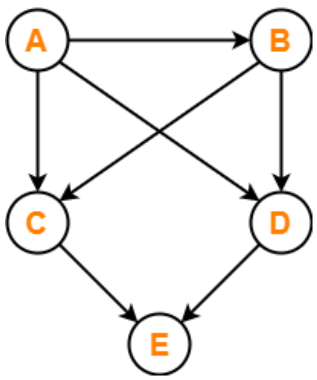
Topological Sorting Examples

Problem-01:

Find the number of different topological orderings possible for the given graph-

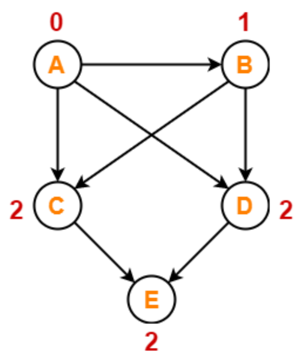


Solution-

The topological orderings of the above graph are found in the following steps-

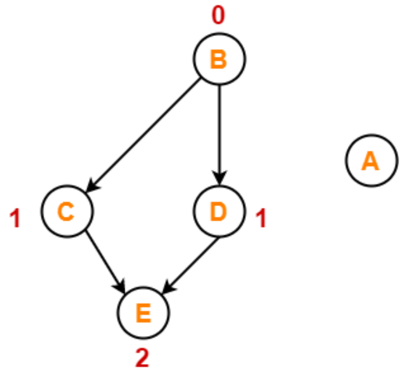
Step-01:

Write in-degree of each vertex-



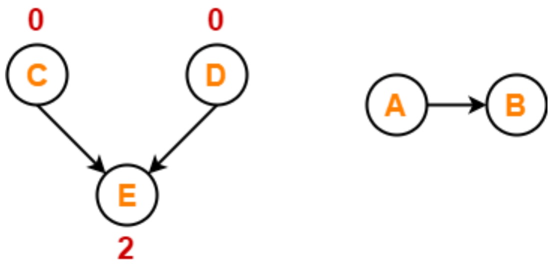
Step-02:

- Vertex-A has the least in-degree.
- So, remove vertex-A and its associated edges.
- Now, update the in-degree of other vertices.



Step-03:

- Vertex-B has the least in-degree.
- So, remove vertex-B and its associated edges.
- Now, update the in-degree of other vertices.



Step-04:

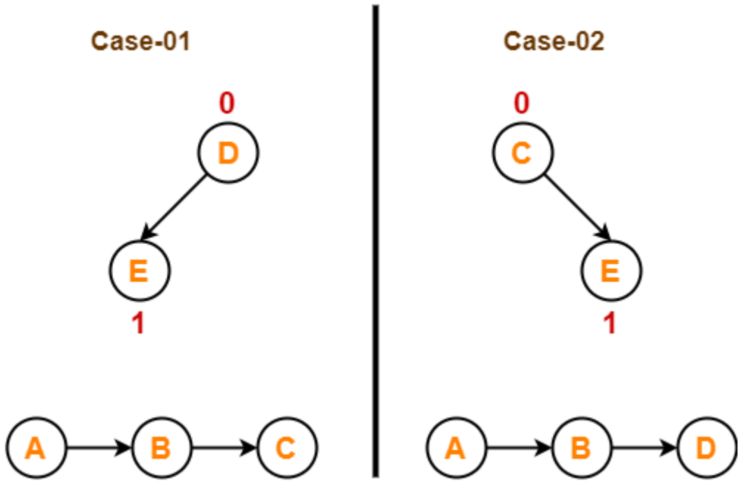
There are two vertices with the least in-degree. So, following 2 cases are possible-

In case-01,

- Remove vertex-C and its associated edges.
- Then, update the in-degree of other vertices.

In case-02,

- Remove vertex-D and its associated edges.
- Then, update the in-degree of other vertices.



Step-05:

Now, the above two cases are continued separately in the similar manner.

In case-01,
- Remove vertex-D since it has the least in-degree.
- Then, remove the remaining vertex-E.

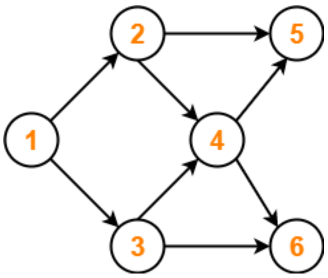
In case-02,
- Remove vertex-C since it has the least in-degree.
- Then, remove the remaining vertex-E.

Conclusion-

For the given graph, following **2** different topological orderings are possible-
- **A B C D E**
- **A B D C E**

Problem-02:

Find the number of different topological orderings possible for the given graph-

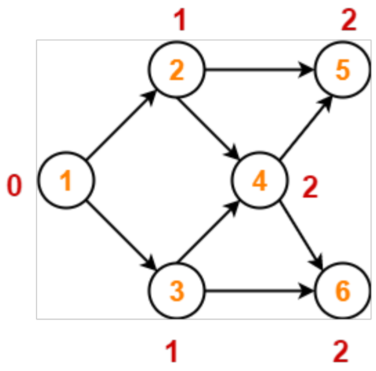


Solution-

The topological orderings of the above graph are found in the following steps-

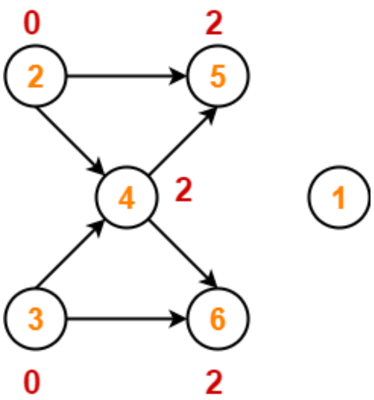
Step-01:

Write in-degree of each vertex-



Step-02:

- Vertex-1 has the least in-degree.
- So, remove vertex-1 and its associated edges.
- Now, update the in-degree of other vertices.

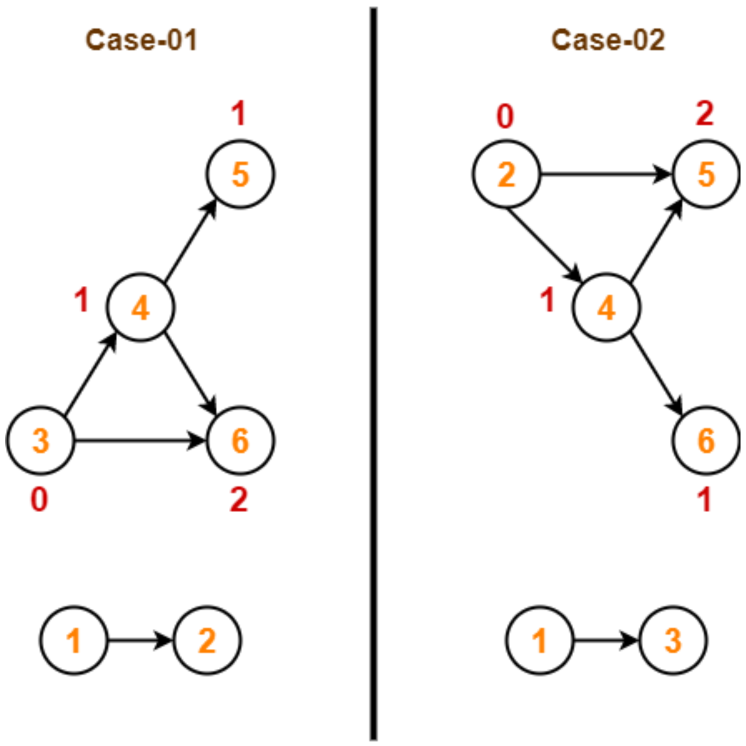


Step-03:

There are two vertices with the least in-degree. So, following 2 cases are possible-

In case-01,
- Remove vertex-2 and its associated edges.
- Then, update the in-degree of other vertices.

In case-02,
- Remove vertex-3 and its associated edges.
- Then, update the in-degree of other vertices.

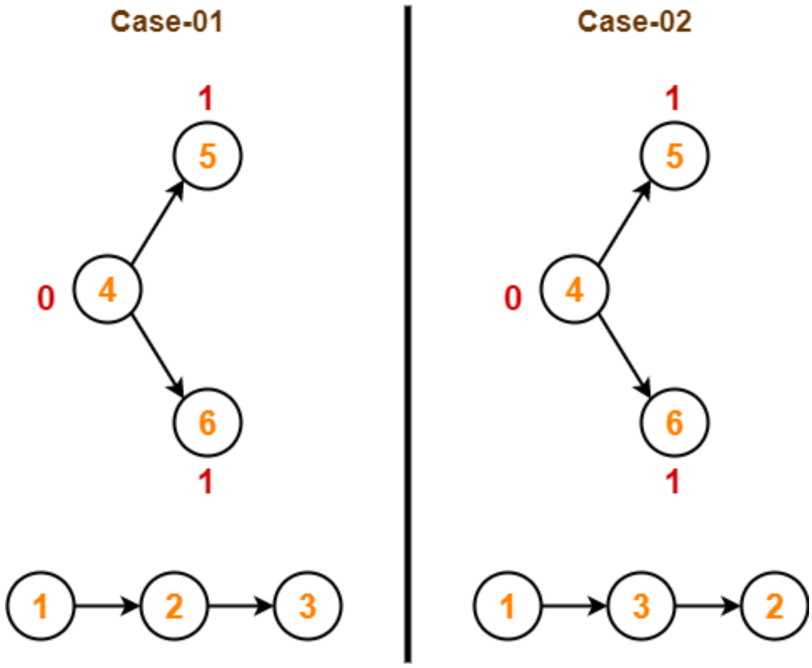


Step-04:

Now, the above two cases are continued separately in the similar manner.

In case-01,
- Remove vertex-3 since it has the least in-degree.
- Then, update the in-degree of other vertices.

In case-02,
- Remove vertex-2 since it has the least in-degree.
- Then, update the in-degree of other vertices.



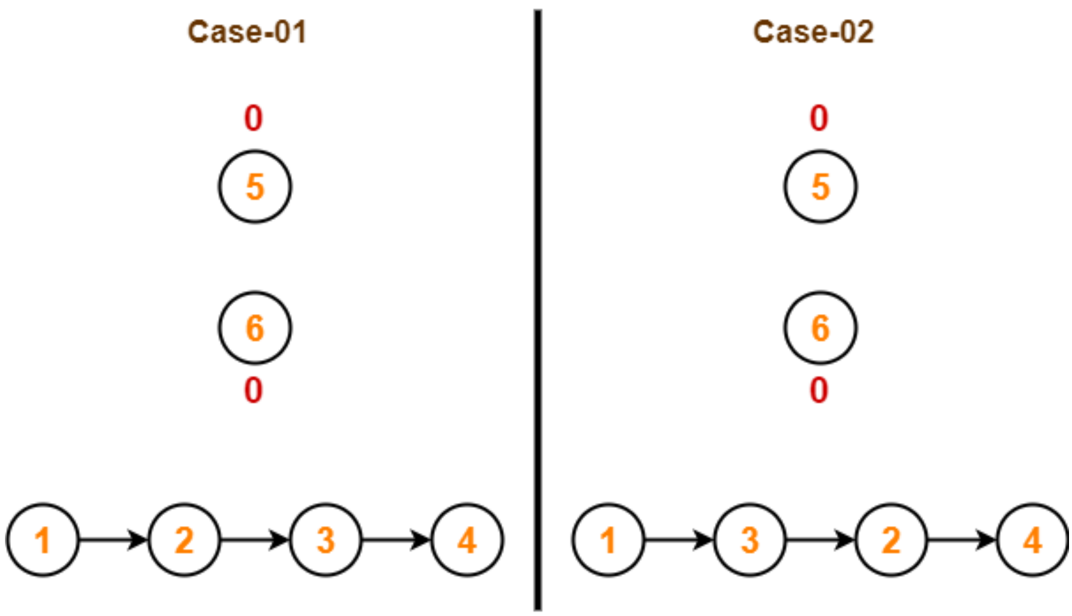
Step-05:

In case-01,

- Remove vertex-4 since it has the least in-degree.
- Then, update the in-degree of other vertices.

In case-02,

- Remove vertex-4 since it has the least in-degree.
- Then, update the in-degree of other vertices.

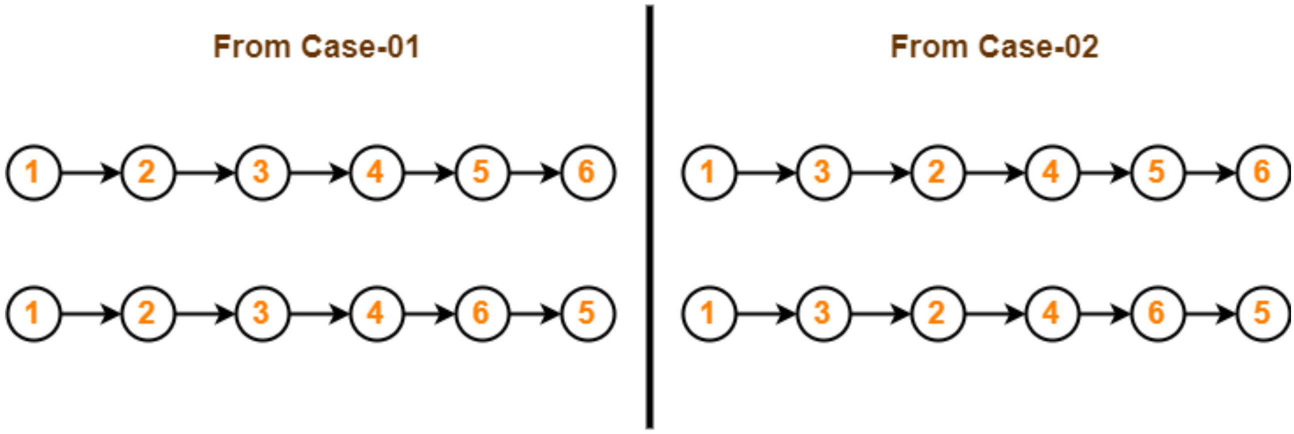


Step-06:

In case-01,

- There are 2 vertices with the least in-degree.
- So, 2 cases are possible.
- Any of the two vertices may be taken first.

Same is with case-02.



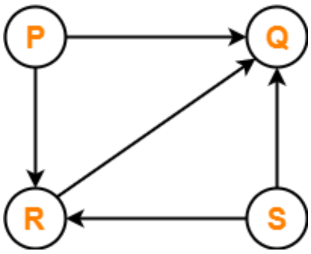
Conclusion-

For the given graph, following **4** different topological orderings are possible-

- **1 2 3 4 5 6**
- **1 2 3 4 6 5**
- **1 3 2 4 5 6**
- **1 3 2 4 6 5**

Problem-03:

Consider the directed graph given below. Which of the following statements is true?



- The graph does not have any topological ordering.
- Both PQRS and SRPQ are topological orderings.
- Both PSRQ and SPRQ are topological orderings.
- PSRQ is the only topological ordering.

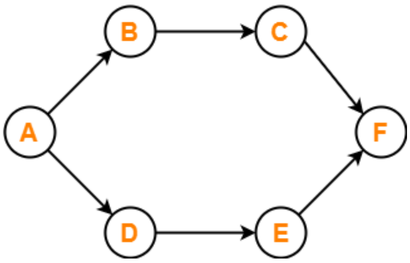
Solution-

- The given graph is a directed acyclic graph.
- So, topological orderings exist.
- P and S must appear before R and Q in topological orderings as per the definition of topological sort.

Thus, Correct option is (C).

Problem-04:

Consider the following directed graph-



The number of different topological orderings of the vertices of the graph is ________ ?

Solution-

Number of different topological orderings possible = 6.
Thus, Correct answer is **6**.