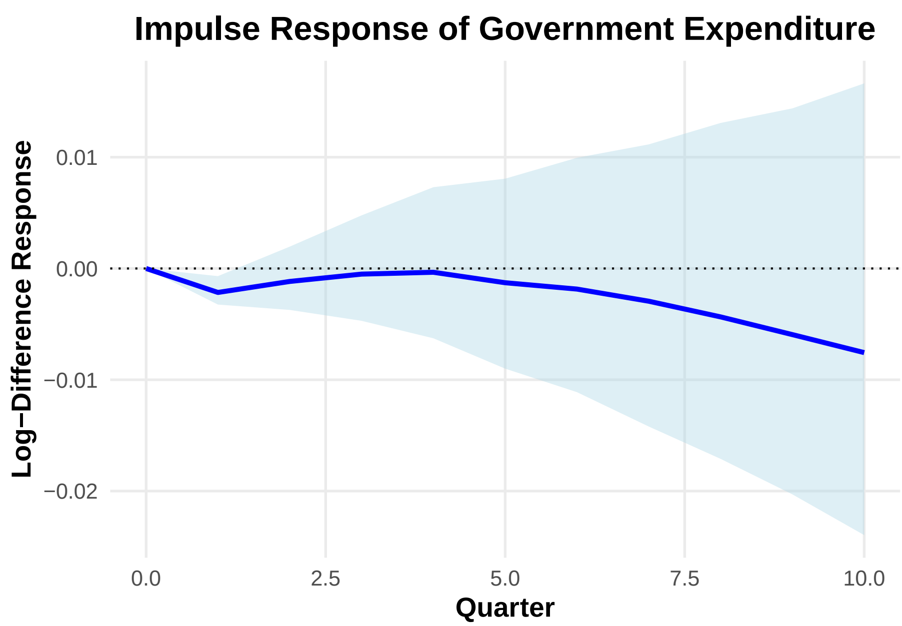
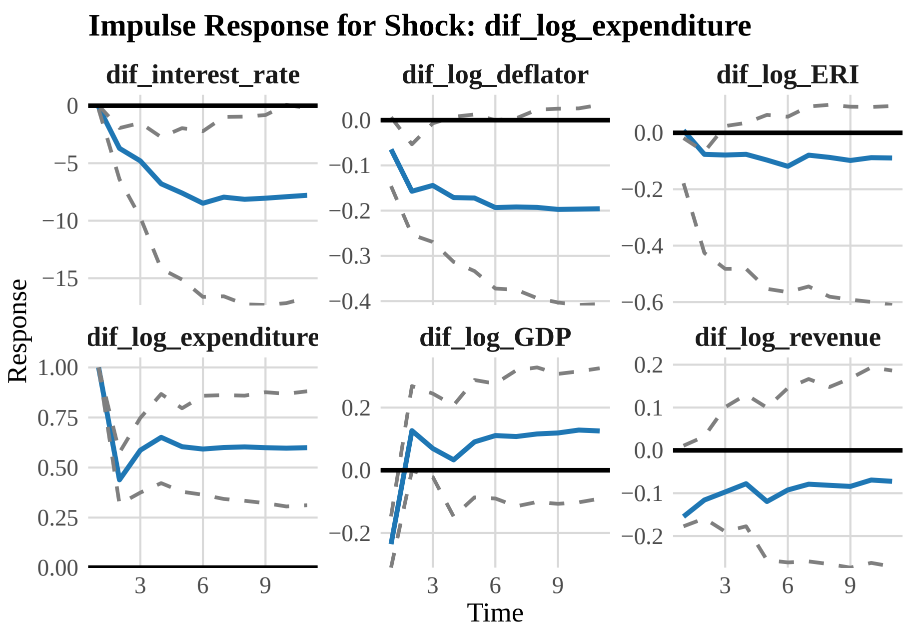
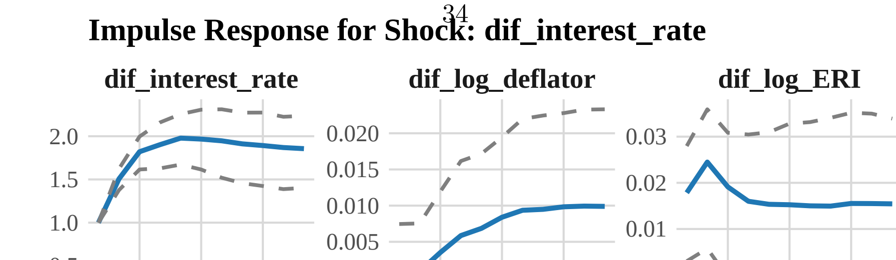


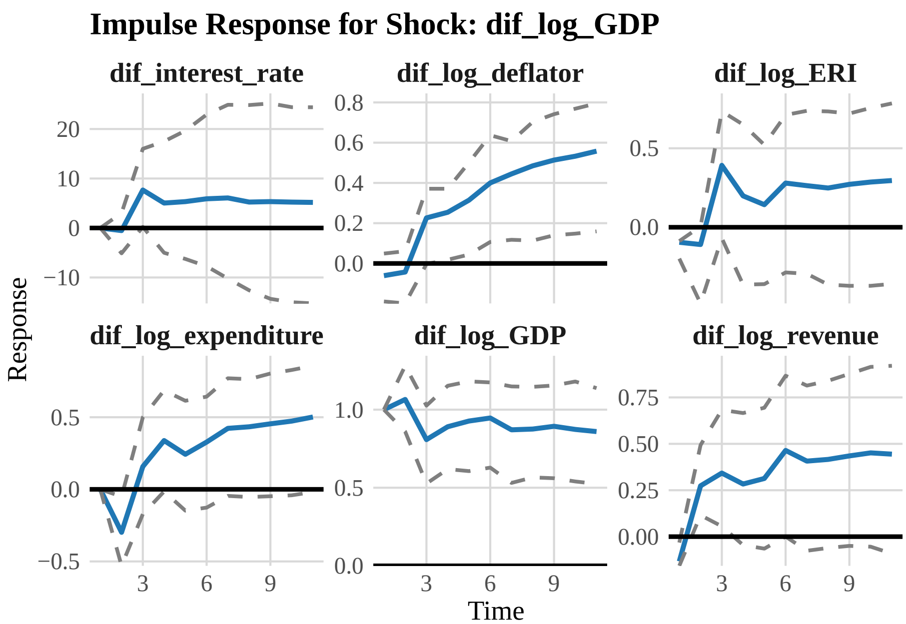
Figure 11: Impulse Response Function of GDP Following a Shock to Expenditure

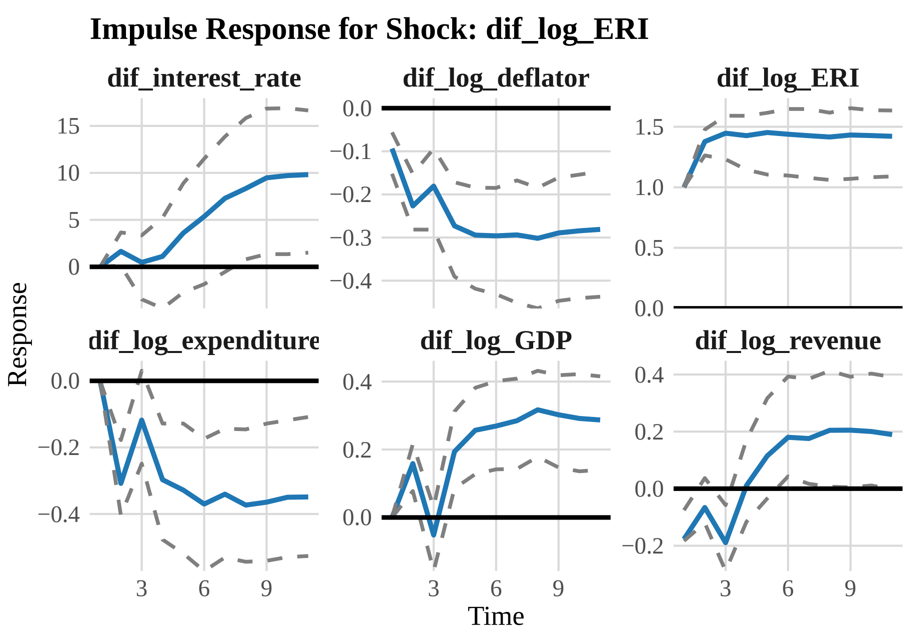
11.2 IRFs

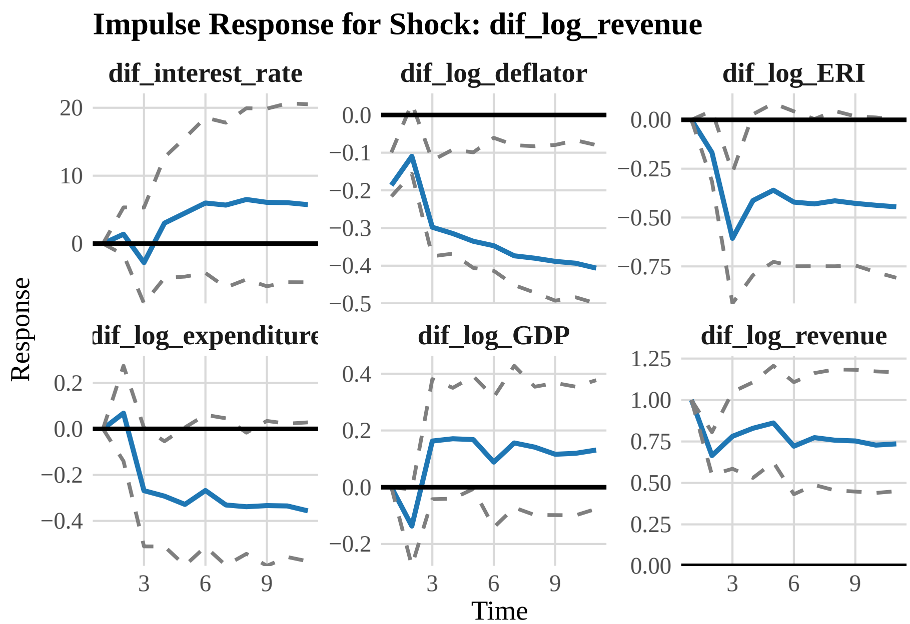
11.2.1 Total Period:

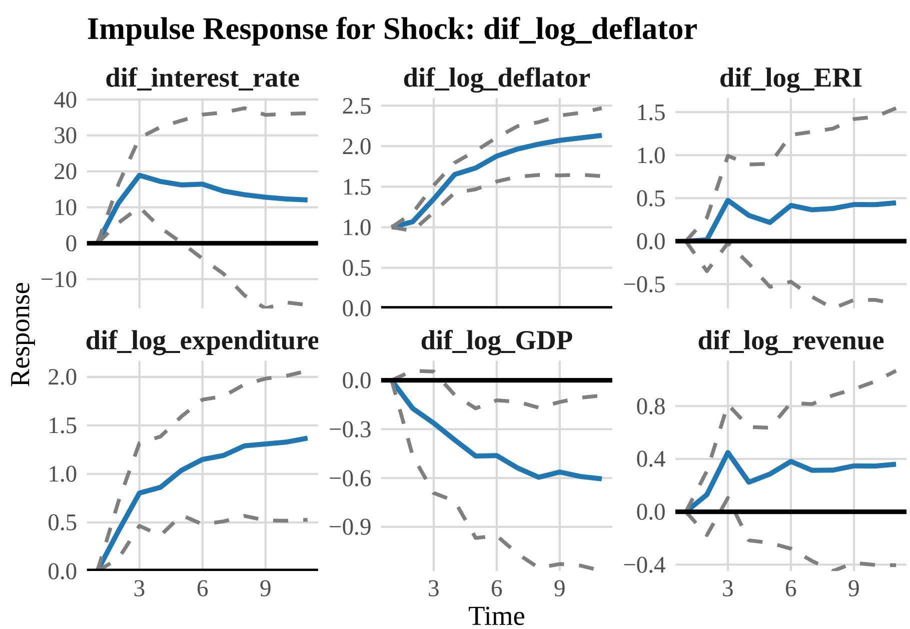


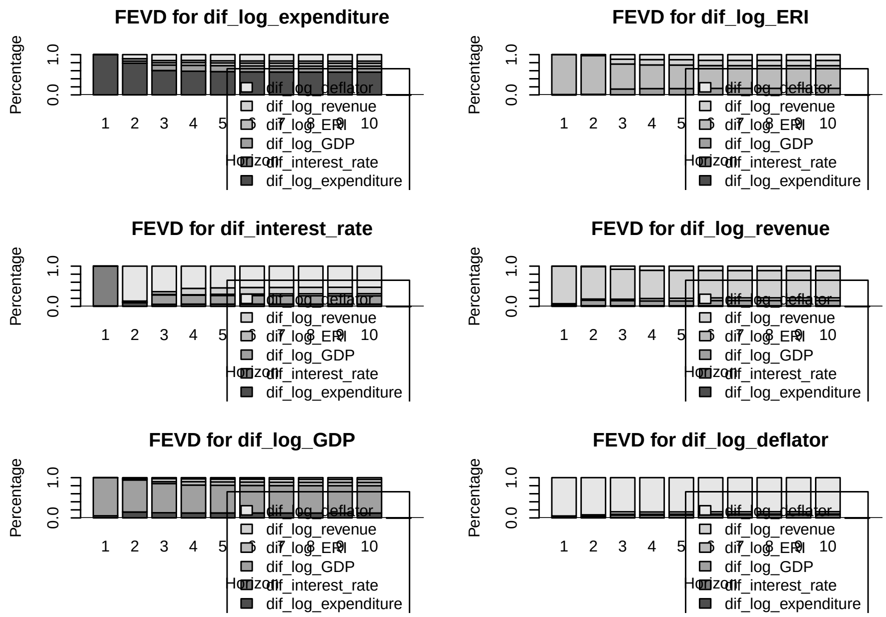



Impulse Response for Shock: dif_log_GDP
dif_interest_rate
dif_log_deflator
dif_log_ERI
dif_log_expenditure
dif_log_GDP
dif_log_revenue
Response
Time


Impulse Response for Shock: dif_log_ERI
dif_interest_rate
dif_log_deflator
dif_log_ERI
dif_log_expenditure
dif_log_GDP
dif_log_revenue
Response
Time


Impulse Response for Shock: dif_log_revenue
dif_interest_rate
dif_log_deflator
dif_log_ERI
dif_log_expenditure
dif_log_GDP
dif_log_revenue
Response
Time


Impulse Response for Shock: dif_log_deflator
dif_interest_rate
dif_log_deflator
dif_log_ERI
dif_log_expenditure
dif_log_GDP
dif_log_revenue
Response
Time




```
## $dif_log_expenditure
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]           1.0000000      0.000000e+00   0.0000000  0.00000000
##  [2,]           0.7858899      1.498593e-05   0.0531387  0.05671181
##  [3,]           0.6060296      1.149908e-05   0.1345979  0.05942105
##  [4,]           0.5884661      1.256998e-05   0.1445961  0.07161017
##  [5,]           0.5781855      1.245491e-05   0.1457549  0.07066532
##  [6,]           0.5720211      1.281501e-05   0.1472479  0.07064805
##  [7,]           0.5682612      1.346754e-05   0.1500985  0.07055343
##  [8,]           0.5656113      1.340877e-05   0.1494468  0.07068020
##  [9,]           0.5654004      1.341331e-05   0.1495695  0.07068607
## [10,]           0.5651726      1.344965e-05   0.1496565  0.07074790
##       dif_log_revenue dif_log_deflator
##  [1,]     0.000000000        0.0000000
##  [2,]     0.002798852        0.1014458
##  [3,]     0.053606461        0.1463335
##  [4,]     0.052142240        0.1431728
##  [5,]     0.051704568        0.1536772
##  [6,]     0.052708014        0.1573621
##  [7,]     0.054037297        0.1570361
##  [8,]     0.053808825        0.1604396
##  [9,]     0.053797793        0.1605329
## [10,]     0.053776738        0.1606328
##
## $dif_interest_rate
```

```
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]          0.01283683       0.987163170 0.000000000  0.00000000
##  [2,]          0.08956994       0.008708351 0.001930672  0.01883713
##  [3,]          0.04844738       0.004650927 0.231666234  0.01409870
##  [4,]          0.05327553       0.004011541 0.219261012  0.01331751
##  [5,]          0.05358573       0.003912543 0.213211258  0.03063672
##  [6,]          0.05483979       0.003842203 0.210260198  0.03871533
##  [7,]          0.05442964       0.003760236 0.205795969  0.04826194
##  [8,]          0.05400094       0.003727400 0.205698632  0.05075637
##  [9,]          0.05373552       0.003708380 0.204615598  0.05397499
## [10,]          0.05373286       0.003706557 0.204462502  0.05408147
##       dif_log_revenue dif_log_deflator
##  [1,]      0.00000000        0.0000000
##  [2,]      0.01328958        0.8676643
##  [3,]      0.06668958        0.6344472
##  [4,]      0.15695882        0.5531756
##  [5,]      0.15870436        0.5399494
##  [6,]      0.16202120        0.5303213
##  [7,]      0.15877703        0.5289752
##  [8,]      0.15910694        0.5267097
##  [9,]      0.15873768        0.5252278
## [10,]      0.15860394        0.5254127
##
## $dif_log_GDP
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]          0.05257084      3.355591e-05   0.9473956  0.00000000
##  [2,]          0.14745504      3.888758e-05   0.7942343  0.01989361
##  [3,]          0.12851326      3.752348e-05   0.7252728  0.04701356
##  [4,]          0.12281671      3.563329e-05   0.6927210  0.08338361
##  [5,]          0.12346526      3.553949e-05   0.6854652  0.08492595
##  [6,]          0.12315544      3.990280e-05   0.6825613  0.08463332
##  [7,]          0.12189822      4.098473e-05   0.6791467  0.08391875
##  [8,]          0.12159426      4.096846e-05   0.6772285  0.08432267
##  [9,]          0.12143383      4.115809e-05   0.6764910  0.08434105
## [10,]          0.12138267      4.121284e-05   0.6761611  0.08434129
##       dif_log_revenue dif_log_deflator
##  [1,]      0.00000000       0.00000000
##  [2,]      0.01480330       0.02357487
##  [3,]      0.07349270       0.02567010
##  [4,]      0.06979273       0.03125032
##  [5,]      0.06898220       0.03712580
##  [6,]      0.07264944       0.03696057
##  [7,]      0.07476012       0.04023518
##  [8,]      0.07468253       0.04213104
##  [9,]      0.07497328       0.04271970
## [10,]      0.07491592       0.04315779
```

```
##
## $dif_log_ERI
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]        7.100536e-05      0.0003155680 0.008975268   0.9906382
##  [2,]        6.115522e-03      0.0003054077 0.007794735   0.9619898
##  [3,]        3.933916e-03      0.0002119366 0.140933489   0.6209106
##  [4,]        3.723738e-03      0.0002054281 0.152388228   0.5874416
##  [5,]        3.907766e-03      0.0002042129 0.152904678   0.5836870
##  [6,]        4.029399e-03      0.0001979253 0.157393249   0.5658017
##  [7,]        4.789287e-03      0.0001975001 0.157164392   0.5645780
##  [8,]        4.818718e-03      0.0001974158 0.157205124   0.5643909
##  [9,]        4.868343e-03      0.0001972639 0.157226291   0.5636373
## [10,]        4.915649e-03      0.0001972236 0.157296058   0.5635323
##       dif_log_revenue dif_log_deflator
##  [1,]       0.0000000     0.0000000000
##  [2,]       0.0236127     0.0001818423
##  [3,]       0.1195160     0.1144940779
##  [4,]       0.1321770     0.1240639959
##  [5,]       0.1326874     0.1266089789
##  [6,]       0.1304336     0.1421441042
##  [7,]       0.1301755     0.1430953136
##  [8,]       0.1302419     0.1431458971
##  [9,]       0.1301238     0.1439469823
## [10,]       0.1301408     0.1439180020
##
## $dif_log_revenue
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]          0.02228984      6.335670e-05  0.01654787  0.02918886
##  [2,]          0.01839771      1.117841e-04  0.13273856  0.03158978
##  [3,]          0.01698691      1.026030e-04  0.12399322  0.03882692
##  [4,]          0.01619935      1.002846e-04  0.11872526  0.06132549
##  [5,]          0.01707536      9.918775e-05  0.11793988  0.06728320
##  [6,]          0.01693391      9.664556e-05  0.12752084  0.06754239
##  [7,]          0.01693330      9.879023e-05  0.12866879  0.06712685
##  [8,]          0.01692589      9.887512e-05  0.12862999  0.06757135
##  [9,]          0.01691541      9.889373e-05  0.12873076  0.06751495
## [10,]          0.01703428      9.888105e-05  0.12880268  0.06748277
##       dif_log_revenue dif_log_deflator
##  [1,]       0.9319101       0.00000000
##  [2,]       0.8050420       0.01212015
##  [3,]       0.7420197       0.07807068
##  [4,]       0.6990335       0.10461610
##  [5,]       0.6918311       0.10577125
##  [6,]       0.6802785       0.10762767
##  [7,]       0.6775571       0.10961513
##  [8,]       0.6772328       0.10954107
```

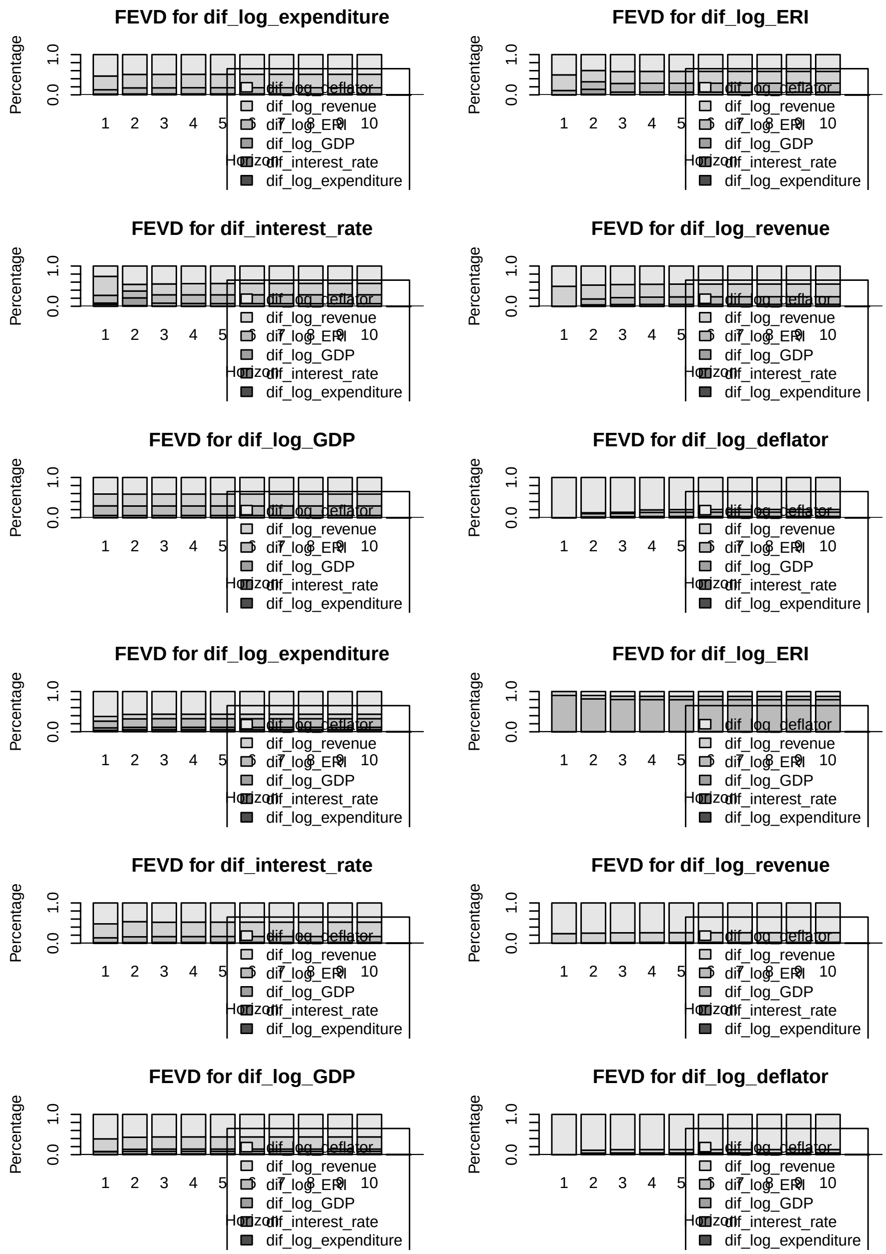
```
##  [9,]       0.6766797       0.11006033
## [10,]       0.6765935       0.10998786
##
## $dif_log_deflator
##       dif_log_expenditure dif_interest_rate dif_log_GDP dif_log_ERI
##  [1,]         0.003950208      1.692645e-06 0.003500227 0.008350581
##  [2,]         0.011708439      1.779006e-06 0.003679210 0.024333443
##  [3,]         0.010110824      6.729717e-06 0.059793419 0.022405748
##  [4,]         0.009880171      1.020779e-05 0.055900032 0.026906142
##  [5,]         0.009803632      1.084293e-05 0.058076705 0.027025976
##  [6,]         0.009913673      1.227891e-05 0.062047554 0.026468638
##  [7,]         0.009842206      1.284029e-05 0.062987857 0.026278150
##  [8,]         0.009806678      1.280486e-05 0.063885787 0.026225273
##  [9,]         0.009798070      1.286077e-05 0.064308068 0.026273736
## [10,]         0.009789318      1.285527e-05 0.064504453 0.026266211
##       dif_log_revenue dif_log_deflator
##  [1,]      0.03306068        0.9511366
##  [2,]      0.03736762        0.9229095
##  [3,]      0.05964350        0.8480398
##  [4,]      0.05540428        0.8518992
##  [5,]      0.05527511        0.8498077
##  [6,]      0.05422931        0.8473285
##  [7,]      0.05433439        0.8465446
##  [8,]      0.05416244        0.8459070
##  [9,]      0.05408928        0.8455180
## [10,]      0.05405611        0.8453710
```

11.3 FEVDs post GFC:

Table 1: Mean Values of Differenced Variables

Variable	Mean
dif_log_expenditure	0.01347
dif_interest_rate	-0.03960
dif_log_ERI	-0.00045
dif_log_GDP	0.00472
dif_log_revenue	0.01281
dif_log_deflator	0.00731

```
## [1] 0.63973648 0.46054164 0.41262905 0.21169834 0.21169834 0.03553232
## [1] 0.6785532 0.4680961 0.3902096 0.3902096 0.3438791 0.1646365
```




```r
# library(knitr)
# library(stargazer)
# library(clipr)
#library(kableExtra)
```

```r
library(ggplot2)
library(knitr)
library(ivreg)
library(ggdag)
library(data.table)
library(dplyr)
library(tidyr)
library(stargazer)
library(clipr)
library(tibble)
library(lubridate)
library(boot)
library(seasonal)
```

```r
lapply(c("ggplot2", "dplyr", "data.table", "lubridate", "janitor", "broom", "tibble", "tidyr
       require, character.only = TRUE)

# knitr::opts_chunk$set(echo = FALSE)
knitr::opts_chunk$set(echo = FALSE, warning = FALSE, message = FALSE)
```

```r
df <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/GDP.csv", skip


# Filter the data frame to exclude rows where the column 'title' matches any of the specific


filtered_df <- df %>%
  # Keep only the quarterly data
  filter(nchar(CDID) == 7 & substr(CDID, 6, 6) == "Q") %>%
  # Select relevant columns and rename them
  dplyr::select(CDID, Deflator = L8GG, GDP = ABMI) %>%
  # Create new columns and convert types
  mutate(
    Year = as.numeric(substr(CDID, 1, 4)),
    Quarter = substr(CDID, 6, 7),
    Q = as.numeric(substr(CDID, 7, 7)),
    Deflator = as.numeric(Deflator),
    GDP = as.numeric(GDP)
  ) %>%
  # Filter by year (can modify for testing)
```

```r
  filter(Year >= 1987)



# fiscal_raw <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Fisc

fiscal_raw <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Fiscal

fiscal_proc <- fiscal_raw %>%
  dplyr::select(Date_ID = Identifier, Revenue = Revenue, Expenditure = Expenditure) %>%
  mutate(Year = as.numeric(gsub("\\D", "", Date_ID)),
         Period = gsub("\\d{4}", "", Date_ID))  %>%
  mutate(
    Q = case_when(
      Period == "Jan to Mar " ~ 1,
      Period == "Apr to Jun " ~ 2,
      Period == "Jul to Sep " ~ 3,
      Period == "Oct to Dec " ~ 4
      ),
    Unique_Period = Year +(Q/4)

  ) %>%
  # Convert to numeric and multiply by 1 million so values as these will later be made into
  mutate(Revenue = as.numeric(gsub(",", "", Revenue) ),
         Expenditure = as.numeric(gsub(",", "", Expenditure )))

# fiscal_proc <- fiscal_raw %>%
#   dplyr::select(Date_ID = Transaction, Revenue = OTR, Expenditure = OTE) %>%
#   subset(Date_ID !=  "Dataset identifier code" & Date_ID != "Identifier") %>%
#   mutate(Year = as.numeric(gsub("\\D", "", Date_ID)),
#          Period = gsub("\\d{4}", "", Date_ID))  %>%
#   mutate(
#     Q = case_when(
#       Period == "Jan to Mar " ~ 1,
#       Period == "Apr to Jun " ~ 2,
#       Period == "Jul to Sep " ~ 3,
#       Period == "Oct to Dec " ~ 4
#       ),
#     Unique_Period = Year +(Q/4)
#
#   ) %>%
#   # Convert to numeric and multiply by 1 million so values as these will later be made in
#   mutate(Revenue = as.numeric(gsub(",", "", Revenue) ),
#          Expenditure = as.numeric(gsub(",", "", Expenditure )))
```

```
# join GDP deflator and GDP data

population <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Popula
                    skip = 4,
                    header = TRUE) %>%
  subset(`Country Name` == "United Kingdom") %>%
  t() %>%
  as.data.frame() %>%
  rownames_to_column(var = "Year") %>%
  rename(Population = V1 ) %>%
  filter(grepl("^\\d{4}$", Year)) %>%
  mutate(Year = as.numeric(Year),
         Population = as.numeric(Population))

Interest_SR <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/3 mon
  mutate(Date = dmy(Date),
         month = month(Date),
         Year = year(Date),
         Q = case_when(
           month %in% 1:3 ~ 1,
           month %in% 4:6 ~ 2,
           month %in% 7:9 ~ 3,
           month %in% 10:12 ~ 4
         )) %>%
  group_by(Year, Q) %>%
  summarize(mean_SR_Rate = mean(Rate, na.rm = TRUE))



SONIA <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Bank of Eng
  mutate(Date = dmy(Date),
         month = month(Date),
         Year = year(Date),
         Q = case_when(
           month %in% 1:3 ~ 1,
           month %in% 4:6 ~ 2,
           month %in% 7:9 ~ 3,
           month %in% 10:12 ~ 4
         )) %>%
  group_by(Year, Q) %>%
  summarize(mean_SONIA = mean(SONIA, na.rm = TRUE))

Policy_Rate <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Polic
  mutate(Date = parse_date_time(`Date Changed`, orders = "dmy"),
         Q = quarter(Date),
```

```
         Year = year(Date)) %>%
  group_by(Year, Q) %>%
  summarise(mean_SR_Rate = mean(Rate, na.rm = TRUE), .groups = "drop") %>%
  complete(Year = full_seq(Year, 1), Q = 1:4) %>%  # Ensure all Year-Quarter combinations
  arrange(Year, Q) %>%
  fill(mean_SR_Rate, .direction = "down")  # Fill missing rates by propagating the previous

Exports <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Exports.c
  mutate(Date = dmy(Month),
         month = month(Month),
         Year = year(dmy(Month)),
         Q = case_when(
           month %in% 1:3 ~ 1,
           month %in% 4:6 ~ 2,
           month %in% 7:9 ~ 3,
           month %in% 10:12 ~ 4
         )) %>%
  group_by(Year, Q) %>%
  summarize(Exports = sum(Exports, na.rm = TRUE))

ERI <- fread("D:/Samid work/University/KCL - Econ and Policy/Dissertation/Data/Boe ERI.csv")
  mutate(
         month = month(Date),
         Year = year(dmy(Date)),
         Q = case_when(
           month %in% 1:3 ~ 1,
           month %in% 4:6 ~ 2,
           month %in% 7:9 ~ 3,
           month %in% 10:12 ~ 4
         )) %>%
  rename(ERI = Value)


data <- fiscal_proc %>%
  left_join(filtered_df, by = c("Q" = "Q", "Year" = "Year")) %>%
  left_join(Interest_SR, by = c("Q" = "Q", "Year" = "Year")) %>%
  left_join(Exports, by = c("Q" = "Q", "Year" = "Year")) %>%
  left_join(population, by = c("Year" = "Year")) %>%
  left_join(ERI, by = c("Q" = "Q", "Year" = "Year")) %>%
# Convert variables to per capita, note revenue, expenditure, and GDP are in £ million so no
  mutate(RevenuePerCapita = (Revenue *10^6) /Population,
         ExpenditurePerCapita = (Expenditure *10^6) /Population,
         GDPPerCapita = (GDP *10^6) /Population) %>%

  # Seasonal Adjustment of data using X-13ARIMA-SEATS
```

```r
  mutate(Revenue_SA = final(seas(ts(Revenue, start = min(Year), frequency = 4))),
         Expenditure_SA = final(seas(ts(Expenditure, start = min(Year), frequency = 4))),
         GDP_SA = final(seas(ts(GDP, start = min(Year), frequency = 4)))) %>%
 # convert back to numeric
    mutate(
    Revenue_SA = as.numeric(Revenue_SA),
    Expenditure_SA = as.numeric(Expenditure_SA),
    GDP_SA = as.numeric(GDP_SA)
  ) %>%


# Convert variables (except interest rate) to logs
  # Note multiplying by 10^6 for variables that are defined in £millions
  mutate(log_revenue = log(Revenue_SA *10^6),
         log_expenditure = log(Expenditure_SA *10^6),
         log_GDP = log(GDP_SA *10^6),
         log_deflator = log(Deflator),
         log_ERI = log(ERI),
         log_exports = log(Exports *10^6)) %>%
  mutate(
    dif_log_revenue = log_revenue - lag(log_revenue),
    dif_log_expenditure = log_expenditure - lag(log_expenditure),
    dif_log_GDP = log_GDP - lag(log_GDP),
    dif_log_deflator = log_deflator - lag(log_deflator),
    dif_log_ERI = log_ERI - lag(log_ERI),
    dif_interest_rate = mean_SR_Rate - lag(mean_SR_Rate),
    dif_log_exports = log_exports - lag(log_exports)
  )


model_data2 <- data %>%
  dplyr::select(Year, CDID, log_expenditure, mean_SR_Rate, log_GDP, log_ERI, log_revenue,  l

# model_data2 <- data %>%
#   dplyr::select(Year, CDID, log_expenditure, mean_SR_Rate, log_GDP, log_exports, log_reven

# model_data <- data %>%
#   dplyr::select(Year, CDID, log_expenditure, mean_SR_Rate, log_exports, log_GDP, log_reven

# model_data <- data %>%
#   dplyr::select(Year, CDID, dif_log_expenditure, dif_interest_rate, dif_log_exports, dif_l

model_data <- data %>%
dplyr::select(Year, CDID, dif_log_expenditure, dif_interest_rate, dif_log_GDP, dif_log_ERI,
```

```
# Adding Exports



GDP_tmp <- data %>%
  dplyr::select(Date_ID, GDP)
# Define a custom theme for consistency
custom_theme <- theme_minimal(base_size = 14) +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title = element_text(face = "bold"),
    panel.grid.minor = element_blank()
  )

debt_GDP <- data %>%
  mutate(debt_GDP_ratio = Expenditure/ GDP,
         Gov_deficit = (Expenditure - Revenue)/GDP)

ggplot(debt_GDP, aes(x = Unique_Period)) +
  geom_line(aes(y = debt_GDP_ratio), color = "blue", size = 1.2) +
  labs(
    title = "Debt to GDP Over Time",
    x = "Year",
    y = "% of GDP"
  ) +
  custom_theme

ggplot(debt_GDP, aes(x = Unique_Period)) +
  geom_line(aes(y = Gov_deficit), color = "blue", size = 1.2) +
  labs(
    title = "Deficit as a Percentage of GDP Over Time",
    x = "Year",
    y = "% of GDP"
  ) +
  custom_theme

clean_data <- na.omit(model_data)

clean_data2 <- na.omit(model_data2)


tmp <- clean_data[,-c(1,2)]

OptimalLag <-  VARselect(clean_data[,-c(1,2)], lag.max = 5, type = "const")
```

```r
OptimalLag2 <-  VARselect(clean_data2[,-c(1,2)], lag.max = 5, type = "const")

# OptimalLag2$selection
# OptimalLag$criteria
# OptimalLag$selection
# OptimalLag$criteria
# OptimalLag2$criteria

# library(vars)

reduced_VAR <- VAR(clean_data[,-c(1,2)], p = 3, type = "const")
# reduced_VAR2 <- VAR(clean_data2[,-c(1,2)], p = 4, type = "both")
# reduced_VAR <- VAR(clean_data[, -1], p = 4)

# roots(reduced_VAR)
# roots(reduced_VAR2)

# summary(reduced_VAR)

# Summary reports the roots of the polynomial.



# Define the 5 dimensional lower triangular matrix, A

# Recover structural VAR using Cholesky decomposition

# Amat <- matrix(c(1, 0, 0, 0, 0,   # Recursive ordering
#                  NA, 1, 0, 0, 0,
#                  NA, NA, 1, 0, 0,
#                  NA, NA, NA, 1, 0,
#                  NA, NA, NA, NA, 1),
#                nrow = 5, byrow = TRUE)


# Variables are already ordered per the recursive identification strategy. Thus create a lo
k <- ncol(clean_data[,-c(1,2)])
Amat <- diag(1, k)
Amat[lower.tri(Amat)] <- NA
# print(Amat)




svar_model <- SVAR(reduced_VAR, Amat = Amat, estmethod = "direct")
```

```r
# svar_model2 <- SVAR(reduced_VAR2, Amat = Amat, estmethod = "direct")
# ?SVAR()

structural_shocks <- residuals(svar_model)

# svar_model


irf_result <- irf(svar_model, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)  # Fo
# irf_result2 <- irf(svar_model2, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = FALSE)
# irf_result <- irf(svar_model, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)  #
# plot(irf_result)
# Visualize IRFs

FEVD_result <- fevd(svar_model, n.ahead = 10)  # Forecast horizons
# FEVD_result2 <- fevd(svar_model2, n.ahead = 10)  # Forecast horizons

# plot(FEVD_result)
# plot(FEVD_result2)



# Post Covid:






# Extract IRFs
exp_irf <- round(irf_result$irf$dif_log_expenditure[, 3],2)
rev_irf <- round(irf_result$irf$dif_log_revenue[, 3],2)

exp_irf_df <- as.data.frame(
  cbind(
    seq(0,10),
    irf_result$irf$dif_log_expenditure[, 3],
    irf_result$Lower$dif_log_expenditure[, 3],
    irf_result$Upper$dif_log_expenditure[, 3]
  )
)

colnames(exp_irf_df) <- c("Quarter","Expenditure", "LB", "UB")

rev_irf_df <- as.data.frame(
```

```r
  cbind(
    seq(0,10),
    irf_result$irf$dif_log_revenue[, 3],
    irf_result$Lower$dif_log_revenue[, 3],
    irf_result$Upper$dif_log_revenue[, 3]
  )
)

colnames(rev_irf_df) <- c("Quarter","Revenue", "LB", "UB")











# Plot for Expenditure IRF
ggplot(exp_irf_df, aes(x = Quarter)) +
  geom_ribbon(aes(ymin = LB, ymax = UB), fill = "lightblue", alpha = 0.4) +
  geom_line(aes(y = Expenditure), color = "blue", size = 1.2) +
  geom_hline(yintercept = 0, linetype = "dotted", color = "black") +
  labs(
    title = "Impulse Response of Government Expenditure",
    x = "Quarter",
    y = "Log-Difference Response"
  ) +
  custom_theme

# Plot for Revenue IRF
ggplot(rev_irf_df, aes(x = Quarter)) +
  geom_ribbon(aes(ymin = LB, ymax = UB), fill = "lightblue", alpha = 0.4) +
  geom_line(aes(y = Revenue), color = "blue", size = 1.2) +
  geom_hline(yintercept = 0, linetype = "dotted", color = "black") +
  labs(
    title = "Impulse Response of Government Revenue",
    x = "Period",
    y = "Log-Difference Response"
  ) +
  custom_theme
```

```r
# structural_shocks <- residuals(svar_model)
# irf_result <- irf(svar_model, n.ahead = 10)  # Forecast horizons
# plot(irf_result)  # Visualize IRFs



# normality.test(reduced_VAR)

serial_test <- serial.test(reduced_VAR, lags.pt = 16, type = "PT.asymptotic")


# serial_test # Don't reject H0 of no autocorrelation

normality_test <- normality.test(reduced_VAR)
# normality_test


hetero_test <- arch.test(reduced_VAR, lags.multi = 5)
# hetero_test


# --- Step 1: Define Restricted Model ---
# reduced_VAR <- VAR(clean_data[,-c(1,2)], p = 3, type = "const")
# --- Step 1: Define Parameters ---
break_year <- 2008
p <- 1
Rep <- 100
T_break <- which((clean_data$Year == break_year))[1]  # First observation of break year


# --- Step 2: Estimate VAR & Prepare Data ---
Y <- clean_data[, -c(1, 2)]  # Drop Year + Quarter columns
var_orig <- VAR(Y, p = p, type = "const")
resids <- residuals(var_orig)
centered_resids <- scale(resids, scale = FALSE)  # Centered residuals
Y_vals <- as.matrix(Y)
presample <- Y_vals[1:p, ]
coef_matrix <- Bcoef(var_orig)
A <- as.matrix(coef_matrix[, -ncol(coef_matrix)])
c <- coef_matrix[, ncol(coef_matrix)]

# --- Step 3: Define Chow Function (Lambda_BP) ---
chow_stat <- function(Y, T_break, p) {
  tryCatch({
```

```r
    T_full  <- nrow(Y)

    # Restricted model: full sample
    model_restricted <- VAR(Y, p = p, type = "const")
    SSR_restricted <- sum(residuals(model_restricted)^2)

    # Unrestricted model: separate pre and post samples
    Y_pre <- Y[1:T_break, ]
    Y_post <- Y[(T_break + 1):T_full, ]

    model_pre <- VAR(Y_pre, p = p, type = "const")
    model_post <- VAR(Y_post, p = p, type = "const")

    SSR_unrestricted <- sum(residuals(model_pre)^2) + sum(residuals(model_post)^2)
    SSR_1 <- sum(residuals(model_pre)^2)
    SSR_tot <- sum(residuals(model_restricted)^2)

    # Extract log-likelihoods
    ll_restricted <- logLik(model_restricted)
    ll_unrestricted <- logLik(model_pre) + logLik(model_post)



    # Count number of coefficients per equation (excluding stats like SE, t)
    k <- nrow(coef(reduced_VAR)[[1]])
    n_eq <- length(coef(reduced_VAR))  # Number of equations

    # Degrees of freedom = number of extra parameters in unrestricted models
    df_chow <<- k * n_eq

    # Compute LR test statistic
    lambda_BP <- -2 * (as.numeric(ll_restricted) - as.numeric(ll_unrestricted))

    return(lambda_BP)
  }, error = function(e) {
    message("Chow statistic failed: ", e$message)
    return(NA)
  })
}

# --- Step 4: Bootstrap Simulation ---
set.seed(1234)
boot_stats <- numeric(Rep)

for (r in 1:Rep) {
```

```r
  # FIX: Sample from available residuals only
  u_star <- centered_resids[sample(1:nrow(centered_resids), size = nrow(Y_vals), replace = T

  Y_boot <- matrix(NA, nrow = nrow(Y_vals), ncol = ncol(Y_vals),
                   dimnames = list(NULL, colnames(Y_vals)))
  Y_boot[1:p, ] <- presample

  for (t in (p + 1):nrow(Y_vals)) {
    Y_lags <- as.vector(t(Y_boot[(t - p):(t - 1), ]))
    Y_boot[t, ] <- c + A %*% Y_lags + u_star[t, ]
  }

  boot_stats[r] <- if (anyNA(Y_boot)) NA else chow_stat(Y_boot, T_break, p)
  # if (r %% 50 == 0) cat("Completed iteration", r, "\n")
}

boot_stats <- na.omit(boot_stats)

# --- Step 5: Actual Statistic & p-value ---
actual_stat <- chow_stat(Y_vals, T_break, p)
p_value_asymptotic <- 1 - pchisq(actual_stat, df_chow)
p_val <- mean(boot_stats > actual_stat)

# --- Step 6: Plot Results ---
if (length(boot_stats) > 0) {
  boot_df <- data.frame(
    Iteration = 1:length(boot_stats),
    Chow_Statistic = boot_stats
  )

 chow_plot <<-  ggplot(boot_df, aes(x = Iteration, y = Chow_Statistic)) +
    geom_line(color = "dodgerblue", alpha = 0.7) +
    geom_hline(yintercept = actual_stat, color = "red", linetype = "dashed", linewidth = 1)
    geom_point(aes(x = 0, y = actual_stat), color = "red", size = 3) +
    labs(
      title = "Bootstrap Chow Test (Recursive)",
      subtitle = paste("Breakpoint:", break_year, "| p-value:", round(p_val, 4)),
      x = "Bootstrap Iteration",
      y = "Chow Test Statistic"
    ) +
    theme_minimal()
} else {
  warning("No valid bootstrap statistics available to plot.")
}
```

```r
# print(actual_stat)
# print(p_val)
# print(p_value_asymptotic)


chow_plot+
  custom_theme

T_break <- which((clean_data$Year == break_year))[1]  # First observation of break year

Y_post_GFC <-  Y[(T_break + 1):nrow(Y), ]


# reduced_VAR <- VAR(clean_data[,-c(1,2)], p = 3, type = "const")


reduced_VAR_post_GFC <- VAR(Y_post_GFC, p = 3, type = "const")

svar_model_post_gfc <- SVAR(reduced_VAR_post_GFC, Amat = Amat, estmethod = "direct")

irf_result_post_gfc <- irf(svar_model_post_gfc, n.ahead = 10, ci = 0.68, boot = 5000, cumula

exp_irf_post_GFC <- round(irf_result_post_gfc$irf$dif_log_expenditure[, 3],2)
rev_irf_post_GFC <- round(irf_result_post_gfc$irf$dif_log_revenue[, 3],2)

# Combine into one table
# Combine IRFs into one table
irf_combined <- cbind(
  Period = 0:(length(exp_irf) - 1),
  Total_Expenditure = exp_irf,
  Total_Revenue = rev_irf,
  PostGFC_Expenditure = exp_irf_post_GFC,
  PostGFC_Revenue = rev_irf_post_GFC
)

# Prepare data for Revenue plot
df_rev <- data.frame(
  Period = 0:(length(rev_irf) - 1),
  Total = rev_irf,
  PostGFC = rev_irf_post_GFC
) %>%
  pivot_longer(cols = -Period, names_to = "Sample", values_to = "IRF")


# Prepare data for Expenditure plot
```

```
df_exp <- data.frame(
  Period = 0:(length(exp_irf) - 1),
  Total = exp_irf,
  PostGFC = exp_irf_post_GFC
) %>%
  pivot_longer(cols = -Period, names_to = "Sample", values_to = "IRF")


# Plot Revenue IRFs
ggplot(df_rev, aes(x = Period, y = IRF, color = Sample)) +
  geom_line(size = 1.2) +
  labs(
    title = "IRF: Revenue Response to Expenditure Shock",
    x = "Periods Ahead",
    y = "Impulse Response"
  ) +
  theme_minimal(base_size = 14) +
  theme(plot.title = element_text(face = "bold", hjust = 0.5)) +
  custom_theme
# Plot Expenditure IRFs
ggplot(df_exp, aes(x = Period, y = IRF, color = Sample)) +
  geom_line(size = 1.2) +
  labs(
    title = "IRF: Expenditure Response to Revenue Shock",
    x = "Periods Ahead",
    y = "Impulse Response"
  ) +
  theme_minimal(base_size = 14) +
  theme(plot.title = element_text(face = "bold", hjust = 0.5)) +
  custom_theme

# T_break <- which((clean_data$Year == break_year))[1]  # First observation of break year
#
# Y_post_GFC <-  Y[(T_break + 1):nrow(Y), ]


# reduced_VAR <- VAR(clean_data[,-c(1,2)], p = 3, type = "const")


reduced_VAR_1_lag <- VAR(clean_data[,-c(1,2)], p = 1, type = "const")


svar_model_1_lag <- SVAR(reduced_VAR_1_lag, Amat = Amat, estmethod = "direct")

irf_result_1_lag <- irf(svar_model_1_lag, n.ahead = 10, ci = 0.68, boot = 5000, cumulative =
```

```
exp_irf_1_lag <- round(irf_result_1_lag$irf$dif_log_expenditure[, 3],2)
rev_irf_1_lag <- round(irf_result_1_lag$irf$dif_log_revenue[, 3],2)


reduced_VAR_post_GFC_1_lag <- VAR(Y_post_GFC, p = 1, type = "const")

svar_model_post_gfc_1_lag <- SVAR(reduced_VAR_post_GFC_1_lag, Amat = Amat, estmethod = "dire

irf_result_post_gfc_1_lag <- irf(svar_model_post_gfc_1_lag, n.ahead = 10, ci = 0.68, boot =

exp_irf_post_GFC_1_lag <- round(irf_result_post_gfc_1_lag$irf$dif_log_expenditure[, 3],2)
rev_irf_post_GFC_1_lag <- round(irf_result_post_gfc_1_lag$irf$dif_log_revenue[, 3],2)

# Combine into one table
# Combine IRFs into one table

# irf_combined <- cbind(
#   Period = 0:(length(exp_irf) - 1),
#   Total_Expenditure = exp_irf,
#   Total_Revenue = rev_irf,
#   PostGFC_Expenditure = exp_irf_post_GFC,
#   PostGFC_Revenue = rev_irf_post_GFC,
#   PostGFC_Expenditure_1_lag = exp_irf_post_GFC_1_lag,
#   PostGFC_Revenue_1_lag = rev_irf_post_GFC_1_lag
#
# )



# Prepare data for Revenue plot
df_rev_all <- data.frame(
  Period = 0:(length(rev_irf) - 1),
  Total = rev_irf,
  PostGFC = rev_irf_post_GFC,
  PostGFC_1_lag = rev_irf_post_GFC_1_lag,
  Total_1_lag = rev_irf_1_lag


) %>%
  pivot_longer(cols = -Period, names_to = "Sample", values_to = "IRF")


# Prepare data for Expenditure plot
df_exp_all <- data.frame(
```

```
  Period = 0:(length(exp_irf) - 1),
  Total = exp_irf,
  PostGFC = exp_irf_post_GFC,
  PostGFC_1_lag = exp_irf_post_GFC_1_lag,
  Total_1_lag = exp_irf_1_lag

) %>%
  pivot_longer(cols = -Period, names_to = "Sample", values_to = "IRF")


# Plot Revenue IRFs
ggplot(df_rev_all, aes(x = Period, y = IRF, color = Sample)) +
  geom_line(size = 1.2) +
  labs(
    title = "IRF: Revenue Response to Revenue Shock",
    x = "Periods Ahead",
    y = "Impulse Response"
  ) +
  theme_minimal(base_size = 14) +
  theme(plot.title = element_text(face = "bold", hjust = 0.5)) +
  custom_theme
# Plot Expenditure IRFs
ggplot(df_exp_all, aes(x = Period, y = IRF, color = Sample)) +
  geom_line(size = 1.2) +
  labs(
    title = "IRF: Expenditure Response to Expenditure Shock",
    x = "Periods Ahead",
    y = "Impulse Response"
  ) +
  theme_minimal(base_size = 14) +
  theme(plot.title = element_text(face = "bold", hjust = 0.5)) +
  custom_theme

ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = Revenue_SA, color = "Revenue"), size = 1) +
  geom_line(aes(y = Expenditure_SA, color = "Expenditure"), size = 1) +
  labs(
    x = "Date ID",
    y = "Amount (? in millions)",
    title = "Seasonally Adjusted Revenue and Expenditure",
    color = "Legend"
  ) +
  scale_color_manual(values = c("Revenue" = "blue", "Expenditure" = "red")) +
  theme_minimal(base_size = 15) +
  theme(
```

```
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
    axis.title.y = element_text(face = "bold"),
    legend.position = "bottom",
    legend.title = element_text(face = "bold")
  )



ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = Exports, color = "Exports"), size = 1) +
  labs(
    x = "Date ID",
    y = "Amount (? in millions)",
    title = "Exports Over Time",
    color = "Legend"
  ) +
  scale_color_manual(values = c("Exports" = "red")) +
  theme_minimal(base_size = 15) +
  theme(
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
    axis.title.y = element_text(face = "bold"),
    legend.position = "bottom",
    legend.title = element_text(face = "bold")
  )



ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = ERI, color = "ERI"), size = 1) +
  labs(
    x = "Date ID",
    y = "Amount (? in millions)",
    title = "Exchange Rate Index Over Time",
    color = "Legend"
  ) +
  scale_color_manual(values = c("ERI" = "red")) +
  theme_minimal(base_size = 15) +
  theme(
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
```

```r
    axis.title.y = element_text(face = "bold"),
    legend.position = "bottom",
    legend.title = element_text(face = "bold")
  )



ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = GDP, color = "GDP"), size = 1) +
  labs(
    x = "Date ID",
    y = "Amount (? in millions)",
    title = "GDP Over Time",
    color = "Legend"
  ) +
  scale_color_manual(values = c("GDP" = "red")) +
  theme_minimal(base_size = 15) +
  theme(
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
    axis.title.y = element_text(face = "bold"),
    legend.position = "bottom",
    legend.title = element_text(face = "bold")
  )



ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = Deflator), color = "blue", size = 1) +
  labs(
    x = "Date ID",
    y = "Deflator",
    title = "Deflator Over Time"
  ) +
  theme_minimal(base_size = 15) +
  theme(
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
    axis.title.y = element_text(face = "bold")
  )

# Define scaling factor
scale_factor2 <- max(data$Deflator, na.rm = TRUE) / max(data$mean_SR_Rate, na.rm = TRUE)
```

```r
ggplot(data, aes(x = Unique_Period)) +
  geom_line(aes(y = mean_SR_Rate * scale_factor2), color = "red", size = 1, linetype = "dash
  scale_y_continuous(
    name = "Mean SR Rate (scaled)",
    sec.axis = sec_axis(~ . / scale_factor2, name = "Mean SR Rate (%)")
  ) +
  labs(
    x = "Date ID",
    title = "Mean Short-Term Interest Rate Over Time"
  ) +
  theme_minimal(base_size = 15) +
  theme(
    axis.text.x = element_text(angle = 45, hjust = 1),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    axis.title.x = element_text(face = "bold"),
    axis.title.y = element_text(face = "bold")
  )
# library(urca)

# urca::cajolst()

# vec2var
# Perform cointegration testing on the (log) levels variables
ca.jo_result_max_eig <- ca.jo(clean_data2[,-c(1,2)], type = "eigen", ecdet = "none", K =4)

ca.jo_result_trace <- ca.jo(clean_data2[,-c(1,2)], type = "trace", ecdet = "none", K =4)

ca.po_result <- ca.po(clean_data2[,-c(1,2)], demean = "const", type = "Pz")

# ca.lut_result <- cajolst(clean_data2[,-c(1,2)], trend = TRUE, K = 2, season = NULL)



summary(ca.jo_result_max_eig)
summary(ca.jo_result_trace)
# summary(ca.po_result)
# summary(ca.lut_result)

# ca.jo()



# scaling_factor <-  T/(T-N*k)
scaling_factor <-  length(clean_data2$Year)/(length(clean_data2$Year)-6*4)
```

```r
# Max Eigenvalue test H0 r=0:
# test statistic: 41.78
# Asymptotic Critical value (5% sig level): 39.43
# scaling_factor = 1.1875
# Finite sample critical value = 90.39 * scaling_factor = 46.82312
# After applying the scaling factor we no longer reject the null of no cointegration. We now

# Trace test H0 r=0:
# test statistic: 115.81
# Asymptotic Critical value (5% sig level): 90.39
# scaling_factor = 1.1875
# Finite sample critical value = 90.39 * scaling_factor = 107.3381
# After applying the scaling factor, we no longer have sufficient evidence to reject the nul

vecm <- vec2var(ca.jo_result_max_eig, r = 1)  # r = cointegration rank

# SVECM <- SVAR(vecm, Amat = Amat, estmethod = "direct")


# irf_result_SVECM <- irf(SVECM, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)  #

LR_mat <- matrix(nrow = k, ncol=k)

svecm <- SVEC(ca.jo_result_max_eig, SR =Amat, LR =LR_mat)

irf_temp <- irf(svecm, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)

# plot(irf_temp)




# plotres(ca.jo_result)



# vecm <- cajorls(ca.jo_result, r = 1)
johansen_test <- cajorls(ca.jo_result_max_eig, r = 1)

# johansen_test

vec2var_model <- vec2var(ca.jo_result_max_eig, r = 1)
vec2var_model
```

```r
class(reduced_VAR)
class(vec2var_model)


# svar_model <- SVAR(vec2var_model, estmethod = "direct", Amat = Amat)
# irf_result <- irf(svar_model, n.ahead = 10, boot = TRUE, ci = 0.68)
# plot(irf_result)

# arch.test(reduced_VAR)
# serial.test(reduced_VAR)
# normality.test(reduced_VAR)









# Extract IRFs
exp_irf_vecm <- round(irf_temp$irf$log_expenditure[, 3],2)
rev_irf_vecm <- round(irf_temp$irf$log_revenue[, 3],2)

exp_irf_df_vecm <- as.data.frame(
  cbind(
    seq(0,10),
    irf_temp$irf$log_expenditure[, 3],
    irf_temp$Lower$log_expenditure[, 3],
    irf_temp$Upper$log_expenditure[, 3]
  )
)

colnames(exp_irf_df_vecm) <- c("Quarter","Expenditure", "LB", "UB")

rev_irf_df_vecm <- as.data.frame(
  cbind(
    seq(0,10),
    irf_temp$irf$log_revenue[, 3],
    irf_temp$Lower$log_revenue[, 3],
    irf_temp$Upper$log_revenue[, 3]
  )
)

colnames(rev_irf_df_vecm) <- c("Quarter","Revenue", "LB", "UB")
```

```r
# Define a custom theme for consistency
# custom_theme <- theme_minimal(base_size = 14) +
#   theme(
#     plot.title = element_text(hjust = 0.5, face = "bold"),
#     axis.title = element_text(face = "bold"),
#     panel.grid.minor = element_blank()
#   )
#




# Plot for Expenditure IRF
ggplot(exp_irf_df_vecm, aes(x = Quarter)) +
  geom_ribbon(aes(ymin = LB, ymax = UB), fill = "lightblue", alpha = 0.4) +
  geom_line(aes(y = Expenditure), color = "blue", size = 1.2) +
  geom_hline(yintercept = 0, linetype = "dotted", color = "black") +
  labs(
    title = "Impulse Response of Government Expenditure",
    x = "Quarter",
    y = "Log-Difference Response"
  ) +
  custom_theme

# Plot for Expenditure IRF
ggplot(rev_irf_df_vecm, aes(x = Quarter)) +
  geom_ribbon(aes(ymin = LB, ymax = UB), fill = "lightblue", alpha = 0.4) +
  geom_line(aes(y = Revenue), color = "blue", size = 1.2) +
  geom_hline(yintercept = 0, linetype = "dotted", color = "black") +
  labs(
    title = "Impulse Response of Government Expenditure",
    x = "Quarter",
    y = "Log-Difference Response"
  ) +
  custom_theme



plot_irf_with_ci <- function(IRF_name) {
```

```r
# Extract response and confidence intervals
irf_data  <- as.data.frame(IRF_name$irf)
irf_lower <- as.data.frame(IRF_name$Lower)
irf_upper <- as.data.frame(IRF_name$Upper)

# Create time index
irf_data$Time  <- seq_len(nrow(irf_data))
irf_lower$Time <- irf_data$Time
irf_upper$Time <- irf_data$Time

# Reshape to long format
irf_long   <- pivot_longer(irf_data, cols = -Time, names_to = "Variable", values_to = "IRF
lower_long <- pivot_longer(irf_lower, cols = -Time, names_to = "Variable", values_to = "Lo
upper_long <- pivot_longer(irf_upper, cols = -Time, names_to = "Variable", values_to = "Up

# Merge and label
irf_combined <- irf_long %>%
  left_join(lower_long, by = c("Time", "Variable")) %>%
  left_join(upper_long, by = c("Time", "Variable")) %>%
  mutate(
    Shock = sub("\\..*", "", Variable),
    Affected_Var = sub(".*\\.", "", Variable)
  )

# Plot IRFs by shock
shock_names <- unique(irf_combined$Shock)

for (shock in shock_names) {
  p <- irf_combined %>%
    filter(Shock == shock) %>%
    ggplot(aes(x = Time, y = IRF)) +
    geom_line(size = 1.2, color = "#1f77b4") +
    geom_line(aes(y = Lower), linetype = "dashed", color = "#7f7f7f", size = 0.9) +
    geom_line(aes(y = Upper), linetype = "dashed", color = "#7f7f7f", size = 0.9) +
    geom_hline(yintercept = 0, color = "black", size = 1.1) +
    facet_wrap(~ Affected_Var, scales = "free_y") +
    theme_minimal(base_family = "serif") +
    theme(
      plot.title = element_text(size = 16, face = "bold"),
      axis.title.x = element_text(size = 14),
      axis.title.y = element_text(size = 14),
      axis.text = element_text(size = 12),
      panel.grid.major = element_line(color = "gray85"),
      panel.grid.minor = element_blank(),
      strip.text = element_text(size = 14, face = "bold")
```

```
      ) +
      labs(
        title = paste("Impulse Response for Shock:", shock),
        x = "Time",
        y = "Response"
      ) +
      scale_y_continuous(expand = expansion(mult = c(0, 0.05)))

    print(p)
  }
}

plot_irf_with_ci(irf_result)
# plot_irf_with_ci(irf_result2)

plot(FEVD_result)


FEVD_result

library(knitr)

# Create named vector of means
means <- c(
  mean(model_data$dif_log_expenditure, na.rm = TRUE),
  mean(model_data$dif_interest_rate, na.rm = TRUE),
  mean(model_data$dif_log_ERI, na.rm = TRUE),
  mean(model_data$dif_log_GDP, na.rm = TRUE),
  mean(model_data$dif_log_revenue, na.rm = TRUE),
  mean(model_data$dif_log_deflator, na.rm = TRUE)
)

# Variable names
variables <- c(
  "dif_log_expenditure",
  "dif_interest_rate",
  "dif_log_ERI",
  "dif_log_GDP",
  "dif_log_revenue",
  "dif_log_deflator"
)

# Assemble and display
mean_table <- data.frame(Variable = variables, Mean = round(means, 5))
kable(mean_table, align = "lr", caption = "Mean Values of Differenced Variables")
```

```
run_var_analysis <- function(model_data_type) {
  # Extract suffix from input name
  input_name <- deparse(substitute(model_data_type))
  suffix <- sub("^[^_]+_", "", input_name)
  suffix <- ifelse(nchar(suffix) == 0, "data", suffix)

  # Split data into pre- and post-2008 samples
  clean_data_a <- model_data_type %>%
    filter(Year < 2008 & complete.cases(.))
  clean_data_b <- model_data_type %>%
    filter(!is.na(Year) & Year >= 2008)

  assign(paste0("clean_data_a_", suffix), clean_data_a, envir = .GlobalEnv)
  assign(paste0("clean_data_b_", suffix), clean_data_b, envir = .GlobalEnv)

  # Choose VAR specification
    var_a <- VAR(clean_data_a[,-c(1,2)], p = 1, type = "const")
    var_b <- VAR(clean_data_b[,-c(1,2)], p = 1, type = "const")


  assign(paste0("reduced_VAR_a_", suffix), var_a, envir = .GlobalEnv)
  assign(paste0("reduced_VAR_b_", suffix), var_b, envir = .GlobalEnv)

  # Check stability
  print(roots(var_a))
  print(roots(var_b))

  # Recursive identification matrix
  k <- ncol(clean_data_a[,-c(1,2)])
  Amat <- diag(1, k)
  Amat[upper.tri(Amat)] <- NA

  # SVAR estimation
  svar_a <- SVAR(var_a, Amat = Amat, estmethod = "direct")
  svar_b <- SVAR(var_b, Amat = Amat, estmethod = "direct")

  assign(paste0("svar_model_a_", suffix), svar_a, envir = .GlobalEnv)
  assign(paste0("svar_model_b_", suffix), svar_b, envir = .GlobalEnv)

  # IRFs
  irf_a <- irf(svar_a, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)
  irf_b <- irf(svar_b, n.ahead = 10, ci = 0.68, boot = 5000, cumulative = TRUE)

  assign(paste0("irf_result_a_", suffix), irf_a, envir = .GlobalEnv)
```

```r
  assign(paste0("irf_result_b_", suffix), irf_b, envir = .GlobalEnv)

  # FEVDs
  fevd_a <- fevd(svar_a, n.ahead = 10)
  fevd_b <- fevd(svar_b, n.ahead = 10)

  assign(paste0("FEVD_result_a_", suffix), fevd_a, envir = .GlobalEnv)
  assign(paste0("FEVD_result_b_", suffix), fevd_b, envir = .GlobalEnv)

  # Plot FEVDs
  plot(fevd_a)
  plot(fevd_b)
}

run_var_analysis(model_data)
# run_var_analysis(model_data2)

# run the chow test



# plot_irf_with_ci(irf_result2_data)

# Apply processing and plotting functions
```