

Homework 1 MATH 250

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Question 48:

If A, B are square matrices, which of the following expressions are guaranteed to equal $(A - B)^2$:

- i) $A^2 - B^2$,
- ii) $A^2 - 2AB + B^2$,
- iii) $A(A - B) - B(A - B)$,
- iv) $A^2 - AB - BA + B^2$

Solution:

Options i and iv are equal to $(A - B)^2$.

Question 49:

True / False

- i) If A^2 is defined, then A is a square matrix.
- ii) If AB and BA are defined, then A, B are square matrices.
- iii) If AB and BA are defined, then AB and BA are square matrices.
- iv) If $AB = B$, then $A = I$

Solution:

Statement i is true.

Statement ii is false.

Statement iii is true.

Statement iv is false.

Question 54:

Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Find an inverse A^{-1} for A .

Solution:

To find the inverse of matrix A , we can use the determinant which is the following formula:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

In our case, $a = 1, b = 2, c = 0, d = 3$. So we'll plug in the following numbers in the formula:

$$\begin{aligned} A^{-1} &= \frac{1}{3 - 0} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} && \text{(simplify)} \\ A^{-1} &= \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix} && \text{(divide all numbers in the matrix by } \frac{1}{3}) \end{aligned}$$

So the final answer is $A^{-1} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}$.