**National Higher School of Nanoscience and Nanotechnologies**

**First Year Preparatory** **2023-2024**

**Algebra 2** **Section A and B**

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# $\mathscr{W}$orksheet #1

## $\mathscr{S}$ystems of Linear Equations

**1.** Use Gaussian Elimination to solve each of the following systems of linear equations. In each case, indicate whether the system is consistent or inconsistent. Give the complete solution set, and if the solution set is infinite, specify three particular solutions.

$$(1)\ \begin{cases} -x_1-2x_2+2x_3=2 \\ 3x_1+x_2-x_3=-1 \\ 2x_1+2x_2-x_3=-4 \end{cases}$$

$$(2)\ \begin{cases} 2x_1-x_2-x_3+2x_4-2x_5=-5 \\ 4x_1+3x_3-x_4+x_5=2 \\ -2x_1+x_2+3x_3-3x_4+x_5=1 \end{cases}$$

$$(3)\ \begin{cases} 4x_1-2x_2-7x_3=5 \\ -6x_1+5x_2+10x_3=-11 \\ -2x_1+3x_2+4x_3=-3 \\ -3x_1+2x_2+5x_3=-5 \end{cases}$$

**2.** Find values of $a, b$, and $c$ such that the system of linear equations has $\begin{cases} x+5y+z=0 \\ x+3y-z=0 \\ 2x+\mathrm{a}y+\mathrm{b}z=\mathrm{c} \end{cases}$

(1) exactly one solution,

(2) infinitely many solutions,

(3) no solution. Explain.

Same questions for $\begin{cases} x-y+2z=\mathrm{a} \\ \mathrm{m}x+\mathrm{m}y-2z=\mathrm{b} \\ x-\mathrm{m}y+2z=\mathrm{c} \end{cases}$

**3.** Suppose that each of the following is the final augmented matrix obtained after Gaussian Elimination. In each case, give the complete solution set for the corresponding system of linear equations.

$$(1)\ \begin{bmatrix} 1 & -5 & 2 & 3 & -2 & -4 \\ 0 & 1 & -1 & -3 & -7 & -2 \\ 0 & 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2)\ \begin{bmatrix} 1 & 4 & -8 & -1 & 2 & -3 & -4 \\ 0 & 1 & -7 & 2 & -9 & -1 & -3 \\ 0 & 0 & 0 & 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**4.** Find the quadratic equation that goes through the points $(3,20), (2,11)$, and $(-2,15)$.

**5.** Find the equation of the circle that goes through the points $(6,8)$, $(8,4)$, and $(3,9)$.

**6.** Use the Gauss-Jordan Method to convert each matrix to reduced row echelon form

$$(1)\ \begin{bmatrix} 5 & 20 & -18 & -11 \\ 3 & 12 & -14 & 3 \\ -4 & -16 & 13 & 13 \end{bmatrix} \qquad (2)\ \begin{bmatrix} -3 & 6 & -1 & -5 & 0 & -5 \\ -1 & 2 & 3 & -5 & 10 & 5 \end{bmatrix}$$

**7.** Use the Gauss-Jordan Method to determine the complete solution set for each system, and give one particular nontrivial solution.

(1) $\begin{cases} -2x_1 - 3x_2 + 2x_3 - 13x_4 = 0 \\ -4x_1 - 7x_2 + 4x_3 - 29x_4 = 0 \\ x_1 + 2x_2 - x_3 + 8x_4 = 0 \end{cases}$ (2) $\begin{cases} 2x_1 + x_2 - 8x_3 - 23x_4 - 43x_5 = 0 \\ -4x_1 + x_2 + 25x_3 + 67x_4 + 155x_5 = 0 \\ 2x_1 + 3x_2 - 2x_3 + 39x_5 = 0 \end{cases}$

**8.** Use the Gauss-Jordan Method to find the minimal integer values for the variables that will balance each of the following chemical equations :

(1) $aC_6H_6 + bO_2 \to cCO_2 + dH_2O$.

(2) $aC_8H_{18} + bO_2 \to cCO_2 + dH_2O$.

(3) $aAgNO_3 + bH_{20} \to cAg + dO_2 + eHNO_3$.

**9.** Use the Gauss-Jordan Method to find the values of $A, B, C$ (and $D$ in part (2)) in the following partial fractions problems :

(1) $\dfrac{5X^2 + 23X - 58}{(X-1)(X-3)(X+4)} = \dfrac{A}{X-1} + \dfrac{B}{X-3} + \dfrac{C}{X+4}$.

(2) $\dfrac{-3X^3 + 30X^2 - 97X + 103}{(X-2)^2(X-3)^2} = \dfrac{A}{(X-2)^2} + \dfrac{B}{X-2} + \dfrac{C}{(X-3)^2} + \dfrac{D}{X-3}$.

**10.** The figure 1, shows the flow of traffic (in vehicles per hour) through a network of streets.

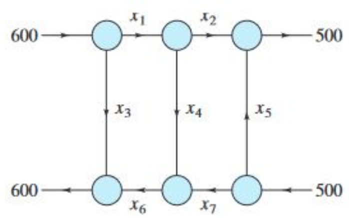


FIGURE 1

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(1) Solve this system for $x_i$, $i = 1, 2, ..., 5$.

(2) Find the traffic flow when $x_2 = 200$ and $x_3 = 50$.

(3) Find the traffic flow when $x_2 = 150$ and $x_3 = 0$.

**11.** Determine the currents $I_1, I_2, I_3, I_4, I_5$, and $I_6$ for the electrical network shown in the figure 2.

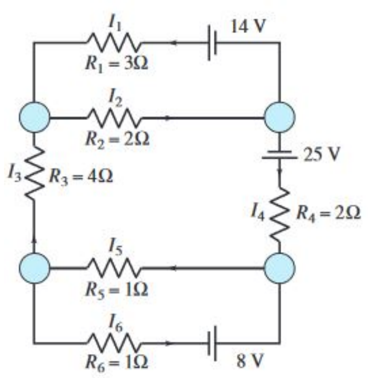


FIGURE 2