Market Power, Banking and Capital: the Deposits Channel*

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Abstract

We develop a monetary model of imperfect competition for bank deposits based on search frictions. The extent of market power is endogenous and a distribution of deposit rates and spreads arises in equilibrium. Banks' market power and the resulting *ex post* heterogeneity of depositors affect the transmission of monetary policy to the real economy. Monetary policy actions affect both the average level and dispersion of the deposit spread. This, in turn, affects production, capital accumulation and ultimately the economy's long-run growth path. A central bank digital currency (CBDC) can affect both banks' market power and the level of deposits, even if the CBDC pays no interest in equilibrium.

JEL codes: E41; E44; E51; E63; G21
Keywords: Banking; Deposit Spreads Dispersion; Market Power; Capital Formation; Liquidity.

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1 Introduction

In this paper, we show how imperfect competition for bank deposits and endogenous *ex-post* heterogeneity in deposit rates affect the transmission of monetary policy to the real economy and its long-run growth prospects. We study a novel model of the *deposits channel* of monetary policy, the basic proposition of which (see Drechsler, Savov and Schnabl, 2017) is: All else equal, if banks do not completely pass through an increase in the monetary policy rate to deposit rates, then this causes an outflow of deposits from the banking system, leading to a contraction in lending and thus to the real economy.

In our theory, households with currently unneeded money search for deposit opportunities offered by banks. These deposit-taking institutions operate under imperfect information and identical deposit products may exhibit dispersion in both posted and transacted rates. In this framework, we highlight the interaction between capital accumulation and money when the distribution of deposit rates is determined in equilibrium. We focus on monetary policy's effects on banks' market power in deposits and their implications for both welfare the path of capital formation and long-run economic growth.

In the model, the Central Bank controls rate on bank loans using the *monetary-policy rate*. Given banks' market power, deposits rates lie below this rate, and with the difference between the lending rate and an individual bank's offered deposit rate being its *deposit spread*, representing an extraction of rent from depositors. An increase in the monetary-policy rate leads to a increase of both the average and dispersion of deposit spreads. The former is a form of imperfect pass-through to the real economy: Higher borrowing costs coupled with relatively lower compensation for unnecessary liquidity. The latter represents and increase in the risk associated with carrying money which may turn out to be unneeded. To the extent that this lowers households real balances, the returns to capital investment are further reduced. Overall, capital accumulation falls by more and the long-run growth path may be lowered, relative relative to an economy with perfect competition for deposits.

The transmission mechanism we study has the novel aspect that extent of pass-through depends on the equilibrium distribution of deposit spreads. Rather than being determined parametrically by the elasticity of deposit demand (see, *e.g.*, Drechsler et al., 2017), it depends on both policy and exogenous shocks.

We focus here on the *liquidity transformation* role of banks as in Berentsen, Camera and Waller (2007) intermediate between households with *ex-post* excessive and insufficient liquidity as a result of household-specific shocks. Effectively, deposits provide insurance against having costly excessive money holdings. Banking of this type is welfare-improving regardless of whether banks are perfectly competitive or not. In the latter case, equilibrium dispersion in deposit-rate spreads and market power erodes, but not completely, the welfare gains from banking.

We provide new empirical evidence supporting our two aspects of our model associated with the deposits channel. Using bank-branch-level data, we consider identical deposit products and control for other factors (*e.g.*, time and geographical fixed effects). We find that the average deposit spread is positive, but not perfectly, affected the monetary-policy rate. This speaks to incomplete pass-through. Second, we find that residual dispersion in deposit-rate spreads is positively correlated with average spread, and thus with the monetary-policy rate.

Our approach is motivated in part by observed relationships between the Federal Funds Rate (FFR) and: (1) the deposit spread as a measure of banking competitiveness; (2) money demand; and (3) capital formation in the United States.[1] Figure 1 depicts these relationships in three panels. In Panel (a), the aggregate deposit spread is positively associated with the FFR. Panel (b), depicts the aggregate money demand relationship, showing that the inverse velocity of money (M/PY) is negatively correlated with the FFR. In Panel (c), it can be seen that the FFR is negatively associated with the investment-to-GDP (I/Y) ratio. Here we associate this ratio with capital formation, from a long-run or steady-state perspective of the neoclassical growth model.

We study the relationships among aggregates summarized in Figure 1 using a New Monetarist model building on Aruoba, Waller and Wright (2011) (AWW). We then show that this model is consistent also with the micro-level evidence on the distribution of deposit rates. Specifically, in Section 2, we show that there is a positive relationship between the dispersion (measured as standard deviation) of the deposit spread and its average level.

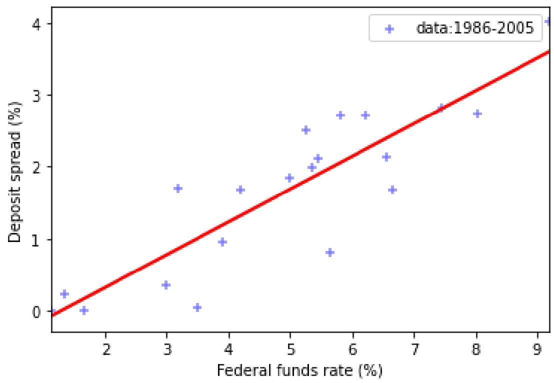
AWW study a tractable framework in which money is essential and nominal variables affect capital accumulation in the real economy. Our contribution is to combine banking with market power on the deposit side into this framework, nesting the original AWW model as a special case. In equilibrium, there is a spread between the average deposit rate and banks' cost of funds. The theory can also account for dispersion in such spreads for identical deposit products. Banking has value in equilibrium here because these institutions can intermediate between (ex-post) heterogeneous liquidity risks, as in Berentsen et al. (2007) (BCW). We will focus on this particular *liquidity transformation* function of banking. In the special case of perfect competition for deposits, banking in our model functions exactly as in BCW.

AWW consider a two-sector economy in the Lagos and Wright (2005) framework, where some economic activities occur in markets with frictions and others in markets without. The authors describe a novel link between the nominal and real sides of the economy regarding monetary policy transmission. Specifically, a higher nominal policy interest rate induces a

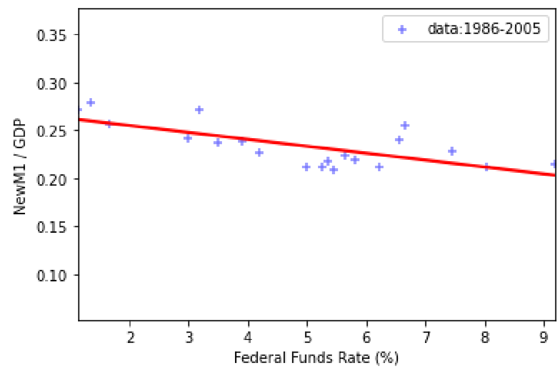
[1] Similar to Drechsler et al. (2017), we use quarterly data from Call Reports to calculate the aggregate deposit spread. The deposit spread is defined as the difference between the FFR and the value-weighted average deposit rate paid by banks. We use the (real) investment-to-GDP ratio as our proxy for capital formation. We obtain the quarterly national accounting data from FRED. The (aggregate) money demand relation uses data from Lucas and Nicolini (2015). The data are from 1986 to 2007. We have also considered a longer time period from 1986 to 2016. The corresponding relationships remain identical.

Figure 1: Monetary policy, bank market power, money and capital

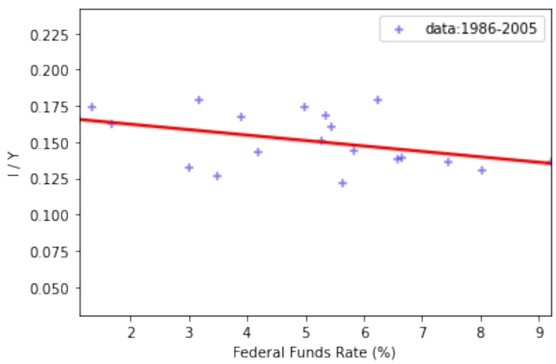
(a) Federal Funds Rate and deposit spread



(b) Federal Funds Rate and M/PY



(c) Federal Funds Rate and capital formation I/Y



lower return on money, reducing agents' incentive to accumulate money balances. This in turn reduces the returns to supplying goods to the frictional goods market, where money must be used. Since capital is an input for producing goods in this market, the return to capital is linked positively to demand in the frictional market. Thus, in equilibrium a higher policy rate reduces real investment and ultimately the capital stock.

Here we study the effects of market power and heterogeneity in deposit pricing in the presence of a similar monetary policy transmission mechanism. In our monetary economy, information frictions render private insurance contracts or promises incentive-infeasible. Fiat money thus has value for supporting goods exchange. Inefficiency arises *ex post*, however, as agents are subject to trading shocks. This creates a role for banks that accept nominal deposits and make nominal loans, reallocating liquidity across *ex post* heterogeneous households as in BCW.[2] Inflation raises the cost of holding idle nominal balances and so exacerbates

[2]We abstract from banks' creation of inside money. (See, for examples *e.g.*, Cavalcanti and Wallace, 1999a,b; Williamson, 1999; He, Huang and Wright, 2005, 2008; Gu, Mattesini, Monnet and Wright, 2013; Chang and Li, 2018). Deposits are held only by agents who do not want to consume and so we focus solely on the implications market power in deposits for capital accumulation and monetary policy.

this inefficiency. Bank deposits provide insurance against these costs. Whereas BCW study perfect competition among banks for deposits, we consider market power of a particular type in this market.

In our environment, agents with idle money balances search among banks for deposit opportunities via an adaptation of the noisy consumer search process of Burdett and Judd (1983). Banks post deposit rates to attract funds to make loans. Potential depositors observe a random selection of the posted deposit rates and may choose that which offers the highest return. Banks post deposit rates taking into account their potential customers' incomplete access to deposit opportunities. They thus face a trade-off: On the one hand a lower deposit rate results in a higher spread between the bank's cost of funds and its loan income. That is, higher profit per unit of deposits, an *intensive margin* effect. On the other hand, a higher deposit rate attracts more depositors, allowing the bank to serve more loan customers profitably (an *extensive margin* effect). This mechanism generates dispersion in deposit rates and spreads as an equilibrium outcome. The dispersion of these spreads depends on the policy interest rate which affects both the opportunity cost of lending for banks and households' incentive to accumulate money balances.

The presence of market power reduces the average return on deposits, diminishes the insurance provided by the banking system and ultimately lowers the return on money affecting goods trades and capital formation in equilibrium. Moreover, dispersion of deposit rates introduces uncertainty and *ex post* heterogeneity across households with regard to the extent to which liquidity risk is insured. Banking, however, improves welfare to an increasing degress as inflation (and the policy interest rate) rises. Banks' market power, however, erodes some of these gains and the extent of this power depends on both the state of the economy and monetary policy.

To understand the role of bank market power in the transmission of monetary policy to capital formation and economic activity, it is useful to start by comparing an economy with perfect competition for deposits (BCW) to another with no banks at all (AWW). Consider the monetary policy instrument in both cases to be a long-run inflation target (or equivalently, the policy nominal interest rate). In both economies, *i.e.* with or without banks, higher inflation lowers the rate of return on money. By paying interest on deposits, banks lower the cost holding idle balances, or the cost of inflation at any given rate. Moreover, such banks can extend credit in the form of nominal loans to buyers who are liquidity constrained. Consumers with higher real balances demand more in the frictional goods market where money is the means of payment and as capital is an input for producing goods in this sector, raise the return to capital. Banking thus increases capital investment relative to that which would occur without them, solely as a result of reallocating liquidity.[3]

[3]Of course, banking could facilitate investment in other ways which we abstract from here.

In our environment with imperfect competition for deposits, agents' decisions regarding both money demand and investment will be distorted due to the imperfect pass-through to deposit rates of changes in the inflation target or nominal policy rate. The extent of this distortion is endogenous: it depends on the level and distribution of the deposit rates.

A higher policy rate increases the value to households of the insurance provided by bank deposits that pay interest at any given rate. Banks thus have incentive to increase the spread between the rate they pay on deposits and the (now higher) nominal loan rate. We think of this as an intensive margin effect as banks now earn a greater return on the funds they take in as deposits due to the incomplete pass-through of changes in the policy rate to nominal deposit rates. In equilibrium, both the average spread between the deposit and loan rates and its dispersion increase with the policy rate. This incomplete pass-through of changes in the policy rate to deposit rates is consistent with empirical evidence presented by Drechsler et al. (2017). It reduces the insurance value of deposits and thus reduces household demand for money balances relative to the case in which there is perfect competition for deposits. Both the real supply of deposits and real demand for loans fall and thus imperfect competition for deposits lowers both output in the frictional goods market and capital formation.

As noted above, in our model the extent of incomplete pass-through reflected in the spread between the deposit and loan rates is endogenous. Monetary policy affects banks' cost of funds, directly. It affects if further through the equilibrium response of the distribution of deposit rates and spreads. Moreover, the model admits three special cases. In one, Bertrand pricing results as a limit resembling perfect competition among banks. This case arises when all depositors have at least two deposit opportunities. In a second case, we have the opposite extreme. If all depositors have only one deposit opportunity, banks pay the monopoly rate on deposits. A third case nests AWW. In this case there are no deposit opportunities and we have essentially a no-bank economy.

The effects of inflation and monetary policy on capital formation is one of the classic questions in macroeconomics (see, *e.g.*, Tobin, 1965; Sidrauski, 1967; Stockman, 1981; Cooley and Hansen, 1989; Gomme, 1993). In the New Monetarist tradition, Waller (2011) investigated the long-run growth effects of this link. Here we revist this question in light of the deposits channel of monetary policy and provide an additional insight. Specifically, we illustrate the effects of market power on the return to deposits and how they distort the long-run growth path of the economy working through this channel.

Others have also studied the connections among money, capital and financial intermediation, see, *e.g.*, Bencivenga and Camera (2011) and Rocheteau, Wright and Zhang (2018). As in Bencivenga and Camera (2011), banks in our model serve as insurers of liquidity risk. We depart, however, from their model of banking in three ways. First, here banks also extend credit to consumers. Second, banks' market power in depostis arises from search frictions.

And third, banks' degree of market power is endogenous and responds to monetary policy in equilibrium. Rocheteau et al. (2018) consider related issues from a corporate finance perspective. Firms choose whether to finance their investment project via internal finance (*i.e.*, their accumulated money balance), trade credit and/or bank funding. They show that the effects of monetary policy depend on the market microstructure and firms' characteristics. The role of financial intermediation in Rocheteau et al. (2018) is to finance investment. In contrast, we focus on the role of banking in intermediating between agents with different liquidity needs. Also, capital here is accumulated through investment as in standard neoclassical models, whereas Rocheteau et al. (2018) consider working capital modelled as a within-period flow.

Several recent papers relate monetary policy to competition in the banking system, including Choi and Rocheteau (2021), Dong, Huangfu, Sun and Zhou (2021), Chiu, Davoodalhosseini, Jiang and Zhu (2019) and Wang (2022). Both Choi and Rocheteau (2021), and we focus on the deposits channel. Banks, however, serve a different purpose in Choi and Rocheteau (2021) as they create inside money that serves as a means of payment in frictional goods trades. Moreover, banks (but not consumers) have access to an investment technology that yields a higher return than money holdings. As such, the gains from banking in Choi and Rocheteau (2021) are due to consumers' access to a cheaper payment instrument. In the baseline model of Choi and Rocheteau (2021) with complete information the transmission of monetary policy through deposits works exclusively through an extensive margin, the total measure of deposit contracts offered by banks. In this environment market power from bargaining is insufficient to generate a transmission channel through deposits. They therefore introduce a second friction, private information about consumers' liquidity needs, to introduce an variable individual demand for bank deposits (an intensive margin). Their principal result is that the outflow of deposits in response to a higher policy interest rate is driven by those with low liquidity needs.

Here, in contrast, banks play a different role, generating a positive return on otherwise idle funds by reallocating them to households with liquidity needs. Banks' market power, driven by consumer search, has implications for capital accumulation and long-run growth. through its effects on the *ex ante* demand for money balances and thus the value of aggregate deposits. Specifically, by limiting the expected return on idle funds, market power lowers money demand, reduces economic activity in the frictional goods market and thereby lowers the return to capital.

Finally, we consider the implications of a Central Bank Digital Currency (CBDC) for the ability of banks to affect the return to deposits through market power. Our focus again is on the connection between role of banking in liquidity reallocation and its implications for capital formation. This analysis contributes to a growing literature on CBDC and is policy implications, mostly focused on the its effects on the aggregate level of bank deposits. See, for examples, the papers by (Andolfatto, 2021; Engert and Fung, 2017; Chapman and Wilkins, 2019; Chiu et al., 2019; Jiang and Zhu, 2021; Fernández-Villaverde, Sanches, Schilling and Uhlig, 2021; Wang and Rahman, 2022; Dong and Xiao, 2022; Keister and Sanches, 2022).

The remainder of the paper is organized as follows. In Section 2 we provide micro evidence on the relationship between the level and dispersion of deposit-rate spreads. Section 3 describes the economic environment in detail and Section 4 defines and characterizes a stationary monetary equilibrium. Section 5 presents a number of analytical results. In Section 6 we present quantitative results using a version of the model calibrated to U.S. macro data. Numerically, we demonstrate first the pass-through of monetary policy to the distribution of deposit rates and aggregate deposits in the presence of bank market power. We then also provide an empirically testable prediction, specific to our theory, for the relationship between the level and dispersion of deposit spreads. Lastly, we illustrate the long-run effects of inflation on money, capital and welfare in equilibrium. In Section 7, we study the implications of inflation, for long-run growth. Section 8 takes up the implications of interest-bearing CBDC for bank market power and capital accumulation. Section 9 concludes.

2 Empirical evidence on deposit rate spreads

In this section, we present new empirical evidence on the level and dispersion of the *deposit spread* between the U.S. federal funds rate and bank-level deposit rates, Using bank-branch level data from *RateWatch*, and controlling for other possible sources of variation in deposit rates, we find a positive relationship between the average level of the spread and its dispersion as measured by the standard deviation.

2.1 Data

Branch-level interest rate data. We obtain weekly interest-rate information on an identical deposit product at each branch from *RateWatch*. Specifically, we use rates for one of the most commonly used time deposit products in the United States, the twelve-month certificate of deposits (CD).[4] This strategy of focusing on posted rates for a class of identical deposit products allows us to rule out any observable (and unobservable) pricing heterogeneity across depositors and deposit products.

Our primary sample includes 1,428,900 branch-weekly observations from 12,381 branches, between January 2001 and December 2007.[5] Our sample covers 49 states and the District of

[4]We focus on fixed-term time deposits in order to be consistent with our theoretical model. In the model, households use time deposits to save idle money balances in contrast to demand deposits, which help to smooth out the consumption expenditures. While we do not report this in the paper, we have also conducted the empirical analysis using other deposit products and have obtained the same results.

[5]We choose not to include observations beyond 2008 to avoid the near-zero-lower-bound interest rate

Columbia. We drop Hawaii due to insufficient branch-level observations to calculate state-level dispersion. To calculate each branch's deposit spread against the federal funds rate, we collect daily effective federal funds data from the U.S. Federal Reserve H15 report.

The deposit spread. We follow Drechsler et al. (2017) and define the deposit spread as the difference between federal funds rate (FF_t) and branch-level deposit rate ($Rate_{b,s,t}$).[6] Specifically, we calculate each bank branch's deposit spread as

$$Spread_{b,s,t} = FF_t - Rate_{b,s,t}, \tag{2.1}$$

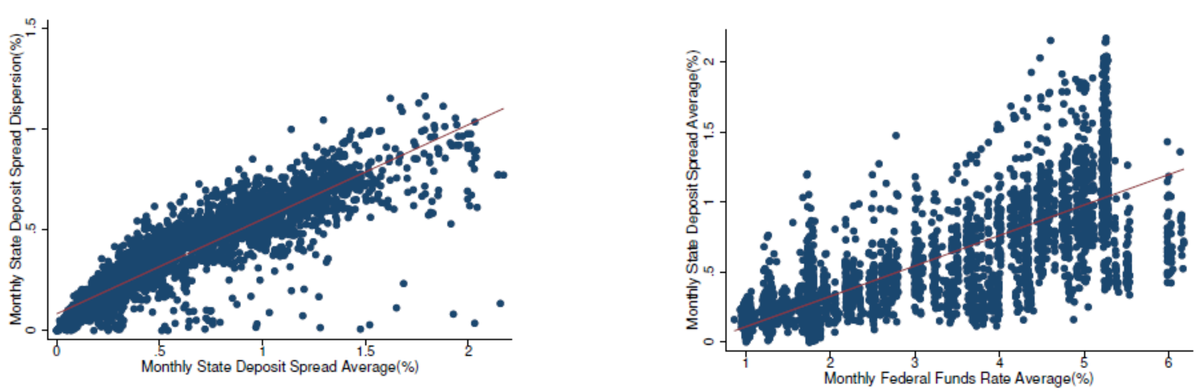
where b denotes the bank branch, s the state, and, t the date for which *RateWatch* reports. We then calculate the mean ($\overline{Spread}_{s,t}$) and the standard deviation ($Dispersion_{s,t}$) of branch-level deposit spreads within a particular state s and a time period t.

Figure 2 depicts the data visually and summarizes our results. Specifically, Panel 2a shows a positive relationship between the monthly standard deviation and the average of deposit spreads at the state level. Panel 2b shows a positive relationship between the average deposit spreads and the federal funds rate. This latter finding is consistent with the findings of both Drechsler et al. (2017) and Choi and Rocheteau (2021).

Figure 2: Dispersion (standard deviation) and average of deposit spreads

(a) Dispersion vs. mean

(b) Mean vs. Federal Funds Rate



environment. As Wang (2022) argues, monetary-policy effectiveness could change even before the zero lower bound binds.

[6] We also use an alternative specification of the deposit spread: $Spread_{b,s,t} = \frac{FF_t - Rate_{b,s,t}}{FF_t}$ following Wang (2022) and find consistent results.

2.2 Regression evidence

To test formally the significance of the relationship observed in Figure 2a, we estimate the following regression equation by OLS:

$$Dispersion_{s,t} = b_0 + b_1 \overline{Spread}_{s,t} + b_2 Z_s + b_3 Z_t + \epsilon_{s,t}, \tag{2.2}$$

where Z_s and Z_t are state and time fixed effects, and standard errors are clustered by state.

Table 1 summarizes the regression results for Equation 2.2. All columns show a positive and statistically significant relationship (b_1) between our measure of the dispersion of the deposit spread and its mean. Column (4) suggests that an increase of 10 basis points in the average of deposit spread is associated with an increase of 3.4 basis points in the standard deviation of the spread after controlling for state fixed effects and time fixed effects.

Table 1: State-Month Regression Results: Dependent Variable: $Dispersion_{s,t}$

	(1)	(2)	(3)	(4)
$\overline{Spread}_{s,t}$	0.482***	0.360***	0.492***	0.335***
	(0.017)	(0.040)	(0.015)	(0.038)
Constant	0.080***	0.148***	0.075***	0.162***
	(0.006)	(0.021)	(0.008)	(0.021)
Month FEs	No	Yes	No	Yes
State FEs	No	No	Yes	Yes
Observations	4155	4155	4155	4155
Adjusted R^2	0.856	0.894	0.885	0.922

These findings complement those of Drechsler et al. (2017) documenting a positive relationship between monetary policy and the average deposit spread. We find a similar relationship and provide additional evidence on it, demonstrating a similar positive relationship between the average spread and its dispersion. These findings, summarized in Figure 2 Table 1 are consistent with our theoretical model, to which we turn now.

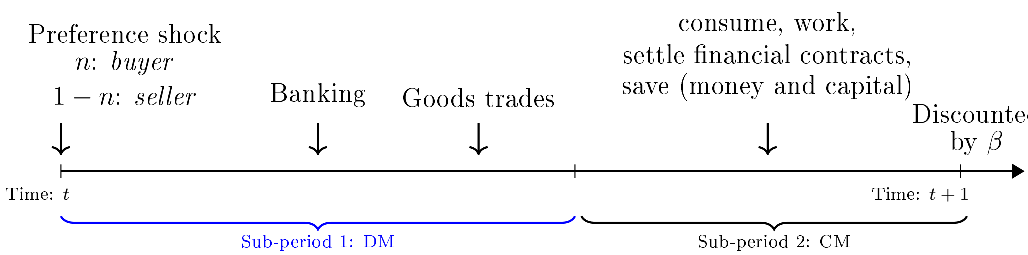
3 The Economy

The economy has four types of agents: a government, and large numbers (*e.g.* unit measures) of households, firms and banks. Time is discrete and infinite, with each period divided into two sub-periods as in Lagos and Wright (2005). A non-storable consumption good is associated with each sub-period of each period. Figure 3 displays the model timeline.

In the first sub-period (DM) households and sellers are anonymous to each other and trade is decentralized. Sellers of goods in the DM cannot observe individual buyers' histories so that exchange cannot be sustained using private credit. Anonymity in the DM thus motivates monetary exchange, which in turn generates a need for banks to reallocated liquidity among *ex post* heterogeneous households. At the beginning of the DM, houses holding excess money balances search for deposit opportunities. Other households may choose to borrow from banks in order to finance goods purchases. In the second sub-period (CM), a centralized market operates frictionlessly. The CM is a metaphor for institutions or markets that allow for agents to trade and rebalance their asset positions without any hindrance. Households consume, repay loans and make investment decisions. Firms produce and sell in the competitive output market. Banks pay interest on deposits and the government engages in various policy actions.

Agents discount payoff flows between periods but not within a period. We use the following notation to denote time-dependent variable outcomes: $X \equiv X_t$ and $X_{+1} \equiv X_{t+1}$, and now describe the sequence of events and actions in the first and second sub-periods.

Figure 3: Timing



3.1 The sequence of events

At the beginning of period t, the government augments the supply of divisible fiat currency, M, by making a uniform lump-sum transfer to all households. The aggregate money stock evolves deterministically according to:$M_{+1} = \gamma M \equiv (1+\tau)M$, where τM is the per household transfer. Immediately the transfer, each household receives an idiosyncratic shock. With probability n the household values consumption in the DM *this period* but cannot produce. Such households are referred to as *buyers* and receive utility flow $u(q)$ from consuming q units of the current DM good. With complementary probability, the household is a *seller*, having no desire for DM consumption this period, but being able to produce it using effort, n, and previously accumulated capital, k, using a CRS technology.

Buyers and sellers are anonymous to each other in the DM and thus the former lack the ability to commitment to private credit. We thus focus on equilibria in which fiat money has value and is used for exchange in the DM.[7] Unlike households, *banks* have access to

[7]The focus of this paper is not on endogeneity of a specific medium of exchange. For simplicity we assume it is costless for to falsify claims on assets other than fiat money, specifically capital. As such, only fiat money is the only possible medium of exchange. For more on this issue see, for example, Lagos and Rocheteau (2008).

a record-keeping technology that enables them both to commit to repay depositors and to enforce loan contracts in the upcoming CM, exactly as in Berentsen et al. (2007).[8]

Banking occurs immediately following the realization of the shock distinguishing buyers and sellers. Banks accept nominal deposits and make nominal loans. Previewing the equilibria we will consider, sellers will deposit their entire money balance for any deposit interest rate $i \geq 0$. Buyers will have no incentive to deposit, but may or may not be interested in augmenting their money balance with a loan.[9] Throughout the paper, we use l and d to denote per household loans and deposits, respectively. The interactions among buyers (potential borrowers), sellers (depositors) and banks are described in detail in Section 3.6.

Following banking, the DM goods market open and buyers and sellers exchange goods in a competitive market. Buyers face a liquidity constraint; they may only spend the money they have, which consists of that which they carried into the period and that which they may have borrowed from a bank. Sellers produce and supply goods to the DM market using labour effort and capital. Let p denote the nominal price of the goods in the DM. Buyers' expenditure then equals pq_b and sellers' revenue pq_s. After goods trade, the DM ends.

Households enter the CM with individual state (m, k, l, d). That is, with money holdings, m, accumulated capital, k, outstanding loan, l, and deposits, d. Recall that households that were buyers (sellers) in the preceding DM will have deposits (loans) equal to zero. Households supply factors in competitive markets to a competitive industry that produces the CM good. They also redeem deposits (if any) plus interest from and repay loans (again with interest) to banks. A representative firm hires capital and labor services to produce the CM good, again using a CRS technology. Households then consume, accumulate money and invest in new capital to carry into the following period, m_{+1} and k_{+1}.

3.2 Preferences and technology

Households' are *ex ante* identical and maximize the discounted sum of utility over their infinite lifetime, discounting future periods geometric with factor $\beta \in (0, 1)$. Their period utility is given by

$$\mathcal{U}(q, k, x, h) = \underbrace{U(x) - \bar{A}h}_{\text{CM}} + \underbrace{Iu(q) - (1 - I)\nu}_{\text{DM}}. \tag{3.1}$$

[8]Various extensions involving limitations on the ability of banks to enforce loan contracts are possible. Since, however, the focus in this paper is on competition for *deposits*, we focus here on the case of full commitment. For more details on default see Berentsen et al. (2007) and Head, Kam, Ng and Pan (2023).

[9]In equilibrium, all households carry money balance M into the DM. Buyers will have no incentive to deposit funds with a bank if they face any prospect of being liquidity or credit constrained. Sellers will never have an incentive to borrow since they already carry idle money. As goods trading occurs only after banking has closed for the period, neither buyers' left over money balances (if any) nor sellers' sales receipts can be deposited in a bank.

where $I = 1$ if the household is a buyer in the current period and $I = 0$ otherwise.

In the CM, households consume good x and incur dis-utility $\bar{A}h$ from working, where $\bar{A}$ is a scaling parameter. For simplicity we assume:

$$U(x) = \bar{B}\ln(x), \qquad \bar{B} > 0. \tag{3.2}$$

In the DM, utility $u(q)$ from consumption of the special good q is assumed to be of the constant-relative-risk-aversion (CRRA) family of functions:

$$u(q) = \begin{cases} \bar{C}\frac{q^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \bar{C}\ln(q) & \text{if } \sigma = 1 \end{cases}, \qquad \bar{C} > 0, \tag{3.3}$$

which satisfies the usual Inada conditions.

A competitive industry produces the CM good using labor and capital, via CRS technology $F(K, H)$. In the DM, sellers produce individually and supply to buyers in a competitive goods market. Sellers' technology is given by

$$f(\nu, k) = \nu^{\psi}k^{1-\psi} \qquad \psi \in (0, 1] \tag{3.4}$$

where ν is labour effort and and k is the sellers' own previously accumulated capital.[10] Sellers have no access to the capital of other households, buyers or sellers. With $q_s = f(\nu, k)$, the utility cost to a seller with capital k of producing q_s be $\nu = c(q, k)$. Given the functional form (3.4), we have

$$c(q, k) = q^{\omega}k^{1-\omega}, \qquad \text{where } \omega = 1/\psi. \tag{3.5}$$

The aggregate state of the economy in the current period, $\mathbf{a} = (M, K, \gamma)$, os given by the total nominal money supply, aggregate capital stock and the inflation rate, which distinguishes the governments committed monetary policy. The relevant states for individual households' decisions change as we move through the various stages on the period as described above. We now turn to the decisions of households, banks and firms, beginning with the CM and working backward to the beginning of the current period.

[10] If $\psi = 1$, capital is not used for DM production thus effectively decoupling the nominal and real sides of the economy ((see Aruoba and Wright, 2003; Aruoba et al., 2011)). In this case, the special good is produced using a seller's own effort e only. For our baseline analysis, we restrict attention to the case where $\psi < 1$ and so $\omega > 1$ Moreover, the technology f satisfies $f_n > 0, f_{nn} < 0$ and $f_k > 0$ and $f_{kk} < 0$. The cost function $c(\cdot)$ satisfies $c_q(q, k) > 0$, $c_{qq}(q, k) > 0$; $c_k(q, k) < 0$, $c_{kk}(q, k) > 0$. Assume k is a normal input, then it follows that $c_{qk}(q, k) < 0$.

3.3 Firms in the CM

There is an industry comprised of a large number of identical firms which produce the CM good using capital and labour. All markets in which these firms interact, output and factor markets, are competitive. A representative firm hires labour and rents capital from households to solve the following static problem:

$$\hat{\Pi} = \max_{k,h} F(k,h) - wh - rk, \tag{3.6}$$

where w and r are the wage and rental rate of capital, respectively. These satisfy the usual conditions:

$$r = F_k(k,h) \quad \text{and} \quad w = F_h(k,h). \tag{3.7}$$

In our baseline economy we will normally assume that $F(\cdot)$ has the Cobb-Douglas form, $F(k,h) = k^{\alpha}h^{1-\alpha}$.

3.4 Households in the CM

A household enters the CM with individual state $(\hat{m}, k, l, d(i_d))$, representing their holdings of money (entering the CM) and capital, their outstanding loans (if any), deposits at promised rate i_d (if any). The household may have been either a buyer or a seller in preceding DM and her lifetime utility going forward is given by

$$W\left(\hat{m}, k, l, d(i_d), \mathbf{a}\right) = \max_{\{x,h,m_{+1},k_{+1}\}} \left\{ U(x) - \bar{A}h + \beta V\left(m_{+1}, k_{+1}, \mathbf{a}_{+1}\right) \right\}, \tag{3.8}$$

subject to

$$\begin{aligned} \underbrace{x}_{\text{consumption}} + \underbrace{k_{+1} - (1-\delta)k}_{\text{net investment}} + \underbrace{\phi(m_{+1} - \hat{m})}_{\text{real money accumulation}} &= \underbrace{rk}_{\text{rental income}} + \underbrace{wh}_{\text{labour income}} + \underbrace{\Pi}_{\text{bank profit}} \\ &+ \underbrace{\phi(1+i_d)d}_{\text{deposit return}} - \underbrace{\phi(1+i_l)l}_{\text{debt repayment}} . \end{aligned} \tag{3.9}$$

where $V(\cdot)$ is the household's value entering the following period and δ is the capital depreciation rate.[11]

[11] With $\alpha = 0$, the CM good is produced one-for-one using only labor and sold at nominal price $1/\phi$. In this case both the real wage and real price of the CM good equal one.

Combining (3.9) and (3.8) by eliminating h we have

$$\begin{aligned} W\left(\hat{m}, k, l, d(i_d), \mathbf{a}\right) = \frac{\phi \bar{A}}{w}\Big[\hat{m} - (1+i_l)l + (1+i_d)d\Big] + \frac{\bar{A}}{w}\Big[(1+r-\delta)k + \Pi\Big] \\ + \max_{x, m_{+1}, k_{+1}} \left\{ U(x) - \frac{\bar{A}}{w}x - \frac{\phi \bar{A}}{w} m_{+1} - \frac{\bar{A}}{w} k_{+1} + \beta V\left(m_{+1}, k_{+1}, \mathbf{a}_{+1}\right) \right\}. \end{aligned} \tag{3.10}$$

The first-order conditions for choices: x, m_{+1} and k_{+1} are respectively

$$\begin{aligned} U_x\left(x\right) &= \frac{\bar{A}}{w} \\ \frac{\phi \bar{A}}{w} &= \beta V_m\left(m_{+1}, k_{+1}, \mathbf{a}_{+1}\right) = \beta V_m(+1) \\ \frac{\bar{A}}{w} &= \beta V_k\left(m_{+1}, k_{+1}, \mathbf{a}_{+1}\right) = \beta V_k(+1). \end{aligned} \tag{3.11}$$

where $V_m(+1)$ and $V_k(+1)$ are the marginal values of money and capital in the next period.

The envelope conditions are

$$\begin{aligned} W_{\hat{m}}(\hat{m}, k, l, d(i_d), \mathbf{a}) &= \frac{\phi \bar{A}}{w} \\ W_k(\hat{m}, k, l, d(i_d), \mathbf{a}) &= \frac{\bar{A}(1+r-\delta)}{w} \\ W_l(\hat{m}, k, l, d(i_d), \mathbf{a}) &= -\frac{\phi \bar{A}}{w}\left(1+i_l\right) \\ W_d(\hat{m}, k, l, d(i_d), \mathbf{a}) &= \frac{\phi \bar{A}}{w}\left(1+i_d\right). \end{aligned} \tag{3.12}$$

As $W(\cdot)$ is linear in all its arguments, all households choose the same m_{+1} and k_{+1}.

3.5 Goods trading the DM

The final interaction in the DM is goods trading. At this point, households have been differentiated into buyers and sellers and have interacted with banks. Previewing the equilibria we will consider, sellers have made deposits d and each has been promised a net return, i_d. Buyers will choose whether and how much to borrow (determining their loan balance, l) depending on the loan interest rate and price, p, of the DM good.

3.5.1 Sellers

A seller who has deposited their entire money balance at deposit rate $i_d \geq 0$ takes the nominal price, p, of the DM good as given and has lifetime value going forward:

$$S(\mathbf{s}, \mathbf{a}; d(i_d)) = \max_{q_s} -c(q_s, k) + W(\hat{m}_s, k, l_s d_s(i_d), \mathbf{a}). \tag{3.13}$$

subject to:

$$\hat{m}_s = m + \tau M - d + pq_s. \tag{3.14}$$

Previewing the equilibria we will consider. First, no seller will borrow, so $l_s = 0$. Second, for any $i_d \geq 0$, $d_s = m + \tau M$ and $\hat{m}_s = pq_s$. That is, sellers deposit their entire money holdings (including the current period transfer and carry their cash sales receipts into the DM. Thus, for sellers

$$W(\hat{m}_s, k, l_s d_s, \mathbf{a}) = W(pq_s, k, 0, m + \tau M). \tag{3.15}$$

The seller will optimally supply the DM good up to the point where its marginal cost of production equals the real price:

$$c_{q_s}(q_s, k) = \frac{p\phi\bar{A}}{w} \tag{3.16}$$

where ϕ is the value of money in units of the CM good as defined in (3.9).

3.5.2 Buyers

A buyer chooses the quantity of DM good, q_b, to purchase and whether and how much to borrow from the bank, l_b, to maximize lifetime utility going forward:

$$B(\mathbf{s}, \mathbf{a}; i_l) = \max_{q_b, l} u(q_b) + W(\hat{m}_b, k, l_b, d_b(i_d), \mathbf{a}) \tag{3.17}$$

subject to:

$$pq_b \leq m + \tau M + lm + \tau M + l \tag{3.18}$$

$$l \leq \bar{l} \tag{3.19}$$

In our baseline model we assume full commitment, implying that banks can enforce loan repayments costlessly. In this case, $\bar{l}$ may be sufficiently high to ensure that (3.19) never binds. Moreover, again previewing equilibrium the maximum deposit rate offered will always be lower than the rate on loans, and buyers will never choose positive deposits: $d_b(i_d) = 0$.

Buyers' demand for the DM good depends not only on its price and the value of money in the upcoming CM, but also on the cost of borrowed funds, i, which may be used to augment the money balance carried into the period, m. A buyer's optimal loan demand is given by

$$l(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w) = \begin{cases} p^{\frac{\sigma-1}{\sigma}} \left[\frac{\phi \bar{A}}{w}(1+i_l)\right]^{-\frac{1}{\sigma}} - (m + \tau_b M) & \text{for}\, p \in (0, \tilde{p}_{i_l}];\ i_l \in (0, \hat{i}_l] \\ 0 & \text{otherwise.} \end{cases} \tag{3.20}$$

where

$$\begin{aligned} \tilde{p}_{i_l} &= (m + \tau_b M)^{\frac{\sigma}{\sigma-1}} \left(\frac{\phi \bar{A}}{w}\right)^{\frac{1}{\sigma-1}} (1+i_l)^{\frac{1}{\sigma-1}} \equiv \hat{p}\,(1+i_l)^{-\frac{1}{\sigma}}, \text{ with} \\ \hat{p} &= (m + \tau_b M)^{\frac{\sigma}{\sigma-1}} \left(\frac{\phi \bar{A}}{w}\right)^{\frac{1}{\sigma-1}}, \text{ and} \\ \hat{i}_l &= p^{\sigma-1} \left(\frac{\phi \bar{A}}{w}\right)^{-1} (m + \tau_b M)^{-\sigma} - 1 > 0. \end{aligned} \tag{3.21}$$

That is, the a buyer borrows to augment their money holdings if the costs of both goods and loans are sufficiently low.

If $\sigma < 1$, then $0 < \tilde{p}_{i_l} < \hat{p} < \infty$, and $0 < \hat{i}_l$. This will be the parametric case of interest in our calibration later. The buyer's demand for the DM good then satisfies

$$q_b(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w) = \begin{cases} \left[\frac{p \phi \bar{A}}{w}(1+i_l)\right]^{-\frac{1}{\sigma}} & \text{if } 0 < p \leq \tilde{p}_{i_l} \text{ and } 0 < i_l \leq \hat{i}_l \\ \frac{m + \tau_b M}{p} & \text{if } \tilde{p}_{i_l} < p < \hat{p} \text{ and } \hat{i}_l < i_l \\ \left(\frac{p \phi \bar{A}}{w}\right)^{-\frac{1}{\sigma}} & \text{if } \hat{p} \leq p \text{ and } \hat{i}_l < i_l. \end{cases}, \tag{3.22}$$

Going forward, we use $l(\mathbf{s}, \mathbf{a}; i_l) = l(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w)$ and $q_b(\mathbf{s}, \mathbf{a}; i_l) = q_b(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w)$ denote the buyers' demands for loans and goods respectively. For $i_l > \hat{i}_l$, the buyer doesn't borrow, but may be liquidity constrained or not depending on the price of the DM good.

3.6 Banking in the DM

Banks lend in a perfectly competitive market but have market power when dealing with depositors. They post a rate i_d at which they commit to repaying depositors with interest in the upcoming CM. Depositors (*i.e.* sellers) search among banks for opportunities to deposit their idle money balances $d \leq m$ in a setting adapted from the non-sequential search model of Burdett and Judd (1983).[12] Each searching seller receives either one or two deposit opportunities (or rate *quotes*) with probabilities α_1 and $\alpha_2 = 1 - \alpha_1$, respectively.[13]

Banks allocate their accumulated deposits (d) to fund their assets, which consist of consumer loans (l) and deposits (b) with the central bank. These deposits earn net rate of return $i = (\gamma - \beta)/\beta$, which represents the policy interest rate.

Let the distribution of posted deposit rates be given by $G(i_d)$. Then, a bank that posts deposit rate $\hat{i}_d$ has expected profit:

$$\Pi(\hat{i}_d) = nl[1 + i_l] + b[1 + i] - (1 - n)[\alpha_1 + 2\alpha_2 G(\hat{i}_d) + \alpha_2 \eta(\hat{i}_d)]d[1 + \hat{i}_d]. \tag{3.23}$$

The bank's profit per depositor is

$$R(\hat{i}_d) = d[i - i_d]. \tag{3.24}$$

The first two terms of (3.23) are the returns on loans and deposits with the central bank, respectively. The final term has three parts associated with the cost of deposits. Given that the bank has posted rate $\hat{i}_d$, it successfully attracts depositors that 1) match with it and have an opportunity to deposit and 2) for whom it is the best deposit opportunity that they have. There are three such types of depositor. First, α_1 is the probability that the depositor has only the option to deposit at this bank. Then, $2\alpha_2 G(\hat{i}_d)$ is the probability that the depositor has two deposit opportunities, of which $\hat{i}_d$ is the single best one. Finally, $\alpha_2\eta(\hat{i}_d)$ is the probability of having two deposit opportunities, both at rate $\hat{i}_d$:

$$\eta(i_d; \gamma) = \lim_{\epsilon \to 0} G(i_d; \gamma) - G(i_d - \epsilon; \gamma), \tag{3.25}$$

Using (3.23)-(3.27), the profit maximization problem is:

$$\max_{l} nl[i_l - i] + \max_{i_d} (1 - n)[\alpha_1 + 2\alpha_2 G(i_d; \gamma) + \alpha_2 \eta(i_d; \gamma)]R(i_d; \gamma)., \tag{3.26}$$

[12] We abstract from imperfect competition here because our focus is on the deposits channel of monetary policy transmission. We could also allow households to search for both loans and deposits in a more complicated setup with interactions following Burdett and Judd (1983) in both markets. A special case of this in which demanders in either market observe only one trading opportunity gives rise to monopoly banking as in *e.g.*, Klein (1971), Monti (1972), and Andolfatto (2021). In contrast, if agents on the demand side always have multiple trading opportunities, the model of Berentsen et al. (2007) effectively arises.

[13] We exclude the possibility of a seller having no deposit opportunity to maintain consistency with Berentsen et al. (2007), in which sellers can always deposit idle funds. The search process can be generalized in many ways without affecting substantively the results on which we focus here. See for examples of such generalizations Head and Kumar (2005) and Wang (2016).

where the bank faces the balance sheet and lending feasibility constraints:

$$\underbrace{nl + b}_{\text{assets}} = \underbrace{(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2 \eta(i_d)]d}_{\text{liabilities}}, \quad \text{and} \quad b \geq 0. \tag{3.27}$$

Note that we exclude the possibility of the bank borrowing from the central bank in order to make loans. Banks' lending and deposit-taking decisions are independent of each other, although both depend on the policy interest rate.[14]

Again previewing the equilibria we will consider, as the loan market is competitive the lending rate will equal the policy rate, $i_l = i$. Thus bank profits depend only on deposits:

$$\Pi(i_d) = \max_{i_d}(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2 \eta(i_d)]R(i_d). \tag{3.28}$$

Observe from (3.28) that the bank faces a trade-off between setting a higher deposit rate to attract more depositors (the extensive margin) and paying a lower rate to earn a larger spread on deposits (the intensive margin).

3.7 Households at the start of the DM

At the beginning of the period before the realization of the shock distinguishing buyers and sellers, each household has individual state $\mathbf{s} = (m, k)$ consisting their own nominal money balance and capital stock. All households have lifetime utility going forward:

$$V(\mathbf{s}, \mathbf{a}) = nB(\mathbf{s}, \mathbf{a}; i_l) + (1-n)\left\{ \int_{\underline{i}_d}^{\bar{i}_d} \Big[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\Big] S(\mathbf{s}, \mathbf{a}; i_d) \mathrm{d}G(i_d;\gamma) \right\} \tag{3.29}$$

Conditional on being a buyer, which occurs with probability n, the household becomes a buyer with value $B(\mathbf{s}, \mathbf{a}; i_l)$, taking prices and the loan interest rate, i_l, as given and optimizing as described above. Alternatively, with probability $1-n$ the household becomes a seller. In this case she searches for a deposit opportunity an obtains either one or two taking the distribution of posted deposit rates $G(i_d;\gamma)$ as given. Following that, she has value $S(\mathbf{s}, \mathbf{a}; d(i_d))$ as described above.

Pushing (3.29) forward one period and differentiating with respect to m_{+1} and k_{+1} we have expressions for the marginal values of money and capital which appear above in (3.11).

[14] We abstract from bankruptcy risk and the role of bank capital and so can preserve the well-known result of independence between deposit and loan rates along the lines of Klein (1971) and Monti (1972). Andolfatto (2021) also uses a similar independence property. The role of bank capital could be introduced in a manner similar to that of Dermine (1986).

First, consider

$$V_m(m,k,\mathbf{a}) = \phi U_x(x)\Bigg[n\Bigg[\underbrace{\mathbb{I}_{\{i_l\le\hat{i}\}}\frac{u_{q_b}(q_b)}{c_{q_s}(q_s,k)}}_{\text{buyer borrows}} + \underbrace{\mathbb{I}_{\{i_l>\hat{i}\}}\frac{u_{\hat{q}_b}(\hat{q}_b)}{c_{\hat{q}_s}(\hat{q}_s,\hat{k})}}_{\text{buyer doesn't borrow}}\Bigg] + (1-n)\int_{\underline{i}_d}^{\bar{i}_d}\Big[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\Big](1+i_d)\mathrm{d}G(i_d;\gamma)\Bigg]. \tag{3.30}$$

The marginal value of money carried into the DM depends on whether the household becomes a buyer or a seller. With probability n it is the former, and the marginal value of a unit of money arises from its use in financing DM consumption. This depends on prices and the loan rate, *i.e.* whether the buyer borrows or not. With probability $1-n$ the household is a seller and the marginal value of money is depends on the rate at which they are able to deposit with a bank.

Similarly, the marginal value of capital carried into the next period is given by

$$V_k(m,k,\mathbf{a}) = U_x(x)\Bigg[n(1+r-\delta) + (1-n)\Bigg[(1+r(k)+1-\delta) - \mathbb{I}_{\{i_l\le\hat{i}\}}\frac{c_k(q_s,k)}{U_x(x)} - \mathbb{I}_{\{i_l>\hat{i}\}}\frac{c_{\hat{k}}(\hat{q}_s,\hat{k})}{U_x(x)}\Bigg]\Bigg]. \tag{3.31}$$

In the event that the household is a buyer, they have no use for capital in the DM and as such its return stems from the CM. For a seller, $c_k(\cdot,k) < 0$ captures the additional return to capital from its use in DM production, depending on whether its customers have borrowed or not.

It is the return to capital in the DM that accounts for the novel links among bank market power, deposits and monetary policy on which we focus. As in Aruoba et al. (2011) money and capital are interdependent because the capital available for use in the DM directly affects the return to money. Imperfect competition for deposits affects the extent of insurance that banks provide, thereby distorting the value of money and affecting in turn the returns to capital accumulation.

4 Stationary Monetary Equilibrium

We focus on stationary monetary equilibria (SME) in which all nominal variables grow at the time-invariant rate of inflation, γ, where $\gamma = 1+\tau = M_{+1}/M = \phi/\phi_{+1}$ and real variables stay constant over time. Let $z = \phi m$ denote the per household real money balance, and $Z = \phi M$, its aggregate counterpart. With the nominal price of the DM good (relative to the CM good) given by p, its real counterpart is $\rho = \phi p$. Similarly, real loans and deposits are, respectively, $\xi = \phi l$ and $\tilde{d} = \phi d$. In characterizing the economy's SME, we focus on cases in which money has value in equilibrium and credit is positive. Our calibrated economy below will have an SME with these characteristics.

4.1 The distribution of posted deposit interest rates

Following Burdett and Judd (1983) we derive analytically the distribution of posted deposit rates, $G(\cdot;\gamma)$, written as such to indicate its dependence on monetary policy. The derivation is well-known, so the details are relegated to Appendix A.1. The following summarizes:

Proposition 1. *Let the growth rate of the money stock satisfy $\gamma > \beta$.*

1. *If $\alpha_1 \in (0,1)$, there is a unique, continuous distribution of posted deposit rates on a connected support:*

$$G(i_d;\gamma) = \frac{\alpha_1}{2\alpha_2}\left[\frac{R(i_d^m;\gamma)}{R(i_d;\gamma)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{i - i_d^m}{i - i_d} - 1\right] \tag{4.1}$$

The support of $G(\cdot)$ is $\left[\underline{i}_d, \bar{i}_d\right]$, where $\underline{i}_d = i_d^m = 0$, $i = (\gamma-\beta)/\beta$ and $\bar{i}_d := \bar{i}_d(\gamma) = \frac{\gamma-\beta}{\beta}\left[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}\right]$ determined by $(\alpha_1 + 2\alpha_2)\, R(\bar{i}_d;\gamma) = \alpha_1 R\left(i_d^m;\gamma\right)$.

2. *If $\alpha_2 = 1$, then G is degenerate at the central bank policy rate $i = i(\gamma)$:*

$$G(i_d;\gamma) = \begin{cases} 0 & \text{if } i_d < i \\ 1 & \text{if } i_d \geq i \end{cases}. \tag{4.2}$$

3. *If $\alpha_1 = 1$, the G is degenerate at the monopoly (i.e. lowest possible) rate $\underline{i}_d$:*

$$G(i_d;\gamma) = \begin{cases} 0 & \text{if } i_d < \underline{i}_d \\ 1 & \text{if } i_d \geq \underline{i}_d \end{cases}. \tag{4.3}$$

The intuition for Proposition 1 also follows Burdett and Judd (1983). Working backwards through the three cases, if all prospective depositors (DM sellers in equilibrium) receive only one deposit opportunity ($\alpha_1 = 1$) then all banks know they are serving their depositors as monopolists and therefore set the lowest rate that sellers will accept. At the opposite extreme, if all DM sellers receive two deposit opportunities ($\alpha_2 = 1$), then Bertrand competition forces the deposit rate to the marginal cost of funds, *i.e.* the bank policy rate. In either case the distribution of deposit rates is degenerate.

When some prospective depositors receive one opportunity and others two, then there is a trade-off between the number of depositors attracted (which falls with the posted interest rate) and bank profit per depositors (which rises with the posted rate). All banks post a rate resulting in the same positive expected profit (see Lemma 6), but the probability that two banks post exactly the same rate equals zero. Moreover the distribution of posted rates, G is continuous with a connected support (see Lemmata 7 and 8).

Note that the distribution of posted rates, $G(\cdot;\gamma)$, does not depend on state variables other than policy γ. This result depends on prospective depositors' asset positions (money holdings) being predetermined when they search for deposit opportunities.

Let the density of $G(\cdot;\gamma)$, the distribution of posted deposit rates, be given by 1 is $\tilde{g}(i_d) = \partial G(i_d;\gamma)/\partial i_d$. The cumulative distribution of *transacted* deposit rates is that of the highest rate observed, given $G(\cdot;\gamma)$, α_1 and α_2:

$$J(i_d;\gamma) = \alpha_1 G(i_d;\gamma) + \alpha_2[G(i_d;\gamma)]^2 \text{ for all } i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d], \tag{4.4}$$

and its associated density by

$$\tilde{j}(i_d;\gamma) = \partial J(i_d;\gamma)/\partial i_d = \alpha_1\tilde{g}(i_d) + 2\alpha_2 G(i_d;\gamma)\tilde{g}(i_d) = [\alpha_1 + 2\alpha_2 G(i_d;\gamma)]\tilde{g}(i_d). \tag{4.5}$$

The average posted rate is

$$g(\gamma) = \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G(i_d;\gamma), \tag{4.6}$$

and the average transacted deposit rate is

$$j(\gamma) := \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}J(i_d;\gamma). \tag{4.7}$$

Below we will focus on the case of a non-degenerate $G(\cdot;\gamma)$, *i.e.* Case 1 of Proposition 1. This is the case that will arise given our calibration in 6. In this case, we have the following results regarding the effect of changes in γ on the distribution of posted rates, G and the average posted and transacted deposit rates. For proofs see Appendices A.2.1 and A.2.2, respectively.

Lemma 1. *Let $\alpha_1 \in (0,1)$. Consider two economies that differ in inflation, γ and γ', such that $\gamma' > \gamma > \beta$. Then distribution $G(\cdot;\gamma')$ first-order stochastically dominates $G(\cdot;\gamma)$.*

Lemma 2. *Assume that $\gamma > \beta$, and $\alpha_1 \in (0,1)$. An increase in the γ leads to:*

1. *An increase in both the average posted and transacted deposit interest rates; and*

2. *An increase in the upper bound of the support of the distribution G, $[\underline{i}_d, \bar{i}_d]$.*

From Lemma 1, an increase in inflation γ (or the nominal policy rate $i = (\gamma - \beta)/\beta$) shifts $G(\cdot;\gamma)$ downward, and prospective depositors are more likely to draw a higher deposit interest rate in the sense of first-order stochastic dominance. Similarly, from Lemma 2 we have that the average posted deposit interest rate is increasing with the anticipated inflation. Also the support of the distribution widens with its upper bound increasing a inflation rises. Higher inflation reduces the value of money balances. Households' incentive to carry money into the DM falls and so banks post higher deposit rates to attract funds as inflation rises.

4.2 Market clearing

In the DM, goods supplied must equal total demand:

$$(1-n)q_s = nq. \tag{4.8}$$

Also, total bank assets must equal total bank liabilities:

$$\underbrace{nl(z,k;i)+b}_{\substack{\text{total assets}\\ \text{loans and central bank deposits}}} = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z+\tau_s Z)\mathrm{d}G(i_d;\gamma)}_{\text{total liabilities } (i.e. \text{ deposits})}. \tag{4.9}$$

In the CM, goods market clearing requires that output equals consumption plus investment:

$$F(K,H) = X + K - (1-\delta)K. \tag{4.10}$$

4.3 The SME defined

Definition 1. For $\gamma > \beta$, a stationary monetary equilibrium (SME) is an allocation $(X^\star, k^\star, K^\star, H^\star)$ in the CM, an allocation $(q^\star, \xi^\star)$ in the DM, real *per capita* and aggregate money holdings, $(z^\star,\ Z^\star)$ and prices $(i, G(i_d), \rho)$ such that

1. Households optimize: (3.11), (4.12), (4.13);
2. Firm optimizes: (3.7);
3. Banks optimize: (4.1);
4. Bank assets equal bank liabilities: (4.9); and
5. Markets clear: (4.8), and (4.10), where $z^\star = Z^\star$, $k^\star = K^\star$.

4.4 An SME with credit

If inflation is not excessive, then an SME will feature positive loans. Let

$$L(q,K)=\frac{u_q(q)}{c_q\left(\frac{n}{1-n}q,K\right)} \tag{4.11}$$

denote the marginal liquidity premium from carrying additional money into the DM, and $q^\star$ and $K^\star$ DM output and aggregate capital, respectively, in an SME. Then, we have the following, with proof in Appendix A.4:

Lemma 3. *If $\alpha_1 \in (0,1)$, $\omega\sigma > \alpha(\omega+\sigma-1)$, and $\gamma \in (\beta, \bar{\gamma}]$ where $\bar{\gamma}=\beta L(q^\star, K^\star)$ in an SME, then:*

1. *Real loan demand is positive, $\xi^\star > 0$, and*
2. *Money demand and capital are characterized respectively by*

$$\frac{\gamma-\beta}{\beta}=(1-n)\int_{\underline{i}_d}^{\bar{i}_d}\left[\alpha_1+2\alpha_2 G(i_d;\gamma)\right]i_d dG(i_d;\gamma)+\mathbb{I}_{\{i=i_l\leq\hat{i}\}}n\left[\frac{u_q(q)}{c_q\left(\frac{n}{1-n}q,K\right)}-1\right], \tag{4.12}$$

and,

$$\frac{1}{\beta}=[1+F_K(K,H)-\delta]-\mathbb{I}_{\{i=i_l\leq\hat{i}\}}(1-n)\left[\frac{c_K\left(\frac{n}{1-n}q,K\right)}{U_X(X)}\right]. \tag{4.13}$$

The upper bound on inflation, $\bar{\gamma}$ has a natural interpretation. For money to be valued in equilibrium, anticipated inflation must not exceed the gross return to money from facilitating goods trade in the DM, $L(q^\star, K^\star)$. In this case, the loan interest rate will not dissuade buyers from borrowing to augment their money balance (see (3.20)).

The left-hand side of (4.12) captures the marginal cost of holding an extra unit of money and the right-hand side the expected (net) marginal benefit (from (3.30)). This benefit has two components regarding different liquidity needs. The first term is the net marginal value of deposits a seller. As in Berentsen et al. (2007), banks thus insure against having idle balances, although here the expected rate of deposit interest is lowered by bank market power. The second term is a buyer's net marginal value of an extra dollar in the DM.

The left-hand side of (4.13) captures the gross risk-free real interest rate, and the right-hand side the expected net marginal value of the capital investment. Again, the right-hand side has two components arising from Equation (3.31). The first is the return to capital in CM production. The second term is the return to a seller's capital in the DM, working here as in Aruoba et al. (2011).

5 Money, credit and capital: analytical results

In this section, we present an existence result and important characteristics of the SME in a number of special cases. We start by discussing the conditions under which the SME is efficient, *i.e.* attains the first-best allocation. Most proofs from this section can be found in Appendices A.4, A.5, and A.6.

5.1 The Friedman Rule and the first best

Consider the case of the *Friedman Rule:* $\gamma = \beta$, or equivalently, $i = 0$. In this case, there is no cost to holding money and the economy attains the first-best allocation in an SME. Moreover, the Friedman Rule eliminates the cost of holding idle money renders banks superfluous. In this case we have

Proposition 2. *If* $1 + \tau \equiv \gamma = \beta$*, then there is no equilibrium with deposit interest rate dispersion. Moreover, the Friedman rule attains the first-best allocation.*

Proof: See Appendix A.4.3.

5.2 An SME with money, banking and capital

From this point on we restrict attention to cases in which $\gamma > \beta$ (or $i > 0$). After some algebra (see Appendix A.4.2) an SME can be characterized by solving the following equation for *per capita* capital, $\hat{k} = K/H$, given policy, $\tau = \gamma - 1$:

$$\frac{1}{\beta} = \underbrace{[1 + \alpha\hat{k}^{\alpha-1} - \delta]}_{R_{CM}(\hat{k})} + \underbrace{\tilde{\theta}\left[\hat{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}} \tilde{f}(\hat{k})}_{R_{DM}(\hat{k},\gamma)}, \tag{5.1}$$

where

$$\tilde{\theta} \equiv \frac{1}{\bar{A}}\left[(\omega - 1)(1 - n)(1 - \alpha)\left(\frac{n}{1-n}\right)^{\omega}\right]\left[\omega\left(\frac{n}{1-n}\right)^{\omega}\right]^{\frac{\omega}{1-\omega-\sigma}} > 0, \tag{5.2}$$

$$\hat{C}(\gamma) \equiv 1 + \hat{g}(\gamma) + \frac{1}{n}\left[\underbrace{i(\gamma) - \hat{g}(\gamma)}_{s(\gamma)}\right], \tag{5.3}$$

$$\hat{g}(\gamma) \equiv \int_{\underline{i}_d}^{\bar{i}_d} \left[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\right] i_d \mathrm{d}G(i_d;\gamma), \tag{5.4}$$

and

$$\tilde{f}(\hat{k}) \equiv \left[\frac{\bar{B}(1-\alpha)}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}\right]^{\frac{\omega\sigma}{1-\omega-\sigma}} \hat{k}^{\frac{\omega\sigma-\alpha(\omega+\sigma-1)}{1-\omega-\sigma}}. \tag{5.5}$$

We now establish, under some regularity conditions, that there is a unique SME with money, credit and capital. These conditions hold and are easily satisfied in the calibrated model considered below.

Proposition 3. *Let $\omega\sigma > \alpha(\omega+\sigma-1)$ and $\gamma > \beta$. Then there exists a unique SME with postive credit, for $\gamma \leq \beta L(q^\star, K^\star)$ defined as in (4.11).*

Proof: See Appendix A.4.4.

5.3 Inflation and capital accumulation: The mechanism

Using the right-hand side of Equation (5.1) the expected return to capital investment can be decomposed into two parts. $R_{CM}(\hat{k})$ captures the return on capital from its use in the CM. It is independent of inflation, γ, as money is not necessary for trading in the CM. This term, of course, appears in the standard neoclassical growth model.

$R_{DM}(\hat{k};\gamma)$ represents the additional return to capital associated with its use in the DM, and can again be decomposed into two parts. The first, captured by $\tilde{f}(\hat{k})$m arises from capital's use in DM production and appears also in the model of Aruoba et al. (2011). As in that environment, inflation here acts as a tax on capital formation in the CM as well as on trades in the DM. The second component relies on DM buyers' access to bank credit. By providing insurance against holding idle balances, banking increases real money balances which depend on DM trade. This directly raises the return to capital in the DM.

This effect is captured by the term $\hat{C}(\gamma)$ in (5.1) and (5.3) and comes from the money demand Euler equation where it measures the gross cost of holding money. As $\omega \geq 1$ and $\sigma > 0$ implying $\omega/(1-\omega-\sigma) < 0$, $[\hat{C}(\gamma)]^{\omega/(1-\omega-\sigma)}$ in (5.1) is positive. That is, it represents an increase of the DM return on capital when bank credit exists. In (5.3) $s(\gamma)$ represents a distortion term reducing the return to capital as the spread between the average deposit rate, $\hat{g}(\gamma)$, and the bank policy rate, $i(\gamma)$ rises.

5.4 Two special cases

We now consider the effects of banking and imperfect competition on capital accumulation and aggregate economic activity considering two special cases associated with particular parameterizations of the economy.

Case 1: An economy without banks

With $\alpha_1 = \alpha_2 = 0$, no household has an opportunity to deposit idle funds with the banking system. In this case the model is a version of that studied by Aruoba et al. (2011), and (5.1) becomes:

$$\frac{1}{\beta} = \underbrace{[1 + \alpha\hat{k}^{\alpha-1} - \delta]}_{R_{CM}(\hat{k})} + \underbrace{\tilde{\theta}\left[\breve{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}} \tilde{f}(\hat{k})}_{R_{DM}^{no-bank}(\hat{k},\gamma)}, \tag{5.6}$$

where

$$\breve{C}(\gamma) = 1 + \frac{1}{n}\left[\frac{\gamma - \beta}{\beta}\right] = 1 + \frac{i}{n}. \tag{5.7}$$

Case 2. Perfectly competitive banking

Next, consider the case in which $\alpha_2 = 1$, *i.e.* all sellers have two deposit opportunities. This case combines the model of Aruoba et al. (2011) model with that of Berentsen et al. (2007) who study a perfectly competitive credit market. In this case the distribution of deposit interest rates, G is degenerate at the policy interest rate (see Proposition 1) and so the spread, $s(\gamma) = 0$. Hence, (5.1) becomes:

$$\frac{1}{\beta} = \underbrace{[1 + \alpha\hat{k}^{\alpha-1} - \delta]}_{R_{CM}(\hat{k})} + \underbrace{\tilde{\theta}\left[\tilde{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}} \tilde{f}(\hat{k})}_{R_{DM}^{PC}(\hat{k},\gamma)}, \tag{5.8}$$

where

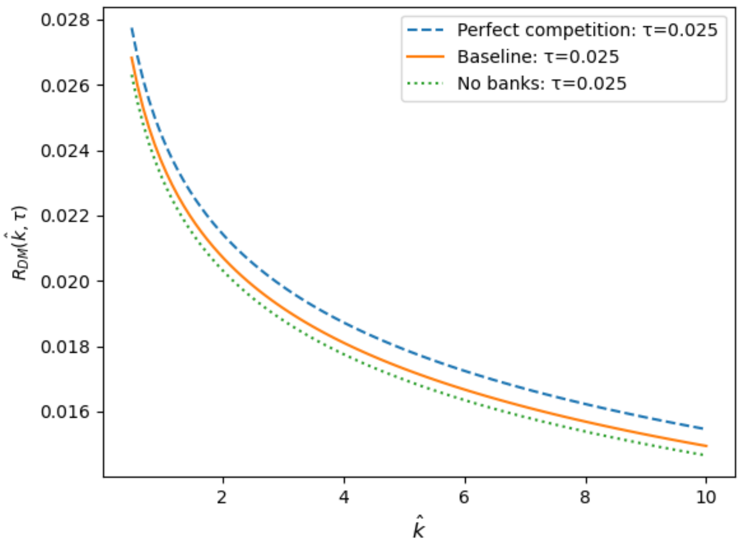
$$\tilde{C}(\gamma) = 1 + \frac{\gamma - \beta}{\beta} = 1 + i. \tag{5.9}$$

Figure 4 compares the component of the gross return to capital emanating from its use in the DM, $R_{DM}(\cdot, \gamma)$, in our baseline economy to these two special cases for the case of inflation at 2.5% (*i.e.* $\gamma = 1 + \tau = 1.025$).

Reflecting the differences among (5.3), (**??**) and (5.9), the figure shows that banking, in general, increases $R_{DM}(\cdot, \gamma)$ up relative to Case 1 (no banking). By insuring households against the risk of carrying costly idle money balances, banking increases production in the DM and thus capital formation. Imperfect competition among banks, however, erodes some of these gains, lowering output and ultimately capital formation relative to Case 2 (perfectly competitive banking).

Comparing the two cases (*i.e.* comparing (5.7) and (5.9)) it can be seen that by paying

Figure 4: Premium on capital return in the DM



interest on deposits and thereby insuring households agains having idle balances in the event that they are sellers the banking system raises the value of real balances and hence output and consumption of the DM good. This raises the return to capital with perfectly competitive banking relative to the case with no banks ($R^{PC}_{DM}(\hat{k};\gamma) > R^{no-bank}_{DM}(\hat{k};\gamma)$). Market power in banking, however, erodes some of these gains by reducing the expected return to deposits, reducing output and consumption in the DM, and thus lowering the return to capital. This accounts for the differences in the capital return depicted in Figure 4.

Overall, monetary policy here works through the channel of agents' decisions with regard to the accumulation of both money and capital, both of which are, in turn, affected by banks' market power with regard to the return on deposits. The following proposition summarizes:

Proposition 4. *Assume $\omega\sigma > \alpha(\omega+\sigma-1)$ and (gross) inflation rate γ satisfies $\beta < \gamma \leq \bar{\gamma} = \beta L(q,K)$, where $L(q,K) := q^{-\sigma}/c_q(\frac{n}{1-n}q, K)$. Financial intermediation improves allocation and welfare relative to a no-bank economy. The economy with perfectly competitive banks Pareto dominates the baseline economy with noisy deposits search:*

$$q^{\star,no-bank} < q^{\star} < q^{\star,PC} \quad \text{and} \quad K^{\star,no-bank} < K^{\star} < K^{\star,PC},$$

where equilibrium allocation $(q^\star, K^\star)$ approaches $(q^{\star,PC}, K^{\star,PC})$ as the baseline economy tends to its perfect-competition limit, i.e., *as $\alpha_2 \to 1$.*

Proof: See Appendix A.5.1.

5.5 The pass-through of the policy rate to deposit-rates

Now consider the effects of changes in the monetary policy rate on the economy working through the banking sector. First, we show that the average deposit rate spread is increasing in the policy rate (*i.e.* in anticipated inflation).

Proposition 5. *Let $\gamma \in (\beta, \bar{\gamma}]$, and $\alpha_1 \in (0,1)$. Then, the average posted deposit rate spread:*

$$s(\gamma) = i(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d dG(i_d), \tag{5.10}$$

is monotone increasing in inflation γ (or equivalently in the policy rate, $i = \frac{\gamma-\beta}{\beta}$).

Proposition 5 indicates that the pass-through of monetary policy to the average deposit rate is incomplete in equilibrium. Moreover, the proposition establishes that the level of inflation affects the extent of market power: banks effectively become *less* competitive—in the sense that they charge a higher spread on deposits—as inflation rises. Thus, banks, in aggregate extract more surplus from depositors when the value of liquidity insurance is high.

Higher inflation induces households to carry smaller real balances and at the same time increases the value of insurance to those that become DM sellers. The latter enables sellers to post lower deposit rates as the marginal value of insurance is high. The former mitigates the extensive margin losses associated with posting relatively low deposit rates. Both effects contribute to lower deposit rates and a higher deposit rate spread. Proposition 5 is consistent with Drechsler et al. (2017) and Choi and Rocheteau (2021). The novelty here is the effect of monetary policy on the extent of market power in banking.

6 Computational Results

In this section, we use a version of the model calibrated U.S. macro-data to illustrate the mechanisms discussed above in a series of computational experiments.

6.1 Calibration

The model has eleven parameters: $(\tau, i, \beta, \overline{A}, \overline{B}, \sigma, \alpha, \psi, n, \delta, \alpha_1)$ and we set the time period to one year.

6.1.1 Parameters determined directly by external targets

Certain parameters can be determined solely from observable statistics. From the Fisher equation, the long-run inflation rate γ and the average effective federal funds rate i determine the discount factor β. Also the measure of DM sellers, $1-n$, corresponds also to the fraction of households that are depositors, where we focus on households with deposits at commercial banks.[15] δ and α are set (as elsewhere in the literature) to match the investment-to-capital ratio and the share of labor income in total output.

Table 2: Calibrated Parameters and Targets

Parameter	Value	Empirical Target	Description
τ	0.0303	Inflation rate[a]	Inflation rate
i	0.0492	Effective federal funds rate[a]	Nominal interest rate
β	0.9820	–	Discount factor, $(1+\tau)/(1+i)$
$\overline{A}$	0.9389	Labor hours	CM labor disutility scale
$\overline{B}$	0.3436	Aux reg. $(i,M/PY)^b$	CM preference scale
$\overline{C}$	1.0000	Normalized	DM preference scale
σ	0.2125	Aux reg. $(i,M/PY)^b$	CRRA (DM q)
α	0.3300	Labor income share	CM technology
ψ	0.7375	I/Y ratio	DM technology
δ	0.0250	I/K ratio	Capital depreciation rate
$\tilde{n}$	0.3500	household depositors[c]	Proportion of DM sellers
α_1	0.2602	Average deposit spread	Prob. $k=1$ bank contacts

[a] Annual nominal interest and inflation rates.
[b] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i) and Lucas and Nicolini (2015) New-M1-to-GDP ratio (M/PY).
[c] Household depositors with commercial banks per 1000 adults for the United States.

6.1.2 Parameters determined jointly to match internal targets

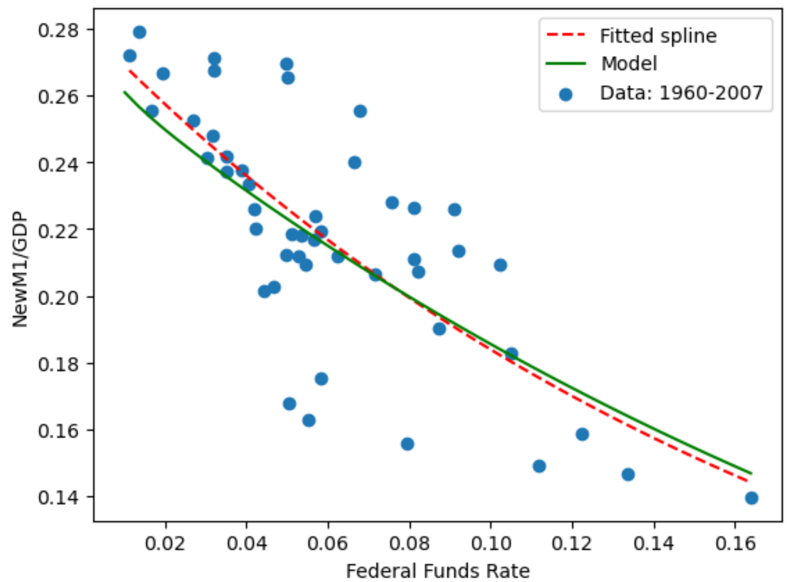
The remaining parameters, $(\overline{A}, \sigma, \overline{B}, \psi, \alpha_1)$, are chosen jointly to match the following targets. $\overline{A}$ is chosen to match average labor hours. The DM technology parameter ψ ($\omega = 1/\psi$) is chosen to match the investment-to-GDP ratio as closely as possible given other parameters. The probability of having exactly one deposit opportunity, α_1, is chosen to match the average deposit spread. Finally, the pair $(\sigma, \overline{B})$ to fit the aggregate relationship between nominal interest rate and the inverse of the velocity of money, as in Lucas and Nicolini (2015).

Table 3: Calibration Fit

Target	Data	Benchmark model
Labor hours	0.333	0.333
Deposit spread (%)	1.646	1.646
I/Y	0.151	0.197
I/K	0.025	0.025

[15] Data source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators".

Figure 5: Aggregate money demand —model and data.



Parameter values and their most closely associated targets are summarized in Table 2. The overall fit of the calibration is shown in Table 3 and Figure 5. The figure depicts the money demand relation through a scatter plot of M1 over GDP and the Fed Funds rate, a spline-fitted to this data and the associated relationship in the calibrated model.

6.2 Comparative steady states

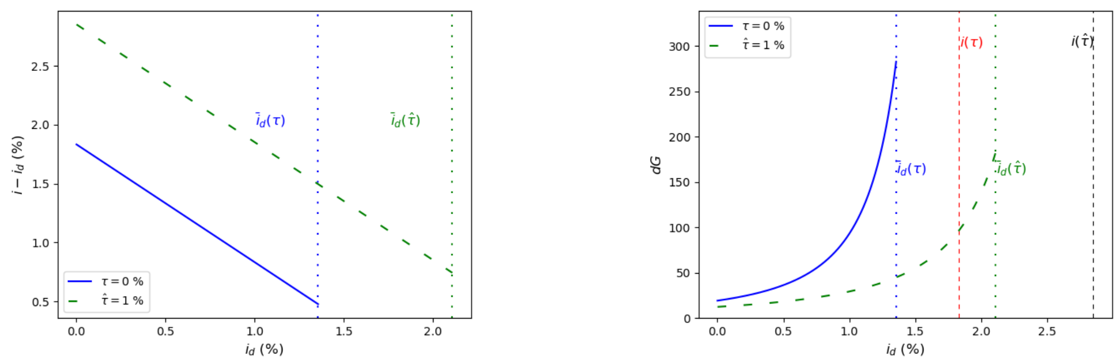
We now consider the effects of changes in the inflation rate on the SME, noting that the net money creation rate, τ, and the nominal interest rate i are equivalent as policy variables. For net money creation rates, $\tau \in [\beta - 1, \bar{\tau} = .1]$, we compute the SME and the links between inflation and banking market power, the equilibrium allocation and welfare.

6.2.1 Deposit spreads and pass-through

Consider first the relationship between the average deposit rate spread and the probability with which individual depositors match with multiple banks. As noted above, a bank which posts a high deposit rate attracts a lot of customers (the extensive margin), but realizes a low deposit rate spread (the intensive margin). Figure 6 depicts the range of deposit rate spreads ($i - i_d$, Panel a) and the densities of posted deposit rates for inflation (Panel b) at net inflation rates of zero and one percent. The red and black dashed lines in Panel b are the competitive interest rates at zero and one percent inflation, respectively. The blue and green dotted lines (in both Panels) are, respectively, the highest posted deposit rates (*i.e.* the upper supports of $G(\cdot)$) at the two inflation rates.

Comparing the two cases, when inflation increases, the range of potential deposit spreads increases, as increased demand for insurance makes depositors willing to accept lower deposit rates. Increased spreads represent higher bank profits along the intensive margin. At the same time, however, the mass of the distribution of posted rates shifts *rightward*. This represents an extensive margin effect. The increase in the competitive rate (and nominal loan rate) induces banks to post higher deposit rates in order to attract more customers. This extensive margin effect mitigates the increase in deposit spreads.

Figure 6: Deposit rates and spreads, with and without inflation.



(a) Deposit spreads

(b) Posted deposit rates (density)

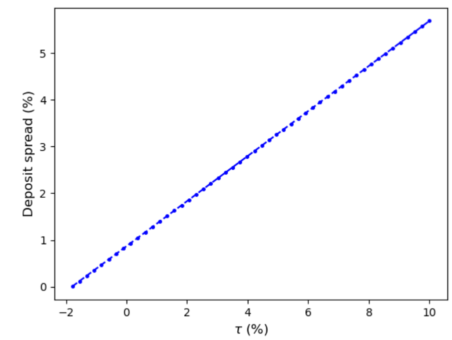
Now consider the extent to which the effects of inflation are *passed-through* to deposit rates. Inflation increases the nominal loan rate and thus the competitive deposit rate. As noted above, the upper support of the distribution of deposit rates shifts upward as well, but not by as much. That is, $\bar{i}_d(\hat{\tau}) - \bar{i}_d(\tau) < i(\hat{\tau}) - i(\tau)$. This incomplete pass-through of an increase in the policy rate to deposit rates indicates an increase in banks' market power in deposits. Banks are effectively able to extract more surplus from depositors when their need for liquidity risk insurance is high.

Figure 7 depicts several aspects of the economy's SME for inflation rates ranging from -2% to 10%. Panels (a) and (b) depict bank market power as reflected in the average level (see (5.10)) and dispersion (*i.e.* standard deviation) of the deposit spread. Panels (c) and (d) reflect aggregate economic activity with the former depicting real money balances and the latter the capital formation via the investment to output ratio.[16]

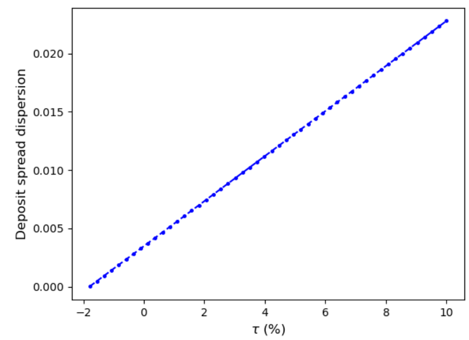
Observe from Figures 7a and 7b the positive relationships between the policy interest rate and the standard deviation and the average deposit rate spread. Imperfect pass-through, consistent with the empirical analysis in Section 2 is evident in Panel (a) as the average deposit rate increases at less than half the rate of inflation. Moreover, Figures 7c and 7d are

[16]The capital-to-output ratio K/Y varies similarly with τ to I/Y as investment I is proportional to capital K in an SME.

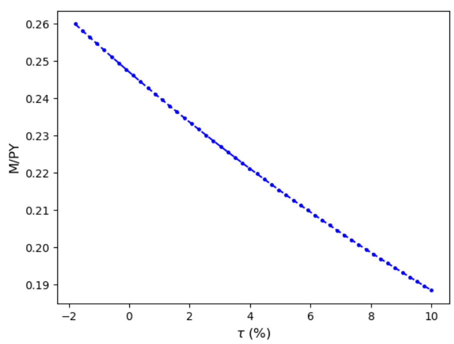
Figure 7: The effects of inflation on money, capital and banks market power for $\tau \in (\beta - 1, \bar{\tau}]$.



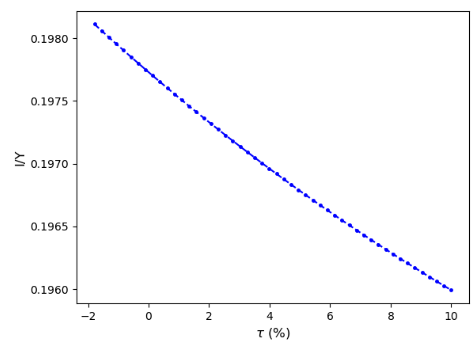
(a) Average deposit spread $s(\tau)$ and τ



(b) Dispersion of deposit spread (SD) and τ



(c) Money demand M/PY and τ



(d) Capital formation I/Y and τ

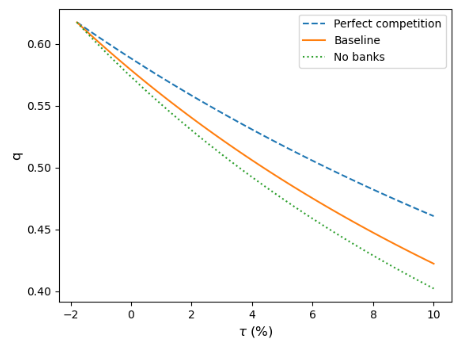
consistent with the long-run dampening effects of increased inflation on both money demand and capital formation as documented for the U.S. by Lucas and Nicolini (2015), Aruoba et al. (2011) and others.

Increases in inflation (and the policy rate) here generate incomplete pass-through via their effect on bank market power. Inflation distort agents' incentives to accumulate money balances and capital (see Proposition 4) with the strength of this channel depending on the interest rate spread as evidenced by its presence (5.1). First, the higher policy rate associated with increased inflation lowers the return to money, inducing households to carry lower real money balances into the DM. This reduces, in turn, the supply of deposits just as the value of insurance against holding idle balances increases. Both of these effects increase banks' market power in the deposits (see Proposition 5) and thus limit pass-through. At the same time, capital formation falls as the policy rate increases because the marginal value of capital falls along with the quantity of goods trade in the DM.

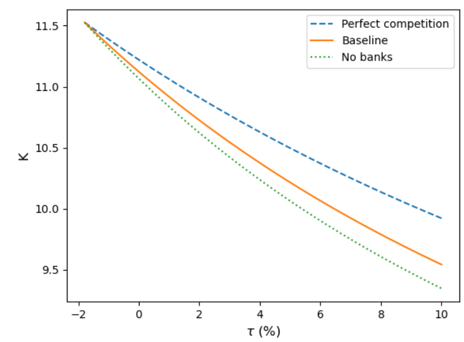
6.2.2 Inflation and welfare

We now illustrate the effects of inflation on aggregate economic activity and welfare. These calculations quantify the analytic results of Proposition 4. Figure 8 depicts the measures of aggregate activity, q, K, X, and H as inflation varies from -2% to 10%. The blue-dashed line is the case of perfectly competitive banking (*i.e.*, $\alpha_2 = 1$). The orange-solid line that of our baseline model with imperfect competition in banking (*i.e.*, $\alpha_2 = 0.7398$). The green-dotted line is the no banking case ($\alpha_1 = \alpha_2 = 0$), effectively that of Aruoba et al. (2011).

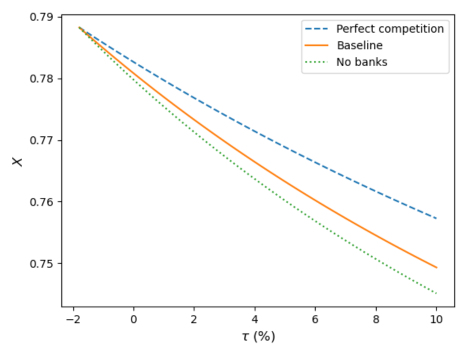
Figure 8: Economic activity for inflation between $\beta - 1$ (-2%) and $\bar{\tau}$ (10%).



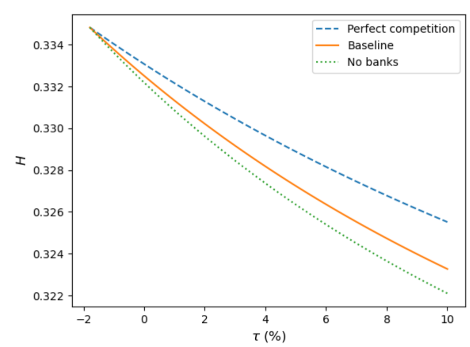
(a) DM output (q)



(b) The Capital Stock (K)



(c) CM output (X)



(d) Employment (H)

Higher inflation reduces output in the DM as inflation is a tax on monetary trade. Since the capital stock is a productive input in both the DM and the CM, reduced DM output lowers the return to capital and thus investment. This nominal-to-real link from monetary policy (inflation) to output is the same as that considered by Aruoba et al. (2011). The novelty here is, again, the endogenous response of banks' market power to changes in the inflation rate.

By insuring the risk of having idle money balances, banking lowers the cost of holding money. This increases activity in the DM and consequently the value of the capital investment. As such, investment, capital and employment are all higher with banking than without.

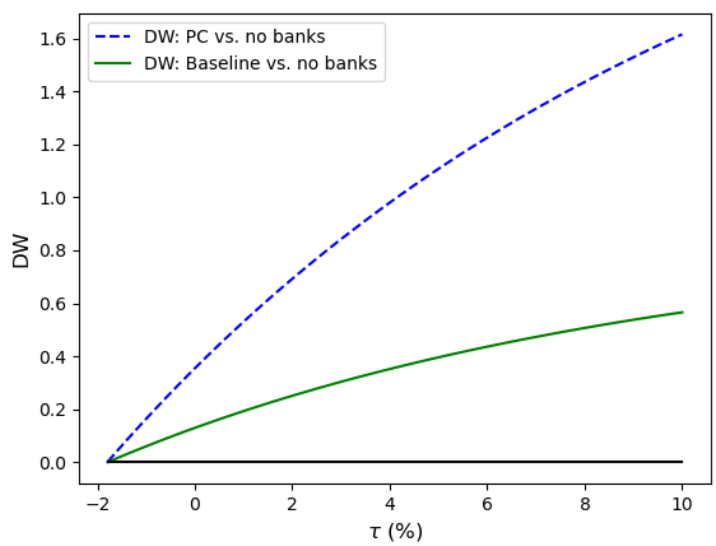
Thus the blue-dashed line lies always above the green-dotted line in Figure 8. Market power in banking, however, erodes these effects by extracting surplus from households in DM trades, lowering the return to holding money. As noted above, this also reduces the investment, the capital stock and employment. Thus the orange-solid line (the baseline economy) lies always below the blue-dashed line which depicts the case of perfectly competitive banking.

Our welfare criterion is the *ex-ante* lifetime utility of (homogenous) households:

$$W(\tau) = \frac{1}{1-\beta}\Bigg[\underbrace{nu[q(K)] - (1-n)c\left(\frac{n}{1-n}q(K), K\right)}_{[A]} + \underbrace{U[X(K)] - \bar{A}H(K)}_{[B]}\Bigg], \tag{6.1}$$

where $[A]$ and $[B]$ capture, respectively, the utility flows from consumption and work in the DM and CM in the baseline economy. Using (5.8) and (5.6), respectively, we derive equilibrium allocations for the perfectly competitive and no-banking economies. We calculate welfare for these economies using the appropriate analogs to (6.1).

Figure 9: Welfare effects of banking: With banks versus without banks



In Figure 9 the green-solid and blue-dashed lines show the differences between welfare in the no-bank economy and our baseline and perfectly competitive banking economies, respectively. Near the Friedman rule banking results in little welfare gain, regardless of the degree of competition, as the value of insuring liquidity risk is very small. As inflation increases, these gains increase, although imperfect competition for deposit erodes them to an increasing extent as the average deposit spread increases (see Figure 7). As inflation continues to rise, the value of the insurance provided by the banking system is eroded by the inflation tax, and eventually returns to zero.[17] Quantitatively, banks' provision of insurance also encourages capital investment by increasing demand in the DM, recalling Proposition 4. Again, market power erodes this effect by reducing real balances carried into the frictional market, again to an increasing extent as inflation rises.

7 Banking Market Power and Growth

Given that monetary policy interacts with banks' market power to affect capital accumulation, it is natural to consider the implications for the economy's long-run growth path. To this end, we augment the model with exogenous labor-augmenting technological progress in an exercise similar in spirit to that of Waller (2011).

The basic structure of the model remains the same as in Section 3, we discuss only the new features here and refer to Appendix B for details. CM production now follows:

$$Y = F(K, AH) = K^{\alpha}(AH)^{1-\alpha}, \tag{7.1}$$

where A represents labor augmenting technical change and evolves via $A_+ = (1+\mu)A$. In the DM, output is produced via the technology

$$q^s = f(k, Ae) = k^{\psi}(Ae)^{1-\psi}, \qquad \text{where} \quad \psi < 1. \tag{7.2}$$

DM sellers again produce using capital k and effort e, and the production cost is:

$$c\left(\frac{q^s}{A}, \frac{k}{A}\right) = \left[\frac{q^s}{A}\right]^{\omega}\left[\frac{k}{A}\right]^{1-\omega}, \quad \text{where} \quad \omega \equiv 1/\psi > 1. \tag{7.3}$$

7.1 Equilibrium

We again restrict attention to equilibria featuring money, credit and capital. As such, we need to rely on mild restrictions such as in Lemma 3. Following Waller (2011), we posit logarithmic utility functions in both the DM and CM. We restrict attention to cases in which the nominal policy interest rate is positive, but not too high. Here the condition $\omega\sigma > \alpha(\omega+\sigma-1)$ reduces to $\alpha < 1$, which is satisfied given CM production technology that we consider, (7.1). Given initial stocks of capital, K_0, and money, M_0, an equilibrium with money, credit and capital satisfies the following equations:

[17] This characteristic of what we might call a *hyperinflationary* regime is consistent with the findings of Berentsen et al. (2007).

1. Money demand with DM goods market clearing ($(1-n)q^s = nq$) imposed:

$$\frac{\phi U_x(x)}{\phi_+ U_x(x_+)} = \beta \left\{ (1-n) \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G_+(i_d)](1+i_d)\mathrm{d}G_+(i_d) + n \left[\frac{u^{'}(q_+)}{c_q\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}} \right] \right\}, \tag{7.4}$$

2. The Euler equation for capital investment:

$$\frac{U_x(x)}{\beta U_x(x_+)} = 1 + F_K(K_+, A_+H_+) - \frac{1}{U_x(x_+)}(1-n)c_k\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}. \tag{7.5}$$

3. Optimal consumption and labour in the CM:

$$U_x(X) = \frac{1}{F_H(K, AH)A}. \tag{7.6}$$

4. CM goods market clearing:

$$F(K, AH) = X + K_+ - (1-\delta)K. \tag{7.7}$$

7.2 Balanced growth

Following Waller (2011), we focus on balanced growth paths (BGP's) with constant labor hours and real variables $(q, X, K_+, \phi M)$ all growing at rate $1+\mu$. Growth of the real money stock on a BGP satisfies

$$\underbrace{1+\tau}_{\text{gross growth of money stock}} = \underbrace{(1+\pi)}_{\text{gross inflation rate}} (1+\mu), \tag{7.8}$$

and the nominal policy interest rate i is

$$1 + i = \frac{(1+\pi)(1+\mu)}{\beta} = \frac{1+\varphi}{\beta}, \tag{7.9}$$

where $1/\beta$ is the gross risk-free real interest rate.[18]

Adjust the deposit rate distribution for growth: $G_+([1+\varphi]i_d) = G(i_d)$, and let $\hat{K} \equiv K/AH$ denote the capital-to-effective-labor ratio. The system (7.4)—(7.7) reduces to the following

[18] As before, banks face perfect competition in the loan market but not in the deposit market and so the lending rate equals the policy interest rate $i^l = i$.

single equation in $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K}), \tag{7.10}$$

where

$$\bar{\theta} \equiv n(1-\alpha)\left(\frac{\omega-1}{\omega}\right), \qquad \omega > 1, \quad n \in (0,1) \tag{7.11}$$

$$\tilde{C}(i) \equiv 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \underbrace{\frac{1}{n}\left[i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)\right]}_{\text{average interest spread}} \tag{7.12}$$

$$\tilde{f}(\hat{K}) \equiv \hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}, \qquad \alpha \in (0,1). \tag{7.13}$$

The left-hand side of (7.10) captures the real gross risk-free interest rate adjusted for growth. The right-hand side of the equation captures the gross return on capital and can be decomposed into two parts relating to trades in the CM and the DM. As the left-hand side is constant and the right-hand side is monotone decreasing in $\hat{K}$, there exists a unique $\hat{K}^\star$ given policy rate i. Having solved for $\hat{K}^\star$, the following equations determine $\{q, X, K, H\}$, *i.e.*, DM consumption, CM consumption, capital and labor along the balanced growth path.

$$X = (1-\alpha)\hat{K}^{\alpha} A \tag{7.14}$$

$$K = \frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}} A \tag{7.15}$$

$$H = \frac{1-\alpha}{1-(\delta+\mu)\hat{K}^{1-\alpha}} \tag{7.16}$$

$$q = A\left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\tilde{C}(i)\right]^{-\frac{1}{\omega}}\left[\frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}\right]^{\frac{\omega-1}{\omega}}. \tag{7.17}$$

Restricting attention to $i > 0$, in showing the effects of market power on the BGP, it is useful first to consider three special cases: perfect competition in banking; and economy without banks; and the neoclassical economy with no monetary distortion. Details can be found in Appendix B.1.2.[19]

[19] As in Waller (2011), the Friedman rule ($i = 0$) achieves the first-best. In the absence of market power in banking, the only distortion in the economy is that associated with the need to use money which is subject to the inflation tax. Moreover, with $i = 0$, banks generate no gain by intermediating liquidity across agents and thus cannot exert market power, regardless of α_1 and α_2.

1. Perfect competition in banking

As in Section 5.3, this is equivalent to setting $\alpha_2 = 1$. The deposit rate distribution is degenerate at the policy interest rate, i, and the interest rate spread on deposits is zero. As such, (7.10) becomes:

$$\begin{aligned}\frac{1+\mu}{\beta} &= (1+\alpha\hat{K}^{\alpha-1}-\delta) \\ &\quad + \left[n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\right]\left[1+i\right]^{-1}\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right],\end{aligned} \tag{7.18}$$

where $\omega > 1$, $0 < n < 1$ and $0 < \alpha < 1$.

2. An economy without banks.

If $\alpha_1 = \alpha_2 = 0$, (7.10) becomes:

$$\begin{aligned}\frac{1+\mu}{\beta} &= (1+\alpha\hat{K}^{\alpha-1}-\delta) \\ &\quad + \left[n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\right]\left[1+\frac{i}{n}\right]^{-1}\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right].\end{aligned} \tag{7.19}$$

3. A non-monetary neoclassical economy.

With either $n = 0$ or $\omega = 1$, there is no trading in the anonymous DM and thus no need for money. Moreover, there is no use for capital in the DM. In this case the analogous expression to (7.10) is

$$\frac{1+\mu}{\beta} = 1+\alpha\hat{K}^{\alpha-1}-\delta. \tag{7.20}$$

Comparing first Economies 1 and 2, note from either (7.18) or (7.19), the gross risk-free real interest rate adjusted for growth is the same in both cases. Comparing the right-hand sides of the two equations, however, we can see that for any policy rate $i > 0$, $\hat{K}$ is higher on the BGP in the economy with banking (Case 1) than in that without (Case 2). This is not surprising. As shown above, banking increases the value of money and thus the return to capital in the DM resulting in more investment and a higher capital-to-effective labor ratio on the BGP.

Comparing (7.10) with (7.18), it can be seen that market power in banking diminishes reduces the incentive to accumulate capital and thus $\hat{K}$ on the BGP relative to the case of perfect competition. By lowering the average deposit rate, market power weakens the insurance provided by the banking system. This lowers the value of real balances, the returns to capital in the DM and thus, ultimately, $\hat{K}$. Comparing, however, (7.10) and (7.20), it can be seen that $\hat{K}$ is nevertheless higher in the monetary economy with imperfect competition (or even without banking) than in the non-monetary neoclassical model, a finding consistent with those of Waller (2011).

7.3 Transitional Dynamics

Setting $\delta = 1$ (so that employment is constant at all times) it is possible to characterize the transitions paths of these economies analytically. Details can be found in Appendix B.1.3. Off the BGP, the growth rate of $\hat{K}_+$ in the baseline model is

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{7.21}$$

where

$$\tilde{C}(i) \equiv 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \frac{1}{n}\left[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average interest rate spread on deposits}}\right]. \tag{7.22}$$

It is again useful to compare the three cases introduced above to our benchmark economy. First, in the neoclassical non-monetary economy inflation and monetary policy do not affect investment. In this case, the growth rate of capital along the transition path is given by

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\alpha\beta\hat{K}^{\alpha-1}\right]. \tag{7.23}$$

As noted by Waller (2011), this growth rate is lower than that arising in monetary model without banks,

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{7.24}$$

owing to the increased return to investment associated with the usefulness of capital in the DM. Moreover, competitive banking further increases the return to investment by raising real balances and expanding goods trade in the DM. Thus, with competitive banks the growth rate along the transition is higher still:

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{7.25}$$

Finally, comparing (7.21) and (7.22) with both (7.25) and (7.24), we have that the growth rate along the transition path in the baseline economy lies between that in the no-bank and perfectly competitive banking cases. Market power lowers deposit rates relative to the competitive case, thus reducing the returns to investment. Even imperfectly competitive banks, however, provide some insurance, and so investment remains higher than in the no-bank case.

8 Central bank digital currency

We now the potential of a central bank digital currency (CBDC) to alleviate some of the effects of banking market power considered above. To this end, we model an interest-bearing CBDC long the lines of that studied by Andolfatto (2021).[20] Here we focus solely on the impact of CBDC on the effectiveness of banks' liquidity transformation activities through the lens of deposit pricing.

We envision the monetary authority issuing an interest-bearing alternative to private bank deposits, and thus having two separate policy tools, trend inflation γ (or nominal policy rate i) and the interest rate paid on CBDC, i^{CBDC}. We continue to consider trend inflation as an exogenous requirement, set independently of the interest (if any) paid on CBDC.

In principle, the central bank could combine any trend inflation rate, γ, with any CBDC interest rate, i^{CBDC}, using appropriate lump-sum taxes in the CM. We rule this out by assuming that CM transfers must be non-negative, so that:

1. trend inflation must be positive, and

2. the net interest rate on CBDC must be less than or equal to the policy rate, $i^{CBDC} \leq i$.

Further, we assume that the central bank faces (unmodeled) costs associated with CBDC, effectively imposing a limit on CBDC interest $i^{CBDC} \leq \bar{i} < i$.[21]

Given these assumptions, the basic structure of the model remains the same as before with the only difference being that CBDC now competes with private bank deposits. While depositors can simultaneously hold both private deposits and CBDC, they will optimally choose only that which pays a higher interest rate. As such, the equilibrium distributions of deposit rates and spreads now depends on both the policy and CBDC interest rates.

In this setting, CBDC increases the outside option to bank deposits and can thus discipline market power in deposits.[22] We summarize our main result regarding the effects of CBDC on capital accumulation in the following Proposition. For the proof see Appendix C.4.

[20] For more details, please see Appendix C.

[21] This rules out a *trivial* second-best policy of setting $i^{CBDC} = i$ and effectively forcing the banking sector to pay the policy rate on deposits and effectively restoring the competitive banking outcome.

[22] For detailed calculations see Appendix C.3

Proposition 6. *Assume $0 < i^{CBDC} \leq \bar{i}$ and $\alpha_1 \in (0,1)$. In the search economy, interest-bearing CBDC increases the growth rate of the capital stock:*

$$g_k < g^{CBDC}(i) \leq g_k^{CBDC}(\bar{i}) < g_k^{PC},$$

and $g_k^{CBDC}(i)$ approaches to g_k^{PC} as $i^{CBDC} \to i$.

In the presence of interest-bearing CBDC, the lower support of the banks' distribution of posted deposit rates becomes i^{CBDC}. To the extent that the CBDC interest exceeds the rate on cash (*i.e.* zero), CBDC thus compresses the distribution of posted deposit rates upward and raises the expected return on deposits. This increases real balances, demand in the DM, and ultimately capital accumulation.

Note that as long as the return on CBDC is less than the policy rate, all potential depositors choose private bank deposits, in spite of the spreads extracted by imperfectly competitive banks. The reason for this is that with probability one each depositor has at least one deposit opportunity that offers a return superior to that on CBDC. As such, CBDC is not held in equilibrium and the central bank effectively pays no interest.

These findings complement those of Chiu et al. (2019) and Andolfatto (2021) in several ways. First, here we show that interest-bearing CBDC can not only reduce the effects of banks' market power, but also can do so at no cost to the central bank in terms of either paying interest or having to adjust its policies to maintain its fixed inflation rate. Moreover, the real value of deposits is increased (as in Andolfatto (2021)) and both the short-term growth rate of capital and long-run capital-labour ratio are increased. The mechanism is related also to the *latent medium of exchange* studied in Lagos and Zhang (2021), who show that even if the use of money in the frictional goods market goes to zero, monetary policy that governs the value of the outside option can still be effective in disciplining equilibrium allocations.

9 Conclusion

We considered and equlibrium model of money, capital and imperfectly competitive banking that sheds new light on the deposit channel of monetary policy recently popularized by Drechsler et al. (2017). Specifically, our version of the channel is one that affects money demand and capital accumulation through an equilibrium distribution of deposit rates and spreads relative to the policy rate.

The theory generates a positive relationship between the average deposit spread. This is consistent with the evidence provided in Drechsler et al. (2017). Moreover, the theory also generates a positive relationship between the average deposit rate spread and its standard deviation which is consistent with empirical evidence using the bank-branch level data for the United States.

In equilibrium, changes in the policy rate are passed through differentially and imperfectly to deposit rates. The extent of market power thus has implications to aggregate activity not only through the value of real money balances, but also through the return to investment and thus the rate of capital accumulation and the economy's long-run growth path. Finally, an interest-bearing CBDC can discipline market power and improve the aggregate outcome even if no households hold it and no interest is paid in equilibrium.

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Appendix

A Omitted proofs

In this online appendix, we provide the intermediate results and proofs for the main results of the model. We lay out the structure of this online appendix as follows.

First, we provide the details for characterizing the posted deposit-rate cumulative distribution function $G(i_d;\gamma)$ in Section A.1. We also provide a discussion on the transacted deposit-rate cumulative distribution function $J(i_d;\gamma)$ afterwards.

Second, we characterize both the steady-state Euler equations for the money demand and the investment demand in Section A.3.

Third, given mild restrictions on the model, we provide intermediate results and proofs for the existence and uniqueness of a stationary monetary equilibrium (SME) co-existing with money, capital and credit in Section A.4.

Fourth, we also provide a proof for the first best allocation result in Section A.4.3. In such a regime, banks generate no additional gains in redistributing liquidity among households.

Fifth, we study how the allocation in an economy with imperfectly competitive banks differs from that with perfect banking competition and no-banks in Section A.5. We then prove that allocation and welfare in a banking equilibrium (with and without market power) always dominate more than that in a no-bank equilibrium. However, allocation and welfare in an imperfectly competitive banking equilibrium are always lower than the equilibrium with perfectly competitive banks.

Sixth, we define measures of deposit-side bank market power by the average deposit-to-policy-rate spreads and the deposit-rate markdown. We then ask how the degree of banking market power on deposits responds to changes in monetary policy. The intermediate results and proofs pertaining to this question are contained in Section A.6.

Seventh, we consider an exogenous growth version of the model for analyses similar to those considered in Waller (2011). We lay out the characterizations of various cases of this model in Section B.

Last, we consider having an interest-bearing central bank digital currency (CBDC) along the lines of Andolfatto (2021). We then study the effects of CBDC on capital accumulation in Section C.

A.1 Deposit interest rate distribution

In this section, we provide the intermediate results and proofs from Section A.1.1 to Section A.1.5. Then we incorporate these results into the characterization of an analytical formula for the equilibrium posted-deposit-rate distribution G. The derived formula G is summarized in Section A.1.6.

A.1.1 Positive monopoly bank profit from deposits

Lemma 4. $\Pi^m(i_d) > 0$ *for any* $i - i_d > 0$.

Proof. Consider a bank's ex-ante problem defined in Equations (3.23) and (3.27). Hypothetically, a monopolist bank's profit can be derived as

$$\Pi^m(i_d) = \underbrace{nR^l(i_l)}_{\text{profit from loans}} + \underbrace{(1-n)\alpha_1 R(i_d)}_{\text{profit from deposits}} = n\underbrace{l[i_l - i]}_{=:\pi_l} + (1-n)\alpha_1 \underbrace{d[i - i_d]}_{=:\pi_d},$$

where l is the amount of loans, d is the amount of (inelastic) deposits, i_l is the loan interest rate, i is the central bank policy interest rate and i_d is the deposit interest rate.

First, consider the profit from loans. The interest rate that the lending bank would have earned by investing funds (with the central bank) is the policy rate i. Since banks are perfectly competitive on the loan side then the equilibrium loan interest rate must equal the opportunity cost of lending, $i_l = i$. It follows that the bank profit from making loans is zero in equilibrium, *i.e.*, $\pi_l = 0$.

We now consider the profit from deposits. Since the bank has market power in the deposits market, it can charge an interest spread on deposits. For any positive spread (or markdown) on deposit, *i.e.*, $0 < i - i_d$, then $\pi_d > 0$ and therefore $\Pi^m(i_d) > 0$. □

Remark. An opportunity for the bank to invest (idle) funds with the central bank at a competitive rate is a convenient modelling choice. This modelling strategy allows us to separate the bank lending and deposit-taking decision independent of each other, but both depend on the policy interest rate. Given the assumption that banks face perfect competition in the loans market, we drop the term π_l for the ease of notation when expressing banks' profit from here onward.

In what follows, we will denote $G(\cdot, \gamma)$ by $G(\cdot)$ or just G.

A.1.2 Monopoly deposit rate

Lemma 5. *The lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$.*

Proof. Consider a hypothetical bank serving depositors who have contacted only this one bank. The bank chooses deposit rate i_d to maximize profit, and the first-order condition is

$$\frac{\partial \Pi^m(i_d)}{\partial i_d} = (1-n)\alpha_1 \left[(i(\gamma) - i_d)\frac{\partial d^\star}{\partial i_d} - d^\star \right] = 0 \implies i_d = \mathcal{M}(i_d) i(\gamma), \tag{A.1}$$

where

$$\mathcal{M}(i_d) = \frac{\epsilon(i_d)}{1+\epsilon(i_d)} \text{ and } \epsilon(i_d) = \frac{i_d}{d^\star}\frac{\partial d^\star}{\partial i_d}.$$

We can think of $\epsilon(i_d)$ in Equation (A.1) as the elasticity of deposit supply, and $\mathcal{M}(i_d)$ captures the markdown for monopoly pricing on deposits. From here, we can see that the monopoly deposit interest rate is proportional to the policy rate depending on $\mathcal{M}(i_d)$. However, the term $\partial d^\star / \partial i_d = 0$ implies $\epsilon(i_d) = 0$ and therefore $i_d^m = 0$. Since the nominal deposit rate cannot go negative, it follows that the lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$. □

Remark. The reason why a hypothetical monopoly bank pays zero deposit interest is as follows. After realizing the households' preference shocks for consumption and production, sellers cannot readjust the amount of money balances they have already carried into the DM. In other words, the households' decision to bring money balances into the DM is sunk because the decision has already been made in the previous CM. Since the bank matches with depositors who have contacted only this one particular bank, the bank acts as a monopoly bank. Consequently, the bank can exercise its full market power to pay no interest on deposits $i_d^m = 0$. The zero–monopoly interest rate does not induce a zero–deposit supply in the banking system. Only a fraction of sellers who happen to be unlucky obtain zero interest on their idle funds.

A.1.3 All banks earn positive expected profit

Lemma 6. $\Pi^\star > 0$.

Proof. The expected profit from posting a deposit interest rate i_d is given by

$$\begin{aligned}\Pi(i_d) &= (1-n)[\alpha_1 + 2\alpha_2 - 2\alpha_2 G(i_d) + \alpha_2\eta(i_d)]R(i_d)\\ &= (1-n)\alpha_1 R(i_d^m)\\ &= \Pi^m(i_d^m) > 0,\end{aligned}$$

where $R(i_d) = \underbrace{d^\star}_{\text{deposit}} \underbrace{(i - i_d)}_{\text{deposit spread}}$, the first two lines are implied by equilibrium equal profit condition, and the last line follows from Lemma 4. In particular, we have

$$\Pi^\star = \max_{i_d} \Pi(i_d) = \Pi^m(i_d^m) > 0 \text{ for all } i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d].$$

□

A.1.4 Distribution is continuous

Lemma 7. *$G(\cdot, \gamma)$ is a continuous distribution function.*

Proof. Suppose there is a $i_d^0 \in \text{supp}(G)$ such that $\eta\left(i_d^0\right) > 0$ and

$$\Pi\left(i_d^0\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^0\right) + \alpha_2\eta\left(i_d^0\right)\right] R\left(i_d^0\right).$$

Given the per-deposit profit function R is continuous in deposit rate i_d, there is a $i_d^1 < i_d^0$ such that $R\left(i_d^1\right) > 0$ and $\Delta \equiv R\left(i_d^0\right) - R\left(i_d^1\right) < \frac{\alpha_2\eta\left(i_d^0\right)R\left(i_d^0\right)}{\alpha_1+2\alpha_2}$. Then

$$\begin{aligned}\Pi\left(i_d^1\right) &= (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^1\right) + \alpha_2\eta\left(i_d^1\right)\right] R\left(i_d^1\right)\\ &\geq (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^0\right) + \alpha_2\eta\left(i_d^0\right)\right]\left[R\left(i_d^0\right) - \Delta\right]\\ &\geq \Pi(i) + (1-n)\left\{\alpha_2\eta\left(i_d^0\right)\left[R\left(i_d^0\right) - \Delta\right] - (\alpha_1 + 2\alpha_2)\Delta\right\},\end{aligned}$$

where the second line follows from $G(i_0^d) - G(i_1^d) \geq \eta(i_0^1)$. Since $R(i_d^0) > \Delta$ and $\Delta < \alpha_2\eta(i_d^0)R(i_d^0)/(\alpha_1 + 2\alpha_2)$, then the last line implies $\Pi\left(i_d^1\right) > \Pi\left(i_d^0\right)$. This contradicts $i_d^0 \in \text{supp}(G)$. □

A.1.5 Support of distribution is connected

Lemma 8. *The support of G, $\text{supp}(G)$, is a connected set.*

Proof. Suppose deposit rates $i_d, i_d^{'} \in \text{supp}(G)$ with $i_d^{'} < i_d$ and $G(i_d) = G(i_d^{'})$. The bank's expected profit evaluated at these two deposits are respectively given by

$$\Pi\left(i_d\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d\right)\right] R\left(i_d\right),$$

and,

$$\Pi\left(i_d^{'}\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G(i_d^{'})\right] R\left(i_d^{'}\right).$$

Since by assumption we have $G\left(i_d\right) = G\left(i_d^{'}\right)$, then the probability weighting function in the two profit evaluations above are identical, *i.e.*,

$$(1-n)\left[\alpha_1 + 2\alpha_2 G(i_d)\right] = (1-n)\left[\alpha_1 + 2\alpha_2 G(i_d^{'})\right].$$

Since $i_d, i_d^{'} \in \text{supp}(G)$ and we have $i_d^m \leq i_d^{'} < i_d \leq \bar{i}_d < i \equiv \frac{\gamma-\beta}{\beta}$. The bank's profit-margin per deposit is then strictly decreasing for all $i_d \in [i_d^m, \bar{i}_d]$, meaning that the bank earns a lower deposit spread if they price closer to the central bank policy interest rate i. Hence, we have $R(i_d^{'}) > R(i_d)$ and therefore $\Pi(i_d^{'}) > \Pi(i_d)$, which violates the equal profit condition $(\Pi(i_d^{'}) = \Pi(i_d) = \Pi^\star)$ for all deposit rates chosen from the support of the distribution in equilibrium. □

A.1.6 Proof of Proposition 1: Deposit-rate distribution

Proof. The probability of obtaining one deposit-rate quote is $\alpha_1 \in (0,1)$. Since G has no mass points from Lemma 8,

$$\Pi = (1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma)]R(i_d;\gamma),$$

and profit is maximized at $i_d^m = \underline{i}_d$ from Lemma 4,

$$\Pi^\star = (1-n)\alpha_1 R(i_d^m;\gamma).$$

By equal profit condition, for any $i_d \in \text{supp}(G) = [i_d^m, \bar{i}_d]$, we have $\Pi(i_d) = \Pi(i_d^m)$ such that

$$(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma)]R(i_d;\gamma) = (1-n)\alpha_1 R(i_d^m;\gamma). \tag{A.2}$$

Solving Equation (A.2) for the cumulative distribution function G, we have an analytical expression in Proposition (1). Specifically, the analytical formula for the case with non-degenerate distribution $G(\cdot, \gamma)$ is:

$$G(i_d; \gamma) = \frac{\alpha_1}{2\alpha_2}\left[\frac{R(i_d^m; \gamma)}{R(i_d; \gamma)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{d^\star(i - i_d^m)}{d^\star(i - i_d)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{i - i_d^m}{i - i_d} - 1\right], \tag{A.3}$$

where $i := i(\gamma) = (\gamma - \beta)/\beta$.

Finally, given the lower support of the distribution G, $\underline{i}_d = i_d^m = 0$ from Lemma 5, and the fact that G is a cumulative distribution function, we can then back out the upper support of G by using the equal profit condition (A.2), and we obtain $\bar{i}_d = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d] < i$.

This establishes the first case in Proposition 1. Proofs for the remaining cases follow directly from Burdett and Judd (1983).

□

Remark. Note that the associate density of the distribution G is characterized by $\tilde{g}(i_d; \gamma) = \partial G(i_d; \gamma)/\partial i_d$. Moreover, a depositor randomly receives deposit-rates quote from banks, which can be one quote or two quotes with probability α_1 and $\alpha_2 = 1 - \alpha_1$ respectively. So, the cumulative distribution function of transacted deposit rates can then be described by

$$J(i_d; \gamma) = \alpha_1 G(i_d; \gamma) + \alpha_2 [G(i_d; \gamma)]^2 \text{ for all } i_d \in \text{supp}(G),$$

and the associate density of $J(i_d; \gamma)$ is given by

$$j(i_d; \gamma) \equiv \partial J(i_d; \gamma)/\partial i_d = \alpha_1 \tilde{g}(i_d; \gamma) + 2\alpha_2 G(i_d; \gamma)\tilde{g}(i_d; \gamma) = [\alpha_1 + 2\alpha_2 G(i_d; \gamma)]\tilde{g}(i_d; \gamma).$$

A.2 Deposit-rates distribution and inflation

A.2.1 Proof of Lemma 1: First-order stochastic dominance and inflation

Proof. Consider the economy away from the Friedman rule: $\gamma > \beta$. The analytical formula for the deposit-rate distribution $G(i_d; \gamma)$ is characterized in Proposition 1. Let $i_\gamma := \partial i(\gamma)/\partial\gamma$ denote the partial derivative of the policy rate with respect to inflation γ.

Now consider how the value of G varies with γ at each fixed i_d such that $0 = \underline{i}_d < i_d < \bar{i}_d$. We have that

$$\frac{\partial G(i_d; \gamma)}{\partial \gamma} = \frac{\alpha_1}{2\alpha_2}\left[\frac{i_\gamma(i - i_d) - i(i_\gamma - i_{d,\gamma})}{(i - \bar{i}_d)^2}\right] = -\frac{\alpha_1}{2\alpha_2}\left[\frac{i_\gamma i_d}{(i - i_d)^2}\right],$$

where $i_\gamma = 1/\beta > 1$ and the second equality obtains since for fixed i_d, $i_{d,\gamma} = 0$.

Since all the other terms are strictly positive, we therefore have, for every fixed $i_d \in (\underline{i}_d, \bar{i}_d) = \text{supp}(G)$, $\partial G(i_d, \gamma)/\partial\gamma < 0$. Thus, we establish that the posted-deposit-rate distribution $G(i_d; \gamma')$ first-order stochastically dominates $G(i_d; \gamma)$ for $\gamma' > \gamma > \beta$.

□

Remark. We have now characterized the relationship between the posted deposit interest rates distribution G and anticipated inflation in Section A.2.1. Since the transacted deposit interest rates distribution J is just a probability re-weighting of the distribution G, the conclusions above regarding inflation and G also apply to J. Hence, we leave out the details here. Instead, we use distribution G for the proof below.

A.2.2 Proof of Lemma 2: Average deposit rate and inflation

Proof. Given monetary policy γ, the nominal policy rate is determined by $i := i(\gamma) = (\gamma - \beta)/\beta$. We first consider the first statement in Lemma 2. First, apply integration by parts to Equation (4.6). This yields

$$g(\gamma) = [i_d G(i_d;\gamma)]_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} \frac{\partial i_d}{\partial i_d} G(i_d;\gamma)\mathrm{d}i_d = \bar{i}_d(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G(i_d;\gamma)\mathrm{d}i_d.$$

We want to show that $\partial g(\gamma)/\partial\gamma > 0$. Using Leibniz' rule, we have

$$g_\gamma(\gamma) = \frac{\partial \bar{i}_d(\gamma)}{\partial\gamma} - \left[\frac{\partial \bar{i}_d(\gamma)}{\partial\gamma} + \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d\right] = -\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d > 0, \tag{A.4}$$

where $G_\gamma(i_d;\gamma) < 0$ follows from the result in Lemma 1.

Observe that the only difference between the average posted deposit rate and the average transacted deposit rate is that an additional probability weighting function appears in the latter. Hence, we can deduce that $\hat{g}(\gamma) \leq g(\gamma)$ holds since the average transacted rate cannot exceed the average posted rate. It follows that the transacted rate cannot grow faster than the posted rate. Therefore, we have $0 < \hat{g}_\gamma(\gamma) \leq g_\gamma(\gamma)$.

Next, we consider the second statement in Lemma 2. Recall that the lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$, which is invariant to inflation change since the "hypothetical" monopoly bank can always pay zero deposit interest. Using the equal profit condition: $i - \bar{i}_d = \frac{\alpha_1}{\alpha_1+2\alpha_2}[i - i_d^m]$, we can back out the upper support of the distribution by $\bar{i}_d = i[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$. Differentiate the upper bound of the support of distribution G with respect to inflation γ. We obtain $\frac{\partial \bar{i}_d(\gamma)}{\partial\gamma} = \frac{1}{\beta}[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$ and it satisfies that $0 < \frac{\partial \bar{i}_d(\gamma)}{\partial\gamma} < \frac{1}{\beta}$.

All together, it establishes that the upper bound of the support of the distribution, $\mathrm{supp}(G) = [\underline{i}_d, \bar{i}_d]$ shifts to the right and it becomes wider at a rate less than $1/\beta$ as inflation γ goes up. □

A.3 Money and Capital

Steady-state money demand Euler equation. Take the partial derivative of the DM value function in Equation (3.29) with respect to money balance. Evaluate this marginal value of money one period ahead, and combine this with the CM first order condition in Equation (3.11). Then rewrite this in terms of stationary variables to obtain the steady-state money demand Euler equation:

$$\begin{aligned}\frac{\gamma-\beta}{\beta} &= (1-n)\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1+2\alpha_2 G(i_d;\gamma)]i_d \mathrm{d}G(i_d;\gamma) \\ &+ \underbrace{n\mathbb{I}_{\{i_l\leq\hat{i}\}}\left[\frac{u_q(q)}{c_{q_s}(q_s,K)}-1\right]}_{\text{borrow}} + \underbrace{n\mathbb{I}_{\{i_l>\hat{i}\}}\left[\frac{u_{\hat{q}}(\hat{q})}{c_{\hat{q}_s}(\hat{q}_s,\hat{K})}-1\right]}_{\text{no borrow}},\end{aligned} \tag{A.5}$$

where $q=(z+\tau_b Z+\xi)/\rho$, $\hat{q}=(z+\tau_b z)/\rho$, $q_s=\frac{n}{1-n}q$ and $\hat{q}_s=\frac{n}{1-n}\hat{q}$.

The left-hand side of Equation (A.5) measures the marginal cost of carrying money balances. The right-hand side of Equation (A.5) measures the marginal value of bringing one extra unit of money balance into the DM, and it has two components. The first term captures the marginal value of depositing an additional unit of idle money balance when a seller does not want to consume in the DM. The second term is the net benefit (marginal utility minus marginal cost) of spending an extra unit money balance when a buyer wants to consume in the DM. However, the buyer may or may not take out a loan from the bank depending on whether her maximum willingness to borrow $\hat{i}$ exceeds the market interest rate i_l. Hence, the liquidity premium is associated with or without bank credit.[23]

Steady-state investment Euler equation. Following a similar procedure for deriving Equation (A.5), the investment Euler equation is characterized by

$$\frac{1}{\beta} = [1+F_K(K,H)-\delta]-(1-n)\mathbb{I}_{\{i_l\leq i\}}\left[\frac{c_K(q_s,K)}{U_X(X)}\right]-(1-n)\mathbb{I}_{\{i_l>i\}}\left[\frac{c_{\hat{K}}(\hat{q}_s,\hat{K})}{U_{\hat{X}}(\hat{X})}\right] \tag{A.6}$$

where $c_K(\cdot;K)<0$, $U_x(\cdot)=\overline{A}/F_H(K,H)$, $q_b=(z+\tau_b Z+\xi)/\rho$, $\hat{q}=(z+\tau_b Z)/\rho$, $q_s=\frac{n}{1-n}q$ and $\hat{q}_s=\frac{n}{1-n}\hat{q}$.

[23] Notice that the banking structure here is slightly different to the one in Berentsen et al. (2007). In a banking equilibrium of BCW, all deposit funds sourced from sellers have to be loaned out to borrowers by banks. Hence, lending is essential there to support a feasible deposit interest payment. Here, even if the banks make zero loans to the buyers in DM, they can invest all of their remaining funds with the central bank to earn a rate of return i. Moreover, the deposit rate $i_d\in\text{supp}(G)$ is always bounded above by the policy rate i. Feasible deposit interest paid to depositors is less of a concern here. However, we need to be careful under what condition bank credit exists in equilibrium and how the banks allocate their deposits between consumer loans and funds they invest with the central bank.

The left-hand side of Equation (A.6) captures the (gross) real interest rate. The right-hand side of Equation (A.6) captures the (gross) value of investing an extra unit of capital. The first component is the return incurred in CM production. The second component is the return incurred in a DM. This term reflects the additional gains from investing capital in reducing the ex-post marginal cost of production in the DM when a seller produces the goods. Also, the DM goods allocation will depend on whether it is associated with or without bank credit.

A.4 Stationary Monetary Equilibrium

In this section, we restrict attention to an SME with money, credit and capital. We prove its existence and uniqueness. The proof for this relies on two mild restrictions regarding parameters and anticipated inflation to be not too high.

A.4.1 Proof of Lemma 3: Money, credit and capital

Proof. First, recall that the technology in the CM is given by $F(K,H) = K^{\alpha}H^{1-\alpha}$, and the technology in the DM is given by $f(e,k) = e^{1-\psi}k^{\psi}$. Also, we can transform the DM production into a (utility) cost representation of the sellers such that $c(q,k) = q^{\omega}k^{1-\omega},\ \omega := 1/\psi$.

Given a restriction on both technology parameters in the CM and the DM, such that $0 < \alpha < 1$ and $0 < \psi < 1$, households always have an incentive to accumulate a positive amount of capital in the economy.[24] Next, we want to show positive ex-ante money demand and ex-post loan demand in the economy.

1. Recall that the buyer takes out a loan from the bank as long as the market interest rate on loans satisfy $0 < i_l \leq \hat{i}_l = \rho^{\sigma-1}(z + \tau_b Z)^{-\sigma}\frac{w(K,H)}{\bar{A}} - 1$. Also, recall that we have $c_q(q,k) = (\rho\bar{A})/w(K,H)$ where $w(K,H) = F_H(K,H)$ from the seller's optimization problem. Hence, we can rewrite the buyer's maximum willingness to borrow, $\hat{i}_l$ as

$$\hat{i}_l = \frac{[(z + \tau_b Z)/\rho]^{-\sigma}}{c_{q_s}(q_s, K)} - 1,$$

where $q_s = (\frac{n}{1-n})[\frac{z+\tau_b Z}{\rho}] \equiv (\frac{n}{1-n})\hat{q}$. Note that the buyer's maximum willingness to borrow $\hat{i}$ is equivalent to the value of an extra dollar spent (*i.e.*, a liquidity premium) in an otherwise pure-monetary economy.

Since banks face perfect competition in the loans market, then $i_l = i = (\gamma - \beta)/\beta$.

[24] Note: if $\alpha = 0$ and $\psi = 1$, then capital is irrelevant for production in both the CM and DM. Hence, what matters in equilibrium is the money demand Euler equation (A.5). In this case, we are back to a model of money and banking credit.

Hence, we can bound gross inflation γ (or equivalently the policy interest rate) by

$$\frac{\gamma-\beta}{\beta} \leq \hat{i}_l \implies \gamma \leq \beta\left[\frac{[(z+\tau_b Z)/\rho]^{-\sigma}}{c_{q_s}(q_s,K)}\right]. \tag{A.7}$$

Hence, if condition (A.7) holds, then there must be ex-post positive loan demand. In other words, if inflation is weakly smaller than the discounted gross value of a dollar spent in an otherwise pure-monetary economy, there is ex-post positive loan demand.

2. Given positive loan demand, combining the agent's first-order condition for money demand and investment, the steady-state money demand Euler Equation (A.5) becomes

$$\frac{\gamma-\beta}{\beta} = (1-n)\int_{\underline{i}_d}^{\bar{i}_d}\left[\alpha_1+2\alpha_2 G(i_d;\gamma)\right]i_d \mathrm{d}G(i_d;\gamma)+n\mathbb{I}_{\{i_l\leq\hat{i}_l\}}\left[\frac{u_q(q)}{c_q(\frac{n}{1-n}q,K)}-1\right], \tag{A.8}$$

where the goods allocation in the DM is supported by both real money balances and real loans.

Also, the steady-state investment Euler Equation (A.6) is given by

$$\frac{1}{\beta} = [1+F_K(K,H)-\delta]-(1-n)\mathbb{I}_{\{i_l\leq\hat{i}_l\}}\left[\frac{c_K(\frac{n}{1-n}q,K)}{U_X(X)}\right]. \tag{A.9}$$

□

A.4.2 Equilibrium

To restrict attention to the case with co-existing money, credit and capital, we need to rely on mild restrictions on parameters and anticipated inflation not to be too high. In particular, using the result in Lemma 3, the system of equations consists of:

1. Money demand Euler equation:

$$\frac{\gamma-\beta}{\beta} = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d}\left[\alpha_1+2\alpha_2 G(i_d;\gamma)\right]i_d \mathrm{d}G(i_d;\gamma)}_{=:\hat{g}(\gamma)}+n\left[\frac{u_q(q)}{c_q(\frac{n}{1-n}q,K)}-1\right], \tag{A.10}$$

where the DM goods market clearing condition $(1-n)q_s = nq$ is imposed.

2. Capital investment Euler equation:

$$\frac{1}{\beta} = [1+F_K(K,H)-\delta]-(1-n)\left[\frac{c_K(\frac{n}{1-n}q,K)}{U_X(X)}\right]. \tag{A.11}$$

3. CM labor market clearing:

$$\underbrace{\frac{\bar{B}}{X}}_{=U_X(X)} = \frac{\bar{A}}{F_H(K,H)}, \tag{A.12}$$

since we have assumed log-utility for CM consumption.

4. CM goods market clearing condition:

$$F(K,H) = X + K - (1-\delta)K. \tag{A.13}$$

Monetary policy works through the channel of agents' money demand and capital investment decisions. These, respectively, are governed by Equation (A.10) and Equation (A.11). This nominal-to-real link regarding the effects of monetary policy transmission is identical to that in Aruoba et al. (2011). The new feature here is the effect of monetary policy pass-through to the banking sector, captured by $\hat{g}(\gamma)$ in Equation (A.10). Hence, banking market power alters agents' incentives on accumulating money and capital.

Let $\hat{k} := K/H$, the system of equations down to one equation in terms of per-capita variable. To do this, we need to use the functional forms: $U(x) = \bar{B}\ln(x)$; $u(q) = \bar{C}\frac{q^{1-\sigma}-1}{1-\sigma}$; $F(K,H) = K^{\alpha}H^{1-\alpha}$ and $c(q,k) = q^{\omega}k^{1-\omega}$, where $\bar{B} > 0, \bar{C} = 1, \sigma < 1, \alpha < 1,$ and $\omega = \frac{1}{\psi} > 1$.

First, we combine Equation (A.12) with Equation (A.13) to obtain

$$K = \frac{\bar{B}(1-\alpha)\hat{k}}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}. \tag{A.14}$$

Next, recall that we have $F_K(K,H) = \alpha\hat{k}^{\alpha-1}$, $F_H(K,H) = (1-\alpha)\hat{k}^{\alpha}$ and $c_K(\frac{n}{1-n}q, K) = (1-\omega)[\frac{n}{1-n}q]^{\omega}K^{-\omega}$. Applying these equations in Equation (A.11). The capital investment Euler equation becomes

$$\frac{1}{\beta} = [1 + \alpha\hat{k}^{\alpha-1} - \delta] - \left[(1-n)(1-\omega)\left(\frac{n}{1-n}\right)^{\omega}(1-\alpha)\overline{A}^{-1}\right]q^{\omega}K^{-\omega}\hat{k}^{\alpha}, \tag{A.15}$$

Next, recall that we have the functional form for the marginal utility of DM consumption and the marginal cost of production. These, respectively, are $u_q(q) = q^{-\sigma}$, and $c_q(\frac{n}{1-n}q, K) = \omega(\frac{n}{1-n})^{\omega-1}q^{\omega-1}K^{1-\omega}$. Substitute these two equations in Equation (A.10) and rearrange, Then, we can express the money demand Euler equation as:

$$q = \left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\hat{C}(\gamma)\right]^{\frac{1}{1-\sigma-\omega}} K^{\frac{1-\omega}{1-\omega-\sigma}}, \tag{A.16}$$

where

$$\hat{C}(\gamma) := 1 + \frac{1}{n}\Big[i(\gamma) - (1-n)\hat{g}(\gamma)\Big] = 1 + \hat{g}(\gamma) + \frac{1}{n}\Big[\underbrace{i(\gamma) - \hat{g}(\gamma)}_{=:s(\gamma)}\Big], \tag{A.17}$$

and the average deposit interest rate $\hat{g}(\gamma)$ is given by Equation (A.10).

Next, we combine Equation (A.16) and Equation (A.14), and then substitute that into Equation (A.15) to get

$$\frac{1}{\beta} = \underbrace{1 + \alpha\hat{k}^{\alpha-1} - \delta}_{=:R^{CM}(\hat{k})} + \underbrace{\tilde{\theta}\Big[\hat{C}(\gamma)\Big]^{\frac{\omega}{1-\omega-\sigma}}\tilde{f}(\hat{k})}_{=:R^{DM}(\hat{k};\gamma)}, \tag{A.18}$$

where

$$\tilde{\theta} := \frac{1}{\bar{A}}\left[(\omega-1)(1-n)(1-\alpha)\left(\frac{n}{1-n}\right)^{\omega}\right]\left[\omega\left(\frac{n}{1-n}\right)^{\omega}\right]^{\frac{\omega}{1-\omega-\sigma}} > 0,$$

$$\tilde{f}(\hat{k}) := \left[\frac{\bar{B}(1-\alpha)}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}\right]^{\frac{\omega\sigma}{1-\omega-\sigma}}\hat{k}^{\frac{\omega\sigma-\alpha(\omega+\sigma-1)}{1-\omega-\sigma}},$$

and $\hat{C}(\gamma)$ is given by Equation (A.17).

In steady-state, the system of equations reduces to one equation in terms of $\hat{k}$ governed by Equation (A.18). This can be decomposed into two components reflecting the return on investing capital associated with the CM and DM trades. Also, the nominal policy interest rate satisfies $i(\gamma) = (\gamma-\beta)/\beta$. The new insight is that there is a policy-dependent interest rate spread on deposits captured by the term $s(\gamma)$ showing up in Equation (A.18). In equilibrium, the effects of monetary policy transmission matter for banking, goods trades and capital formation.

A.4.3 Proof of Proposition 2: First-best allocation

Proof. Suppose the economy is at the Friedman rule such that the real money stock grows at $1+\tau \equiv \gamma = \beta$. By the Fisher equation $1+i = \gamma/\beta$, it is then equivalently set to the nominal policy interest rate of zero at the Friedman rule, $i = 0$.

Also, suppose that the probability of obtaining two deposit interest rate quotes is less than one, $\alpha_2 < 1$. From Proposition 1, every deposit rate i_d in the support of the deposit interest rate distribution is lower than the policy interest rate i, *i.e.*, $\underline{i}_d \leq i_d \leq \bar{i}_d < i$. Since $i = 0$ holds at the Friedman rule, and the nominal interest rate cannot go negative, it follows that the integral term of Equation (A.18) collapses to zero. That is, the Friedman rule cannot support deposit interest dispersion in equilibrium.

Next, given that the integral term of Equation (A.18) collapses to zero and $i = 0$, it follows that we have a condition of $\hat{C}(\gamma) = 1$. This condition reflects that it is costless for agents to carry money balances across periods at the Friedman rule. Equivalently, this condition coincides with efficient trades in the DM such that $u_q(q) = c_q(q, k)$ holds at equilibrium.

Since $\hat{C}(\gamma) = 1$ holds at $\gamma = \beta$, then Equation (A.18) becomes:

$$\frac{1}{\beta} = [1 + \alpha\hat{k}^{\alpha-1} - \delta] + \tilde{\theta}\tilde{f}(\hat{k}), \tag{A.19}$$

where $\tilde{\theta}$ and $\tilde{f}(\hat{k})$ are identical to that shown in Equation (A.18).

Next, let $\tilde{\gamma} > \gamma = \beta$. Evaluate Equation (A.18) at $\tilde{\gamma}$, and compare this with Equation (A.19), we can deduce that $\hat{k}^{\star,FB} > \hat{k}^{\star}$ for any policy $\tilde{\gamma} > \beta$. From Equation (A.14) and Equation (A.16), both capital stock K and DM consumption q are positively related to $\hat{k}$. It follows that $q^{\star,FB} > q^{\star}$ and $K^{\star,FB} > K^{\star}$ for any policy $\tilde{\gamma} > \beta$. □

A.4.4 Proof of Proposition 3: Unique SME with money and credit

We are now ready to establish the existence and uniqueness of a class of SME featuring the co-existence of money, credit and capital.

Proof. Fix long-run inflation target γ such that $\bar{\gamma} \geq \gamma > \beta$ where $\bar{\gamma}$ is defined in the proof of Lemma 3 in Section A.4.1.

Observe the first term on the right-hand-side of Equation (A.18) is continuous and monotone decreasing in $\hat{k}$ since $\alpha - 1 < 0$. Second, the parameter in the DM cost function is assumed to be $\omega > 1$. As such, the product of the second term is non-negative. Third, observe that the term C is a constant with respect to $\hat{k}$. Forth, it is assumed that $\omega\sigma > \alpha(\omega + \sigma - 1)$ holds. It follows that $\tilde{f}(\hat{k})$ is monotone decreasing in $\hat{k}$ since $1 - \omega - \sigma < 0$. Fifth, the left-hand-side of Equation (A.18) is a constant with respect to $\hat{k}$. Therefore, there exists a unique solution $\hat{k}^{\star}$ to Equation (A.18).

Given $\hat{k}^{\star}$, Equation (A.14) pins down $K^{\star}$. Given $K^{\star}$, Equation (A.16) pins down $q^{\star}$. Given $\hat{k}^{\star}$ and $K^{\star}$, we can back out $H^{\star}$ using the definition of $\hat{k} = K/H$. Finally, we can back out $X^{\star}$ using the CM goods market clearing condition. Likewise, we can back out the other variables including real money balances $z^{\star}$, and real loans $\xi^{\star}$. Moreover $z^{\star} = Z^{\star}$ and $k^{\star} = K^{\star}$ in equilibrium. Details are omitted here.

Next, we want to establish the uniqueness of an SME with co-existing money, capital and credit. What remains is to show that aggregate banking feasibility is satisfied such that

$$\underbrace{n\xi^{\star}(z^{\star}; i_l, Z^{\star}, \gamma)}_{\text{total loans}} + b = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z^{\star} + \tau_b Z^{\star}) \mathrm{d}G(i_d;\gamma)}_{\text{total deposits}}.$$

First, notice that banks cannot lend more to borrowers than the amount they have sourced from depositors. That is, the total amount of loans demanded by borrowers (*i.e.*, buyers) cannot exceed the total amount of deposits supplied by depositors (*i.e.*, sellers) such that

$$n\xi^{\star}(z; i_l, Z, \gamma) \leq (1-n)\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z^{\star} + \tau_b Z^{\star}) \mathrm{d}G(i_d;\gamma).$$

If the above condition does not hold with equality, then the banks can invest any remaining funds b with the central bank to earn a rate of return i. As such, the aggregate banking-feasibility constraint is always balanced. Note: In an otherwise perfectly competitive banking sector, the above condition will always hold at equality given a market clearing interest rate. In such a case, the extra channel of investing idle funds with the central bank is redundant. The reason is that the amount of deposits will eventually be loaned out to borrowers in a perfectly competitive banking market.

Finally, we need to check whether a deposit interest payment is feasible. First, the loan market's zero-profit condition implies that the equilibrium loan interest rate is equal to the policy interest rate, $i_l = i$. Second, all deposit rate i_d in the distribution G, *i.e.*, $i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d]$ is strictly less than policy interest rate i. Third, banks source deposit funds to fund their assets. The cost of funds is lower than what the banks can earn via their assets. Hence, deposit interest paid to the depositors in the CM is always feasible. □

A.5 Equilibrium with banks versus without banks

In this section, we study how the allocation in an economy with banks differs from that without banks. We first discuss two special cases: (1) an economy with perfectly competitive banks and (2) an economy without banks. Then we provide the proof for Proposition 4 in Section A.5.1.

An economy with perfectly competitive banks. This case is equivalent to setting $\alpha_2 = 1$, so that G is degenerate on the singleton set $\{i = i_d = (\gamma - \beta)/\beta\}$ by Proposition 1. We then obtain

$$\frac{1}{\beta} = [1 + \alpha\hat{k}^{\alpha-1} - \delta] + \tilde{\theta}\left[\tilde{C}(\gamma)\right]^{\frac{\omega}{1-\sigma-\omega}} \tilde{f}(\hat{k}), \tag{A.20}$$

where $\tilde{C}(\gamma) := 1 + \frac{\gamma - \beta}{\beta} \equiv 1 + i(\gamma)$, and the rest of the terms are identical to Equation (A.18).

An economy without banks. In this case, we have $\alpha_1 = \alpha_2 = 0$ in which agents earn zero interest on their idle money balances, *i.e.*, $i_d = 0$. The system of equations is then reduced to

$$\frac{1}{\beta} = [1 + \alpha \hat{k}^{\alpha - 1} - \delta] + \tilde{\theta} \left[\breve{C}(\gamma) \right]^{\frac{\omega}{1 - \sigma - \omega}} \tilde{f}(\hat{k}), \tag{A.21}$$

where $\breve{C}(\gamma) := 1 + \frac{1}{n} \left(\frac{\gamma - \beta}{\beta} \right) \equiv 1 + \frac{i(\gamma)}{n}$, and the rest of the terms are identical to Equation (A.18).

A.5.1 Proof of Proposition 4: Banking versus no-banking allocations

Proof. Fix an anticipated inflation γ such that $\beta < \gamma \leq \bar{\gamma}$, where $\bar{\gamma}$ is defined in Section A.4.1. Given inflation $\gamma > \beta$, the nominal policy interest rate in steady state satisfies $i := i(\gamma) = (\gamma - \beta)/\beta > 0$.

Case 1. We first compare a perfectly competitive banking equilibrium to a no-bank equilibrium. Observe that from Equation (A.20) and Equation (A.21), the only difference across these two economies is due to the gross cost of accumulating money balances.

Since the measure of DM buyers satisfies $0 < n < 1$, then it follows that $\breve{C}(\gamma) = 1 + \frac{i}{n} > 1 + i = \tilde{C}(\gamma)$. This follows that the right-hand side of Equation (A.21) must be smaller than the right-hand side of Equation (A.20) due to these terms are raised to a negative power, $1 - (\sigma + \omega) < 0$. In words, the extra return on capital associated with DM trades is smaller in an economy without access to banks. Moreover, the right-hand side of Equations (A.20) and (A.21) are both monotonically decreasing in $\hat{k}$ using the result established in Proposition 3. Hence, the following order must hold

$$\hat{k}^{\star, No-bank} < \hat{k}^{\star, PC}, \tag{A.22}$$

for any given policy $\beta < \gamma$.

Recall that both DM consumption q and capital stock K are both positively related to $\hat{k}$. Given Condition (A.22), we can deduce that the following ranking holds:

$$q^{\star, No-bank} < q^{\star, PC} \quad \text{and} \quad K^{\star, No-bank} < K^{\star, PC}, \tag{A.23}$$

for any given policy $\beta < \gamma$.

In summary, Case 1 establishes that banking improves goods trades by increasing the additional return on capital associated with DM trades, relative to the economy without banks. The reason is that banking reduces the gross cost of accumulating money balances for households.

Case 2. Now we compare our baseline economy $\alpha_2 \in (0, 1)$ to an economy with a perfectly competitive banking sector ($\alpha_2 = 1$).

We will show the following: As long as $\alpha_2 \in (0, 1)$, we can show that the gross cost of accumulating money balances is higher in our baseline economy than that with perfectly competitive banks, *i.e.*, $\hat{C}(\gamma) > \tilde{C}(\gamma)$.

Suppose to the contrary that $\hat{C}(\gamma) \leq \tilde{C}(\gamma)$. Using the expressions of $\hat{C}(\gamma)$ and $\tilde{C}(\gamma)$, respectively, from Equation (A.18) and (A.20), we have

$$\hat{C}(\gamma) = 1 + \frac{1}{n}[i - (1-n)\hat{g}(i_d; \gamma)] \leq 1 + i = \tilde{C}(\gamma)$$
$$\implies i - \hat{g}(i_d; \gamma) \leq n[i - \hat{g}(i_d; \gamma)].$$

Since it is required that the n measure of DM buyers satisfies that $0 < n < 1$, there is a contradiction to the weak inequality. Thus, we have $\hat{C}(\gamma) > \tilde{C}(\gamma)$.

Applying a similar reasoning as in Case 1, we can conclude that

$$\hat{k}^{\star} < \hat{k}^{\star,PC}, \quad \text{and} \quad \hat{k}^{\star} \to \hat{k}^{\star,PC} \quad \text{as } \alpha_2 \to 1, \tag{A.24}$$

for any given policy $\beta < \gamma$.

Likewise, we can deduce that the following order must also hold:

$$q^{\star} \leq q^{\star,PC} \ \text{ and } \ K^{\star} \leq K^{\star,PC}, \tag{A.25}$$

given policy $\beta < \gamma$. The equality in Condition (A.25) holds when $\alpha_2 = 1$.

Case 3. Finally, we compare our baseline economy to a no-bank economy. From Equation (A.18) and Equation (A.21), it follows immediately that households face a higher gross cost of accumulating money balances in a no-bank equilibrium than the case with imperfectly competitive banks. The reason is that depositors, on average, can still benefit from (imperfectly competitive) banks by receiving a positive interest on idle money balances. This is better than being stuck with idle balances being subject to inflation tax. Using similar reasoning as above, we can also verify that the following relationships also hold

$$q^{\star,No-bank} \leq q^{\star} \ \text{ and } K^{\star,No-bank} \leq K^{\star}, \tag{A.26}$$

for any given policy γ such that $\beta < \gamma \leq \bar{\gamma}$. The equality in Condition (A.26) holds when $\alpha_1 = \alpha_2 = 0$. In other words, Case 3 says that having imperfectly competitive banks is welfare-improving relative to the no-bank equilibrium. This is because households still receive liquidity risk insurance through banking. Hence, imperfectly competitive banks can still improve goods trades, augmenting capital investment value relative to the no-bank equilibrium.

In summary, by Conditions (A.23)-(A.26), we have established the following order

$$q^{\star,No-bank} < q^{\star} < q^{\star,PC} \text{ and } K^{\star,No-bank} < K^{\star} < K^{\star,PC},$$

for any given policy γ such that $\beta < \gamma \leq \bar{\gamma}$. Moreover, we have $(q^{\star}, k^{\star}) \to (q^{\star,PC}, K^{\star,PC})$ as $\alpha_2 \to 1$ by Proposition 1. □

A.6 Deposit-rates spread, markdown and inflation

In this section, we consider two measures of bank market power in the deposit market: the average interest rate spread on deposits and the average deposit-rates markdown. We then study how bank market power responds to the change in the anticipated inflation, γ. We provided intermediate results and proofs in Section A.2.1 and Section A.2.2. We will apply these results in the proofs in Section A.6.1 and Section A.6.2 to see how monetary policy affects the degree of banking market power in deposits.

A.6.1 Proof of Proposition 5: Deposit-rates spread and inflation

Deposit-rates spread. Let the average interest rate spread on deposits be defined as the difference between the central bank policy interest rate and the average of deposit interest rates across banks. As such, the average posted interest rate spread on deposits is defined as

$$s(\gamma) = i(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d; \gamma), \tag{A.27}$$

where the distribution G is characterized in Proposition 1.

Proof. We first consider the average posted deposit-rates spread and make a few observations before we show how it changes with respect to the change in inflation. Recall that all deposit interest rate i_d in the support of the distribution G must be smaller than the policy interest rate i in a banking equilibrium. This means that $i(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d; \gamma)$, since all banks earn positive expected profit in equilibrium by marking down the deposit rate that they post from the result in Lemma 6.

Bank market power arises from the noisy search frictions in the deposit market. Therefore, we can first establish that the deposit spread is positive. That is, $i(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$ implies that $s(\gamma) > 0$, for any γ such that $\bar{\gamma} \geq \gamma > \beta$ and $\bar{\gamma}$ is defined in Section A.4.1.

Next, we consider how the average posted deposit-rates spread $s(\gamma)$ moves with respect to the change in inflation. Let the function $\hat{g}(\gamma)$ to denote the average posted deposit rates, *i.e.*, $\hat{g}(\gamma) := \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$.

Differentiate Equation (A.27) with respect to γ, we obtain

$$s_\gamma(\gamma) = i_\gamma(\gamma) - \hat{g}_\gamma(\gamma) \equiv i_\gamma(\gamma) - \left[- \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d \right]. \tag{A.28}$$

We show that the average deposit rate is increasing with respect to inflation from the result in Lemma 2, *i.e.*, $\hat{g}_\gamma(\gamma) > 0$ since $G_\gamma(\cdot) < 0$. We also show that the growth rate of the support of the distribution G is less than $1/\beta$ in Lemma 2. It follows that the integral function $\hat{g}_\gamma(\gamma)$ must be also less than $1/\beta$. Hence, we have $\frac{1}{\beta} > \hat{g}_\gamma(\gamma) > 0$.

Next, recall that the growth rate of the policy interest rate is given by $i_\gamma(\gamma) = 1/\beta$. Combining this result with the inequality above, then $i_\gamma(\gamma) > \hat{g}_\gamma(\gamma)$ implies that $s_\gamma(\gamma) = i_\gamma(\gamma) - \hat{g}_\gamma(\gamma) > 0$. This establishes that the average posted deposit-rates spread is increasing with inflation. Moreover, it follows that growth rate of the average posted deposit-rates spread is also bounded such that $\frac{1}{\beta} > s_\gamma(\gamma) > 0$ since $i_\gamma(\gamma) > i_\gamma(\gamma) - \hat{g}_\gamma(\gamma)$ holds.

□

Remark. Proposition 5 shows that:

1. There is an imperfect pass-through of monetary policy to the deposit rates; and
2. Banks are less competitive as they charge a higher deposit-rates spread when inflation goes up.

In other words, banks with market power in the deposits market extract more surplus from depositors when their need for liquidity insurance is high.

The intuition is as follows. First, an increase in the anticipated inflation (*i.e.*, equivalent to a rise in the nominal risk-free policy interest rate in a stationary equilibrium) makes it more costly for households to carry money balances across periods. As such, households need more banking to insure against the risk of holding idle balances. Second, the supply of deposits (from $1-n$ measure of DM sellers who have idle money balances) falls as inflation goes up. Consequently, banks can exploit their intensive-margin channel more (*i.e.*, a higher deposit-rate spread) to compensate for the losses from trading with fewer depositors with smaller money balances. Hence, higher inflation gives more market power to the banks in pricing their deposit rates. This distorts the gains from financial intermediation by more.

A.6.2 Proof of Proposition 7: Deposit-rate markdown and inflation

Alternatively, we can consider deposit-rates markdown as another measure of bank market power in the deposits market. Following the notation from previous sections, we first let the average posted-deposit rates be denoted by $\hat{g}(\gamma) = \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d, \gamma)$.

Since we focus on linear pricing strategies, then we can define a (gross) markdown $\mathcal{M}$ on average (gross) posted deposit-rates $1 + \hat{g}(\gamma)$ over the (gross) policy interest rate $1 + i(\gamma)$:

$$\mathcal{M} := \mathcal{M}(\gamma) = \frac{1 + \hat{g}(\gamma)}{1 + i(\gamma)}, \tag{A.29}$$

In this section, we study the relationship between $\mathcal{M}$and anticipated inflation γ. We now summarize our discussion formally in Proposition 7 and provide the proof below.

Proposition 7. *Assume $\bar{\gamma} \geq \gamma > \beta$, and $\alpha_1 \in (0, 1)$. Then, both the average posted- and transacted-deposit-markdowns are monotonically decreasing in inflation γ.*

Proof. Assume anticipated inflation γ satisfies $\bar{\gamma} \geq \gamma > \beta$. From the result in Proposition 5, we show that the increase in the average posted deposit rate is always less than the increase in the policy interest rate as inflation goes up, *i.e.*, $i_\gamma(\gamma) > \hat{g}_\gamma(\gamma) > 0$. It follows that the increase in the numerator of Equation (A.29) must be smaller than that in the denominator given an increase in inflation γ. Hence, the gross markdown on the average posted deposit rates is monotonically decreasing in inflation γ. □

Remark. The average posted deposit-rates markdown $\mathcal{M}$ measures the deviation away from the policy interest rate. As such, banks are more competitive if $\mathcal{M}$ is closer to one (*i.e.*, banks markdown less on deposits in which the average posted deposit rates are closer to the policy rate). Conversely, they are less competitive if $\mathcal{M}$ is closer to zero (*i.e.*, markdown more).

Proposition 7 shows that banks tend to markdown more on deposit rates as inflation increases. The intuition is that the supply of deposits falls when it is more costly for households to carry money balances across periods. So banks exploit more on their intensive-margin channel to make up the trading loss with fewer depositors with lower money balances as inflation increases.

In summary, Proposition 5 and Proposition 7 both measure the degree of banking market power responding to the change in monetary policy. It has the following implications for the welfare effects of banking. As inflation increases, the need for liquidity insurance against the risk of holding idle money balances is high. This effect gives more market power to the banks in which they can charge a larger deposit-rates spread (or markdown more). As such, banks extract more surplus from depositors, further distorting the gains from financial intermediation. Moreover, this distortion induces a lower allocation in the goods market as agents carry fewer money balances to trade, negatively impacting capital investment. Overall, the degree of banking market power distorts equilibrium allocation, and economic welfare varies with monetary policy changes (*i.e.*, anticipated inflation, or equivalently, the risk-free nominal policy interest rate).

B Long-run growth path

Here, we provide the details of the equilibrium description under exogenous growth found in Section 7. The basic structure of the model remains the same as in Section 3. We only lay out the new features here to avoid repetition. The difference here is that the aggregate production function in the CM is given by $Y = F(K, AH) = K^{\alpha}(AH)^{1-\alpha}$ where $\alpha < 1$, and A is a labor-augmenting technology factor. Moreover, A evolves according to the process $A_{+} = (1+\mu)A$. In the DM, output is given by $q^s = f(k, Ae) = k^{\psi}(Ae)^{1-\psi}$ where $\psi < 1$. Sellers produce q^s using capital k and effort e. The disutility cost of production for the sellers can also be expressed as $c(\frac{q^s}{A}, \frac{k}{A}) = (\frac{q^s}{A})^{\omega}(\frac{k}{A})^{1-\omega}$, where $\omega := 1/\psi > 1$.

B.1 SME with growth

To restrict attention to an equilibrium with money, credit and capital, we need to rely on mild restrictions on parameters, *i.e.*, $\omega\sigma > \alpha(\omega + \sigma - 1)$, and, the requirement that the nominal policy interest rate be positive and not too high. This restriction is similar to that discussed in Lemma 3. Since we consider log utility ($\sigma = 1$) in both the DM and the CM in order to compare results with Waller (2011) (and also for existence of a balanced growth path), the only restriction is to have a positive nominal policy interest rate that is not too high. This is because the restriction on parameters, $\omega\sigma > \alpha(\omega+\sigma-1)$, simplifies to the requirement $\alpha < 1$ when $\sigma = 1$. This requirement is automatically satisfied by the Cobb-Douglas CM production technology. For the ease of presentation, we normalize the preferences scale parameters to $\overline{A} = \overline{B} = \overline{C} = 1$ in what follows.

Given the formula for G, the equilibrium system of equations are as follows.

1. Money demand Euler equation:

$$\phi U_x(x) = \beta\phi_{+}U_x(x_{+})\left\{(1-n)\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G_{+}(i_d)](1+i_d)\mathrm{d}G_{+}(i_d) + n\left[\frac{u'(q_{+})}{c_q\left(\frac{n}{1-n}\frac{q_{+}}{A_{+}}, \frac{k_{+}}{A_{+}}\right)\frac{1}{A_{+}}}\right]\right\}, \tag{B.1}$$

where the DM goods market clearing condition $(1-n)q^s = nq$ is imposed.

2. Capital investment Euler equation:

$$U_x(x) = \beta U_x(x_+)\left[1 + F_K(K_+, A_+H_+) - \frac{1}{U_x(x_+)}(1-n)c_k\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}\right]. \tag{B.2}$$

3. CM labor market clearing:

$$U_x(X) = \frac{1}{F_H(K, AH)A}. \tag{B.3}$$

4. CM goods market clearing condition:

$$F(K, AH) = X + K_+ - (1-\delta)K. \tag{B.4}$$

An equilibrium with money, credit and capital solves Equations (B.1)-(B.4) given initial capital stock K_0 and money stock M_0.

B.1.1 Balanced-growth steady state: Baseline model

We now derive the conditions describing a steady state in our model. As in Waller (2011), we assume constant labor hours and the real variables, $(q, X, K_+, \phi M)$, all grow at the constant rate of $1+\mu$. The growth of real money stock satisfies

$$1 + \tau = (1+\pi)(1+\mu), \tag{B.5}$$

and the nominal policy interest rate i satisfies the growth-adjusted Fisher equation:

$$1 + i = \frac{(1+\pi)(1+\mu)}{\beta}. \tag{B.6}$$

Let $1+\varphi := (1+\pi)(1+\mu)$. In a steady state equilibrium under balanced growth, we have the deposit-rate distribution be such that $G_{+1}([1+\varphi]i_d) = G(i_d)$. Let $\hat{K} := K/AH$ denote the capital-to-effective-labor ratio.

Next, we derive the balanced-growth steady state using similar steps to that for the model without growth (see Appendix A.4.4). Given policy i, we can reduce the system of equations down to one equation solving for the steady-state point $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K}), \tag{B.7}$$

where

$$\tilde{C}(i) := 1 + \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d) + \frac{1}{n}\left[i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)\right],$$

$\bar{\theta} := n(1-\alpha)[(\omega-1)/\omega]$, and $\tilde{f}(\hat{K}) := \hat{K}^{\alpha-1}[(1-\alpha)/(1-[\delta+\mu]\hat{K}^{1-\alpha})]^{-1}$.

The left-hand side of Equation (B.7) captures the (gross) risk-free interest rate adjusted for growth. The two terms on the right-hand side of Equation (B.7), respectively, capture the (gross) return on capital used in the CM and that in the DM. Again, note that the return to capital in the DM is augmented by the deposit-side market power distortion term $\tilde{C}(i)$.

Proposition 8. *There is a unique balanced growth steady state equilibrium $\hat{K}^\star > 0$.*

Proof. The left-hand side of Equation (B.7) is constant with respect to $\hat{K}$. The right-hand side is monotone decreasing in $\hat{K}$: First, CM production exhibits diminishing returns to capital—*i.e.*, the term $\alpha\hat{K}^{\alpha-1} + (1-\delta)$ is a continuous and strictly decreasing function of $\hat{K}$. Second, the additional return to capital in the DM, $\bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K})$ is strictly decreasing in $\hat{K}$. Note that $\tilde{C}(i)$ is independent of $\hat{K}$ by virtue of Proposition 1. It is straightforward to verify that $\tilde{f}' < 0$. Therefore, there is a unique $\hat{K}^\star > 0$ satisfying Condition (B.7). $\square$

We can then back out the other endogenous variables (q, K, X, H) at the balanced-growth steady state $\hat{K}^\star$. In particular, we have

$$X = (1-\alpha)\hat{K}^\alpha A, \quad K = \frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}A, \quad H = \frac{1-\alpha}{1-(\delta+\mu)\hat{K}^{1-\alpha}},$$

and $q = A\left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\tilde{C}(i)\right]^{-\frac{1}{\omega}}\left[\frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}\right]^{\frac{\omega-1}{\omega}}$.

B.1.2 Balanced-growth steady state: Three special limits

Our steady-state characterization from Section B.1.1 nests three specials cases: A neoclassical growth model; a version of our monetary environment with perfectly competitive banks, *i.e.*, a combination of Aruoba et al. (2011) and Berentsen et al. (2007); and, a version of the monetary model without banks (Waller, 2011).

1. **Neoclassical growth model.** To obtain this case, we can set $n = 0$ or $\omega = 1$. Setting $n = 0$ shuts down the DM. Alternatively, setting $\omega = 1$ eliminates the additional return to capital associated with DM goods production. In the first setting, money plays no role in the CM and that makes the limit economy a neoclassical one. In the second, we have the result akin to Aruoba and Wright (2003) where nominal activity is

decoupled from the real economy. Either way, the limit economies imply that money is inconsequential to capital accumulation and growth. Either case imply the neoclassical growth model's steady state condition:

$$\frac{1+\mu}{\beta} = 1 + \alpha\hat{K}^{\alpha-1} - \delta. \tag{B.8}$$

2. **Perfect competition among banks.** This case, is equivalent to requiring $\alpha_2 = 1$ (depositors contact at most one bank) in our model. By Proposition 1 the deposit rate distribution is degenerate at the policy interest rate i and the interest rate spread on deposits is zero. The steady state fixed-point condition becomes

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\Big[1+i\Big]^{-1}\tilde{f}(\hat{K}), \tag{B.9}$$

where $\bar{\theta}$ and $\tilde{f}(\hat{K})$ are identical as in Equation (B.7).

3. **No-bank.** This case obtains if we set $\alpha_1 = \alpha_2 = 0$. It reduces to the (price-taking) setup in Waller (2011). In particular, we have

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\Big[1+\frac{i}{n}\Big]^{-1}\tilde{f}(\hat{K}), \tag{B.10}$$

where $\bar{\theta}$ and $\tilde{f}(\hat{K})$ are identical as in Equation (B.7).

From the right-hand-side of Equations (B.7)—(B.10), we can deduce the following order: $\hat{K}^{\text{neoclassical},\star} \underbrace{\leq}_{\text{“=” if } n=0 \text{ or } \omega=1} \hat{K}^{\text{no-bank},\star} < \hat{K}^{\star} < \hat{K}^{\text{PC},\star}$, given policy $i > 0$.

Suppose we let $n > 0$ and $\omega > 1$. In that case, we can see that the capital-per-effective-labor ratio in a monetary economy (with or without banks) is always higher than that in a neoclassical growth model. The reason is that capital has an additional value from reducing the cost of DM production. Moreover, having access to banks, in general, improves such benefits relative to the setting without banks. However, bank market power distorts some of these gains and this reduces the capital-to-effective-labor ratio in equilibrium relative to the case with perfect competition among banks.

B.1.3 Dynamics: Baseline model

We derive an analytical special case for the model's balanced-growth-path dynamics. This case will be comparable to the results in Waller (2011). To do so, we set the rate of depreciation for capital as $\delta = 1$. The equilibrium dynamical system simplifies to

$$\frac{q_{+1}}{\hat{K}_{+1}} = \left(\frac{1}{A_{+1}}\right)^{-\frac{1}{\omega}} \left[\omega \left(\frac{n}{1-n}\right)^{\omega-1} \tilde{C}(i)\right]^{-\frac{1}{\omega}} \hat{K}_{+1}^{-\frac{1}{\omega}}, \tag{B.11}$$

$$\left(\frac{X_{+1}}{X}\right)\frac{1}{\beta} = \alpha \hat{K}_{+1}^{\alpha-1} + X_{+1}(1-n)\left(\frac{\omega-1}{A_{+1}}\right)\left(\frac{n}{1-n}\right)\left(\frac{q_{+1}}{\hat{K}_{+1}}\right)^{\omega}, \tag{B.12}$$

and,

$$\hat{K}_{+1} = \frac{1}{1+\mu}\left[\frac{H}{H_{+1}} - \frac{1-\alpha}{H_{+1}}\right]\hat{K}^{\alpha}, \tag{B.13}$$

where

$$\tilde{C}(i) := 1 + \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d) + \frac{1}{n}\left[i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)\right].$$

Equation (B.11) is obtained by rearranging the money demand Euler equation. Equation (B.12) is the capital Euler equation. We then combine the CM labor- and goods-market-clearing conditions—Equation (B.3) and Equation (B.4)—to obtain Equation (B.13).

Combining Equation (B.11) and Equation (B.12) yields

$$\hat{K}_{+1} = \frac{\beta}{1+\mu}\left[\alpha + \frac{n(1-\alpha)(\frac{\omega-1}{\omega})}{H_{+1}}[\tilde{C}(i)]^{-1}\right]\hat{K}^{\alpha}. \tag{B.14}$$

Equations (B.13) and (B.14) jointly pin down the transitional dynamics of capital ($\hat{K}$) and labor (H).

Following Waller (2011), we consider constant hours (H) along a balanced growth path. The constant labor hours can be derived as

$$H = \frac{1-\alpha}{1-\alpha\beta}\left[1 + n\beta\left(\frac{\omega-1}{\omega}\right)\left[\tilde{C}(i)\right]^{-1}\right]. \tag{B.15}$$

This further yield the restriction on the dynamics of capital per effective worker:

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{B.16}$$

where

$$\tilde{C}(i) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \frac{1}{n}\left[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)}_{\text{average interest rate spread on deposits}}\right]$$

is the price-dispersion distortion effect in our model. This is the additional feature affecting capital growth dynamics relative to Waller (2011). Recall that this distortion arises from endogenous market power on the deposit side of banking.

B.1.4 Dynamics: Three special limits

As before, we can also derive three special parametric limits of our model:

1. **Neoclassical growth model.** This case obtains in our model if we set $n = 0$ or $\omega = 1$. The corresponding growth rate of capital in such an economy is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{neoclassical}}} = \frac{1}{1+\mu}\left[\alpha\beta\hat{K}^{\alpha-1}\right]. \tag{B.17}$$

2. **Perfect competition among banks.** Set $\alpha_2 = 1$. The corresponding growth rate of capital is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{PC}}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{B.18}$$

3. **No-bank.** Set $\alpha_1 = \alpha_2 = 0$. The corresponding growth rate of capital reduces to the (price-taking) setup in Waller (2011), which is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g^{\text{no-bank}_k}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{B.19}$$

From the right-hand-side of Equations (B.16)-(B.19), and given policy $i > 0$, we can deduce the following order: $g_k^{\text{neoclassical}} \underbrace{\leq}_{\text{“=”if } n=0 \text{ or } \omega=1} g_k^{\text{no-bank}} < g_k < g_k^{\text{PC}}$. This ranks the growth rate of capital across the different economies.

The following is similar to the reasoning in Section B.1.2. If $n > 0$ and $\omega > 1$, then starting from the same value of $\hat{K}$, capital in a monetary economy (with or without banks) is always accumulated at a faster rate than the neoclassical growth model. The reason is that capital investment in a monetary economy has an additional value in reducing the DM cost of production. Banking, in general, improves such benefits than the economy without banks. However, bank market power lowers the growth rate of capital relative to the economy with perfectly competitive banks.

C Central bank digital currency (CBDC)

In this section, we consider having an interest-bearing central bank digital currency (CBDC) made available to the public along the lines of Andolfatto (2021). The central bank now has two separate policy tools. One that targets the trend inflation γ (equivalently, the nominal rate $i = (\gamma - \beta)/\beta$) and one that controls the interest rate on CBDC, i^{CBDC}. Both γ and i^{CBDC} are exogenous parameters. The nominal policy rate i and the CBDC rate i^{CBDC} can differ.

The purpose here is to study how the presence of CBDC affects the endogenous market power on the deposit side of (private) banking arising from informational frictions. We then study the implications for deposit rate markdowns (and dispersion), capital formation and long-run growth.

For our purpose, we assume that private bank deposits and CBDC have no technological advantage over each other, as in Andolfatto (2021). The central bank provides lending and deposit facilities that private banks can borrow and lend at the same policy rate. Consequently, private bank deposit and lending decisions are separate, as presented in the main text. We also keep private banks' lending side of operations competitive. As before, the loan rate equals the policy rate and is not affected by the CBDC rate. However, the deposit rate is affected by both the policy rate and CBDC rate, when CBDC serves as an alternative depository facility for the households.

C.1 Overview

The model's basic structure remains the same as we have discussed thus far. The only differences are the (ex-post) sellers' problem and (private) banks' profit-maximization problem. We lay out the implications as follows.

First, $(1 - n)$ sellers can now choose where to deposit their unproductive idle funds m. Sellers can deposit their idle money balances m at the private bank saving account or the central bank CBDC account (or both). The such decision depends on the deposit rate offered by private banks and the CBDC rate set by the central bank. Since households can choose to deposit at the private or central banks, they optimally deposit at the one that offers a higher interest rate. That is, households optimally deposit zero (all) idle funds at the private banks (central bank) if $i_d < i^{CBDC}$, and vice versa. They are indifferent between saving at the private banking system and the central bank if $i_d = i^{CBDC}$.

Second, the central bank can use interest-bearing CBDC to discipline the distribution of deposit interest rates and associated markdowns arising from informational frictions in the private banking deposit market. Next, we briefly discuss the intuition behind this effect.

Recall that depositors can now switch to depositing their idle funds with CBDC at a rate of i^{CBDC} offered by the central bank. Suppose the depositor has only one contact with the private bank (in the event of α_1). In this case, the private monopoly bank has to match their interest rate i_d^m to the CBDC rate. Otherwise, the private bank cannot source resources to fund its assets. As a consequence, the lower support of the distribution G^{CBDC} is now disciplined by the CBDC rate such that $\underline{i}_d = i_d^m = i^{CBDC} \geq 0$. As before, we can back out the upper support of the distribution G^{CBDC} using the equal profit condition. In this case, the upper support is determined by $\bar{i}_d = i(\gamma) - \frac{\alpha_1}{\alpha_1+2\alpha_2}[i(\gamma) - \underline{i}_d]$, depending on both trend inflation γ and CBDC rate i^{CBDC}. Consequently, the CBDC rate (as an additional policy tool) matters for the distribution $G^{CBDC}(i_d; \gamma, i^{CBDC})$ and equilibrium allocations.

In summary, an interest-bearing CBDC as an outside option can help to discipline the market power (both markdowns and dispersion) of private banks in the deposit market, improving capital accumulation. This mechanism is related to the insight of *latent medium of exchange* shown in Lagos and Zhang (2021).

In what follows, we use $G^{CBDC}(i_d; i, i^{CBDC})$, and, $G(i_d; i)$ to respectively denote the posted deposit interest rate distribution in an economy with CBDC and without CBDC. For short-hand notation, we use $G^{CBDC}(\cdot)$, $G(\cdot)$ or just G^{CBDC} and G.

C.2 Baseline model with interest-bearing CBDC

We derive the balanced-growth steady state in an economy with interest-bearing CBDC using similar steps to that for the model without CBDC (see Appendix B). Given policies $i = [\gamma(1+\mu)]/\beta$ and i^{CBDC}, we can reduce the system of equations down to one equation solving for the steady-state point $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right]\left[\tilde{C}(i, i^{CBDC})\right]^{-1}, \tag{C.1}$$

where

$$\tilde{C}(i, i^{CBDC}) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G^{CBDC}(i_d; i, i^{CBDC})] i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})}_{\text{average deposit rate}}$$

$$+ \frac{1}{n}\Bigg[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G^{CBDC}(i_d; i, i^{CBDC})] i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})}_{\text{average interest rate spread on deposits}}\Bigg].$$

Similar to Section B.1.3 and setting $\delta = 1$, dynamics of capital per effective worker in an economy with CBDC is given by:

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{CBDC}}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i, i^{CBDC})]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i, i^{CBDC})]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{C.2}$$

where the central bank can use i^{CBDC} to affect the posted deposit rates distribution G^{CBDC}, and hence the term $\tilde{C}(i, i^{CBDC})$. This is the additional feature affecting capital growth dynamics relative to Waller (2011) and discussion in Section B.1.3. Recall that endogenous market power on the deposit side of private banking arises from informational frictions.

Note: If we set the CBDC rate to be $i^{CBDC} = 0$, then Equations (C.1) and (C.2) are identical to the baseline model discussed in Section B.1.3.

C.3 Analysis

In this section, we study the effects of interest-bearing CBDC on the equilibrium outcome of the economy.

Posted deposit-rate distribution. Assuming $\alpha_1 \in (0, 1)$, the analytical formula for the distribution of deposit rates posted by private banks in an economy with CBDC is characterized by:

$$G^{CBDC}(i_d; i, i^{CBDC}) = \frac{\alpha_1}{2\alpha_2}\left[\frac{i - \underline{i}_d}{i - i_d} - 1\right], \tag{C.3}$$

where the lower support of the distribution is $\underline{i}_d = i_d^m = i^{CBDC}$ and the upper support of the distribution is $\bar{i}_d$ and $\bar{i}_d := \bar{i}_d(i, i^{CBDC}) = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$.

The associated density of the posted deposit-rate distribution $G^{CBDC}(i_d; i, i^{CBDC})$ is $\tilde{g}^{CBDC}(i_d) = \partial G^{CBDC}(i_d; i, i^{CBDC})/\partial i_d = \frac{\alpha_1}{2\alpha_2}[\frac{i - \underline{i}_d}{(i - i_d)^2}]$.

Remark.

- In general, the nominal policy rate i and the CBDC rate i^{CBDC} can be different.
- If $0 < i^{CBDC} < i$, then G^{CBDC} is non-degenerate with a connected support $[\underline{i}_d, \bar{i}_d]$, and every $i_d \in [\underline{i}_d, \bar{i}_d]$ is below i.
- If $0 < i^{CBDC} = i$, then G^{CBDC} is degenerate at $i = i^{CBDC} = i_d$.
- If $0 < i < i^{CBDC}$, then the private banks' profits earned from deposits is negative, $\pi^d < 0$. Since we have focused on perfect competition on the loans side, then profit is zero, $\pi^l = 0$. Overall, the private banks' profit is $\pi = \pi^l + \pi^d < 0$. In this case, private banks will not operate, and there is no such G in this particular economy.

In what follows, we exclude the possibility that $0 < i < i^{CBDC}$.

First-order stochastic dominance and CBDC

Lemma 9. *Consider the economy away from the Friedman rule: $i > 0$. Assume $\alpha_1 \in (0, 1)$. Consider any two CBDC interest rates i_1^{CBDC} and i_2^{CBDC} such that $0 < i_1^{CBDC} < i_2^{CBDC} < i$. The induced deposit rate distribution $G^{CBDC}(i_d; i, i_2^{CBDC})$ first-order stochastically dominates $G^{CBDC}(i_d; i, i_1^{CBDC})$.*

Proof. Now consider how the value of G^{CBDC} varies with i^{CBDC} at each fixed i_d such that $i^{CBDC} = \underline{i}_d < i_d < \bar{i}_d$. We have that $\partial G^{CBDC}(i_d; i, i^{CBDC})/\partial i^{CBDC} = -\frac{\alpha_1}{2\alpha_2(i-i_d)} < 0$ since all the other terms are strictly positive. Hence, $G^{CBDC}(i_d; i, i_2^{CBDC})$ first-order stochastically dominates $G^{CBDC}(i_d; i, i_1^{CBDC})$ for $0 < i_1^{CBDC} < i_2^{CBDC} < i$. □

Average posted deposit rates and CBDC

Lemma 10. *Fix $i > i^{CBDC} > 0$. Assume $\alpha_1 \in (0, 1)$. An increase in the CBDC rate leads to:*

1. *an increase in the average deposit interest rates posted by private banks;*
2. *an increase in the lower and upper bound of the support of the distribution G^{CBDC}, $[\underline{i}_d, \bar{i}_d]$.*

Proof. Suppose $i > i^{CBDC} > 0$. Let $g^{CBDC}(i, i^{CBDC}) = \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})$ to denote the average deposit interest rates posted by private banks in an economy with CBDC. We first consider the first statement in Lemma 10. Apply integration by parts to $g^{CBDC}(i, i^{CBDC})$, and this yields

$$g^{CBDC}(i, i^{CBDC}) = [i_d G(i_d; i, i^{CBDC})]_{\underline{i}_d}^{\bar{i}_d} - \int_{\underline{i}_d}^{\bar{i}_d} \frac{\partial i_d}{\partial i_d} G(i_d; i, i^{CBDC}) \mathrm{d}i_d = \bar{i}_d - \int_{\underline{i}_d}^{\bar{i}_d} G(i_d; i, i^{CBDC}) \mathrm{d}i_d.$$

Next, we want to show that $\partial g^{CBDC}(i, i^{CBDC})/\partial i^{CBDC} > 0$. Using Leibniz' rule, we have

$$\begin{aligned} \frac{\partial g^{CBDC}(i, i^{CBDC})}{\partial i^{CBDC}} &= \frac{\partial \bar{i}_d}{\partial i^{CBDC}} - \left[\frac{\partial \bar{i}_d}{\partial i^{CBDC}} + \int_{\underline{i}_d}^{\bar{i}_d} G_{i^{CBDC}}(i_d; i, i^{CBDC}) \mathrm{d}i_d \right] \\ &= - \int_{\underline{i}_d}^{\bar{i}_d} \underbrace{G_{i^{CBDC}}(i_d; i, i^{CBDC})}_{<0} \mathrm{d}i_d > 0, \end{aligned} \tag{C.4}$$

where the last equality follows from the result in Lemma 9 meaning that higher CBDC rate shifts the distribution G^{CBDC} downward.

Next, we consider the second statement in Lemma 10. Recall that the lower support of the distribution G^{CBDC} is given by $\underline{i}_d = i_d^m = i^{CBDC} > 0$. The lower support is a one-to-one to change in the CBDC rate since the monopoly private banks has to match their deposit rate up to the CBDC rate.

Using equal profit condition, $R(\bar{i}_d; i, i^{CBDC}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\underline{i}_d; i, i^{CBDC})$, we can back out the upper support of the distribution G^{CBDC} by $\bar{i}_d := \bar{i}_d(i, i^{CBDC}) = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$. Differentiate $\bar{i}_d$ with respect to i^{CBDC}, we have $\frac{\partial \bar{i}_d}{\partial i^{CBDC}} = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} > 0$ and it is less than one. Hence, all together establishes that the support of the distribution, $\text{supp}(G^{CBDC}) = [\underline{i}_d, \bar{i}_d]$ shifts to the right in response to a higher CBDC rate. □

Deposit-rates spread and CBDC As before, the average interest rate spread on deposits to be defined as the difference between the central bank policy interest rate and the average of deposit interest rates across banks. As such, the average posted interest rate spread on deposits in an economy with interest-bearing CBDC is defined as

$$\tilde{s}(i, i^{CBDC}) = i - \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC}), \tag{C.5}$$

where the distribution G^{CBDC} is determined by Equation (C.3).

If we set $i^{CBDC} = 0$, then $\tilde{s}(i, i^{CBDC})$ is identical to the baseline model without CBDC, i.e., $s(i)$ defined in Equation (A.27).

Lemma 11. *Suppose the nominal policy interest rate, $i > 0$, and $\alpha_1 \in (0, 1)$, are fixed in both economies (with CBDC and without CBDC) featuring noisy deposit search. Then, we* *have*

1. *If* $0 = i^{CBDC} < i$*, then* $0 < \tilde{s}(i, i^{CBDC}) = s(i)$.
2. *If* $0 < i^{CBDC} < i$*, then* $0 < \tilde{s}(i, i^{CBDC}) < s(i)$.
3. *If* $0 < i^{CBDC} = i$*, then* $0 = \tilde{s}(i, i^{CBDC}) < s(i)$.

Proof. We first consider the first statement in Lemma 11. Suppose the central bank sets the CBDC rate to be zero, $i^{CBDC} = 0$. It then follows that the support of the deposit rate distribution in an economy with CBDC is identical to that without CBDC, i.e., $\text{supp}(G^{CBDC}) = \text{supp}(G)$. Hence, $\int_{\underline{i}_d}^{\bar{i}_d} i_d \text{d}G^{CBDC}(i_d; i, i^{CBDC}) = \int_{\underline{i}_d}^{\bar{i}_d(i)} i_d \text{d}G(i_d; i)$. Since i and $\alpha_1 \in (0, 1)$ are fixed the same in these two economies, it then follows that $\tilde{s}(i, i^{CBDC}) = s(i) > 0$. Hence, the conclusion of the first statement in Lemma 11.

Next, we consider the second statement in Lemma 11. Suppose the central bank sets the CBDC rate above zero but below the nominal policy rate, $0 < i^{CBDC} < i$. In an economy with CBDC, the lower and upper bound of the support of the distribution G^{CBDC} are respectively given by $\underline{i}_d = i_d^m = i^{CBDC} > 0$ and $\bar{i}_d = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$. Let $\underline{i}_d^{'}$, and, $\bar{i}_d^{'}$ to respectively denote the lower and upper bound of the support of the distribution G in an economy without CBDC. In this case, we have $\underline{i}_d^{'} = i_d^m = 0$, and $\bar{i}_d^{'} = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} i$. Since both economies have an identical nominal policy rate i, and the same degree of noisy search frictions, we have the ordering: $\underline{i}_d^{'} < \underline{i}_d < \bar{i}_d^{'} < \bar{i}_d$. It follows that $\int_{\underline{i}_d}^{\bar{i}_d} i_d \text{d}G^{CBDC}(i_d; i, i^{CBDC}) > \int_{\underline{i}_d}^{\bar{i}_d} i_d \text{d}G(i_d; i)$. That is, the average deposit rate posted by private banks in an economy with CBDC is higher than the economy without CBDC. Also, $\alpha_1 \in (0, 1)$ implies each i_d drawn from G^{CBDC} (or G) must be lower than the policy rate, implying a positive deposit spread. All together, we then have $0 < \tilde{s}(i, i^{CBDC}) < s(i)$ as stated in the second statement in Lemma 11.

Finally, we consider the third statement in Lemma 11. Suppose the central bank sets the CBDC rate equals to the nominal policy rate, $0 < i^{CBDC} = i$. In this case, the distribution G^{CBDC} is degenerate at $i = i^{CBDC} = i_d$. Hence, it follows that the average deposit spread collapse to zero in this economy, i.e., $\tilde{s}(i, i^{CBDC}) = 0$. For the case without CBDC, using result established above, we have $s(i) > 0$. Hence, we have the conclusion of the third statement in Lemma 11.

□

C.4 Proof of Proposition 6: Effects of CBDC on capital growth

Proof. Recall that the growth rate of capital per effective worker in (1) an economy with CBDC and noisy deposits search, (2) an economy without CBDC and noisy deposits search, and, (3) an economy with perfectly competitive banks but no CBDC, are respectively determined by Equations (C.2), (B.16) and (B.18). Let g_k^{CBDC}, g_k, and g_K^{PC} to respectively denote the corresponding growth rate.

Suppose $0 < i^{CBDC} < i$ and $\alpha_1 \in (0, 1)$. We then have $0 < \tilde{s}(i, i^{CBDC}) < s(i)$ by the result established in Lemma 11. Comparing Equations (C.2), (B.16) and (B.18), we have that $(1 + i)^{-1} > [\tilde{C}(i, i^{CBDC})]^{-1} > [\tilde{C}(i)]^{-1}$. Hence, the right-hand side of Equation (C.2) must be lower that the right-hand side of Equation (B.18) and higher than the right-hand side of Equation (B.16). All together, it establishes that $g_k < g_k^{CBDC} < g^{PC}$.

Next, suppose $0 < i^{CBDC} = i$ and $\alpha_1 \in (0, 1)$. By the result established in Lemma 11, the distribution G^{CBDC} degenerates at $i = i^{CBDC} = i_d$ in this case. It then follows that $(1 + i)^{-1} = [\tilde{C}(i, i^{CBDC})]^{-1} > [\tilde{C}(i)]^{-1}$. Thus, we have that $g_k < g_k^{CBDC} = g_k^{PC}$. □

In summary, Proposition 6 highlights that having an interest-bearing CBDC (as an alternative depository facility) can help to reduce the positive deposit spread that arises from informational frictions in the private banking deposit market. When the central bank ties the CBDC rate to the nominal policy rate, the effectiveness of banking liquidity transformation and capital accumulation can be restored as in a perfectly competitive banking equilibrium.