Homework 0 MATH 250

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Problem 1.

Question:

This example will be useful next time. Recall that the mirror matrix was $M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and that shift matrix was $S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Suppose you start with the vector $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Part (a):

What vector do you get if you apply the shift matrix S to v first, and then apply the mirror M matrix to what you got? In terms of diagrams, I am asking for

$$v \to [^S] \to [^M] \to ? \tag{1}$$

or in matrix-vector notation, I am asking for the vector $M(Sv)$.

Solution:

We'll first start by applying the shift matrix:

$$v \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = v \begin{pmatrix} 0+0 \\ 1+0 \end{pmatrix} = v \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Now we'll apply the mirror matrix:

$$v \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = v \begin{pmatrix} 0+0 \\ 0-1 \end{pmatrix} = v \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

So our final answer is $v\begin{pmatrix}0\\-1\end{pmatrix}$.

Part (b):

What vector do you get if you apply the mirror matrix M to v first, and then apply the shift matrix S to what you got? In terms of diagrams, I am asking for

$$v \to [^M] \to [^S]? \tag{2}$$

or in matrix-vector notation, I am asking for the vector $S(Mv)$.

Remark: you should find different answers! In fact, we will learn to define the product of two matrices, and this shows that $MS \neq SM$, in other words, the order of the factors for matrix multiplication matters!

Solution:

We'll start by applying the mirror matrix:

$$v = \begin{pmatrix}1\\0\end{pmatrix} \to \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} = v\begin{pmatrix}1+0\\0+0\end{pmatrix} = v\begin{pmatrix}1\\0\end{pmatrix}$$

Now we'll apply the shift matrix:

$$v\begin{pmatrix}1\\0\end{pmatrix} \to \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = v\begin{pmatrix}0+0\\1+0\end{pmatrix} = v\begin{pmatrix}0\\1\end{pmatrix}$$

So our final answer is $v = \begin{pmatrix}0\\1\end{pmatrix}$

Problem 2.

Question:

Consider the matrix $A = \begin{bmatrix}1 & -1\\2 & 3\\4 & 5\end{bmatrix}$ and the vectors $u = \begin{pmatrix}1\\2\\3\end{pmatrix}, v = \begin{pmatrix}-1\\1\end{pmatrix}$.

One of the operations Au or Av is well defined, while the other can't be done. Enter the entries of the vector that you get for the operation that can be made.

Solution:

We can only perform the dot product between Av because the dimensions are compatible. The dimensions of A are $3x2$ and the dimensions of v are $2x1$ The inner dimensions for both A and v match, which tells us that we can perform the dot product. The inner dimension for u are $3x1$ which does not match the inner dimension of 2.

Now we'll calculate Av:

$$Av = v\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} \qquad \text{(multiply } A \text{ and } v\text{)}$$

$$Av = \begin{bmatrix} (-1 \cdot 1) + (1 \cdot -1) \\ (-1 \cdot 2) + (1 \cdot 3) \\ (-1 \cdot 4) + (1 \cdot 5) \end{bmatrix} \qquad \text{(multiply first column by } -1 \text{ and second column by 1)}$$

$$Av = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \qquad \text{(simplify)}$$

Our final answer is $Av = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.