Money and Imperfectly Competitive Credit*

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Abstract

We develop a monetary economy in which banking market power and its associated dispersion in deposit and loan rates are equilibrium phenomena. The theory accounts for the dispersion of loan and deposit interest rates, and incomplete pass-through of monetary policy to them. This is a distinguishing feature of our search-based theory of market power and is consistent with new micro-level evidence on U.S. consumer loans and deposits. Imperfect pricing competition among banks also creates a novel channel from monetary policy to interest rate spreads, and thus, to real consumption and welfare. The overall welfare effect of financial intermediation depends on the interplay between banking market power, individual liquidity risk, and monetary policy. Under a given inflation target, welfare gains arise if a central bank uses state-contingent monetary injections to reduce lenders' market power in response to fluctuations in aggregate demand.

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1 Introduction

In this paper, we study liquidity reallocation by imperfectly competitive financial intermediaries. A conventional view holds that the reallocation of excess liquidity from one party to another that needs them more increases economic activity and improves welfare. Here we evaluate this view in a monetary economy where the extent of market power in the markets for both loans and deposits is endogenous and responds in equilibrium to monetary policy.

Building on the model of perfectly competitive financial intermediaries developed by Berentsen, Camera and Waller (2007) (BCW), we introduce imperfect competition for both loans and deposits and characterize distributions of loan and deposit rates that arise in equilibrium. We present evidence that these equilibrium features are empirically relevant and show that they both depend on monetary policy in the short and long run and have important implications for output and welfare. The novelty in our approach is that the endogenous distributions of deposit and lending rates affect the degree of the pass-through of monetary policy to the real economy. The extent of surplus extraction by banks depends on policy, affects the equilibrium distribution of deposit and lending rates, and thus has direct welfare consequences.

Our framework maintains the information friction of BCW. That is, in certain decentralized markets promissory claims are not incentive feasible. This renders money valuable as a medium of exchange and connects monetary policy to banking. As in BCW, financial intermediaries here potentially improve welfare by paying interest on idle funds to depositors when the opportunity cost of holding money is positive. We refer to this benefit of financial intermediation as the *liquidity-risk (alleviation) channel*.

We contribute to the body of theories of imperfect banking competition by building a model where the degree of imperfect competition and dispersion in deposit and loan rates are jointly determined as equilibrium phenomena. We do so by adapting the noisy search model of Burdett and Judd (1983). In our setting, banks compete by posting interest rates for both loans and deposits. In each market, they face a trade-off between earning a higher return per customer (*i.e.*, pricing higher (lower) on loans (deposits)) and attracting a higher number of customers. Households, whether borrowers or depositors (depending on individual liquidity needs), observe only a random number of these rate offers with their perceived distributions of loan and deposit rates being consistent with equilibrium imperfect competition among banks.

While one might think that the underlying noisy search theory is not new, its adaption to a monetary banking model is novel, as this allows us to identify a new and opposing force to the existing *liquidity-risk (alleviation) channel*. We prove and quantitatively demonstrate how noisy search gives rise to banks' ability to extract trade surpluses from households on both sides of the banking business. Moreover, the dispersion of interest rates is both a result and conduit of such market power. We refer to this feature of our model as the *banking market power channel*.

These two channels act as countervailing forces rendering the welfare effects of financial intermediation ambiguous and non-monotone in monetary policy. Specifically, depending on policy there are potential welfare losses through the banking market power channel to banking activity *per se*. We demonstrate this analytically in Proposition 3 and Corollary 1, characterizing conditions under which financial intermediation improves or worsens welfare relative to a monetary economy without banks. We prove these theoretical results for the (notation-wise) less cumbersome case of the model with noisy loan search and Bertrand deposits pricing. We also verify this numerically in a calibrated benchmark model with noisy search on both the deposit and loan side (see Section 5.2).

As noted above, our theory implies equilibrium distributions in rates for both identical loan and deposit products. Using U.S. data at the bank branch level, we document evidence of loan and deposit interest rate dispersion for identical products in each market.[1] We measure dispersion in *posted* loan rates of identical consumer loan products, controlling for geography and other confounding factors.[2] We refer to the remainder or unexplained dispersion as *residual* or *orthogonalized dispersion* in loan rates. We follow a similar methodology for analyzing the deposit rate dispersion for identical time deposit products. Empirically we find positive relationships between the standard deviations of both bank loan and deposit rate spreads and their averages at both the national and state levels. This is consistent with the predictions of our model that is calibrated to macro-level U.S. data.

We view the interest rate dispersion and the associated interest spreads as indications of banking market power on both the loan and deposit services. (See also, Allen, Clark and Houde, 2014, 2019; Clark, Houde and Kastl, 2021, for more evidence using Canadian mortgage data and analysis of interest rate dispersion.) We thus consider optimal policy design in the presence of banking market power. As we nest Bertrand pricing as a parametric limit, we can replicate the competitive banking setting of BCW as a special case. We decompose money demand into several components, one capturing the liquidity insurance role of banks that is identical to that arising in BCW. We also, however, identify new marginal benefit and cost terms that capture the effects of equilibrium market power and its attendant loan rate risk on agents' money accumulation decisions.

This decomposition of money demand is then used to illustrate the effects of cyclical monetary policies that redistribute liquidity in a version of the model with shocks to aggregate demand, an exercise in the spirit of Berentsen and Waller (2011) and Boel and Waller (2019). In this analysis, however, we shut down the fluctuations in the deposit rate that are the focus of Berentsen and Waller (2011) and isolate welfare improvements arising from the effects of policy on market power in bank lending. As such, our optimal policy exercise is wholly different (although complementary) to theirs.

While both in Berentsen and Waller (2011) and here optimal policy redistributes liquidity among *ex-post* heterogeneous agents and is akin to the maintenance of an "elastic currency" it does so through different channels in the two settings.[3] With perfect competition in lending, redistributive tax instruments do not directly affect individual agents' money demand in equilibrium although they are useful for counteracting sub-optimal interest rate movements by raising the deposit rate when aggregate demand is low (Berentsen and Waller, 2011). Here, in contrast, the optimal stabilization policy exploits the endogeneity of market power in banking and counteracts movements in interest rate spreads. Specifically, it reduces lenders' market power (lowering the average spread) in periods of high aggregate demand and allows it to increase when demand is low.

Recently there has been increased policy interest in the link between market power in the financial sector and monetary policy in both the academic literature—see, *e.g.*, Scharfstein and Sunderam (2016), Duval, Furceri, Lee and Tavares (2021), Godl-Hanisch (2022), Bellifeime, Jamilov and Monacelli (2022), Wang, Whited, Wu and Xiao (2022) and Wang (2022)—and policy circles—see, respectively, Sims (2016), Productivity Commission (2018), Wilkins (2019), and Executive Order 14036 (2021) for Australia, Canada

[1]We find similar evidence for various loan and deposit products. In what follows, we restrict attention to a specific class of loan and deposit products that are consistent with our theoretical model description.

[2]Martín-Oliver, Vicente and Saurina (2007) and Martín-Oliver, Salas-Fumás and Saurina (2009) also find price dispersion in the loan rates for identical loan products in the case of Spanish banks.

[3]This harks back to the *Aldrich-Vreeland Act* of 1908. The Act was enacted to implement elastic or emergency currency in response to the Bankers' Panic or Knickerbocker Crisis of 1907. The Act also led to the creation of a decentralized Federal Reserve model under the *Federal Reserve Act* of 1913. In the official title of the *Federal Reserve Act*, one finds the phrase: "[A]n Act to provide for the establishment of Federal reserve banks, to furnish an elastic currency ... [*etc*] (*sic*)." (We thank Randy Wright for suggesting this interpretation.)

and the U.S.

In that vein, a large literature has studied aspects of the nature and implications of market power in banking. Several authors have identified substantial spreads in both lending (in the form of loan-rate spreads over the policy rate) and in bank funding (in the form of deposit "markdowns"). See, for example, Wang et al. (2022). Others have studied links between market power in the financial sector and macroeconomic stability (see, *e.g.*, Allen and Gale, 2004; Brunnermeier and Sannikov, 2014; Coimbra, Kim and Rey, 2022; Corbae and Levine, 2022). Frictions in the interbank market are studied in detail by Bianchi and Bigio (2022). Bencivenga and Camera (2011) and Head, Kam, Ng and Pan (2022) study the connection of banking to capital accumulation. This paper focuses on the *dispersion* of *both* loan and deposit rates specifically and its links to market power. Our work is thus distinguished from most of the theoretical literature, which generally focuses on degenerate distributions of lending and deposit rates.

Chiu, Dong and Shao (2018) also document a potentially negative effect of financial intermediation, but in a different setting and arising from a different mechanism. In their model, both banks and firms are competitive, as in BCW. A pecuniary externality (contributing to the losses of financial intermediation) may arise if "too many" agents have access to credit and the cost of goods production is convex. More agents with access to credit results in more goods demand, raising the marginal cost and thus prices. Boel and Camera (2020) use a similar model and generate a negative welfare effect of banking through a policy-varying operating cost for banks in providing loans. In contrast to both of these papers, our result is driven by banks' market power in lending, the extent of which is determined in equilibrium, and depends on monetary policy.[4]

Several other recent papers also consider imperfect competition in banking. Drechsler, Savov and Schnabl (2017) show that banks' ability to mark down deposits is empirically important. Choi and Rocheteau (2023b) assume depositors have private information in deposit contract bargaining. Banks may thus second-degree price discriminate among depositors, leading to a bank-deposit channel of monetary policy along the lines of Drechsler et al. (2017), in which the deposit outflow from the banking system is concentrated on those with low liquidity needs.

Others have studied oligopoly in the banking industry. Corbae and D'Erasmo (2021) model big banks interacting with small fringe banks and other non-bank lenders. Their model generates an empirically relevant bank-size distribution and illustrates the effects of regulatory policies on banking stability. Altermatt and Wang (2024) show how oligopoly among banks affects both the monetary policy transmission mechanism and bank defaults. Dong, Huangfu, Sun and Zhou (2021), endogenize the number of banks and Chiu, Davoodalhosseini, Jiang and Zhu (2023) consider oligopolistic competition for deposits to study the effects of central bank digital currency. Our approach complements these papers by accounting for interest rate dispersion on both the lending and deposit side. Finally, our model effectively endogenizes the costly credit of Wang, Wright and Liu (2020).

The remainder of the paper is organized as follows. In Section 2, we present the details of the environment and the decision problems of households, sellers, banks, and the government. In Section 3, we describe a

[4] Of course, it is not our claim that banking is bad for welfare overall, as there are other, possible positive effects. Chang and Li (2018) consider the same mechanism for banks but extend it to incorporate fractional reserves and liquidity buffers (see also, Kashyap, Rajan and Stein, 2002). This gives rise to a non-neutral liquidity channel of monetary policy in their model. Gu, Mattesini, Monnet and Wright (2013) consider a setting with limited commitment in exchange. In their model, banks improve welfare since limited commitment to private contractual obligations inhibits more efficient allocations in the absence of banks. Also, bank liabilities can serve as payment instruments. He, Huang and Wright (2008) consider the safe-keeping role of banks when there is a risk of asset theft. These various reasons imply that banks can support a more efficient allocation in equilibrium. We eschew these factors in our model and focus solely on the role of banks as potential institutions for insuring private liquidity risk.

stationary monetary equilibrium. Section 4 provides analytical results and a discussion of the novel features of the model. In Section 5, we calibrate the model to U.S. data and illustrate numerically the quantitative effects of equilibrium market power in the banking sector. Here we identify a relationship between the dispersion and level of loan rate spreads implied by the theory. In Section 6, we provide micro evidence on this relationship that lends support to our theoretical mechanism. In Section 7, we study an optimal monetary stabilization policy in response to aggregate demand shocks and their attendant fluctuations in money and loan demand. We conclude in Section 8.

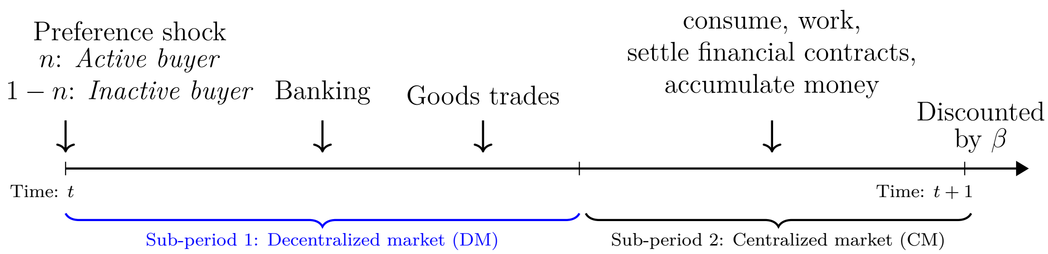
2 The model

We build on the perfectly competitive banking model of BCW and nest it as a special case. As do they, we focus solely on the role banks play in insuring individuals against liquidity risk: Banks take deposits from *ex-post* holders of idle money and make loans to those who require additional liquidity.[5] We depart from BCW by introducing imperfect competition for both deposits and loans by integrating noisy search along the lines of Burdett and Judd (1983). Banking market power (measured by interest rate spreads and dispersions) is thus endogenous and determined in equilibrium, forming a conduit for the transmission of monetary policy.

2.1 Overview

The economy has four types of agents: a government (central bank) and unit measures of households, sellers, and banks. Time is discrete and infinite, with each period divided into two sub-periods as in Lagos and Wright (2005). A non-storable consumption good is associated with each sub-period of each period. Agents discount payoff flows between periods but not within a period by a discount factor $\beta \in (0, 1)$. We use the following notation to denote time-dependent variable outcomes: $X \equiv X_t$ and $X_{+1} \equiv X_{t+1}$. Figure 1 displays the model timeline.

Figure 1: Timing



Let $\boldsymbol{\tau} = (\tau_b, \tau_s, \tau_1^e, \tau_2^e, \tau_2)$ denote a list of policy actions (taxes and/or transfers), where τ_b, τ_s and τ_1^e are imposed respectively on households, sellers and banks in the decentralized market (DM), and τ_2 and τ_2^e are imposed respectively on households/sellers and banks in the subsequent centralized market (CM). The aggregate state is given by a list consisting of an aggregate stock of money and policies denoted by the

[5] Following BCW, we abstract from means of consumption smoothing other than banks and individually held money. In general, we could allow agents to own other assets (*e.g.*, claims to private equity or bonds). In order to rationalize the equilibrium coexistence of fiat money alongside other asset claims, we could introduce costly asset liquidation in the frictional secondary asset market. This could be modeled, for example, as frictional over-the-counter trades as in Rocheteau and Rodriguez-Lopez (2014), Geromichalos and Herrenbrueck (2016) and Duffie, Gârleanu and Pedersen (2005). This would render demand for multiple assets that have different liquidity premia in equilibrium. For the purposes of this paper, these additional features would not alter our main insights.

vector $\mathbf{s} \equiv (M, \boldsymbol{\tau})$. An individual household with a money balance m will have a state vector $(m, \mathbf{s})$. The sequencing of events and actions are as follows:

1. Households enter the period each knowing $\mathbf{s}$ and carrying individual money balance, m. Nominal government transfers to households and sellers are $\tau_b M$ and $\tau_s M$.

2. Each household observes the outcome of an individual shock:

   (a) With probability n, the household becomes an *active buyer*. That is, the household wishes to consume good q in the DM of the current period. As agents are anonymous in the DM, they cannot trade with sellers using promises of future repayment. Goods exchange will thus be supported by fiat money. Active buyers also search among the lending banks for loans and may match with one, two, or no lenders. Those who have matched with two lenders can borrow additional money balance from the one that offers a lower interest rate.

   (b) With probability $1 - n$, the household becomes an *inactive buyer* and does not want to consume in the current DM. Inactive buyers hold idle money and search among banks for depositing opportunities. They receive a random selection of the deposit rates posted by banks, which can be zero, one, or two quotes. Those with two deposit rate quotes can deposit with the one offering a higher return.

3. After the realization of households' status but before the exchange of goods, the banking market opens. As in BCW, there is a continuum of institutions (banks) that have access to a record-keeping technology. This technology enables them to commit to repay depositors and enforce loan contracts in the upcoming CM. Here, however, rather than interacting in competitive markets, banks post loan and deposit rates in markets characterized by noisy search frictions as in Burdett and Judd (1983).[6] The interactions among borrowers/depositors and banks are described in detail in Section 2.5. At this point, banks also have access to a competitive interbank market. They may borrow and lend funds at a spot rate i_f which is effectively set by the central bank.[7]

4. Goods trades take place after the banking arrangements. Sellers in the DM produce non-storable goods on the spot using their own effort. They trade with active buyers who are heterogeneous *ex-post* with regard to their access to loans at potentially different rates. These buyers' demands vary with their money holdings and borrowing costs.

5. In the subsequent CM, markets are perfectly competitive. Banks enforce loan repayments from borrowers and repay depositors. Given i_f, any aggregate deficit (surplus) in deposits incurred in the DM (denoted by e) is met by the central bank's lump-sum money injection (extraction) via a transfer (tax) $\tau_1^e M$. The central bank then taxes (transfers to) the banks the same amount given to (taken from) them in the preceding DM, $\tau_2^e M = -\tau_1^e M$.[8]

[6]The loan market has two well-defined limits: Bertrand pricing and monopoly pricing, likewise, for the deposit market. In both markets, if households always receive multiple trading opportunities then we have Bertrand competition. This parametric limit yields equivalent competitive outcomes of Berentsen et al. (2007). Similarly, our model also nests Lagos and Wright (2005), and monopoly banking (e.g., along the lines of Klein (1971) and Monti (1972)), as special cases.

[7] See Rocheteau, Wright and Zhang (2018) for details on formalizing the interbank interest rate. They show that the interbank interest rate in equilibrium is also the policy rate, which is equivalent to the opportunity cost of holding money. It can also be interpreted as the interest rate on a risk-free bond. See also Choi and Rocheteau (2023a). We thank Guillaume Rocheteau for suggesting this extension on an earlier version of the model.

[8]An alternative interpretation is as follows. The central bank sets a price-level target (by choosing a path of money stock

6. In the CM, all households consume and can produce a homogeneous good using labor. Heterogeneity at this point is due to different DM experiences only. Depending on their individual state, households may collect interest on deposits, pay interest on loans, work, and/or consume. Finally, households accumulate a money balance, m_{+1}, to carry into the following period.

This sequence of events repeats with households carrying m_{+1} at the start of date $t+1$.

2.2 Preferences

Following Lagos and Wright (2005) and BCW we assume households' utilities are quasi-linear with period utility given by

$$\mathcal{U}(q,x,h) = u(q) + U(x) - h, \tag{2.1}$$

where $u(q)$ denotes the utility flow from consumption of the DM good q, $U(x)$ is the utility of consumption good x in the CM, and $-h$ is the disutility of CM labor. We assume that $u' > 0$, $u'' < 0$, and that u satisfies the usual Inada conditions. We make the same assumptions for U. For concreteness now, and anticipating the quantitative analysis under the calibration later, we restrict attention to the constant-relative-risk-aversion (CRRA) family of functions:[9]

$$u(q) = \lim_{\hat{\sigma}\to\sigma} \frac{q^{1-\hat{\sigma}} - 1}{1-\hat{\sigma}}, \qquad \text{with } \sigma < 1. \tag{2.2}$$

We now turn to detailed descriptions of the decision problems of each type of agent. In each case, we work backward from the CM to the DM.

2.3 Households in the Centralized Market

Consider a household at the beginning of the CM, with money holdings, loan and deposit balances (m,l,d). Households discount payoffs between time periods using subjective discount factor $\beta \in (0,1)$. In the preceding DM, the agent may have been an active buyer (with $m \geq 0$, $l \geq 0$ and $d = 0$) or an inactive buyer (with $l = 0$ and $m \geq d \geq 0$). Let $V(\cdot)$ denote the value function of the household at the beginning of the next period. The household's value at the beginning of the CM is

$$W(m,l,d,\mathbf{s}) = \max_{x,h,m_{+1}} \left[U(x) - h + \beta V(m_{+1},\mathbf{s}_{+1})\right], \tag{2.3}$$

subject to

$$x + \phi m_{+1} = h + \phi m + \phi(1+i_d)\, d - \phi(1+i)\, l + \pi + T, \tag{2.4}$$

where ϕ is the date-t value of a unit of money in units of CM good x, i_d is the interest rate on deposits d, i is the interest rate on the buyer's outstanding loan l, π is aggregate profit from bank ownership, and

in the CM). Depending on the aggregate liquidity status of private banks due to interaction with borrowers or depositors, the central bank can also borrow or lend money to the banking system in the DM. The interbank settlement occurs in the upcoming CM with interest payment at a rate of i_f. The interbank settlement we consider is also in the spirit of Berentsen and Waller (2011).

[9] This restriction is empirically consistent with our calibration later, as comes out of our fitting of long-run money-demand data. It is not required theoretically. We also consider the case of $\sigma > 1$ but we do not discuss it here for brevity. The knife-edge case of $\sigma = 1$ is not well-defined in terms of equilibrium characterization. The restriction with $\sigma < 1$ is also the case studied by Head, Liu, Menzio and Wright (2012).

$T = \tau_2 M$ is any lump-sum tax or transfer from the government in the CM.

Using Equations (2.4) and (2.3), the problem may be rewritten as

$$W(m, l, d, \mathbf{s}) = \phi [m - (1+i) l + (1+i_d) d] + \pi + T + \max_{x, m_{+1}} \{U(x) - x - \phi m_{+1} + \beta V(m_{+1}, \mathbf{s}_{+1})\}. \quad (2.5)$$

The first-order conditions with respect to the choices of x and m_{+1}, respectively, are

$$U_x(x) = 1, \quad (2.6)$$

and,

$$\beta V_m(m_{+1}, \mathbf{s}_{+1}) = \phi, \quad (2.7)$$

where $V_m(m_{+1}, \mathbf{s}_{+1})$ is the marginal value of an additional unit of money taken into period $t+1$. The envelope conditions are

$$W_m(m, l, d, \mathbf{s}) = \phi, \quad W_l(m, l, d, \mathbf{s}) = -\phi(1+i), \quad \text{and,} \quad W_d(m, l, d, \mathbf{s}) = \phi(1+i_d). \quad (2.8)$$

Note that $W(\cdot, \mathbf{s}_{+1})$ is linear in (m, l, d) and optimal decisions characterized by Equations (2.6) and (2.7) are independent of the agent's wealth. Moreover, each household supplies labor in the CM exactly sufficient to produce enough of the CM good to repay their loan (if necessary) and acquire the optimal money balance m_+ and consumption of x from (2.7) and (2.6), respectively.

2.4 Goods trading in the DM

We now describe households' problems in the DM after having realized their status as either inactive or active and after having potentially matched with zero, one, or more banks.

2.4.1 Sellers

In the DM, there is a unit measure of sellers of goods. They behave much like households, except that they can produce the DM good on demand and do not value consuming it. DM sellers are analogous to Walrasian price-taking producers in Rocheteau and Wright (2005). Each DM seller has value:

$$S(m, \mathbf{s}) = \max_{q_s} \{-c(q_s) + W(m + \tau_s M + p q_s, 0, 0, \mathbf{s})\}. \quad (2.9)$$

Here, $c(q)$ represents the cost of producing quantity q of goods, where $c(0) = 0$, $c_q(q) > 0$ and $c_{qq}(q) \geq 0$. The sellers' optimal production plan satisfies

$$c_q(q_s) = p\phi. \quad (2.10)$$

That is, DM sellers produce to the point where the marginal cost of producing q_s equals its relative price. It is straightforward to show that in equilibrium their valuation will be $S(0, \mathbf{s})$ at the start of each DM—*i.e.*, sellers optimally carry no money into the DM.

2.4.2 Inactive buyers

Inactive buyers without a deposit opportunity. Consider an inactive buyer who has failed to match with a bank. This buyer exits the DM with her beginning of period money holdings plus the transfer, $\tau_b M$. Her *ex-post* value of continuing to the CM is: $W(m+\tau_b M, 0, 0, \mathbf{s})$.

Inactive buyers with one or more deposit opportunities. Conditional on having contacted at least one depository agent, and inactive buyer with money ex-transfer, $m\tau_b M$, can deposit with the bank at the lowest of their observed deposit rates, i_d. She then has the *ex-post* value of continuing to the CM: $W(m+\tau_b M - d, 0, d(i_d), \mathbf{s})$. Previewing the equilibrium we will consider, such inactive buyers will deposit their entire money holdings as long as this lowest deposit rate is non-negative: For any $i_d \geq 0$, $d = m + \tau_b M$.

2.4.3 Active buyers

Active buyers with no borrowing opportunity. Consider an active household that has met no lending bank and thus cannot borrow money in addition to her money holdings ex-transfer, $m + \tau_b M$. Given p the money price of the DM good, the buyer has the following value:

$$B^0(m, \mathbf{s}) = \max_{0 \leq q_b \leq \frac{m+\tau_b M}{p}} \left\{ u(q_b) + W(m + \tau_b M - pq_b, 0, 0, \mathbf{s}) \right\}. \tag{2.11}$$

Using (2.2), the buyer's demand for the DM good is

$$q_b^{0,\star}(m, \mathbf{s}) = \begin{cases} \frac{m+\tau_b M}{p} & \text{if } p < \hat{p} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases}, \tag{2.12}$$

where $\hat{p}$ is the price below which the agent spends all of their money and in equilibrium is

$$\hat{p} \equiv \hat{p}(m, \mathbf{s}) = \phi^{\frac{1}{\sigma-1}} (m + \tau_b M)^{\frac{\sigma}{\sigma-1}}. \tag{2.13}$$

Active buyers with at least one borrowing opportunity. Next, consider the post-match value of a buyer who has contacted at least one lending agent:[10]

$$B(m, \mathbf{s}) = \max_{q_b \leq \frac{m+l+\tau_b M}{p}, l \in [0, \bar{l}]} \left\{ u(q_b) + W(m + \tau_b M + l - pq_b, l(i), 0, \mathbf{s}) \right\}. \tag{2.14}$$

Using the Karush-Kuhn-Tucker conditions from (2.14), we obtain the demands for the DM goods and loans. The demand for the DM good is given by:

$$q_b^{\star}(i, m, \mathbf{s}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \text{ and } 0 \leq i \leq \hat{i} \\ \frac{m+\tau_b M}{p} & \text{if } \tilde{p}_i < p < \hat{p} \text{ and } i > \hat{i} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \text{ and } i > \hat{i} \end{cases}, \tag{2.15}$$

[10] Under the assumption that loan contracts are perfectly enforceable as in the baseline case of BCW, the borrowing limit, $\bar{l}$, can be set sufficiently high so as never to bind in equilibrium.

where

$$\hat{p} \equiv \hat{p}(m, \mathbf{s}) = \phi^{\frac{1}{\sigma-1}} (m + \tau_b M)^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i = \hat{p}\,(1+i)^{\frac{1}{\sigma-1}}\,, \tag{2.16}$$

respectively, correspond to a maximal DM price at which the household will use both her own liquidity and also borrowed funds, and, a maximal price at which her purchase results in her being liquidity-constrained. Since $\sigma < 1$, we have: $0 < \tilde{p}_i < \hat{p} < +\infty$.

The maximal interest rate at which a buyer is willing to borrow is given by

$$\hat{i} \equiv \hat{i}(m, \mathbf{s}) = (p\phi)^{\sigma-1} \left[\phi(m + \tau_b M)\right]^{-\sigma} - 1. \tag{2.17}$$

For any interest rate $i \in [0, \hat{i}]$, the buyer's loan demand is:

$$l^{\star}(i, m, \mathbf{s}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}} \left[\phi\,(1+i)\right]^{-\frac{1}{\sigma}} - (m + \tau_b M) & p \in (0, \tilde{p}_i];\ i \in [0, \hat{i}] \\ 0 & p \in (\tilde{p}_i, \hat{p});\ i > \hat{i} \\ 0 & p \geq \hat{p};\ i > \hat{i}. \end{cases} \tag{2.18}$$

From the respective first cases of (2.15) and (2.18), we can see that if the DM good's relative price ($p\phi$) and interest on bank loans (i) are sufficiently low, the agent borrows to augment her money balance and her goods and loan demands are decreasing in both i and $p\phi$. If, however, the DM good's relative price and interest on borrowing are higher (*i.e.*, the intermediate case), the agent prefers not to borrow, but rather to spend all her money on the DM good and be liquidity-constrained. In this case, the loan rate does not matter for demand. Finally, if $p\phi$ and i are sufficiently high, the buyer not only doesn't borrow but does not spend all her money balance on the DM good. The cutoff prices $(\hat{p}, \tilde{p}_i, \hat{i})$ are all determined in equilibrium.

2.5 Banking

The banking system consists of a continuum of private banks with access to a financial record-keeping technology that enables them to accept deposits and extend loans. Banks compete in both the deposit and loan markets by posting rates and meeting demand, following the noisy search model of Burdett and Judd (1983).

Banks contract with prospective borrowers before the latter trade with sellers in the DM and can enforce loan contracts in the upcoming CM.[11] Each bank, taking others' posted loan rates as given, posts a loan rate of i and commits to satisfying the demand for loans at that rate. This commitment is credible because each lending bank can access both household (inactive buyer) deposits and an interbank market, if necessary. Active buyers, who randomly receive zero, one, or two borrowing opportunities (or loan rate quotes), are able to borrow at the lowest rate they observe. Let α_k for $k \in \{0, 1, 2\}$ denote the probability that an active buyer has k borrowing opportunities.

A bank also posts deposit rate i_d to attract deposits, $d \leq m + \tau_b M$, from inactive buyers with idle money balances.[12] Each bank takes the opportunity cost of loans and the deposit rates posted by others as given.

[11] Various extensions involving limitations on the ability of banks to enforce loan contracts are possible. See Berentsen et al. (2007) and Li and Li (2013). We have also considered a setup with an exogenous loan default for robustness checks. The result regarding pricing competition for *deposits* and *loans* is qualitatively the same. It effectively induces banks to post higher interest rates and extend fewer loans. For presentation brevity, we focus here on the case of full commitment.

[12] This is a slight variation of the original setup of BCW. In BCW, there is an equivalent measure n of agents who become buyers in the DM. The remaining $1 - n$ become sellers in the DM. Here, we fix a unit measure of agents as always being sellers

Again, prospective depositors receive randomly zero, one, or two deposit opportunities and can deposit at the highest rate they observe. Let α_j^d for $j \in \{0,1,2\}$ to denote the probability that an inactive buyer has j deposit opportunities.[13]

At this stage, banks have access to a competitive interbank market in which they can borrow or lend excess funds at interest rate i_f (the *policy rate*) which is effectively set by the central bank. To begin with, we associate the long-run monetary policy with this rate: $i_f = (\gamma - \beta)/\beta$, where $\gamma = 1 + \tau$ is the gross growth in the money supply.

We denote the distributions of posted deposit and loan rates by $G(i_d, m, \mathbf{s})$ and $F(i, m, \mathbf{s})$, respectively. A bank posting loan and deposit rates i and i_d, thus has expected profit:

$$\begin{aligned}\Pi(m,\mathbf{s}) = \max_{i,i_d} & \, n\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i,m,\mathbf{s}\right)\right) + \alpha_2\zeta\left(i,m,\mathbf{s}\right)\right] l\left(i,m,\mathbf{s}\right)[1+i] \\ & - (1-n)[\alpha_1^d + 2\alpha_2^d G(i_d,m,\mathbf{s}) + \alpha_2^d\eta(i_d,m,\mathbf{s})]\, d\,[1+i_d] + (1+i_f)e(m,\mathbf{s},i_f,i,i_d),\end{aligned} \tag{2.19}$$

where $e(m,\mathbf{s},i_f,i,i_d) \equiv (1-n)\left[\alpha_1^d + 2\alpha_2^d G(i_d,m,\mathbf{s})\right] d - n\left[\alpha_1 + 2\alpha_2(1 - F(i,m,\mathbf{s}))\right] l^\star(i,m,\mathbf{s})$.

The existence of a competitive interbank market renders the loan-rate posting and deposit-rate posting problems for each bank independent of one another except for their dependence on the policy rate i_f and households' DM money holdings which influence both borrowers' loan demand and the supply of deposit funds.[14] As such, substituting out each bank's fund deficit or surplus term, $e(m,\mathbf{s},i_f,i,i_d)$, we rewrite expected profit as

$$\begin{aligned}\Pi(m,\mathbf{s}) &= \max_{i}\Pi_l(i,m,\mathbf{s}) + \max_{i_d}\Pi_d(i_d,m,\mathbf{s}) \\ &= \max_{i} n\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i,m,\mathbf{s}\right)\right) + \alpha_2\zeta\left(i,m,\mathbf{s}\right)\right] R_l\left(i,m,\mathbf{s}\right) \\ &+ \max_{i_d}(1-n)[\alpha_1^d + 2\alpha_2^d G(i_d,m,\mathbf{s}) + \alpha_2^d\eta(i_d,m,\mathbf{s})]R_d\left(i_d,m,\mathbf{s}\right),\end{aligned} \tag{2.20}$$

where

$$\zeta\left(i,m,\mathbf{s}\right) = \lim_{\varepsilon \searrow 0}\left\{F\left(i,m,\mathbf{s}\right) - F\left(i-\varepsilon,m,\mathbf{s}\right)\right\}, \tag{2.21}$$

$$R_l\left(i,m,\mathbf{s}\right) = l^\star\left(i,m,\mathbf{s}\right)[i - i_f] \tag{2.22}$$

$$\eta(i_d,m,\mathbf{s}) = \lim_{\epsilon \searrow 0} G(i_d,m,\mathbf{s}) - G(i_d - \epsilon,m,\mathbf{s}), \tag{2.23}$$

$$R_d(i_d,m,\mathbf{s}) = d[i_f - i_d]. \tag{2.24}$$

$\Pi_l(\cdot)$ is expected profit from lending, with $R_l\left(i,m,\mathbf{s}\right)$ the profit per loan, $l^\star\left(i,m,\mathbf{s}\right)$ the demand for loans, and $n\alpha_2\zeta\left(i,m,\mathbf{s}\right)$ the measure of consumers that contact this bank and another posting the same loan rate, i. Likewise, $\Pi_d(\cdot)$ is expected profit from deposits. This has three parts associated with the types

and re-label the $1-n$ measure of agents as "inactive buyers". Substantively, the role of banking is still of the same form as BCW.

[13]In our empirically calibrated model, we relax the restriction that a depositor has no deposit opportunity to maintain consistency with Berentsen et al. (2007), in which inactive buyers can always meet a bank to deposit idle funds. Here, we present a setup where there could potentially be more friction in the deposit market. Either way, this will not alter the main insight of the model. Moreover, the search process in both loan and deposit markets can be generalized in many ways without substantively affecting the results we focus on here. See for examples of introducing a cost of search in Head and Kumar (2005), and Wang (2016).

[14] The external clearing house (via borrowing/lending in the competitive interbank market) also allows us to preserve a well-known result of independence between deposit and loan rates along the lines of Klein (1971) and Monti (1972). Andolfatto (2021) also makes use of a similar independence property.

of depositors the bank serves, those with only one deposit opportunity (with this bank), those with two, of which this bank's offered deposit rate is higher, and those with two opportunities at the same rate.[15]

Banks earn profit from both loan and deposit operations and in both cases face a similar trade-off. In lending, a bank can raise its profit *per loan* by raising its posted rate relative to the policy rate, i_f (that is, by increasing its *loan spread*). Alternatively, it can increase the *measure* of borrowers it serves by *lowering* this spread. On the deposit side, the bank can post a higher deposit rate to attract a larger number of depositors, or by lowering its deposit rate relative to i_f (*i.e.* increasing its *deposit spread*) it can earn a higher return per deposit.

As banks here are *ex-ante* identical, we may think of the distribution $F(\cdot, m, \mathbf{s})$ (and $G(\cdot, m, \mathbf{s})$) as representing different pure-strategy choices or of banks as mixing symmetrically over a range of loan (deposit) interest rates that yield the same expected profit. In either interpretation, each borrower and depositor faces distributions $F(\cdot, m, \mathbf{s})$ and $G(\cdot, m, \mathbf{s})$ of random loan and deposit rates, respectively. The existence of dispersion in either deposit or loan rates does not, however, depend on banks being homogeneous and/or earning equal expected profits from a range of posting strategies. For examples of equilibrium dispersion with price-posting and noisy search see Herrenbrueck (2017) and Baggs, Fung and Lapham (2018).

2.6 Households at the start of DM

Now consider the beginning of period t prior to both the realization of whether or not the household is active and the subsequent matching with lending and depository agents. Given money balance m, *all* households have *ex ante* value:

$$\begin{aligned} V(m,\mathbf{s}) = n\Big\{&\alpha_0 B^0(m,\mathbf{s}) + \int_{[\underline{i},\bar{i}]} [\alpha_1 + 2\alpha_1(1-F(i,m,\mathbf{s}))] B(i,m,\mathbf{s})\,\mathrm{d}F(i,m,\mathbf{s})\Big\} \\ &+ (1-n)\Big\{\alpha_0^d W^0(m+\tau_b M, 0, 0, \mathbf{s}) \\ &+ \int_{[\underline{i}_d,\bar{i}_d]} [\alpha_1^d + 2\alpha_1^d G(i_d,m,\mathbf{s}))] W(m+\tau_b M - d, 0, d(i_d), \mathbf{s})\,\mathrm{d}G(i_d,m,\mathbf{s})\Big\} \end{aligned} \tag{2.25}$$

Conditional on being active in the DM (with probability n), a buyer matches with zero, one, or two lenders and then behaves as described above. Similarly, with probability $1-n$, the buyer is inactive, has zero, one, or two deposit opportunities, and behaves as described above. All buyers take the distributions of posted loan and deposit rates, $F(\cdot, m, \mathbf{s})$ and $G(i_d, m, \mathbf{s})$, as given.[16]

2.7 Government

The government/monetary authority can effect nominal lump-sum transfers and taxes in both the CM and DM. Transfers made directly to households are denoted τ_1 (DM) and τ_2 (CM). Transfers to *banks* in the DM and CM are denoted τ_1^e and τ_2^e, respectively. In each period the *total* change to the money supply is $(\gamma-1)M \equiv \tau M$ is effected as follows:

$$M_{+1} - M = (\gamma - 1)M = \tau_1 M + \tau_2 M + \tau_1^e M + \tau_2^e M, \tag{2.26}$$

[15] We assume that in such cases, prospective borrowers (depositors) randomize between the two identical loan (deposit) rates posted by banks. In equilibrium, the probability of a borrower (depositor) observing two identical lending (deposit) rates goes to zero.

[16] We assume for now a compact supports for both distributions and show later that this is an equilibrium outcome. See Online Appendix A.

where

$$\tau_1 = n\tau_b + (1-n)\tau_b + \tau_s, \quad \text{and} \quad \tau_2^e = -\tau_1^e. \tag{2.27}$$

That is, in the DM transfers to households may be targeted separately to buyers and sellers, but not on the basis of a buyer being active or inactive. The last two terms on the right-hand side of (2.26) reflect interbank settlement. The total surplus or deficit of liquidity in the banking system, $e \equiv (1-n)\left[\alpha^d 1 + 2\alpha^d 2G(i_d, m, \mathbf{s})\right] d - n\left[\alpha_1 + 2\alpha_2(1 - F(i, m, \mathbf{s}))\right] l^\star(i, m, \mathbf{s})$, is met by a lump-sum injection or extraction of money made by the government. Specifically, if $e < 0$, indicating a total liquidity deficit among private banks in the DM, the government injects liquidity $\tau_1^e M = e$ into the banks on the spot via a lump-sum transfer. In the subsequent CM, the government must then extract money from the economy by taxing the banks the same amount, $\tau_2^e M = -\tau_1^e M$. The opposite occurs if there is a total surplus of liquidity.

Alternatively, we can interpret interbank settlement as follows. The central bank sets a price-level target by choosing a path for the money stock or pegging the policy interest rate, i_f, in the CM. Private banks can borrow from or lend to the central bank at the same interest rate i_f, depending on whether they have a surplus or deficit of liquidity. If $e > 0$, private banks lend this surplus in total, $e = \tau_1^e M$, to the central bank in the DM. The corresponding debt balance for the central bank, to be repaid to the private banks in the CM, is $\tau_2^e M = -\tau_1^e M$ associated with an interest rate i_f. The reverse occurs if private banks borrow from the central bank. This competitive interbank settlement we consider is analogous to the central bank's liquidity management studied in Berentsen and Waller (2011).

3 A Stationary Monetary Equilibrium

We focus on a *stationary monetary equilibrium* (SME), in which the price level and money supply grow at the same constant rate: $\phi/\phi_{+1} = M_{+1}/M = \gamma \equiv 1 + \tau$. In this section, we characterize the components of an SME, focusing on an equilibrium with valued money and positive credit. This equilibrium configuration will also emerge in the calibrated economy later. Moreover, the interesting cases will be when $\gamma > \beta$, *i.e.*, when the economy is away from the Friedman rule.[17]

As the price level $(1/\phi)$ is non-stationary, to obtain a well-defined stationary equilibrium we multiply nominal variables by ϕ. Let $z = \phi m$ and $Z = \phi M$ denote individual and aggregate real balances, respectively. Also, let $\rho = \phi p$ denote the real relative price of DM goods and $\xi = \phi l$ the real value of a loan. The stationary counterpart to the state-policy vector $(m, \mathbf{s})$ will now be $(z, \mathbf{z})$, where $\mathbf{z} = (Z, \boldsymbol{\tau})$.

3.1 The distribution of posted loan rates

In an SME, DM sellers neither accumulate money in the CM nor borrow. Inactive DM households deposit all their money with banks. Thus, we focus first on the loan demand of active buyers. In Online Appendix A.7 (Lemma 14), we show that if inflation above the Friedman rule prescription $(\gamma > \beta)$ and agents have a non-zero and interior probability of seeing more than one loan rate quote, then there is a continuous cumulative probability distribution of posted loan rates:

$$F(i, z, \mathbf{z}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\bar{i}, z, \mathbf{z})}{R(i, z, \mathbf{z})} - 1\right]. \tag{3.1}$$

[17] There are many reasons why long-run inflation at β may not be implementable. For our purposes, we take this as an institutional constraint on monetary policy.

Here $R(i, z, \mathbf{z}) \equiv R(i, z, \mathbf{z}) = \left[\rho^{\frac{\sigma-1}{\sigma}} (1+i)^{-\frac{1}{\sigma}} - (z + \tau_b Z)\right] (i - i_f)$ the real bank profit per loan at the rate of i. The distribution has support $\left[\underline{i}(z, \mathbf{z}), \bar{i}(z, \mathbf{z})\right]$, where $\underline{i}(z, \mathbf{z})$ solves $R(\underline{i}, z, \mathbf{z}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R\left(\bar{i}, z, \mathbf{z}\right)$,. The upper support satisfies $\bar{i}(z, \mathbf{z}) = \min\{i^m(z, \mathbf{z}), \hat{i}(z, \mathbf{z})\}$. For the case of $\alpha_2 = 1$, the $F(\cdot)$ is degenerate at the central bank policy rate, i_f, effectively a Bertrand pricing equilibrium. Alternatively, if $\alpha_2 = 0$, $F(\cdot)$ is degenerate at $i^m(z, \mathbf{z})$, the monopoly outcome. These results are akin to those characterizing the original notion of "firm equilibrium" in Burdett and Judd (1983, Lemma 2) and in the monetary version of Head and Kumar (2005, Proposition 3).

We now have the following useful comparative static result regarding the relationship between household-level real balances and the distribution of posted lending rates:

Lemma 1. *Fix a long-run money growth rate $\gamma > \beta$, and let $\alpha_0, \alpha_1 \in (0, 1)$. Consider any two real money balances z and z' such that $z < z'$. The induced loan-price distribution $F(\cdot, z, \mathbf{z})$ first-order stochastically dominates $F(\cdot, z', \mathbf{z})$.*

The proof can be found in Online Appendix C.1. In short, in an SME where households carry higher (lower) real balances into the DM, they are more (less) likely to draw lower loan-rate quotes, *ceteris paribus*. This reflects the fact that when potential borrowers carry low real balances into the period, demand for loans will be relatively high. All else equal, given strong loan demand, lending agents' optimal loan rates and spreads rise. Hence, the conclusion of Lemma 1.

3.2 The distribution of posted deposit rates

We derive the distribution of posted deposit rates, $G(\cdot)$, in Lemma 15 in Online Appendix A.8. In particular, if $\alpha_1^d \in (0, 1)$, there is a unique, continuous distribution of posted deposit rates:

$$G(i_d; \gamma) = \frac{\alpha_1^d}{2\alpha_2^d} \left[\frac{R(i_d^m, z, \gamma)}{R(i_d, z, \gamma)} - 1\right] = \frac{\alpha_1^d}{2\alpha_2^d} \left[\frac{(z + \tau_b Z)[i_f - i_d^m]}{(z + \tau_b Z)[i_f - i_d]} - 1\right], \tag{3.2}$$

where the support of $G(i_d; \gamma)$ is $\left[\underline{i}_d, \bar{i}_d\right]$, $\underline{i}_d = i_d^m = 0$, $i_f = (\gamma - \beta)/\beta$ and $\bar{i}_d = \frac{\gamma-\beta}{\beta}\left[1 - \frac{\alpha_1^d}{\alpha_1^d + 2\alpha_2^d}\right]$. If $\alpha_2^d = 1$, the Bertrand outcome for the deposit rate (for all banks) is the policy rate, i_f. Alternatively, if $\alpha_2^d = 0$, we again have the monopoly case where all banks offer a zero rate of return on deposits. Note that because prospective depositors' real money is predetermined when they search for deposit opportunities, the distribution of posted deposit rates, $G(\cdot; \gamma)$, does not depend on state variables other than the policy γ.

3.3 The demand for money and bank credit

We now derive an equation for households' optimal money demand in the CM. Again, we restrict attention to an SME in which both *ex-ante* demand for money balances and *ex-post* demand for loans in the DM are positive.[18]

Lemma 2. *Fix the long-run money growth rate $\gamma \equiv 1 + \tau > \beta$. Assume $\alpha_0, \alpha_1 \in (0, 1)$, and $\alpha_0^d, \alpha_1^d \in (0, 1)$. Assume that there is an SME in which real balances, $z^\star \in \left(0, \left(\frac{1}{1+\bar{i}(z^\star, \mathbf{z})}\right)^{\frac{1}{\sigma}}\right)$. Then,*

[18] This will turn out to be the equilibrium configuration that emerges under our calibration. This is also the case when we consider a range of computational experiments later. These properties of the SME rely on sufficient conditions that are *per se* not purely characterized by model primitives. (See Proposition 2 further below for the details.) However, the sufficient conditions are satisfied automatically in the computational experiments.

1. *the relative price of DM goods satisfies*

$$\rho = 1 < \tilde{\rho}_i(z^\star, \mathbf{z}) \equiv (z^\star)^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}, \tag{3.3}$$

for any $i \in \text{supp}(F(\cdot; z^\star, \mathbf{z}))$*;* $\tilde{\rho}_i = \phi\tilde{p}_i$ *is the stationary transform of cut-off pricing function* $\tilde{p}_i$*, defined in* (2.16)*;*

2. *loan demand is always positive; and,*

3. *real money demand, satisfies the equation:*

$$\frac{\gamma-\beta}{\beta} = \underbrace{(1-n)\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d[\alpha_1^d + 2\alpha_2^d G(i_d;\gamma)]\,dG(i_d;\gamma)}_{[A]} + \underbrace{n\alpha_0\left(u'[q_b^0(z^\star,\mathbf{z})]-1\right)}_{[B]}$$
$$+ \underbrace{n\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} i\left[\alpha_1 + 2\alpha_2(1-F(i,z^\star,\mathbf{z}))\right] dF(i,z^\star,\mathbf{z})}_{[C]}. \tag{3.4}$$

In the CM of each period, an agent anticipates that in the following DM, they will be an active buyer with probability n. In this case, the household has an incentive to carry money given the potential cost of borrowing.[19] The left-hand side of (3.4) is the forgone nominal risk-free interest rate due to demanding money–*i.e.*, the marginal cost of real balances.

The terms on the right-hand side of (3.4) constitute the expected marginal benefit of carrying money into the next DM. Term A captures the benefit of banking in reducing the cost of holding money balances in the event that the agent is inactive and does not want to spend. As in BCW, banks here insure households against carrying money while inactive. In contrast to BCW, here households must take into account the effect of the search process on the expected return on deposits.

Term B is the marginal return on own money in the event that the household is an active buyer and makes no contact with a lender. Term C represents the marginal return on money for an active buyer that contacts at least one lender and borrows. A higher beginning-of-period money balance in this case economizes on loan interest. This term is also present in BCW. However, in this case, it reflects the effect of the search process on the expected loan rate.

Berentsen et al. (2007) show that by providing insurance the banking system increases the demand for money, thus raising real balances, DM consumption, and welfare. Here, imperfect competition among banks both raises loan rates and *lowers* deposit rates to an extent determined by the search process in equilibrium. This reduces the demand for money and lowers real balances and DM consumption relative to the BCW case as indicated by the presence of the interest spread in (3.4). As a result, whether or not the presence of banking improves household welfare in this environment is not clear. We return to this issue and explore the welfare characteristics of the SME further in Sections 4 and 5.2.

[19]Here as in BCW, households are insured against the cost of carrying idle balances by the availability of bank deposits. Their incentive to carry money is increased here relative to BCW by 1) the existence of positive loan spreads (loans are expensive) and 2) the possibility of having no access to loans (if $\alpha_0 > 0$).

3.4 Equilibrium in the DM and CM goods markets

Sellers in the DM are Walrasian price takers, and so in equilibrium, the real price of the DM good equals its marginal cost: $\rho = c'(q_s)$. Supply, q_s, equals demand for the DM good:

$$\begin{aligned} q_s(z,\mathbf{z}) \equiv c'^{-1}(\rho) = n\alpha_0 q_b^{0,\star}(z,\mathbf{z}) \\ + n\left[\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,z,\mathbf{z})]\, q_b^\star(i,z,\mathbf{z})\, \mathrm{d}F(i,z,\mathbf{z})\right]. \end{aligned} \tag{3.5}$$

Given $x^\star = 1$, we can also verify that aggregate CM labor equals $x^\star$ due to the assumption that all households have access to a linear production technology in the CM.

3.5 Equilibrium in banking

In equilibrium, banks posted loan and deposit rates must earn non-negative expected profits given the search processes in both markets:

$$\Pi^\star(z,\mathbf{z}) = \Pi_l^\star(z,\mathbf{z}) + \Pi_d^\star(z,\mathbf{z}) = \max_{i\in\mathrm{supp}(F(\cdot,z,\mathbf{z}))} \Pi_l(i,z,\mathbf{z}) + \max_{i_d\in\mathrm{supp}(G(\cdot,z,\mathbf{z}))} \Pi_d(i_d,z,\mathbf{z}) \geq 0. \tag{3.6}$$

Banks have access to a competitive interbank market and thus may borrow if they face a shortfall in liquidity and if they have a surplus. The resource constraint is balanced via the adjustment of e in the interbank market:

$$\begin{aligned} &(1-n)\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} [\alpha_1^d + 2\alpha_2^d G(i_d;\gamma)](z+\tau_b Z)\mathrm{d}G(i_d;\gamma) \\ &= e + n\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,z,\mathbf{z})]\, \xi^\star(i,z,\mathbf{z})\, \mathrm{d}F(i,z,\mathbf{z}), \end{aligned} \tag{3.7}$$

where $e = \tau_1^e Z$ and $z = Z$ at equilibrium. As additional borrowing/lending is permitted at policy rate i_f, total interest earned on assets weakly exceeds that paid on total liabilities in equilibrium.

3.6 Summary: an SME with valued money and positive credit

Definition 1. *A* stationary monetary equilibrium with money and credit *is a steady-state allocation* $(x^\star, z^\star, Z)$, *allocation functions* $\{q_b^{0,\star}(z^\star,\mathbf{z}), q_b^\star(\cdot,z^\star,\mathbf{z}), \xi^\star(\cdot,z^\star,\mathbf{z}\}$, *and pricing functions* $(\rho, F(\cdot;z^\star,\mathbf{z}), G(i_d,\mathbf{z}))$ *such that given government policy* $\boldsymbol{\tau}$ *satisfying* (2.27),

1. $x^\star = 1$;
2. $z^\star \equiv z^\star(\boldsymbol{\tau}) = Z$ *solves* (3.4);
3. *given* $z^\star$, $q_b^{0,\star}(z^\star,\mathbf{z})$ *and* $q_b^\star(\cdot,z^\star,\mathbf{z})$, *respectively, satisfy*

$$q_b^{0,\star}(z^\star,\mathbf{z}) = \frac{z^\star + \tau_b Z}{\rho}, \qquad \text{for } \rho < \hat{\rho}(z^\star,\mathbf{z}), \tag{3.8}$$

and,

$$q_b^\star(i,z^\star,\mathbf{z}) = [\rho(1+i)]^{-\frac{1}{\sigma}}, \qquad \text{for } 0 < \rho \leq \tilde{\rho}_i(z^\star,\mathbf{z}) \text{ and } 0 \leq i < \hat{i}(z^\star,\mathbf{z}); \tag{3.9}$$

4. $\xi^\star(\cdot, z^\star, \mathbf{z})$ *satisfies:*

$$\xi^\star(i, z^\star, \mathbf{z}) = \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z^\star + \tau_b Z), \quad \text{for } \rho \in (0, \tilde{\rho}_i(z^\star, \mathbf{z})], \quad i \in [0, \hat{i}(z^\star, \mathbf{z})); \tag{3.10}$$

5. ρ *solves* (3.5);

6. $F(\cdot; z^\star, \mathbf{z})$ *is determined by* (3.1);

7. $G(i_d; \gamma)$ *is determined by* (3.2), *and*

8. *Banking is feasible according to* (3.6) *and* (3.7).

Remarks. In an SME, we may set $z = z^\star(\boldsymbol{\tau}) = Z$, and collapse the state-policy vector $(z, \mathbf{z})$ into $\mathbf{z}$ for the purposes of the discussion in the next section. The policy parameter τ_s does not materially affect equilibrium determination, so we set $\tau_s = 0$ with no substantive effects. For now and in our baseline calibration below, we also set $\tau_b = 0$, so that there is no redistributive policy in place. Later we consider counterfactual exercises involving differential tax policies.

4 Analytical characterization of the SME

We start with the first-best allocation and then discuss the existence and uniqueness of an SME. We then illustrate the workings of the lending market power channel using a number of special cases. We then consider the welfare effects of banking activity and provide conditions under which an economy with banks achieves lower welfare in equilibrium than it would if there were no banking whatsoever. Lastly, we consider the pass-through of changes in the policy rate to the distributions and average levels of both the loan and deposit rates.

4.1 The Friedman Rule attains the first-best

Proposition 1. *If $1 + \tau \equiv \gamma = \beta$ (the Friedman Rule), then there is no SME with dispersion in loan and deposit interest rates. Moreover, the unique SME attains the first-best allocation, $q^{\star,FB}$.*

The proof can be found in Appendix B. With $\gamma = \beta$, it is costless to carry money across periods. Banks, as a facility for reallocating ex-post liquidity needs, are redundant. Households have no need of insurance against the risk of being inactive and there is no gain to redistributing liquidity in an SME. From this point on, we restrict attention to $\gamma > \beta$.

4.2 SME with money and credit

Under sufficient conditions, away from the Friedman Rule there exists a unique SME with money and credit:

Proposition 2. *Assume loan contracts are perfectly enforceable. If $1 + \tau \equiv \gamma > \beta$, $z^\star \in (0, \overline{z})$, where $\overline{z} = \left[1 + \overline{i}(z^\star, \mathbf{z})\right]^{-\frac{1}{\sigma}}$, then there exists a unique SME with both money and credit.*

For a proof, see Appendix C.4, with formal proofs of intermediate results found in Appendices C.1, C.2, and C.3. Here we sketch the basic idea. Fix $\gamma > \beta$. First, note that the distribution of posted deposit-rate distribution $G(i_d; \mathbf{z})$ is invariant to z as households' real money balance is predetermined in the current banking session, having been chosen in the preceding CM.

Second, we show that lending banks' posted loan-price distribution $F(\cdot, z, \mathbf{z})$ is decreasing (in the sense of first-order stochastic dominance) in households' real balance, z. As households carry more money into the DM, the marginal benefit of bank credit falls. See Lemma 1 for details. As such, when households have higher real balances, given the distribution of posted rates, they are more likely to be able to borrow at a lower interest rate.

Third, with probability α_0, a household contacts no lending agent and so its marginal benefit from holding an extra dollar falls as real balances rise. Together, these factors establish that the right-hand side of (3.4) is a continuous and monotone decreasing function of z. Since the left-hand side of (3.4) is constant in z, there exists a unique real money balance $z^\star$ for a given $\gamma > \beta$.

The condition $z^\star \in (0, \bar{z})$ ensures that real balances are low enough and that the maximal loan interest is not too high. This guarantees positive loan demand. Although the upper bound $\bar{z}$ is not determined solely by model primitives (*i.e.*, it depends on the equilibrium object, $\bar{i}(\cdot)$), we have verified that it holds throughout our numerical results below.

4.3 The monetary policy rate and banking market power

To illustrate the effects of monetary policy on the extent of market power in banking, we consider several special cases of the general economy presented in Section 2.[20]

A pure monetary economy without banks. Suppose $\alpha_0 = 1$ and $\alpha_0^d = 1$. The economy will then resemble a monetary economy with no banks. In particular, the DM consumption in this economy is determined by

$$\hat{q} = \left[1 + \underbrace{\frac{1}{n}}_{>1} \frac{\gamma - \beta}{\beta}\right]^{-\frac{1}{\sigma}}. \tag{4.1}$$

Bertrand competition for both loans and deposits. Suppose $\alpha_2 = \alpha_2^d = 1$ so that all active (inactive) households have two loan (deposit) opportunities. Then, the distributions of both loan and deposit rates degenerate at the policy rate, which determines the opportunity cost of holding money in the SME (see Lemmas 14 and 15 in the Online Appendix). Moreover, with $i_f = (\gamma - \beta)/\beta$, it is equal to the equilibrium market rate with perfectly competitive banking. Thus, in this case, the SME is equivalent to the competitive outcome of BCW. In particular, DM consumption is

$$q^{BCW} = \left[1 + \frac{\gamma - \beta}{\beta}\right]^{-\frac{1}{\sigma}}. \tag{4.2}$$

A comparison of (4.2) and (4.1) highlights the distortion that banking overcomes in BCW (by raising DM output and welfare). As the probability of being active in the DM (n) falls, the opportunity cost of holding money is amplified as households are more likely to hold idle money. Banks insure against this risk by paying interest on deposits of idle funds. In our economy, we refer to this as the *liquidity risk* channel. As in BCW, this channel is operative at any policy rate away from the Friedman Rule.

Monopoly lending and Bertrand competition for deposits. Suppose $\alpha_1 = 1$ and $\alpha_2^d = 1$. In this case, the distribution of loan rates is degenerate at the highest possible rate that borrowers will accept, *i.e.*

[20] More details on these special cases can be found in Appendix D.

the *monopoly rate*. The distribution of deposit rates remains, however degenerate at the policy rate (again, see Lemmas 14 and 15). The equilibrium loan rate now is

$$i^m = i_f \left[\frac{\epsilon(i_m, \mathbf{z})}{1 + \epsilon(i_m, \mathbf{z})} \right] = \frac{\gamma - \beta}{\beta} \left[\frac{\epsilon(i_m, \mathbf{z})}{1 + \epsilon(i_m, \mathbf{z})} \right], \tag{4.3}$$

where $\epsilon(i_m, \mathbf{z}) = (\partial \xi(i_m, \mathbf{z}) / \partial i_m)[i_m / \xi(i_m, \mathbf{z})]$ is the elasticity of loan demand, $\xi(i_m, \mathbf{z})$.[21]

Let $\mu^m(\mathbf{z}) = \epsilon(\mathbf{z})/(1 + \epsilon(\mathbf{z}))$ denote the monopoly loan rate spread over the opportunity cost of holding money. This term captures how much more expensive it is to borrow from the monopoly lending bank. We label this factor as the *lending market power* channel. In this case, DM consumption in the SME is

$$q^m = \left[1 + \underbrace{\mu^m(\mathbf{z})}_{>1} \frac{\gamma - \beta}{\beta} \right]^{-\frac{1}{\sigma}}. \tag{4.4}$$

Comparing (4.4) and (4.1) in this case we can see that two mechanisms increase the (gross) cost of accumulating real money balances. One is due to the *lending market power channel*, the other to the *liquidity risk channel*. In this case, banks provide liquidity risk insurance to the same extent as in BCW by paying deposit interest at a rate equivalent to the opportunity cost of holding money. Thus, the term $1/n$ does not appear in (4.4). There is, however, now an additional friction. Banks can charge a loan rate spread $\mu^m(\mathbf{z})$ over the policy rate i_f. This friction manifests (through monopoly lender market power) similarly to the market power of the part of sellers in goods trades with the difference being that buyers' forgone surplus shows up as banks' profits. Overall, the welfare implications of banks' liquidity reallocation depend on which of these channels dominates.

Noisy search for loans and Bertrand competition for deposits. Next, we consider an economy with noisy search for loans while maintaining the assumption of Bertrand competition for deposits. Parametrically, let $\alpha_0 = 0$, so all active buyers either receive one or two loan-rate quotes and let $\alpha_2^d = 1$. In this case, banks insure households against liquidity risk to the same extent as in BCW.

Let $J(i, \mathbf{z}) = \alpha_1 F(i, \mathbf{z}) + \alpha_2[1 - (1 - F(i, \mathbf{z}))^2]$ denote distribution of *transacted* loan rates and $\mu(i, \mathbf{z}) = i/i_f$ the loan rate spread associated with $i \in \text{supp}(F(i, \mathbf{z}))$. Hence, $\mu(\mathbf{z}) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \mu(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z})$ is the average transacted loan rate spread in an SME, given policy $\gamma > \beta$. The average loan rate spread here lies between the limiting cases of Bertrand and monopoly loan pricing (above the former, below the latter). In this case, the expected DM consumption for an active buyer is

$$q = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [1 + \underbrace{\mu(i, \mathbf{z})}_{>1} i_f]^{-\frac{1}{\sigma}} \mathrm{d}J(i, \mathbf{z}), \tag{4.5}$$

where the lending market power channel is now captured by the average and dispersion of the loan rate spread.

Market power in lending, arising here from the search (*i.e.*, imperfect informational) friction, allows banks to extract surplus from the use of the medium exchange, that is, from *goods* trades. Consequently, banking does not necessarily increase real balances and improve welfare, even though banking insures liquidity risk to

[21] The elasticity (and the monopoly rate) depend on preferences, σ, the DM goods price, ρ, and the real money balance z. It thus varies with inflation, γ. We focus on the case where $\sigma < 1$, in which the demand for loans is elastic: $\epsilon(\cdot)/(1 + \epsilon(\cdot)) > 1$ when $\gamma > \beta$. The bank thus charges a finite positive interest spread over the policy rate. Moreover, if $\epsilon(\cdot) \to -\infty$, then $i^m \to i_f$. For brevity, we have used $\mathbf{z}$ only to reference its dependence on the state-policy vector.

the same extent as in BCW. Rather, the welfare implications of banking activity (relative to that achieved by an economy without banks), depend on whether the *lending market power* channel dominates the *liquidity risk* channel. This, in turn, depends on monetary policy through its impact on $\mu(\mathbf{z})$.

Noisy search for both deposits and loans. Next, consider a case with a noisy search in the markets for both loans and deposits. Effectively, imperfect competition for deposits amplifies the lending market power channel, but not does not alter the main qualitative results. The principal difference is that banks no longer completely insure households' liquidity risk *ex-ante*, thus affecting money demand.

Inactive households (depositors) in this case are paid less than the competitive deposit rate, discouraging money accumulation. (See Equation (3.4).) Lower real money balances, in turn, increase market power in lending (see Lemma 1). Consequently, active households (borrowers) face a distribution of less favorable loan rates, resulting in lower DM consumption than they would realize with Bertrand deposit pricing. We turn next to the welfare consequences of the lending market power channel.

4.4 Welfare analysis

We focus here on the case of noisy search in the loan market and Bertrand competition for deposits, as this is sufficient to illustrate our principal results. Later, we allow for imperfect competition for both loans and deposits in our calibration and numerical/quantitative analysis in Section 5. For now we set $\alpha_0 = 0$, $\alpha_1 \in (0, 1)$, $\alpha_2^d = 1$. A household's lifetime expected value is then given by:

$$(1-\beta)W(\gamma) = U(x^\star) - x^\star + \underbrace{n\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b^\star(i,\mathbf{z})]\mathrm{d}J(i,\mathbf{z}) - c[q_s^\star(\mathbf{z})],}_{W^{DM}(\gamma)\text{: net trading surplus in DM,}} \tag{4.6}$$

where the functions $q_b^\star(\cdot)$ and $q_s^\star(\cdot)$ are characterized by (3.9) and (3.5), respectively. The monopoly limit is equivalent to setting $\alpha_1 = 1$ in (4.6).

Let $\alpha_0 = \alpha_0^d = 1$. Lifetime expected utility in the no-bank economy is given by:

$$(1-\beta)\hat{W}(\gamma) = U(x^\star) - x^\star + \underbrace{nu(\hat{q}) - c(n\hat{q})}_{\hat{W}^{DM}(\gamma)}, \tag{4.7}$$

where $\hat{q}$ is determined by (4.1). Note that since $x^\star$ is always constant and identical across the different regimes or economies we compare, it suffices to consider welfare as the ex-ante, indirect utility induced by DM activity. In the no-banking regime, this is the term labeled $\hat{W}^{DM}(\gamma)$ in Equation (4.7).

From (4.6) and (4.7), we have that the difference in welfare across the two economies is that of the net DM trading surpluses. Denote the welfare under our noisy loan search equilibrium as $W^{DM}(\gamma)$. Thus, given $\gamma > \beta$, welfare is higher in a pure monetary economy without banking than in the economy with noisy loan search if:

$$\underbrace{n\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} u[q_b(i,\mathbf{z})]\mathrm{d}J(i,\mathbf{z}) - c\left(n\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} q_b(i,\mathbf{z})\mathrm{d}J(i,\mathbf{z})\right)}_{W^{DM}(\gamma)} < \underbrace{nu(\hat{q}) - c(n\hat{q})}_{\hat{W}^{DM}(\gamma)} \tag{4.8}$$

If lending market power is sufficiently strong it may dominate the liquidity risk channel and *ex-ante* welfare will be lower in an economy with imperfectly competitive credit than in an economy with no banking at all.

The following proposition characterizes sufficient conditions for this to be the case.

Proposition 3. *Let $\gamma > \beta$, $\alpha_1 \in (0,1)$ and $c(\cdot)$ be linear. If $z^\star \in (0, \bar{z})$, where $\bar{z} = (1 + \bar{i}(z^\star, \mathbf{z})]^{-\frac{1}{\sigma}}$, and there exists a n such that $1/\mu(\gamma) < n < 1$, then (4.8) holds: An equilibrium under noisy search banking is inessential or not welfare improving over a no-bank equilibrium.*

For the proof, see Appendix D.5, with detailed proofs of intermediate results in Appendices D.1, D.2, D.3, and D.4. Proposition 3 shows that the gains from banking depend on whether the *lending market power* or the *liquidity risk alleviation* channel dominates. When inflation is low, the latter channel is weak and the former is strong. In such cases, banking may lower welfare compared to a monetary economy without banks, contrary to BCW. Based on this result, we also have a corollary (proved in Appendix D.6) relating the result to the rate of inflation.

Corollary 1. *Assume $\gamma > \beta$ and $0 < \alpha_1 < 1$. If $z^\star \in (0, \bar{z})$, where $\bar{z} = (1 + \bar{i}(z^\star, \mathbf{z})]^{-\frac{1}{\sigma}}$. There exists a cut-off value on inflation $\tilde{\gamma} \in (\beta, \infty)$ such that $\mu(\tilde{\gamma}) = 1/n$. If, for $\gamma \geq \tilde{\gamma}$, and it holds that $1/n \geq \mu(\gamma)$, then welfare under noisy-search banking can dominate that under the no-bank equilibrium: $W^{DM}(\gamma) \geq \hat{W}^{DM}(\gamma)$.*

4.5 Pass-through of the policy rate to loan rates

In this section, we study the effect of long-run monetary policy on the distribution of loan rates. We summarize the results here and provide formal proofs in Appendix E.

Lemma 3. *Let $\alpha_0, \alpha_1 \in (0,1)$. Consider two economies that differ in inflation, γ and γ', such that $\gamma' > \gamma > \beta$. The induced loan-price distribution $F(\cdot, \gamma', \mathbf{z})$ first-order stochastically dominates $F(\cdot, \gamma, \mathbf{z})$.*

Lemma 4. *Assume that $\gamma > \beta$ and $\alpha_1 \in (0,1)$. Let the average posted loan rate and the average transacted loan rate, respectively, be*

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i dF(i, \mathbf{z}), \text{ and } \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i dJ(i, \mathbf{z}). \tag{4.9}$$

Both the average posted and transacted loan interest rates are monotone increasing in inflation γ.

Proposition 4. *Assume $\gamma = 1 + \tau > \beta$, and $\alpha_1 \in (0,1)$. Let the average posted loan rate spread and the average transacted loan rate spread, respectively, be*

$$\hat{\mu}(\gamma) \equiv \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \frac{i}{i_f(\gamma)} dF(i, \mathbf{z}), \text{ and } \mu(\gamma) \equiv \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \frac{i}{i_f(\gamma)} dJ(i, \mathbf{z}) \tag{4.10}$$

where $i_f(\gamma) = (\gamma - \beta)/\beta$. If (1): $\bar{i}(\mathbf{z}) - \underline{i}(\mathbf{z}) < 1/\beta$, and, (2): $\underline{i}(\mathbf{z}) - i_f(\gamma) < \hat{\epsilon}(\gamma)$, where

$$\hat{\epsilon}(\gamma) := \sqrt{\frac{1}{\beta} \frac{\alpha_1}{2\alpha_2} \frac{\xi(\bar{i}(\mathbf{z}), \mathbf{z})}{\xi(\underline{i}(\mathbf{z}), \mathbf{z})} \frac{1}{\hat{\mu}(\gamma)}} > 0 \tag{4.11}$$

then both the average loan rate spread and the transacted loan rate spread are monotone decreasing in inflation γ. That is, $\hat{\mu}_\gamma(\gamma) < 0$ and $\mu_\gamma(\gamma) < 0$.

Proposition 4 indicates incomplete pass-through of changes in the long-run inflation rate to the loan market, working through its effect on lenders' market power in loan pricing. Borrowers' demand for liquidity falls as inflation increases. As each prospective borrower demands a smaller loan, lenders lower their spreads to attract more customers. Effectively, the extensive margin response (the number of customers the lender successfully serves) dominates the intensive margin (profit per customer) in the lenders' rate posting decision. This leads to lower rate spreads over the policy rate, which is the opportunity cost of holding money. In this sense, loan rate pricing becomes more competitive when the opportunity cost of money is high and the need for additional liquidity is low.

4.6 Pass-through of the policy rate to deposit rates

Now consider the effect of a change in the long-run inflation rate on the distribution of deposit rates. We measure the deposit spread following the convention of Drechsler et al. (2017) and Choi and Rocheteau (2023a) and summarize our results below. Formal proofs are in Appendix F.

Lemma 5. *Let $\alpha_0^d, \alpha_1^d \in (0,1)$. Consider two economies that differ in inflation, γ and γ', such that $\gamma' > \gamma > \beta$. Then distribution $G(\cdot;\gamma')$ first-order stochastically dominates $G(\cdot;\gamma)$.*

Lemma 6. *Assume that $\gamma > \beta$, and $\alpha_1^d \in (0,1)$. Let the average posted deposit rate and the average transacted deposit rate, respectively, be*

$$\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d dG(i_d,\gamma), \text{ and } \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d d\hat{G}(i_d,\gamma),$$

where $\hat{G}(i_d,\gamma) \equiv \alpha_1^d G(i_d;\gamma) + \alpha_2[G(i_d,\gamma)]^2$ denotes the distribution of transacted deposit rates. Both the average posted and transacted deposit interest rates are monotone increasing in inflation γ.

Proposition 5. *Assume that $\gamma > \beta$, and $\alpha_1^d \in (0,1)$. Then, the average posted deposit rate spread:*

$$s^d(\gamma) = i_f(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d dG(i_d;\gamma),$$

is monotone increasing in inflation γ. Likewise, for the average transacted deposit rate spread.

Proposition 5 indicates that the pass-through of monetary policy to the average deposit rate is also incomplete. As for pass-through to the loan rates, this effect works through banks' market power, in this case for deposits. Higher inflation induces households to carry smaller real balances and at the same time increases the value of bank deposits (as an insurance against liquidity risk) to those that turn out inactive (depositors). The increased demand for deposits enables banks to post lower deposit rates as the marginal value of insurance is high, mitigating the extensive margin losses associated with posting relatively low deposit rates. Banks become effectively *less* competitive for deposits and thus extract more surplus from depositors.

4.7 Summary

In this section we have illustrated the effect of trend inflation (long-run monetary policy) on the extent of price competition among banks (represented by the level and dispersion of loan and deposit rate spreads) and its associated welfare implications. As outlined in Proposition 3 and Corollary 1, the *lending market power* channel may or may not dominate the *liquidity risk alleviation* channel. Specifically, welfare implications of imperfectly competitive banks' liquidity reallocation activities depend on the relative sizes of two frictions in the loan and deposit markets.

The lending market power channel tends to dominate the liquidity risk channel when inflation (and thus liquidity risk) is low and/or the search friction is strong (α_1 is high). The effects are reversed when the relative strength of these frictions is reversed. From Lemmata 5 and 6, and Proposition 5, we can also deduce that this non-monotone welfare gain of banking under the noisy search for loans friction will be further worsened when there is also a noisy search for deposits. As this is a quantitative issue, we now consider the relative strengths of these channels in numerical exercises using a calibrated version of the model.

5 Quantitative Analysis

We now discipline our *baseline model*—the full version with noisy search and non-degenerate deposit and loan rate distributions—by calibrating its parameters to macro-level data. Some parameters can be externally pinned down. For internally-determined parameter calibrations, our principal targets are the empirical money demand and the average loan spread. Full details of the calibration are provided in Appendix G; we omit the details here for brevity.

In our analysis here we investigate the effects of various parameters and alternative policies.[22] In Section 6 (below), we relate the model's testable predictions to micro-level empirical evidence on the dispersion and levels of loan and deposit rate spreads. We characterize an optimal cyclical policy for our calibrated economy in Section 7.

5.1 Comparative steady states

In an SME fixing the trend inflation rate at $\gamma = 1 + \tau$ and setting the nominal policy interest rate at $i_f = (1+\tau-\beta)/\beta$ are equivalent. From here on, we consider trend inflation the monetary policy instrument and study SMEs indexed by different net inflation rates, τ. We ask first, what mechanisms affect the distributions of loan and deposit rate spreads and the pass-through of monetary policy? Second, we derive testable empirical predictions of these mechanisms. Finally, we ask, under what circumstances are agents *ex-ante* better off in an economy with banks than in one without them?

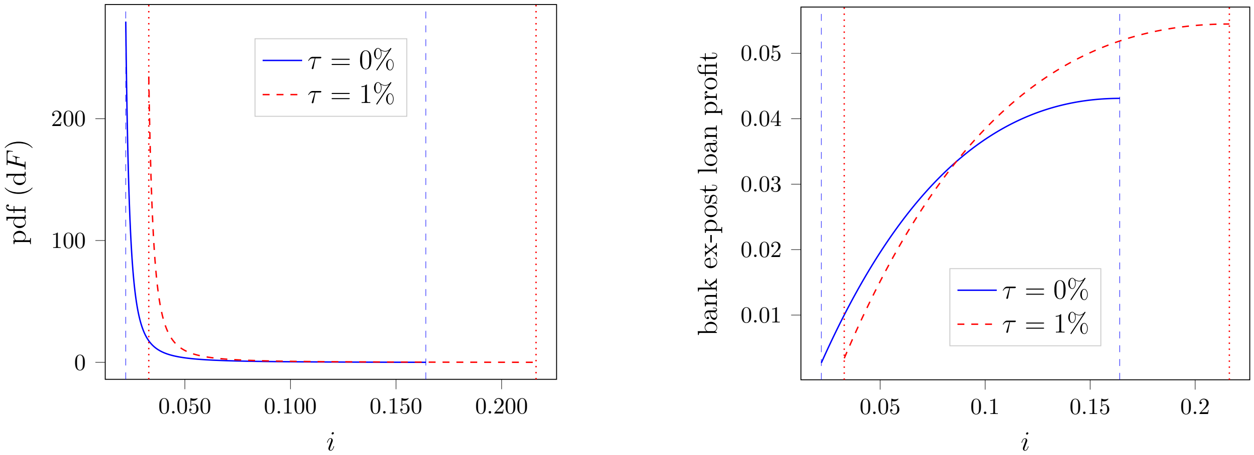
5.1.1 Loan and Deposit Pricing: intensive-extensive profit margin trade-off

Figure 2 depicts posted loan rate densities and realized profit per loan customer for steady-state inflation rates at zero (*solid-blue*) and one percent (*dashed-red*). Note both the shift and increase in the support of the distribution as inflation increases. The figure depicts the trade-off between profit per customer (the intensive margin) which is increasing in the posted loan rate, and the number of customers that it successfully serves (the extensive margin) which is *decreasing* in the posted rate. As inflation, τ, rises, not only does the equilibrium support of F shift to the right, but the mass of the density also shifts rightward relative to the lower bound. We identify this latter effect as the extensive margin: As inflation rises lenders raise their loan rates relative to the lower bound, increasing profit per loan but reducing their expected number of loan customers.

Similarly, Figure 3 depicts the densities of posted *deposit* rates and realized profit per depositor (represented here as the spread over the policy rate) for zero (*solid-blue*) and one percent (*dashed-red*) trend

[22] We have also studied a version of the baseline model augmented to allow for exogenous random default on loans targeting the national average percentage of consumers with new bankruptcies in the United States. Our data source for that is the *Quarterly Report on Household Debt and Credit*, May 2022, Federal Reserve Bank of New York. While default of this type has some quantitative implications the results are qualitatively the same as those of our baseline economy presented here.

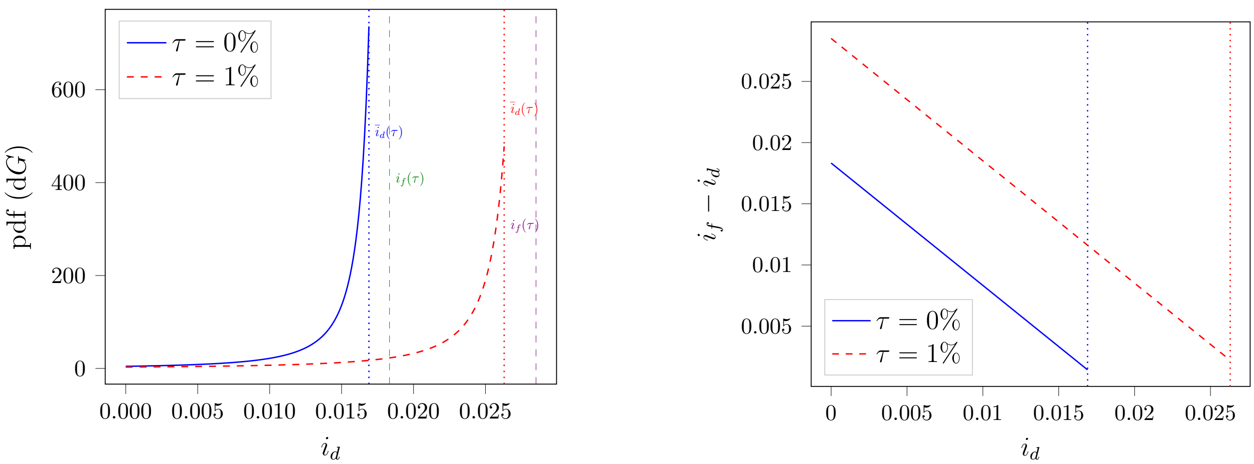
Figure 2: Posted loan rates and bank profit per loan.



(a) Density of posted loan rates

(b) Loan profit per customer

Figure 3: Posted deposit rates and spreads.



(a) Density of posted deposit rates

(b) Deposit spread (profit per depositor)

inflation. The blue and purple dashed lines in Panel (a) of Figure 3 are the policy rates at zero and one percent inflation, respectively. Banks face a similar trade-off in deposit pricing to that described above for loans. A bank that posts a high deposit rate attracts more customers (the extensive margin) at the expense of realizing a low deposit rate spread (the intensive margin).

As inflation (and the policy rate) rises banks have the incentive to raise deposit rates to attract more customers. At the same time, however, higher inflation increases the demand for insurance against liquidity risk, making depositors willing to accept lower deposit rates. As a result, while deposit rates rise on average, spreads increase, generating incomplete pass-through and higher bank profits at any given rate.

5.1.2 The dispersion of loan and deposit rate spreads

The distribution of posted loan rates, $F(i, \mathbf{z})$, gives rise to an associated distribution of loan rate spreads, with the average loan rate spread given by

$$\bar{\mu}(\tau) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \left[\frac{i}{i_f(\tau)} - 1 \right] \mathrm{d}F(i, \mathbf{z}). \tag{5.1}$$

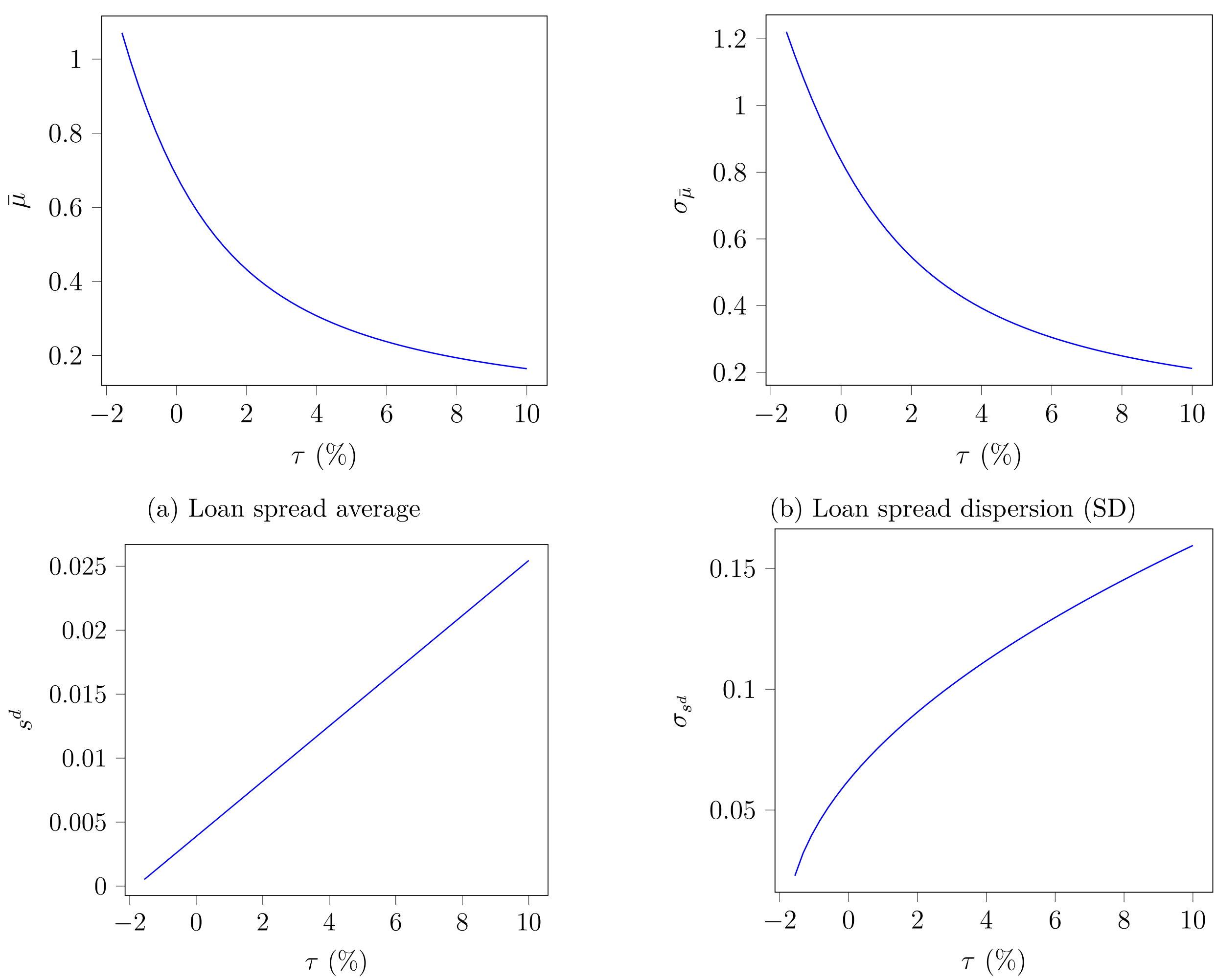
We measure the dispersion of the spreads by their standard deviation and coefficient of variation. Let $\breve{\mu}(i,\mathbf{z}) \equiv \frac{i}{i_f(\tau)} - 1$. The standard deviation of the loan rate spread is:

$$\sigma_{\bar{\mu}} = \left[\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\breve{\mu}(i,\mathbf{z}) - \bar{\mu}]^2 \mathrm{d}F(i,\mathbf{z}) \right]^{\frac{1}{2}}, \tag{5.2}$$

The associated coefficient of variation is $\breve{\mu}(i,\mathbf{z})/\bar{\mu}(\tau)$. We define measures of the dispersion of the deposit rate spread analogously.

Figure 4 illustrates both the means and standard deviations of the loan and deposit rate spreads for trend inflation rates ranging from -2% to 10%. Note that the monotonic relationships between these measures and inflation align with the analytical results in Propositions 4 and 5. Moreover, the relationships between inflation and the average spreads are consistent with both the theoretical and empirical results of Drechsler et al. (2017), Wang et al. (2022), and Choi and Rocheteau (2023a). We also show in Section 6 that these relationships (for both the average and dispersion of the spreads) are consistent with the U.S. micro-level evidence.[23]

Figure 4: The effects of inflation on banks market power for $\tau \in (\beta - 1, \bar{\tau}]$



(a) Loan spread average

(b) Loan spread dispersion (SD)

(c) Deposit spread average

(d) Deposit spread dispersion (SD)

[23]Similar results to those in Figure 4 arise in the U.S. data (for the sample period consistent with our use of *RateWatch* data on bank-level loan rates). See Online Appendix I.

As trend inflation (equivalently, the policy rate) rises, the average loan spread declines sharply, especially at low inflation. The average spread in (5.1) is the ratio of two parts that are both increasing in the inflation rate. First, the policy rate in the denominator rises, increasing the opportunity cost of holding money and putting upward pressure on loan rates. Second, higher inflation reduces real money balances and lowers consumption, raising marginal utility for active buyers who are thus willing to pay more for loans. This results in the shift in the distribution of loan rates established in Lemma 3 and underlying the conclusion of Lemma 4.

For the average loan rate *spread* to fall with inflation, the average loan rate itself must rise by less than the policy rate (*i.e.* the former effect must dominate the latter). In Proposition 4, we identify sufficient conditions for this to be the case, and these conditions hold in the calibrated model. Borrowers demand smaller loans when the rates they face are higher. Lenders are thus less willing to lose customers and "compete harder" as inflation rises, mitigating the *pass-through* to loan rates of the increase in the policy rate.

Increases in inflation are also passed through incompletely to deposit rates. First, the higher policy rate associated with increased inflation lowers the return to money, inducing households to carry lower real money balances into the DM. This reduces, in turn, the supply of deposits just as the value of insurance against holding idle balances increases. Both of these effects increase banks' market power in the deposits. As such, deposit rates rise by less than the policy rate. (See Proposition 5.)

The pass-through of monetary policy to deposit rates differs from that to loan rates. Banks are effectively *less* competitive for deposits when households' need for liquidity risk insurance is high. As such, the incomplete pass-through to deposit rates indicates an increase in banks' market power in deposits. In contrast, banks' market power in lending *falls* with inflation as this reduces households' need for additional liquidity is low.

5.2 The welfare effects of banking and inflation

As noted above in Sections 3.3 and 4 the welfare benefits of banking are non-monotonic in the rate of inflation. The banking system offers insurance against the cost of idle money balances, a benefit that increases with inflation but is mitigated by market power in deposit rates. At the same time, market power in lending reduces active buyers' surplus in goods trades, even when the DM goods market is perfectly competitive. This distortion can offset the insurance benefits of banking and may even outweigh them in some cases. We now consider quantitatively the welfare effects of banking in the presence of trend inflation in our calibrated economy.

In the presence of a monetary distortion (*i.e.* $\gamma \equiv 1+\tau > \beta$), imperfect competition among banks affects money demand through several channels that are distinct from those present in the competitive banking model of BCW. First, deposit rate spreads and dispersion discourage money accumulation relative to the competitive benchmark as deposit interest no longer fully offsets the cost of holding money. Second, loan rate spreads tighten the liquidity constraint of active buyers.[24] Third, as the trend rate of inflation (or the policy rate) changes, the bank passes these changes to their customers only partially and to extents that depend on both the elasticity of loan demand (deposit supply) and the nature of the search process.

We calculate welfare using the consumption equivalent variation (CEV) measure. A negative (or positive) CEV indicates how much additional (less) consumption would be needed to compensate a household from a pure monetary economy (without banks) to move to an economy with banks. In addition to our

[24] This effect is particularly strong for households that have no access to credit but it exists even if $\alpha_0 = 0$.

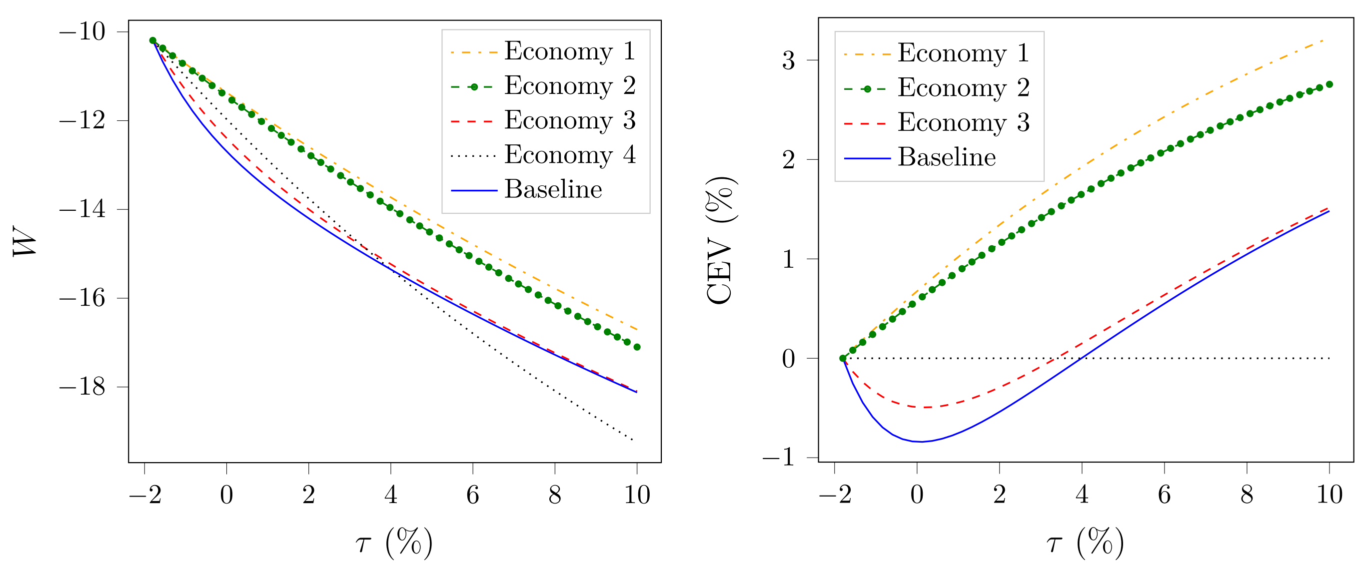
baseline calibrated economy, we consider the following alternatives: (1) Bertrand loan and deposit market competition (equivalent to BCW), where $\alpha_2 = \alpha_2^d = 1$; (2) Bertrand loan pricing and noisy search for deposit opportunities ($\alpha_2 = 1$ and $\alpha_2^d < 1$); and (3) noisy search for loan opportunities and Bertrand competition for deposits($\alpha_2 < 1$ and $\alpha_2^d = 1$). Expected lifetime welfare is defined in Section 4 (see, *e.g.*, (4.7) and (4.6)). We use $W^e(\tau)$, for $e \in \{1, 2, 3\}$ to denote expected lifetime welfare in each of the three cases above and use $W^{HKNP}(\tau)$ to denote this for our baseline calibrated economy.

A CEV is a wedge or factor Δ on DM consumption in an SME of any of the four banking scenarios above ($e \in \{1, 2, 3, HKNP\}$) such that its induced ex-ante utility $W^e_\Delta(\tau)$ equals that which is induced by a pure monetary (no-bank) economy, $W^{no-banks}(\tau)$. For example, consider the comparison of our baseline economy to the pure monetary economy. In this case, the Δ satisfies

$$\begin{aligned} W_\Delta^{HKNP}(\tau) = & \frac{1}{1-\beta}\left[U(\Delta x^\star) - x^\star - c[q_s^\star(\mathbf{z})]\right] \\ & + \frac{n}{1-\beta}\left[\alpha_0 u[\Delta q_b^{0,\star}(\mathbf{z})] + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,\mathbf{z})] u[\Delta q_b^\star(i,\mathbf{z})] \mathrm{d}F(i,\mathbf{z})\right] \\ & = W^{no-banks}(\tau). \end{aligned} \tag{5.3}$$

Similarly, we calculate Δ for economies $e = 1, 2, 3$, using (5.3) subject to the parametric restrictions on the α's discussed above.

Figure 5: Welfare Comparisons: Banks, competitive and non-competitive versus no banks.



(a) Welfare: level

(b) Banking versus no banks: CEV

Notes. Economy 1: Bertrand loan and deposit market competition (BCW-equivalent); Economy 2: Bertrand loan pricing and noisy search for deposit opportunities; Economy 3: Noisy search for loan opportunities and Bertrand competition for deposits; Economy 4: A monetary economy without banking.

Figure 5 compares welfare for five economies at inflation rates between -2% and 10%. The five cases are the four banking economies described above and a pure monetary economy without banking. Figure 5a illustrates how welfare declines with inflation in all five cases. Figure 5b shows the contribution of banking for the four banking economies relative to the pure monetary economy as inflation changes.

Gains from banking are always positive and increase in trend inflation when the loan market is competitive, regardless of market power in deposits. The contribution of banking to welfare is highest in the BCW-equivalent economy (the orange dashed-dotted line in Figure 5b). As described above, inflation represents the cost of carrying money into the DM for consumption purchases, a cost exacerbated by the possibility of being inactive in the DM. Bank deposits provide insurance against this additional cost, raising welfare relative to a pure monetary economy without insurance. Banking thus improves welfare, more so the higher the inflation rate. Market power in deposits alone erodes some of these insurance gains, but not all of them (the green dotted-solid line in Figure 5b).

Imperfect competition in lending, however, may result in welfare losses more than sufficient to offset the insurance benefits of interest on deposits. This is the case for both of our banking economies with market power in lending at relatively low rates of inflation (the red dashed and blue solid lines in Figure 5b). At a given inflation rate, banking raises the nominal price level even as it increases real balances by providing insurance through the deposit rate. With competitive banking, this effect is compensated for by loans which are no more costly than carrying money that is spent into the DM. With imperfect competition in lending, however, the loan rate spread renders borrowed *nominal* balances more costly than cash carried into the DM and effectively extracts surplus from active DM buyers in goods markets, lowering welfare. Households must either carry excessive nominal balances into the DM or pay lenders a high rate on loans in the event that they are liquidity-constrained. For households, this is akin to facing DM sellers who exercise market power. As the DM goods market is competitive, however, this surplus goes to lenders, rather than to DM sellers.

The overall effect is strongest at low inflation when the insurance value of deposits is low and loan spreads are high. While it always reduces welfare relative to the competitive lending case, this effect can only dominate if banks have sufficient market power and inflation is sufficiently low (see Proposition 3). As inflation rises, loan spreads fall and deposit interest rises. For any configuration of parameters consistent with an SME, at some inflation rate banking raises welfare. The welfare gains, however, are always lower than they would be if lending were competitive.[25]

To summarize, banking has two opposing welfare effects. First, it improves welfare by paying interest on deposits, thus providing insurance against holding idle money in the DM as an inactive buyer. Second, market power in the loan market reduces household surplus from goods trades in the DM, lowering the value of real balances. This happens as banks both increase the nominal price level and raise the cost of additional funds. With constant marginal cost of production, this can only happen in the presence of a loan spread and occurs even if all active buyers have access to banks. The overall welfare effect of banking depends on the relative sizes of these effects. In our baseline calibrated economy when inflation is sufficiently low, loan spreads are high enough and the gains to insurance are low enough that banking of this type reduces household welfare.

6 Inflation, interest pass-through and dispersion: empirics

At least two distinguishing features of our theory warrant empirical consideration. First, there is an imperfect pass-through of the policy rate to both lending and deposit rates that increases with inflation. Second, the equilibrium average loan (deposit) spread and the standard deviation of spreads in the loan (deposit) market

[25] We have considered a *hyperinflationary regime* as a robustness check. In this case, in all four banking economies, the relationship between trend inflation and welfare gain from banking is non-monotonic. This result is consistent with Berentsen et al. (2007). As $\tau \to \infty$, the welfare gains from banking approaches zero. The reason for this is that at sufficiently high inflation, the value of real balances tends towards zero.

are positively correlated. That is, as the policy rate rises, banks pass through the increase in costs of funds *differentially* to their lending (deposit) rates in a manner analogous to that described by Head, Kumar and Lapham (2010).

Since the first feature (imperfect pass-through *per se*) is already well-known, we focus here on the second. The model's equilibrium dispersion of loan (deposit) rates for an identical loan (deposit) product suggests the need to document new empirical relationships between dispersion measures and the average loan and deposit spread, controlling for other possible sources of variation in rates.

To maintain as close a match as possible between our model and the data, we focus on consumer loan rates in U.S. data (obtained from *RateWatch*). Likewise, using the same data set, we focus on fixed-term time deposits.[26] While we have information starting from the granular bank-branch level, we aggregate to the national level in our main regression results but find similar results at the state level.[27] We measure the dispersion of the spreads with their standard deviations.

Our main empirical finding is that there are positive relationships between the standard deviations and average levels of both the loan and deposit rate spreads at monthly frequency. Second, there is a positive relationship between the standard deviation and the average level of the deposit spread at monthly frequency. These empirical results corroborate the theoretical predictions of the model. Detailed regression results and correlations are in Appendices H and I. Appendix J contains the state-level analysis and considers alternative loan product classes, including mortgages. We find that our main results hold also for these alternative products.

7 Optimal stabilization policy

To this point we have focused on the effects of monetary policy in the long run; specifically a steady-state policy rate or (equivalently) rate of trend inflation. We now consider an optimal stabilization policy in response to a type of aggregate demand shock. Specifically, we solve a version of the Ramsey problem considered also by Berentsen and Waller (2011). Details of the problem setup and solution can be found in Appendix K.

In this exercise, we abstract from imperfect competition for deposits. The deposit rate distribution in equilibrium does not depend on state variables (and hence state-contingent policy) other than the trend inflation rate, γ. Here we consider stabilization policy within the context of a long-run price-level targeting regime. Effectively, policy actions taken in the DM are undone in the subsequent CM, thus maintaining a path of price-level growth at rate γ. Consequently, a state-contingent policy has no effect on the deposit rate distribution G and so for simplicity we assume Bertrand competition ($\alpha_2^2 = 1$) for deposits.

We associate random fluctuations in the fraction of households that are active buyers in the DM as shifts in aggregate demand. We then consider the problem of a central bank choosing state-contingent injections of liquidity in the DM optimally in response to these shocks. To maintain its commitment to the long-run price path associated with γ, the central bank commits to the DM liquidity injections to households and

[26] See https://www.rate-watch.com/. We have also checked that similar results are obtained on the loan side when we consider alternative classes of loan products (*e.g.*, mortgages) and different borrower risk groups. Since the simple model is about liquidity risk at the consumer level, it is appropriate here to present results for the consumer-loan case. On the other hand, in the model, households use time deposits to save idle money balances in contrast to demand deposits, which help to smooth out consumption expenditures. We have also conducted the empirical analysis using other deposit products and have obtained the same results. These extended results are available from the authors upon request.

[27] In theory, one could perform the empirical analysis at the bank branch or county level. In practice, however, the information is too sparse at many branches and/or counties to be informative at such levels.

the extraction of any excess liquidity associated with these injections in the subsequent CM.

As was in Berentsen and Waller (2011), the effect of the optimal policy is to redistribute liquidity among *ex-post* heterogeneous households in a manner akin to the maintenance of an "elastic currency". Policy here, however, works through completely different channels than in the analysis of Berentsen and Waller (2011). In their setting with perfectly competitive lending, while state-contingent liquidity injections do not directly affect households' money demand in equilibrium, they are useful for counteracting sub-optimal deposit interest rate movements by lowering the rate when aggregate demand is high (and deposits low). We shut down this channel here by assuming that the central bank maintains a constant policy rate. The optimal stabilization policy here, in contrast, exploits the endogeneity of market power in banking, counteracting movements in interest rate spreads. Specifically, it reduces lenders' market power (lowering the average spread) in states of high aggregate demand and allows it to increase when demand is low. We illustrate this using a numerical example in Online Appendix L.

8 Conclusion

We construct and study a monetary economy in which banking market power in both loan and deposit markets are endogenous and responds to policy. The theory rationalizes empirically measurable (residual) dispersions of loan and deposit interest rates. The model predicts positive relationships between the dispersion of loan (deposit) rate spreads as measured by their standard deviation and the average spread level. We provide new evidence from micro-level data on U.S. consumer loans and deposits to support the model's insight. In the model, these symptoms are also associated with incomplete pass-through of monetary policy to deposit and lending.

We also show that imperfect pricing competition among banks may render an otherwise useful banking system detrimental to welfare when inflation is sufficiently low. That is, a no-banking monetary economy may achieve higher welfare than one in which banks provide a benefit in the form of insurance against idiosyncratic liquidity risks. In our model, the negative welfare effect of financial intermediation arises solely from banks' market power (especially in lending). It suggests that this power may be of particular concern in states of low inflation.

We also study an optimal monetary policy in which the central bank reallocates liquidity differentially in response to aggregate demand shocks under the constraint of a long-run inflation target. For a given inflation target, the optimal stabilization policy reduces loan spreads in states of high demand and allows them to increase when aggregate demand is low. Policy makers' ability to erode market power, both under stabilization policy and in the long run is limited by the need to maintain the inflation target.

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Appendix

A Omitted proofs: Lending with noisy search for loans

In this section, we collect the intermediate results and proofs that lead to the characterization of an equilibrium distribution of loan rates in the noisy-search model for loans. Most of the proofs in this section are standard in the Burdett and Judd (1983) model. We revisit them here for completeness.

Remark on notation. Here, we use functions such as Π, Π^m, R, $l^\star$—respectively, to denote *ex-ante* loan profit, monopoly loan profit, per-customer loan profit, and optimal loan demand. These functions all depend on a vector of individual state m, aggregate state M, and policies $\boldsymbol{\tau}$, which we summarize as $(m, \mathbf{s}) = (m, (M, \boldsymbol{\tau}))$. Since the noisy-search banking equilibrium is an intratemporal or static one, in the proofs below, we dispense with explicit dependencies on $(m, \mathbf{s})$ to keep proofs more readable. For example, we will write $l^\star(i)$ in place of the explicit notation $l^\star(i, m, \mathbf{s})$.

Summary of results. The main results summarizing the distributions of loan rates and deposit rates, respectively, can be found in Lemma 14 (Section in Section A.7) and Lemma 15 (Section A.8). Since the essence of the proof in both characterizations are similar, we will only provide the detailed proof of Lemma 14.

To get to Lemma 14, some intermediate results and objects will need to be established. First, we show any bank faced with just one loan customer ex-post will earn a strictly positive profit (Lemma 7). Second, we show that banks that ex-post face more than one customer will also earn a strictly positive profit (Lemma 8). Third, we show that there is a unique upper bound on loan prices (Lemma 9). Fourth, if the upper bound loan rate is the monopoly rate, we show that this rate is uniquely determined as a function of the state of the economy (Lemma A.4). There is a natural lower bound on loan rates, which is the policy rate i_f. These results help establish that the equilibrium support on the distribution of loan rate F is bounded. The lemmata in Sections A.5 and A.6 tell us the following: In a noisy search equilibrium, the banks will be indifferent between a continuum of pure-strategy loan price posting outcomes. For example, a bank can choose a lower loan rate in return for attracting a larger measure of borrowers. Or, it can post a higher loan rate to increase its profit per loan but attract a smaller measure of borrowers. It can also charge a monopoly price. The intermediate results establish that the distribution is continuous and its support is a connected set.

A.1 Positive monopoly bank profit

Lemma 7. $\Pi^m(i) > 0$ *for* $i > i_f$.

Proof. For any positive loan spread $i - i_f$,

$$\begin{aligned}\Pi^m(i) &= n\alpha_1 R(i) \\ &= n\alpha_1 l^\star(i)\left[(1+i) - (1+i_f)\right].\end{aligned}$$

Since $l^\star(i) > 0$ and $i - i_f > 0$, then $\Pi^m(i) > 0$. □

A.2 All banks earn positive expected profit

Now, we prove that banks will earn strictly positive expected profits:

Lemma 8. $\Pi^{\star} > 0$.

Proof. Since pricing rules are linear then if any loan rate exceeds the marginal cost of funds, $\mu > 1$, the profit from posting $i = \mu i_f$ is $\Pi(\mu i_f) = n[\alpha_1 + 2\alpha_2(1 - F(\mu i_f)) + \alpha_2 \xi(\mu i_f)] R(\mu i_f) > n\alpha_1 R(\mu i_f) = \Pi^m(\mu i_f) > 0$, where $R(i) = l^{\star}(m; i, p, \phi, M, \tau_b)[(1+i) - (1+i_f)]$. The last inequality is from Lemma 7. From the definition of the max operator in (3.6), $\Pi^{\star} = \max_{i \in \text{supp}(F)} \Pi(i) \geq \Pi(\mu i_f) > \Pi^m(\mu i_f) > 0$. $\square$

A.3 Maximal loan pricing

Third, we can also show that:

Lemma 9. *The largest possible price in the support of F is the smaller of the monopoly price and ex-post borrower's maximum willingness to pay: $\bar{i} := \min\{i^m, \hat{i}\}$.*

Although the monopoly rate i^m is the maximal possible price in defining an arbitrary support of F, it may be possible in some equilibrium that this exceeds the maximum willingness to pay by households, $\hat{i}$. We condition on this possibility when characterizing an *equilibrium* support of F later.

Proof. First assume the case that $\hat{i} \geq i^m$. Suppose there is a $\bar{i} \neq i^m$ which is the largest element in $\text{supp}(F)$. Then $\Pi^m(\bar{i}) = n\alpha_1 R(\bar{i})$. Since $F(i^m) \geq 0$ and $\zeta(i^m) \geq 0$, then

$$\Pi(i^m) = n[\alpha_1 + 2\alpha_2(1 - F(i^m)) + \alpha_2 \zeta(i^m)] R(i^m) \geq n\alpha_1 R(i^m) = \Pi^m(i^m) > \Pi^m(\bar{i}).$$

The last inequality is true by the definition of a monopoly price i^m. Therefore $\Pi(i^m) > \Pi^m(\bar{i})$. The equal profit condition would require that, $\Pi^m(\bar{i}) = \Pi^{\star} \geq \Pi^m(i^m)$. Therefore $\bar{i} = i^m$ if $\hat{i} \geq i^m$.

Now assume $\hat{i} < i^m$. In this case, the most that a bank can charge for loans is $\hat{i}$, since at any higher rate, no ex-post buyer will execute his line of credit (i.e., he will not borrow). Thus trivially, $\bar{i} = \hat{i}$ if $\hat{i} < i^m$. $\square$

A.4 Unique monopoly loan rate

Fourth, under a mild parametric regularity condition on preferences, we show that there is a unique monopoly loan rate.

Lemma 10. *Assume $\sigma < 1$. For an arbitrarily small constant bounded below by zero, i.e., $\epsilon > 0$, if $\sigma \geq \epsilon/(2+\epsilon)$, then there is a unique monopoly-profit-maximizing price i^m that satisfies the first-order condition $\frac{\partial \Pi^m(i)}{\partial i} = n\alpha_1 \left[\frac{\partial l^{\star}(i)}{\partial i}(1+i) + l^{\star}(i) - \frac{\partial l^{\star}(i)}{\partial i}(1+i_f)\right] = 0$.*

Proof. Assume $\hat{i} > i^m$. Using the demand for loans from (2.18) the first-order condition at $i = i^m$ is explicitly

$$\underbrace{-\frac{m + \tau_b M}{p^{\frac{\sigma-1}{\sigma}} \phi^{-\frac{1}{\sigma}}}}_{f(i)} + \underbrace{\frac{1}{\sigma}(1+i)^{-\frac{1}{\sigma}}\left[(\sigma - 1) + \frac{1+i_f}{1+i}\right]}_{g(i)} = 0. \tag{A.1}$$

Note that given individual state m, aggregate state M, and policy/prices (τ_b, p, ϕ), the term $f(i)$ is constant for all i. Given i_f, the term $g(i)$ has these properties: (1) $g(i)$ is continuous in i; (2) $\lim_{i \searrow 0} g(i) = +\infty$; (3) $\lim_{i \nearrow +\infty} g(i) = 0$, and, (4) the RHS is monotone decreasing, $g'(i) < 0$.

The first three properties are immediate from (A.1). Since $\Pi^m(i)$ is twice-continuously differentiable, the last property can be shown by checking for a second-order condition: For a maximum profit at $i = i^m$, we must have $\left.\frac{\partial^2 \Pi^m(i)}{\partial i^2}\right|_{i=i^m} \leq 0$. Observe that the second-derivative function is

$$\frac{\partial^2 \Pi^m(i)}{\partial i^2} = g'(i) = -\underbrace{\frac{1}{\sigma^2}(1+i)^{-\frac{1}{\sigma}-1}}_{>0}\left[(\sigma-1) + \frac{(1+\sigma)(1+i_f)}{(1+i)}\right]. \tag{A.2}$$

For (A.2) to hold with ≤ 0, we would require $\frac{(1+\sigma)(1+i_f)}{(1+i)} \geq 1-\sigma$ for all $i \geq i_f$. Let $1+i \equiv (1+\epsilon)(1+i_f)$ since $i^m \geq i > i_f$. The above inequality can be re-written as $\frac{1}{1+\epsilon} \geq \frac{1-\sigma}{1+\sigma}$, which implies $1 > \sigma \geq \frac{\epsilon}{2+\epsilon}$. This is a sufficient condition on parameter σ to ensure that a well-defined and unique monopoly profit point exists with monopoly price $i^m \geq \underline{i} > i_f$.

□

A.5 Distribution is continuous

In the next two results, we show that the loan pricing distribution is continuous with connected support.

Lemma 11. *F is a continuous distribution function.*

We will prove Lemma 11 in two parts. First, we document a technical observation that the per-customer profit difference is always bounded above:

Lemma 12. *Assume there is an $i' < i$ and an $i'' < i'$, with $\zeta(i) = \lim_{i' \nearrow i}\{F(i) - F(i')\} > 0$, and $\zeta(i') = \lim_{i'' \nearrow i'}\{F(i') - F(i'')\} > 0$, and that $R(i') > 0$. The per-customer profit difference is always bounded above: $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$.*

Proof. The expected profit from posting i is

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)\right] R(i).$$

The expected profit from posting i' is

$$\Pi(i') = n\left[\alpha_1 + 2\alpha_2(1 - F(i')) + \alpha_2 \zeta(i')\right] R(i').$$

A firm would be indifferent to posting either price if $\Pi(i) - \Pi(i') = 0$. This implies that

$$(\alpha_1 + 2\alpha_2)\left[R(i) - R(i')\right] + \alpha_2 \zeta(i) R(i) - \alpha_2 \zeta(i') R(i') \\ - 2\alpha_2\left[F(i) R(i) - F(i') R(i')\right] = 0.$$

Rearranging and using the definition of $\zeta(i) = \lim_{i' \nearrow i}\{F(i) - F(i')\} > 0$:

$$\begin{aligned}(\alpha_1 + 2\alpha_2)\left[R(i) - R(i')\right] &= \alpha_2\left[F(i) R(i) - F(i') R(i')\right] - \alpha_2 \zeta(i') R(i') \\ &< \alpha_2\left[F(i) R(i) - F(i') R(i')\right] \\ &\leq \alpha_2 \lim_{i' \nearrow i}\left\{F(i) - F(i')\right\} R(i).\end{aligned}$$

The strict inequality is because $R(i') > 0$ and $\zeta(i') > 0$. The subsequent weak inequality comes from the fact that $R(i)$ is continuous, so that we can write

$$\lim_{i' \nearrow i} \left\{ F(i) R(i) - F\left(i'\right) R\left(i'\right) \right\} = \lim_{i' \nearrow i} \left\{ F(i) - F\left(i'\right) \right\} R(i).$$

Since $\zeta(i) = \lim_{i' \nearrow i} \{F(i) - F(i')\}$, the last inequality implies that $R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. □

The following is the proof of Lemma 11.

Proof. Assume $i \in \text{supp}(F)$ such that $\zeta(i) > 0$ and $\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2\zeta(i)] R(i)$. R is clearly continuous in i. Hence there is a $i' < i$ such that $R(i') > 0$ and from Lemma 12, $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. Then

$$\begin{aligned}
\Pi\left(i'\right) &= n\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i'\right)\right) + \alpha_2\zeta\left(i'\right)\right] R\left(i'\right) \\
&\geq n\left[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2\zeta(i)\right]\left[R(i) - \Delta\right] \\
&\geq \Pi(i) + n\left\{\alpha_2\zeta(i)\left[R(i) - \Delta\right] - (\alpha_1 + 2\alpha_2)\Delta\right\}.
\end{aligned}$$

The first weak inequality is a consequence of $F(i) - F(i') \geq \zeta(i)$. Since $R(i) > \Delta$ and $\Delta < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$, then the last line implies $\Pi(i') > \Pi(i)$. This contradicts $i \in \text{supp}(F)$. □

A.6 Support of distribution is connected

Lemma 13. *The support of F, $\text{supp}(F)$, is a connected set.*

Proof. Pick two prices i and i' belonging to the set $\text{supp}(F)$, and suppose that $i < i'$ and $F(i) = F(i')$. The expected profits are, respectively, $\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i))] R(i)$ and $\Pi(i') = n[\alpha_1 + 2\alpha_2(1 - F(i'))] R(i')$. Since $F(i) = F(i')$, then the first terms in the profit evaluations above are identical: $n[\alpha_1 + 2\alpha_2(1 - F(i))] = n[\alpha_1 + 2\alpha_2(1 - F(i'))]$. However, since i and i' belonging to the set $\text{supp}(F)$, then clearly, $i_f < i < i' \leq i^m$. From Lemma 10, we know that $R(i)$ is strictly increasing for all $i \in [i_f, i^m]$, so then, $R(i) < R(i')$. From these two observations, we have $\Pi(i) < \Pi(i')$. This contradicts the condition that if firms are choosing i and i' from $\text{supp}(F)$ then F must be consistent with maximal profit $\Pi(i) = \Pi(i') = \Pi^\star$ (viz. the equal profit condition must hold). □

A.7 Distribution of posted loan rates

Lemma 14. *Suppose that the aggregate money stock grows by the factor $\gamma > \beta$.*

1. *If $\alpha_1 \in (0,1)$, each borrower $(z, \mathbf{z})$ faces a unique non-degenerate, posted-loan-rate distribution $F(\cdot, z, \mathbf{z})$. This distribution is continuous with connected support:*

$$F(i, z, \mathbf{z}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R\left(\bar{i}, z, \mathbf{z}\right)}{R(i, z, \mathbf{z})} - 1\right], \tag{3.1}$$

where $\text{supp}(F) = \left[\underline{i}(z, \mathbf{z}), \bar{i}(z, \mathbf{z})\right]$, $\underline{i}(z, \mathbf{z})$ solves

$$R\left(\underline{i}, z, \mathbf{z}\right) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R\left(\bar{i}, z, \mathbf{z}\right), \qquad \bar{i}(z, \mathbf{z}) = \min\{i^m(z, \mathbf{z}), \hat{i}(z, \mathbf{z})\}, \tag{A.3}$$

and,

$$R(i,z,\mathbf{z}) = \left[\rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z+\tau_b Z)\right](i-i_f) \tag{A.4}$$

is the real bank profit per loan customer served.

2. *If $\alpha_2 = 1$, then $F(\cdot,z,\mathbf{z})$ is degenerate at i_f:*

$$F(i,z,\mathbf{z}) = \begin{cases} 0 & \text{if } i < i_f \\ 1 & \text{if } i \geq i_f \end{cases}. \tag{A.5}$$

3. *If $\alpha_1 = 1$, $F(\cdot,z,\mathbf{z})$ is degenerate at the largest possible loan rate $\bar{i}$ such that*

$$F(i,z,\mathbf{z}) = \begin{cases} 0 & \text{if } i < \bar{i}(z,\mathbf{z}) \\ 1 & \text{if } i \geq \bar{i}(z,\mathbf{z}) \end{cases}. \tag{A.6}$$

The intuition for Lemma 14 follows Burdett and Judd (1983). Working backward through the three cases, if all prospective borrowers (active buyers in equilibrium) receive only one borrowing opportunity ($\alpha_1 = 1$) then all banks know they are serving their customers as monopolists and therefore set the highest rate that borrowers will accept. At the opposite extreme, if all borrowers receive two borrowing opportunities ($\alpha_2 = 1$), then Bertrand competition forces the loan rate to the opportunity cost of holding money, *i.e.*, the policy rate. In either case, the distribution of loan rates is degenerate.

Proof. Consider the case where $\alpha_1 \in (0,1)$. We can verify that the distribution F has no mass points and is continuous. Then expected profit from any $i \in \text{supp}(F)$ is a continuous function over $\text{supp}(F)$,

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2(1-F(i))\right]R(i),$$

where the image $\Pi[\text{supp}(F)]$ is also a connected set. The monopoly loan profit is maximized at $\Pi^m(i^m) = n\alpha_1 R(i^m)$. For any $i \in \text{supp}(F)$, the induced expected profit must also be maximal, (equal profit condition on the loan side must hold), i.e.,

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2(1-F(i))\right]R(i) = n\alpha_1 R(i^m).$$

Solving for F and rewriting it in terms of stationary variables, we get the analytical expression for the loan rate distribution in (3.1).

Proofs for the remaining Case 2 and Case 3 in Lemma 14 follow directly from Lemma 1 and Lemma 2 in Burdett and Judd (1983). The pricing outcomes, $\bar{i}$ and i_f are, respectively, the upper bound (the monopoly price) and the lower bound (Bertrand price) on the support of F. □

A.8 Distribution of posted deposit rates

Lemma 15. *Let the growth rate of the money stock satisfy $\gamma > \beta$.*

1. *If $\alpha_1^d \in (0,1)$, there is a unique, continuous distribution of posted deposit rates on a connected support:*

$$G(i_d;\gamma) = \frac{\alpha_1^d}{2\alpha_2^d}\left[\frac{R(i_d^m,z,\gamma)}{R(i_d,z,\gamma)} - 1\right] = \frac{\alpha_1^d}{2\alpha_2^d}\left[\frac{(z+\tau_b Z)[i_f - i_d^m]}{(z+\tau_b Z)[i_f - i_d]} - 1\right], \tag{3.2}$$

where the support of $G(i_d;\gamma)$ *is* $[\underline{i}_d, \bar{i}_d]$, $\underline{i}_d = i_d^m = 0$, $i_f = (\gamma-\beta)/\beta$ *and* $\bar{i}_d = \frac{\gamma-\beta}{\beta}\left[1 - \frac{\alpha_1^d}{\alpha_1^d + 2\alpha_2^d}\right]$.

2. *If* $\alpha_2^d = 1$, *then* G *is degenerate at the central bank policy rate* i_f:

$$G(i_d;\gamma) = \begin{cases} 0 & \text{if } i_d < i_f \\ 1 & \text{if } i_d \geq i_f \end{cases}. \tag{A.7}$$

3. *If* $\alpha_1^d = 1$, *the* G *is degenerate at the monopoly (i.e. lowest possible) rate* $\underline{i}_d$:

$$G(i_d;\gamma) = \begin{cases} 0 & \text{if } i_d < \underline{i}_d \\ 1 & \text{if } i_d \geq \underline{i}_d \end{cases}. \tag{A.8}$$

Note that the distribution of posted deposit rates, $G(\cdot;\gamma)$, does not depend on state variables other than policy γ. This result depends on prospective depositors' asset positions (real money holdings) being predetermined when they search for deposit opportunities. Likewise, on the deposit side, the posted deposit rate distribution is also sandwiched between the two well-defined extremes: A Bertrand equilibrium and a monopoly-price equilibrium.

B Friedman Rule and the first-best: Proof of Proposition 1

Proof. Suppose that $\gamma = \beta$ but that there is an SME with a non-degenerate distribution of loan interest rates, $F(\cdot, z, \mathbf{z})$.

Since we focus on $\alpha_1 \in (0,1)$, from Lemma 14 (part 1), we know that if there is an SME, then the posted loan-rate distribution $F(\cdot, z, \mathbf{z})$ is non-degenerate and continuous with connected support, $\text{supp}(F(\cdot, z, \mathbf{z})) = [\underline{i}(\cdot, z, \mathbf{z}), \bar{i}(\cdot, z, \mathbf{z})]$.

If there is an SME, then the Euler condition for money demand holds. However, the marginal cost of holding money—i.e., LHS of the Euler condition—is zero at the Friedman rule ($\gamma = \beta$). Also, the liquidity premium of carrying more real money balance at the margin into the next period is always non-negative—*i.e.*, for any $q > 0$, $u'(q)/c'(q) - 1 \geq 0$. What remains on the RHS of the Euler condition is all the (net) marginal benefit of borrowing less at the margin when one has additional real balance, *i.e.*, the integral terms. These terms are also non-negative measures. Thus, for an SME to hold, it must be that $F(\cdot, z, \mathbf{z})$ is degenerate on a singleton set, likewise, for the deposit rate distribution $G(\cdot, z, \mathbf{z})$.

Since the Euler condition must hold in an SME, then our previous reasoning must further imply that the integral terms reduce to the condition $u'(q^f) = c'(q^f)$. We can compare this with the first best allocation. Given our CRRA preference representation assumption, the first-best allocation solving $u'(q^*) = c'(q^*)$ will yield $q^* = 1$.

Thus if there is an SME at the Friedman rule, then both $F(\cdot, z, \mathbf{z})$ and $G(\cdot, z, \mathbf{z})$ must be degenerate. Moreover, at the Friedman rule, the allocation is Pareto efficient: $q^f = q^* = 1$. $\square$

C Omitted proofs: SME

We provide the intermediate results and proofs for establishing the existence and uniqueness of a stationary monetary equilibrium with co-existing money and credit. The conclusion is arrived at in a few intermediate steps. First, in Section C.1 we show that a posted loan-price distribution with lower real money balance first-order stochastic dominance a distribution with higher real money balance, given a monetary policy rule $\gamma > \beta$. Second, in Section C.2 we show that the money demand Euler Equation simplifies to Condition (3.4), and the candidate real money balance solution to the money demand Euler equation is bounded. Third, we use results from Section C.1 and Section C.2 together in section C.3 to show there exists a unique real money balance that solves the money demand Euler (3.4). This establishes existence. Finally, we prove the uniqueness of an SME with co-existing money and credit in Section C.4.

C.1 Proof of Lemma 1: First-order stochastic dominance

Proof. The analytical formula for the loan-price distribution $F(i, z, \mathbf{z})$ is characterized in (3.1). Suppose we fix $\bar{i}(z, \mathbf{z}) = \bar{i}(z', \mathbf{z})$, and denote it as $\bar{i}$. In general, the lower and upper support of the distribution F is changing with respect to z and policy γ. By fixing the upper support at both z and z' here, we are checking whether the curve of the cumulative distribution function, $F(\cdot, z, \mathbf{z})$, is lying on top or below for z relative to z'. We have $\frac{\partial F(i,z,\mathbf{z})}{\partial z} = \underbrace{\frac{\alpha_1}{2\alpha_2}}_{>0} \left[\frac{(\bar{i}-i_f)R(i,z,\mathbf{z}) - (i-i_f)R(\bar{i},z,\mathbf{z})}{\underbrace{(R(i,z,\mathbf{z}))^2}_{>0}} \right]$. For $\partial F(i, z, \mathbf{z})/\partial z > 0$ to hold, one needs to show the numerator is positive. Suppose this were not the case. Then we have

$$
\begin{aligned}
&(\bar{i} - i_f)R(i, z, \mathbf{z}) - (i - i_f)R(\bar{i}, z, \mathbf{z}) \leq 0 \\
\implies & (\bar{i} - i_f) \underbrace{\left[(1+i)^{\frac{-1}{\sigma}} - z\right](i - i_f)}_{=R(i,z,\mathbf{z})} \leq (i - i_f) \underbrace{\left[(1+\bar{i})^{\frac{-1}{\sigma}} - z\right](\bar{i} - i_f)}_{=R(\bar{i},z,\mathbf{z})} \\
& \qquad \implies \left[(1+i)^{\frac{-1}{\sigma}} - z\right] \leq \left[(1+\bar{i})^{\frac{-1}{\sigma}} - z\right]
\end{aligned}
$$

The last inequality contradicts the fact that the loan demand curve is downward sloping in i, and $\bar{i}$ is the highest possible loan price posted by banks (lending agents). Thus, the numerator must be positive and $\partial F(i, z, \mathbf{z})/\partial z > 0$. This shows that a loan-price distribution $F(\cdot, z, \mathbf{z})$ first-order stochastically dominates $F(\cdot, z', \mathbf{z})$, for $z < z'$. □

C.2 Proof of Lemma 2: Money and credit

Proof. We want to show equivalence in the three claims in Lemma 2. The proof relies on a CRRA(σ) preference representation and linear cost of producing the DM good $c(q) = q$.

1. We say that the DM relative price ρ is sufficiently low if real money balance z is such that

$$\rho = 1 < \tilde{\rho}_i(z, \mathbf{z}) \equiv (z)^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}, \qquad 0 < \sigma < 1. \tag{C.1}$$

The following is a sufficient requirement: If $z < \left(\frac{1}{1+i}\right)^{\frac{1}{\sigma}}$, then inequality (C.1) holds. From Lemma 14, if $\alpha_1 \in (0,1)$, the distribution $F(\cdot, z, \mathbf{z})$ is non-degenerate and $\text{supp}(F(\cdot, z, \mathbf{z})) = [\underline{i}(z, \mathbf{z}), \bar{i}(z, \mathbf{z})]$

exists. This implies that for all $i \in \text{supp}(F(\cdot, z, \mathbf{z}))$, the inequality $z < \left(\frac{1}{1+\bar{i}(z,\mathbf{z})}\right)^{\frac{1}{\sigma}}$ is also true. Since SME $z = z^\star$ exists and $z^\star < \left(\frac{1}{1+\bar{i}(z^\star,\mathbf{z})}\right)^{\frac{1}{\sigma}}$, then ρ is sufficiently low and satisfies inequality (C.1).

2. From Claim 1 above, the DM relative price ρ satisfies inequality (C.1). From (3.10), there is ex-post positive loan demand by the active DM buyers who meet at least one bank. In the opposite direction: If there is ex-post positive loan demand, then condition (C.1) must hold, thus implying Claim 1.

3. Combining Claim 2 with agents' first-order condition for optimal money demand, their money-demand Euler Equation reduces to (3.4). In reverse, (3.4) implies that there is a positive demand for loans and money (Claim 2).

□

C.3 Unique real money balance

Lemma 16. *Fix long-run inflation as $\gamma = 1 + \tau > \beta$. Assume $\alpha_0, \alpha_1 \in (0, 1)$. In any SME, there is a unique real money demand, $z^\star \equiv z^\star(\boldsymbol{\tau})$.*

Proof. Consider the case where the long-run inflation target is set away from the Friedman rule, i.e., $\gamma > \beta$. From Lemma 2, the money demand Euler equation is characterized by

$$\begin{aligned}\frac{\gamma - \beta}{\beta} &= \underbrace{(1-n)\left\{\alpha_0^d + \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d[\alpha_1^d + 2\alpha_2^d G(i_d;\gamma)]\mathrm{d}G(i_d;\gamma)\right\}}_{=:A} \\ &\quad + \underbrace{n\alpha_0\left(u'[q_b^0(z^\star,\mathbf{z})] - 1\right)}_{=:B} + \underbrace{n\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} i\mathrm{d}J\left(i, z^\star, \mathbf{z}\right)}_{=:C}\end{aligned}, \tag{C.2}$$

where

$$\begin{aligned}\mathrm{d}J(i, z^\star, \mathbf{z}) &= \underbrace{\{\alpha_1 + 2\alpha_2(1 - F(i; z^\star))\}\, f(i, z^\star, \mathbf{z})}_{=:j(i,z^\star,\mathbf{z})}\mathrm{d}i \\ &\equiv \alpha_1 + 2\alpha_2(1 - F(i, z^\star, \mathbf{z}))\mathrm{d}F(i, z^\star, \mathbf{z}).\end{aligned}$$

First, the term A is constant in z for a given policy $\gamma > \beta$. Next, recall that $1 \equiv \rho < \tilde{\rho}_i(z^\star, \mathbf{z})$ from Lemma 2, the ex-post DM goods demand function for the event where the active DM buyer failed to meet with a lending bank is given by $q_b^0 = \frac{z}{\rho}$, i.e., she is liquidity constrained with own money balance. Thus, $\partial q_b^0/\partial z > 0$. Since $u'' < 0$, then $u' \circ q_b^0(z, \mathbf{z})$ is continuous and decreasing in z. Thus, the term B is continuous and decreasing in z.

Next, let $H(z, \mathbf{z}) := \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} i\mathrm{d}J(i, z, \mathbf{z})$. Applying integration by parts, we obtain $H(z, \mathbf{z}) = \bar{i}(z, \mathbf{z}) - \tilde{H}(z)$, where $\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} J(i, z, \mathbf{z})\mathrm{d}i$. Applying Leibniz' Rule to $\tilde{H}(z)$, we have $\tilde{H}'(z, \mathbf{z}) = \bar{i}'(z, \mathbf{z}) + \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \frac{\partial J(i,z,\mathbf{z})}{\partial z}\mathrm{d}i$. Overall, we have $H'(z, \mathbf{z}) = \bar{i}'(z, \mathbf{z}) - \tilde{H}'(z, \mathbf{z}) = -\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \frac{\partial J(i,z,\mathbf{z})}{\partial z}\mathrm{d}i$. From Lemma 1, we know that $J(\cdot, z, \mathbf{z})$ first-order stochastically dominates $J(\cdot, z', \mathbf{z})$ for all $z < z'$. Thus, $\partial J(i, z, \mathbf{z})/\partial z > 0$, which implies $H'(z, \mathbf{z}) < 0$. Thus, both terms B and C on the RHS of (C.2) are continuous and monotone decreasing in z. Moreover, the LHS of (C.2) is constant with respect to z. Therefore, there exists a unique real money demand $z^\star(\boldsymbol{\tau})$ that solves the money-demand Euler (C.2). Moreover, $z^\star(\boldsymbol{\tau})$ is bounded, by Lemma 2. □

C.4 SME with money and credit: Proof of Proposition 2

Proof. From Lemmata 1, 2, and 16, we have established the existence of a solution to both money and credit. In particular, we have shown that there exists a unique money demand $z^\star \equiv z^\star(\boldsymbol{\tau})$ such that $z^\star \in \left(0,\ [1+\bar{i}(z^\star)]^{\frac{-1}{\sigma}}\right)$, for a given $\gamma > \beta$. This condition ensures that the optimal real money balance $z^\star$ is bounded and that the maximal loan interest of the posted loan-price distribution is not too high. Moreover, this guarantees positive loan demand.

To establish a unique SME with both money and credit, what remains is to show that the following equilibrium requirements also hold, when evaluated at $z = z^\star$. That is,

1. Total bank assets must equal total bank liabilities:

$$\begin{aligned}&\underbrace{(1-n)\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)}[\alpha_1^d + 2\alpha_2^d G(i_d;\gamma)](z+\tau_b Z)\mathrm{d}G(i_d;\gamma)}_{=:D} \\ &= e + n\underbrace{\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})}\left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F\left(i,z,\mathbf{z}\right)\right]\xi^\star\left(z,\mathbf{z}\right)\mathrm{d}F\left(i,z,\mathbf{z}\right)}_{=:L},\end{aligned} \tag{C.3}$$

   where $e = \tau_1^e Z$ and $z = Z$ at equilibrium.

2. The bank earns non-negative expected profit condition:

$$\Pi^\star(z,\mathbf{z}) = \max_{i\in(\mathrm{supp}(F(i,z,\mathbf{z}))}\Pi_l(i,z,\mathbf{z}) + \max_{i_d\in(\mathrm{supp}(G(i_d;\gamma))}\Pi_d(i_d,\gamma) \geq 0. \tag{C.4}$$

3. DM (competitive price-taking) goods market clears:

$$\begin{aligned}q_s\left(z,\mathbf{z}\right) &= n\alpha_0 q_b^{0,\star}\left(z,\mathbf{z}\right) \\ &\quad + n\left[\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})}\left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F\left(i,z,\mathbf{z}\right)\right] q_b^\star\left(z;\rho,Z,\gamma\right)\mathrm{d}F\left(i,z,\mathbf{z}\right)\right].\end{aligned} \tag{C.5}$$

4. Both CM goods and labor market clear.

We first consider Condition 1. Recall that banks have access to a competitive interbank market on the sport to borrow excess funds when they face a shortfall in liquidity, or lend a surplus of liquidity. The total surplus or deficit of liquidity in the banking system is met by a lump-sum injection or extraction of money made by the government. If $D < L$, indicating a total liquidity deficit in the banking system, the government injects liquidity into the banks on the spot in the DM, $e = \tau_1^e Z$, via a lump-sum transfer. In the subsequent CM, the government extracts money from the economy by taxing the banks the same amount, $\tau_2^e Z = -\tau_1^e Z$. Effectively, this maintains the overall price-level target (i.e., price level and money supply grow at the same constant rate of $\gamma = 1 + \tau$) while satisfying the resource constraint. The opposite occurs if there is a total surplus of liquidity.

Now, we turn to the banks' expected profit in Condition 2. Given that $\alpha_1 \in (0, 1)$, it follows that $\Pi_l(i, z, \mathbf{z}) > 0$ for all i in the support of the distribution $F(i, z, \mathbf{z})$. Hence, the profit from the loan side is positive. Likewise, we can show the profit from the deposit side $\Pi_d(i_d; \gamma)$ is also positive given $\alpha_1^d \in (0, 1)$. Moreover, we know that $\Pi_l(i, z, \mathbf{z}) \to 0$ as $\alpha_1 \to 0$, and $\Pi_d(i_d; \gamma) \to 0$ as $\alpha_1^d \to 0$. Since additional borrowing/lending is permitted at policy rate i_f, we can also verify that the total interest earned on assets weakly exceeds that paid on total liabilities in equilibrium. Hence, the details are omitted here.

Next, we turn to the DM goods market clearing requirement in Condition 3. Since the DM firms' optimal production rule is pinned down by a constant marginal cost (due to linear production technology), then the aggregate supply equals the aggregate demand in the DM goods market.

Finally, we consider Condition 4. In any equilibrium, we have constant optimal CM consumption $x^\star$ (due to quasi-linear preference). Given real money balance $z^\star \equiv z^\star(\boldsymbol{\tau})$ and DM allocations $(q_b^{0,\star}(z^\star, \mathbf{z}), q_b^\star(\cdot, z^\star, \mathbf{z}))$, we can verify that the CM goods and labor market also clear. Hence, the details are omitted here. In equilibrium $z = z^\star(\boldsymbol{\tau}) = Z$, so we could further reduce the characterizations above by rewriting $(z, \mathbf{z})$ as just $\mathbf{z}$ in an SME. □

D Omitted proofs: Banking market power and welfare

In this section, we highlight and illustrate the mechanism of the lending market power channel using a number of special cases. Following this, we provide conditions under which an economy with banks can achieve lower welfare than an economy without banks.

Case 1: Monopoly loan and Bertrand deposits. Suppose $\alpha_1 = 1$ and $\alpha_2^d = 1$. The economy will then resemble a case where the bank is a monopoly in loan-side operation while keeping the deposit side competitive. In particular, the equilibrium loan rate is determined by

$$i^m = i_f \left[\frac{\epsilon(i_m, z, \gamma)}{1 + \epsilon(i_m, z, \gamma)} \right] = \frac{\gamma - \beta}{\beta} \left[\frac{\epsilon(i_m, z, \gamma)}{1 + \epsilon(i_m, z, \gamma)} \right], \tag{D.1}$$

where $\epsilon(i_m, z, \gamma) = (\partial \xi(i_m, z, \gamma)/\partial i_m)[i_m/\xi(i_m, z, \gamma)]$ captures the elasticity of loan demand, $\xi(i_m, z, \gamma)$. The elasticity term (and so the monopoly rate) depends on preference σ, goods price ρ, and real money balance z. Moreover, the monopoly loan rate markup varies with inflation γ.[28]

Suppose, for now, the elasticity of loan demand is fixed for a given $\gamma > \beta$. Note that we focus on the case where $\sigma < 1$, we will have an elastic demand for loans. Consequently, we have $\epsilon/(1 + \epsilon) > 1$. This implies that the bank charges a positive interest spread over the policy rate (the opportunity cost of holding money implied by inflation), and hence $i_m > i_f$. Moreovr, if $\epsilon(\cdot) \to -\infty$, then $i^m \to i_f$.

Let $\mu^m(\gamma) := \epsilon(\mathbf{z})/(1 + \epsilon(\mathbf{z}))$ to denote the monopoly loan spread over the policy rate i_f for a given γ in

[28] Note: In this economy, a stationary monetary equilibrium is determined by solving a system of two equations with two unknowns (z^m, i_m). In particular, given policy γ, and monopoly rate i_m the money demand is determined by

$$z^{m,\star} \leftarrow \frac{\gamma - \beta}{\beta} = u'[q^m(z, i_m)] - 1, \tag{D.2}$$

assuming linear cost function in the DM, and given γ and z^m, the monopoly rate is pinned down

$$i_m^\star \leftarrow \frac{\partial \pi}{\partial i_m} = \frac{\partial \xi(i_m, z^m, \gamma)}{\partial i_m}[i_m - i_f] + \xi(i_m, z^m, \gamma) = 0. \tag{D.3}$$

After some algebra, we can express Equation (D.3) as (D.1). Notice that i_m actually shows up on both sides of the Equation, which requires the numerical method to determine the monopoly rate. Once z and i_m are pinned down, we can back out the DM consumption $q^m = (1 + i_m)^{-\frac{1}{\sigma}} \iff u'(q^m) - 1 = i^m$. This equation is from the buyers' FOC, which governs his optimal goods demand strategy. This says he borrows up to the marginal benefit (LHS) equal to the marginal cost (RHS).

an SME. Combining Equation (D.1) with the buyers' FOC, we have

$$q^m = \left[1 + \mu^m(\gamma)\frac{\gamma-\beta}{\beta}\right]^{-\frac{1}{\sigma}}. \tag{D.4}$$

The (lifetime) welfare in this economy is given by

$$(1-\beta)W^m(\gamma) = nu(q^m) - c(q_s^m) + U(x) - x, \tag{D.5}$$

where $q_s^m = nq^m$ (by DM goods market clearing).

Case 2. A monetary economy without banks. Suppose $\alpha_0 = 1$ and $\alpha_0^d = 1$. The economy will then resemble a monetary economy with no banks. In particular, the DM consumption in this economy is determined by

$$\frac{\gamma-\beta}{\beta} = n[u'(\hat{q}) - 1] \implies \hat{q} = \left[1 + \frac{1}{n}\frac{\gamma-\beta}{\beta}\right]^{-\frac{1}{\sigma}}. \tag{D.6}$$

The lifetime welfare in this economy is determined by:

$$(1-\beta)\hat{W}(\gamma) = nu(\hat{q}) - c(\hat{q}_s) + U(x) - x, \tag{D.7}$$

where $\hat{q}_s = n\hat{q}$ (by DM goods market clearing).

Case 3. Noisy loan search and Bertrand deposits. Next, we consider an economy with noisy loan search while keeping the deposit operation competitive. That is, we let $\alpha_0 = 0$, so all buyers either receive one or two loan-rate quotes. Moreover, assume $\alpha_2^d = 1$, so households are insured against the liquidity risk to the same extent as BCW. We will return to discuss this assumption at the end of this Appendix. Overall, it would not alter the main economic insight of the lending market power channel that we want to highlight.

Let the function $J(i, \mathbf{z}) := \alpha_1 F(i, \mathbf{z}) + \alpha_2[1-(1-F(i, \mathbf{z}))^2]$ to denote the transacted loan rate distribution. Let $\mu(i, \mathbf{z}) := i/i_f$ be the loan rate spread function for $i \in \text{supp}(F(i, \mathbf{z}))$. Hence, we use the function $\mu(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \mu(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z})$ to denote the average transacted loan rate spread for a given policy $\gamma > \beta$.

In this case, the average DM (transacted) consumption in the economy with a one-sided noisy loan search is given by

$$q := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [1 + \mu(i, \mathbf{z})i_f]^{-\frac{1}{\sigma}} \mathrm{d}J(i, \mathbf{z}). \tag{D.8}$$

Lifetime welfare in this economy is given by

$$W(\gamma) = \frac{1}{1-\beta}\left[n\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] \mathrm{d}J(i, \mathbf{z}) - c(q_s) + U(x) - x\right], \tag{D.9}$$

where $q_s = n\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [1 + \mu(i, \mathbf{z})i_f]^{-\frac{1}{\sigma}} \mathrm{d}J(i, \mathbf{z})$.

D.1 Monopoly banking versus no banks

Lemma 17. *Assume $\alpha_1 = 1$ and $\alpha_2^d = 1$. If $\gamma > \beta$ and there exists a n that satisfies $1/\mu^m(\gamma) < n < 1$, then $q^m < \hat{q}$, and $u(q^m) < u(\hat{q})$.*

Proof. For a given $\gamma > \beta$, under the assumption that $n > 1/\mu^m(\gamma)$, the term inside the bracket of Equation (D.4) is larger than that in Equation (D.6). Since these two terms are raised to the same negative power, it follows that $q^m < \hat{q}$, and therefore $u(q^m) < u(\hat{q})$ at a given $\gamma > \beta$. □

D.2 Average transacted loan rate and spread

Lemma 18. *Assume $\gamma > \beta$ and $0 < \alpha_1 < 1$. If $z^\star \in (0, \bar{z})$, where $\bar{z} = (1 + \bar{i}(z^\star, \mathbf{z})]^{-\frac{1}{\sigma}}$, and there exists a n such that $1/\mu^m(\gamma) \leq 1/\mu(\gamma) < n < 1$, then the following holds:*

1. *$i_f < \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i dJ(i, \mathbf{z}) \leq i_m$, and $\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i dJ(i, \mathbf{z}) \to i_m$ as $\alpha_1 \to 1$.*
2. *$1 < \mu(\gamma) \leq \mu^m(\gamma)$, and $\mu(\gamma) \to \mu^m(\gamma)$ as $\alpha_1 \to 1$, where $\mu(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} (i/i_f) dJ(i, \mathbf{z})$.*

This also holds for the average posted loan rate and markup.

Proof. First, from Lemma 14, we have established that there exists a unique non-degenerate loan-rate distribution $F(i, \mathbf{z})$, with connected support $\text{supp}(F(i, \mathbf{z})) = [\underline{i}(\mathbf{z}), \bar{i}(\mathbf{z})]$, given $\gamma > \beta$, and $\alpha_1 \in (0, 1)$, Second, the first sufficient condition on real money balance ensures that there will always be positive loan demand at a given $\gamma > \beta$, by the result established in Lemma 2.

In this case, the upper support (i.e., the general monopoly rate) of the loan-rate distribution is given by

$$\bar{i}(\mathbf{z}) = \min\{i_m(\mathbf{z}) \equiv \mu^m(\gamma) i_f, \hat{i}(\mathbf{z})\},$$

where $\hat{i}(\mathbf{z})$ denotes the buyer's willingness to borrow, $i_m := i_m(\mathbf{z})$ is the monopoly loan rate, and $i_f := i_f(\gamma)$ is the policy rate.

Using the equal profit condition, the lower support of the loan rate distribution, $\underline{i}(\mathbf{z})$ is determined by solving:

$$\xi(i, \mathbf{z})[i - i_f] = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \xi(\bar{i}, \mathbf{z})(\bar{i}(\mathbf{z}) - i_f).$$

Under the assumption that $0 < \alpha_1 < 1$, it follows that all i in the support of the distribution lies between the Bertrand limit (i_f) and the Monopoly limit (i_m), i.e., $i_f < \underline{i} < i < \bar{i}$. Consequently, the average (transacted) loan rate satisfies that

$$i_f < \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \mathrm{d}J(i, \mathbf{z}) \leq i_m,$$

where the function $J(i, \mathbf{z}) := \alpha_1 F(i, \mathbf{z}) + \alpha_2[1 - (1 - F(i, \mathbf{z}))^2]$ denotes the transacted loan rate distribution.

Next, from Lemma 14, we also know that $\lim_{\alpha_1 \to 1} \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \mathrm{d}J(i, \mathbf{z}) = \bar{i}(\mathbf{z})$ exits since the distribution F degenerates at the highest possible rate, i.e., $F(i \geq \bar{i}(\mathbf{z})) = 1$ and $F(i < \bar{i}(\mathbf{z})) = 0$ given $\alpha_1 = 1$. Notice that $\bar{i}(\mathbf{z}) \leq i_m$. Let $\mu(i, \mathbf{z}) := i/i_f \equiv i[\frac{\gamma - \beta}{\beta}]$ be the loan rate spread function for all $i \in \text{supp}(F(i, \mathbf{z}))$. From the reasoning above, and for all $i \in \text{supp}(F(i, \mathbf{z}))$, it follows that

$$1 < \mu(i, \mathbf{z}) \leq \frac{i_m}{i_f}.$$

Consequently, integrating $\mu(i, \mathbf{z})$ over the entire support of the distribution, the implied average transacted loan-rate markup in the economy with noisy search on loans relative to the monopoly bank satisfies:

$$1 < \underbrace{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \mu(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z})}_{=: \mu(\gamma)} \leq \frac{i_m}{i_f} = \mu^m(\gamma).$$

Moreover, following the result established in Lemma 14, we know that $\mu(\gamma) \to \mu^m(\gamma)$ as $\alpha_1 \to 1$ also holds. The analysis above follows through for the average posted loan rate and spread. □

D.3 Allocation: Imperfectly competitive banking versus no banks

Lemma 19. *Assume $\gamma > \beta$ and $0 < \alpha_1 < 1$. If $z^\star \in (0, \bar{z})$, where $\bar{z} = (1 + \bar{i}(z^\star, \mathbf{z})]^{-\frac{1}{\sigma}}$, and there exists a n such that $1/\mu^m(\gamma) \leq 1/\mu(\gamma) < n < 1$, then the following holds:*

1. *$q < \hat{q}$, where $q := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) dJ(i, \mathbf{z})$.*
2. *$q^m \leq q$, and $q \to q^m$ as $\alpha_1 \to 1$.*

Proof. First, from Lemma 14, we have established that there exists a unique non-degenerate loan-rate distribution $F(i, \mathbf{z})$, with connected support $\mathrm{supp}(F(i, \mathbf{z})) = [\underline{i}(\mathbf{z}), \bar{i}(\mathbf{z})]$, given $\gamma > \beta$, and $\alpha_1 \in (0, 1)$, Second, the first sufficient condition on real money balance ensures that there will always be positive loan demand at a given $\gamma > \beta$, by the result established in Lemma 2. Let $\mu(i, \mathbf{z}) := i/i_f \equiv i[\frac{\gamma - \beta}{\beta}]$ be the loan rate spread function.

In this case, the buyers' optimal goods demand function is given by

$$q_b(i, \mathbf{z}) = [1 + i]^{-\frac{1}{\sigma}} \equiv [1 + \underbrace{\mu(i, \mathbf{z}) i_f}_{=i}]^{-\frac{1}{\sigma}} \quad \text{for all} \quad i \in \mathrm{supp}(F(i, \mathbf{z})). \tag{D.10}$$

First, integrating $q_b(i, \mathbf{z})$ for all i over the entire support of the distribution. The average DM (transacted) consumption is then given by

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [1 + \mu(i, \mathbf{z}) i_f]^{-\frac{1}{\sigma}} \mathrm{d}J(i, \mathbf{z}). \tag{D.11}$$

Given $n > 1/\mu(\gamma)$, the allocation pinned down by Equation (D.11) must be smaller than that determined by Equation (D.6) in the no-bank economy, i.e.,

$$q := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) < \left[1 + \frac{1}{n} \frac{\gamma - \beta}{\beta}\right]^{-\frac{1}{\sigma}} = \hat{q}. \tag{D.12}$$

This is because the number inside the square bracket on the RHS is smaller in a no-bank economy and is raised to a negative number $(-1/\sigma)$. (The sufficient condition says that n is larger than the inverse of the average transacted loan rate spread.)

Next, from the result in Lemma 18, we can also deduce that

$$q^m = [1 + i_m]^{-\frac{1}{\sigma}} = [1 + \mu^m i_f]^{-\frac{1}{\sigma}} \leq q_b(i, \mathbf{z}),$$

for all $i \in \text{supp}(F(i, \mathbf{z}))$ where it holds at equality if $\bar{i}(\mathbf{z}) = i_m$. Since the DM consumption function is decreasing in loan rate, then it follows that $q^m \leq q$.

Next, by the result established in Lemma 14, we also know that $\lim_{\alpha_1 \to 1} \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) = q^m$ exists since the distribution F degenerates at the monopoly rate, i.e., $F(i \geq \bar{i}(\mathbf{z})) = 1$ and $F(i < \bar{i}(\mathbf{z})) = 0$ given $\alpha_1 = 1$. In summary, we have the order: $q^m \leq q < \hat{q}$. The same reasoning applies if we instead consider the posted loan rate distribution. □

D.4 Utility: Imperfectly competitive banking versus no banks

Lemma 20. *Assume $\gamma > \beta$ and $0 < \alpha_1 < 1$. If $z^\star \in (0, \bar{z})$, where $\bar{z} = (1 + \bar{i}(z^\star, \mathbf{z})]^{-\frac{1}{\sigma}}$, and there exists a n such that $1/\mu^m(\gamma) \leq 1/\mu(\gamma) < n < 1$, then the following holds:*

1. $\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z}) < u(\hat{q})$.
2. $u(q^m) \leq \int_{\underline{i}}^{\bar{i}} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z})$, *and* $\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z}) \to u(q^m)$ *as* $\alpha_1 \to 1$.

Proof. Consider the first statement. From the result established in Lemma 19, and given the property of the utility function u, it follows that

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) dJ(i, \mathbf{z}) < \hat{q} \implies u\left[\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) dJ(i, \mathbf{z})\right] < u(\hat{q}). \tag{D.13}$$

Since the utility function (u) is concave, and the fact that $q_b(\cdot)$ is a random variable with respect to the transacted loan rate distribution J, applying Jensen's Inequality, then we have

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z}) \leq u\left[\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) dJ(i, \mathbf{z})\right]. \tag{D.14}$$

Combining Conditions D.13 and D.14, it follows that $\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z}) < u(\hat{q})$.

Next, consider the second statement. From the analysis in Lemma 19, we know there also exists $\lim_{\alpha_1 \to 1} \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] dJ(i, \mathbf{z}) = u(q^m)$. Hence, we can rank the order:

$$u(q^m) = \lim_{\alpha_1 \to 1} \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] \mathrm{d}J(i, \mathbf{z}) < \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] \mathrm{d}J(i, \mathbf{z}) < u(\hat{q}).$$

□

D.5 Proof of Proposition 3: Imperfectly competitive banking versus no-banking

Proof. For the inequality stated in Proposition 3 holds, it suffices to check whether the following holds:

$$\underbrace{n\, u\left[\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z})\right]}_{\equiv u(q)} - n u(\hat{q}) < c\left[n \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} q_b(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z})\right] - c(n\hat{q}), \tag{D.15}$$

where $c(\cdot)$ is a linear cost function in the DM and we use a short-hand expression q to denote the expected transacted DM consumption as defined in Equation (D.8).

We can then rewrite Condition D.15 as

$$n[u(q) - u(\hat{q})] < n[q - \hat{q}]. \tag{D.16}$$

Let the left-hand side of Condition (D.16) be denoted by $\Delta u = u(q) - u(\hat{q})$ and the right-hand side by $\Delta c = q - \hat{q}$. The term Δu represents the net change in utility when consumption moves from $\hat{q}$ to q. Using a first-order approximation around q, we estimate this change as $\Delta u \approx u'(q)(q - \hat{q})$. Similarly, the term Δc captures the net change in DM cost of production by moving consumption from $\hat{q}$ to q. The marginal change in cost can also be approximated by: $\Delta c \approx c'(q)(q - \hat{q})$. Recall that the cost function is linear in its production. So, the slope of it satisfies $c^{'}(q) = 1$.

These two terms capture how the utility of consuming and the cost of producing the DM goods vary at the margin as consumption changes from $\hat{q}$ to q. First, recall that at $\gamma = \beta$, $u^{'}(\cdot)/c^{'}(\cdot) = 1$ must hold at the Friedman rule (the slope of the utility function equals the slope of the cost function) where the liquidity premium is zero. Hence, at the Friedman rule, the net change in utility and cost is zero (moving from a monetary economy without banks to an economy with imperfectly competitive lending banks).

Next, recall that under the same sufficient conditions, we have $u(q) < u(\hat{q})$ (and $q < \hat{q}$) for a given $\gamma > \beta$ established in Lemma 20 and Lemma 19. Moreover, we know that both q and $\hat{q}$ are below the first-best allocation, $q^{\star} = 1$. Then, it must be the case that $u^{'}(\cdot)/c^{'}(\cdot) > 1$ in an SME (where the liquidity premium is positive).

Since the partial derivative of u with respect to q is positive and $q - \hat{q}$ is negative, then the product of it will be negative. Moreover, the slope of the cost function is a constant, $c^{'}(q) = 1$. It follows that the magnitude of Δu will be larger than that of Δc, i.e., Δu becomes more negative when moving from $\hat{q}$ to q. Hence, Condition (D.16) holds. Rearranging it, we have

$$u(q) - q < u(\hat{q}) - \hat{q}. \tag{D.17}$$

Since u is a concave function and the fact that $q_b(\cdot)$ is a random variable with respect to the transacted loan rate distribution J, then we get:

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] \mathrm{d}J(i, \mathbf{z}) \leq u(q),$$

by Jensen's inequality.

Next, we add and subtract $q > 0$ to the weak inequality above, and combine that with Condition (D.17). We get

$$\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} u[q_b(i, \mathbf{z})] \mathrm{d}J(i, \mathbf{z}) - q < u(\hat{q}) - \hat{q}.$$

Finally, we multiply both sides by n to get the desired inequality in the proposition. □

D.6 Proof of Corollary 1: When banks improve welfare

Proof. Consider the case where $0 < \alpha_1 < 1$ (noisy search on loans) and $\alpha_2^d = 1$ (resembles the Bertrand-pricing on deposits). From the result established in Lemma 15, it follows that the deposit rate distribution $G(i_d; \gamma)$ degenerates at the policy rate, $i_f := i_f(\gamma) = (\gamma - \beta)/\beta$ in an SME. Next, we know that $i_f(\gamma) \to 0$ as $\gamma \to \beta$. Since $0 < \alpha_1 < 1$, and by the result established in Lemma 14, it also follows that all loan interest rates in the support of the loan rate distribution are non-zero, i.e., $0 < i$ for all $i \in \text{supp}(F) = [\underline{i}(\mathbf{z}), \bar{i}(\mathbf{z})]$. Recall that we let $\mu(i, \mathbf{z}) = i/i_f$ be the loan rate spread function for all $i \in \text{supp}(F)$. The above two results imply $\mu(i, \mathbf{z}) \to +\infty$ as $\gamma \to \beta$. Then, integrating $\mu(i, \mathbf{z})$ with respect to the transacted loan rate distribution $J(i, \mathbf{z})$, will also be unbounded as the monetary policy approaches the Friedman rule, i.e., $\mu(\mathbf{z}) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \mu(i, \mathbf{z}) \mathrm{d}J(i, \mathbf{z}) \to +\infty$ as $\gamma \to \beta$.

Since we have assumed the same sufficient conditions, then by the previous result established in Proposition 4, $\mu(\mathbf{z})$ is monotone decreasing in γ and it is bounded below by unity, it follows that $\mu(\mathbf{z}) \to 1$ as $\gamma \to +\infty$. Moreover, $1/n$ is constant with respect to γ. Using these two relationships, we can then find an inflation threshold $\beta < \tilde{\gamma} < +\infty$ such that $1/n \geq \mu(\gamma)$ holds for $\gamma \geq \tilde{\gamma}$. Following the similar steps as in the proof of Proposition 3, we can then show $W^{DM}(\gamma) \geq \hat{W}^{DM}(\gamma)$ for $\gamma \geq \tilde{\gamma} > \beta$. In words, if inflation is sufficiently high, and the liquidity risk channel dominates the loan market power channel, i.e., $1/n \geq \mu(z)$, an economy with imperfectly competitive banks can also improve welfare relative to the pure currency economy. $\square$

E Omitted proofs: Monetary policy and loan market power

Recall that gross inflation is $\gamma = 1+\tau$. How does the average, posted loan spread ($\mu(\gamma)$) change with respect to inflation γ? Also, from a household's perspective, how does the *ex-ante* loan spread ($\hat{\mu}(\gamma)$) change with respect to inflation γ? We will show below that successively higher-inflation SME economies have higher average loan rates and policy rates (the opportunity cost of holding money). However, in our comparative stationary monetary equilibrium (SME) experiments, higher inflation is associated with successively lower average interest spread over the policy rate in the banking (loans) sector.

For the result that the average loan-rate spread falls with inflation, it must be that the average loan rate itself is rising slower than the policy rate. In this part, we prove this result under quite mild regularity conditions. It requires that if the support of an SME loan-rate distribution is not too wide, and, the gap between the lowest posted loan rate and policy rate is not too large, then one can show that the average loan spread measure is a decreasing function of long-run inflation.

We should point out that the sufficient conditions behind Proposition 4 are perhaps not the most general ones, but they suffice practically: For plausible experiments around the empirically calibrated model, the sufficient conditions always hold. For extremely high, hyperinflationary scenarios, these specific sufficient conditions may not hold. Nevertheless, we will see that the average loan spread is still decreasing with inflation in our numerical experiments.

We will use the notation $f_x(x; y) := \frac{\partial f(x,y)}{\partial x}$ to denote the partial derivative of function $f(x, y)$ with respect to argument x. The results below are with regard to an equilibrium, so we have $z = Z = z^\star(\boldsymbol{\tau})$ and we can also write $\mathbf{z} = (z^\star, \mathbf{z})$.

E.1 Proof of Lemma 4: Average loan rate and inflation

Proof. Let the average posted loan rate be $\tilde{i}^l(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \mathrm{d}F(i, \mathbf{z})$ and the average transacted loan rate be $\hat{i}^l(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\alpha_1 + 2\alpha_2(1 - F(i, \mathbf{z}))] i \mathrm{d}F(i, \mathbf{z})$. Applying integration by parts, we can rewrite the average posted loan rate $\tilde{i}^l(\gamma)$ as

$$\tilde{i}^l(\gamma) = [iF(i,\mathbf{z})]_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} - \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \frac{\partial i}{\partial i} F(i,\mathbf{z})\,\mathrm{d}i \;= \bar{i}(\mathbf{z}) - \underbrace{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F(i,\mathbf{z})\,\mathrm{d}i}_{=:\tilde{f}(\gamma)}. \tag{E.1}$$

Differentiating Expression (E.1) with respect to γ yields

$$\tilde{i}^l_\gamma(\gamma) = \bar{i}_\gamma(\gamma) - \tilde{f}_\gamma(\gamma) \;= \bar{i}_\gamma(\gamma) - \left[\bar{i}_\gamma(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i\right] = -\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i, \tag{E.2}$$

where

$$F_\gamma(i,\mathbf{z}) = \frac{\alpha_1}{2\alpha_2}\frac{1}{\beta}\left\{\frac{\xi(\bar{i},\mathbf{z})R(i,\mathbf{z}) - \xi(i;\mathbf{z})R(\bar{i},\mathbf{z})}{[R(i,\mathbf{z})]^2}\right\} \;= \frac{1}{\beta}\frac{\alpha_1}{2\alpha_2}\frac{\xi(\bar{i},\mathbf{z})}{\xi(i,\mathbf{z})}\frac{i-\bar{i}(\mathbf{z})}{[i-i_f(\gamma)]^2} < 0, \tag{E.3}$$

The last term $\tilde{f}_\gamma(\gamma)$ in (E.2) is obtained by Leibniz' rule: $\tilde{f}_\gamma(\gamma) \;= \bar{i}_\gamma(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i$. Observe that $F_\gamma(\cdot,\mathbf{z})$ has negative values for all i in the equilibrium support of $F(\cdot,\mathbf{z})$, since $i < \bar{i}$ and since the event that two banks post the same interest rate, $\{i\}$, has zero probability measure in any SME. Hence, the conclusion of Lemma 3. Moreover, from Equations (E.2) and (E.3), we have also that the average posted loan rate is increasing with inflation.

Next, observe that the only difference between $\tilde{i}^l(\gamma)$ and $\hat{i}^l(\gamma)$ is that in the latter, an additional probability weighting function appears in the definition of the ex-ante or mean transaction rate buyers face. It is immediate that $\hat{i}^l(\gamma) \leq \tilde{i}^l(\gamma)$. Moreover, the integrand in the integral function $\hat{i}^l(\gamma)$ is dominated by that in $\tilde{i}^l(\gamma)$, then $\hat{i}^l(\gamma)$ can grow no faster than $\tilde{i}^l(\gamma)$ with respect to γ. □

E.2 Proof of Proposition 4: Average loan rate spread and inflation

Proof. Let the average loan rate spread be

$$\hat{\mu}(\gamma) := \frac{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i\mathrm{d}F(i,\mathbf{z})}{i_f(\gamma)} =: \frac{g(\gamma)}{h(\gamma)},$$

and let the average transacted loan rate spread be

$$\mu(\gamma) := \frac{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i\cdot[\alpha_1 + 2\alpha_2(1-F(i,\mathbf{z}))]\,\mathrm{d}F(i,\mathbf{z})}{i_f(\gamma)} =: \frac{\hat{g}(\gamma)}{h(\gamma)},$$

where $i_f(\gamma) = (\gamma - \beta)/\beta$ is the policy rate (i.e., the opportunity cost of holding money in an SME).

Fix $\gamma > \beta$ (i.e., inflation target away from the Friedman rule) and $\alpha_1 \in (0,1)$ (i.e., agents can meet more than one lending agent). Consider an SME with co-existence of money and bank loans at the given γ. In such an equilibrium, the distribution of loan rates is non-degenerate.

The average posted loan rate spread. First, we prove this for $\hat{\mu}(\gamma)$. At each $\gamma > \beta$, $g(\gamma) > h(\gamma)$, since average spread is strictly greater than unity $\hat{\mu}(\gamma) > 1$.

Since the average loan spread function $\hat{\mu}$ is differentiable with respect to γ, then we have

$$\hat{\mu}_{\gamma}(\gamma) = \frac{g_{\gamma}(\gamma)h(\gamma) - g(\gamma)h_{\gamma}(\gamma)}{[h(\gamma)]^2}. \tag{E.4}$$

To show that the average loan spread is decreasing in inflation, $\hat{\mu}_{\gamma}(\gamma) < 0$, it suffices to verify that $\frac{g_{\gamma}(\gamma)}{g(\gamma)} < \frac{h_{\gamma}(\gamma)}{h(\gamma)}$. This requires that the percentage change in average loan rate with respect to inflation is strictly smaller than that of banks' marginal cost of funds.

Using the definition of g and h, we can also rewrite the last inequality as $g_{\gamma}(\gamma) < \frac{1}{\beta}\hat{\mu}(\gamma)$. Applying integration by parts, we can rewrite the average loan rate $g(\gamma)$ as

$$g(\gamma) = [iF(i, \mathbf{z})]_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} - \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \frac{\partial i}{\partial i} F(i, \mathbf{z})\, \mathrm{d}i \; = \bar{i}(\mathbf{z}) - \underbrace{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F(i, \mathbf{z})\, \mathrm{d}i}_{=:\tilde{g}(\gamma)}. \tag{E.5}$$

Differentiating Expression (E.5) with respect to γ yields

$$g_{\gamma}(\gamma) = \bar{i}_{\gamma}(\gamma) - \tilde{g}_{\gamma}(\gamma) \; = \bar{i}_{\gamma}(\gamma) - \left[\bar{i}_{\gamma}(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_{\gamma}(i, \mathbf{z})\, \mathrm{d}i\right] = -\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_{\gamma}(i, \mathbf{z})\, \mathrm{d}i, \tag{E.6}$$

where

$$F_{\gamma}(i, \mathbf{z}) = \frac{\alpha_1}{2\alpha_2}\frac{1}{\beta}\left\{\frac{\xi(\bar{i}, \mathbf{z})R(i, \mathbf{z}) - \xi(i, \mathbf{z})R(\bar{i}, \mathbf{z})}{[R(i, \mathbf{z})]^2}\right\} \; = \frac{1}{\beta}\frac{\alpha_1}{2\alpha_2}\frac{\xi(\bar{i}, \mathbf{z})}{\xi(i, \mathbf{z})}\frac{i - \bar{i}(\mathbf{z})}{[i - i_f(\gamma)]^2} < 0. \tag{E.7}$$

The last term $\tilde{g}_{\gamma}(\gamma)$ in (E.6) is obtained by Leibniz' rule: $\tilde{g}_{\gamma}(\gamma) \; = \bar{i}_{\gamma}(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_{\gamma}(i, \mathbf{z})\, \mathrm{d}i$.

Observe that $F_{\gamma}(\cdot, \mathbf{z})$ is negatively valued for all i in the equilibrium support of $F(\cdot, \mathbf{z})$, since $i < \bar{i}$ and since the event that two banks post the same interest rate, $\{i\}$, has zero probability measure in any SME. Thus, from Equations (E.6) and (E.7), we have that the average loan rate is increasing with inflation, or, $g_{\gamma}(\gamma) > 0$.

Consider Expression (E.7). Since loan demand ξ is decreasing in i, $\bar{i}(\mathbf{z}) > \underline{i}(\mathbf{z})$, and, $\underline{i}(z, \mathbf{z}) - i_f(\gamma) < i - i_f(\gamma)$, then the relative demand terms are always bounded in $(0, 1)$:

$$0 < \frac{\xi(\bar{i}(\mathbf{z}), \mathbf{z})}{\xi(\underline{i}(\mathbf{z}), \mathbf{z})} < \frac{\xi(\bar{i}(\mathbf{z}), \mathbf{z})}{\xi(i, \mathbf{z})} < 1, \tag{E.8}$$

and,

$$0 < \frac{1}{[i - i_f(\gamma)]^2} < \frac{1}{[\underline{i}(z, \mathbf{z}) - i_f(\gamma)]^2} < 1, \tag{E.9}$$

for all $i \in \left(\underline{i}(\mathbf{z}), \bar{i}(\mathbf{z})\right)$.

The bounds in Inequalities (E.8) and (E.9) allow us to look at the extreme case by setting $i = \bar{i}(z, \mathbf{z})$ so that the sufficient bound is independent of the endogenous i. From sufficient condition (1), we can deduce

$$0 < \frac{\bar{i}(z, \mathbf{z}) - i}{\beta} < \frac{\bar{i}(z, \mathbf{z}) - \underline{i}(z, \mathbf{z})}{\beta} < 1. \tag{E.10}$$

Using Inequalities (E.8), (E.9) and (E.10), $0 < \alpha_1/2\alpha_2 < 1$, Sufficient Conditions (1) and (2) and (E.7), we have an upper bound on how fast the average loan rate varies with inflation:

$$0 < g_\gamma(\gamma) := -\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i < \left[\bar{i}(z,\mathbf{z}) - \underline{i}(z,\mathbf{z})\right]\hat{\mu}(\gamma) < \frac{1}{\beta}\hat{\mu}(\gamma). \tag{E.11}$$

The result above says that the upper bound on $g_\gamma(\gamma)$ is given by the rate of change in the deposit rate with respect to inflation, $1/\beta$, times the average loan spread, $\hat{\mu}(\gamma)$. Therefore, we have that the average loan spread decreases with inflation, $\hat{\mu}_\gamma(\gamma) < 0$.

Note that at any $\gamma > \beta$, the second last term in Condition (E.11) gives the area of a rectangle whose height is $\bar{\mu}(\gamma)$, and width is $\left[\bar{i}(\mathbf{z}) - \underline{i}(\mathbf{z})\right]$. Under sufficient conditions (1) and (2), and the fact that $F_\gamma(i,\mathbf{z})$ is monotone decreasing in i, we have that the maximal value of $F_\gamma(i,\mathbf{z})$ is bounded above by $\hat{\mu}(\gamma)$. Sufficient condition (1) bounds the limits of the integral above by $1/\beta$. Hence the definite integral $g_\gamma(\gamma)$ is bounded: $0 < g_\gamma(\gamma) < \frac{1}{\beta}\hat{\mu}(\gamma)$. This suffices for the conclusion that the average spread is decreasing with inflation, i.e., $\hat{\mu}_\gamma(\gamma) < 0$ as desired.

The average (transacted) loan spread. We now prove the second part. Observe that the only difference between $\hat{\mu}(\gamma)$ and $\mu(\gamma)$ is that in the latter, an additional probability weighting function, $\alpha_1 + 2\alpha_2(1 - F(i,\mathbf{z}))$ appears in the definition of the mean transaction rate buyers face. Let this be $\hat{g}(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \cdot [\alpha_1 + 2\alpha_2(1 - F(i,\mathbf{z}))]\,\mathrm{d}F(i,\mathbf{z})$. It is immediate that $0 < \hat{g} \leq g$. Under the same sufficient conditions above, we also have $\frac{\hat{g}_\gamma(\gamma)}{\hat{g}(\gamma)} \leq \frac{g_\gamma(\gamma)}{g(\gamma)} < \frac{h_\gamma(\gamma)}{h(\gamma)}$. Since the integrand in the integral function $\hat{g}$ is dominated by the integrand in g, then $\hat{g}(\gamma)$ can grow no faster than $g(\gamma)$ with respect to inflation γ. Finally, since we concluded that $g(\gamma)$ grows slower than the deposit rate $h(\gamma)$ as γ increases, then so must $\hat{g}(\gamma)$. Thus, $\hat{g}(\gamma)$ is also decreasing with γ under the same sufficient condition. □

F Omitted proofs: Monetary policy and deposit market power

In this section, we consider how monetary policy affects banks' market power in deposit pricing: the average interest rate spread on deposits. Following the convention in Drechsler et al. (2017) and Choi and Rocheteau (2023b), we also define the average deposit spread by the difference between the policy rate and deposit rate. We then study how bank market power responds to the change in the anticipated inflation, γ. Intermediate results and proofs are provided in Section F.1 and Section F.2. We will apply these results in the proof in Section F.3 to see how monetary policy affects the degree of banking market power in deposit operation.

F.1 Proof of Lemma 5: Deposit first-order stochastic dominance and inflation

Proof. Consider the economy away from the Friedman rule: $\gamma > \beta$. The analytical formula for the deposit-rate distribution $G(i_d;\gamma)$ is characterized in Lemma 15. Let $i_{f,\gamma} := \partial i_f(\gamma)/\partial\gamma$ denote the partial derivative of the policy rate with respect to inflation γ.

Now consider how the value of G varies with γ at each fixed i_d such that $0 = \underline{i}_d < i_d < \bar{i}_d$. We have that

$$\frac{\partial G(i_d;\gamma)}{\partial\gamma} = \frac{\alpha_1^d}{2\alpha_2^d}\left[\frac{i_{f,\gamma}(i_f - i_d) - i_f(i_{f,\gamma} - i_{d,\gamma})}{(i_f - i_d)^2}\right] = -\frac{\alpha_1^d}{2\alpha_2^d}\left[\frac{i_{f,\gamma} i_d}{(i_f - i_d)^2}\right],$$

where $i_{f,\gamma} = 1/\beta > 1$ and the second equality obtains since for fixed i_d, $i_{d,\gamma} = 0$.

Since all the other terms are strictly positive, we, therefore, have, for every fixed $i_d \in \left(\underline{i}_d, \bar{i}_d\right) = \mathrm{supp}(G)$,

$\partial G(i_d,\gamma)/\partial\gamma < 0$. Thus, we establish that the posted-deposit-rate distribution $G(i_d;\gamma^{'})$ first-order stochastically dominates $G(i_d;\gamma)$ for $\gamma^{'} > \gamma > \beta$.

□

Remark. Note that the associate density of the distribution G is characterized by $g(i_d;\gamma) = \partial G(i_d;\gamma)/\partial i_d$. Moreover, a depositor randomly receives deposit-rate quotes from banks, which can be one quote or two quotes with probability α_1^d and $\alpha_2^d = 1-\alpha_1$ respectively. So, the cumulative distribution function of transacted deposit rates can then be described by

$$\hat{G}(i_d;\gamma) = \alpha_1^d G(i_d;\gamma) + \alpha_2^d[G(i_d;\gamma)]^2 \text{ for all } i_d \in \text{supp}(G),$$

and the associate density of $\hat{G}(i_d;\gamma)$ is given by

$$\hat{g}(i_d;\gamma) \equiv \partial\hat{G}(i_d;\gamma)/\partial i_d = \alpha_1^d g(i_d;\gamma) + 2\alpha_2^d\hat{G}(i_d;\gamma)g(i_d;\gamma) = [\alpha_1^d + 2\alpha_2^d G(i_d;\gamma)]g(i_d;\gamma).$$

We have characterized the relationship between the posted deposit interest rates distribution G and anticipated inflation in Section F.1. As highlighted above, the transacted deposit interest rates distribution $\hat{G}$ is a probability re-weighting of the distribution G. The conclusions above regarding inflation and G also apply to $\hat{G}$. Hence, we leave out the details here. Instead, we use distribution G for the proof below.

F.2 Proof of Lemma 6: Average deposit rate and inflation

Proof. Given monetary policy γ, the nominal policy rate is determined by $i_f := i_f(\gamma) = (\gamma-\beta)/\beta$. First, apply integration by parts to the average posted deposit rate: $\tilde{i}^d(\gamma) := \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$. This yields

$$\tilde{i}^d(\gamma) := [i_d G(i_d;\gamma)]_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} \frac{\partial i_d}{\partial i_d} G(i_d;\gamma)\mathrm{d}i_d = \bar{i}_d(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G(i_d;\gamma)\mathrm{d}i_d.$$

For the average posted deposit rate to be increasing in γ, we want to show that $\partial\tilde{i}^d(\gamma)/\partial\gamma > 0$. Using Leibniz's rule, we have

$$\tilde{i}_\gamma^d(\gamma) = \frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} - \left[\frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} + \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d\right] = -\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d > 0, \tag{F.1}$$

where $G_\gamma(i_d;\gamma) < 0$ follows from the result in Lemma 5.

Observe that the only difference between the average posted deposit rate, $\tilde{i}^d(\gamma)$, and the average transacted deposit rate, $\hat{i}^d(\gamma) := \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}\hat{G}(i_d;\gamma)$, is that an additional probability weighting function appears inside $\hat{G}$. Hence, we can deduce that $\hat{i}^d(\gamma) \leq \tilde{i}^d(\gamma)$ holds since the average transacted rate cannot exceed the average posted rate. It follows that the transacted rate cannot grow faster than the posted rate. Therefore, we can verify $0 < \hat{i}_\gamma^d(\gamma) \leq \tilde{i}_\gamma^d(\gamma)$.

Next, we consider how the support of the distribution G changes. Recall that the lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$, which is invariant to inflation change since the "hypothetical" monopoly bank can always pay zero deposit interest. Using the equal profit condition: $i - \bar{i}_d = \frac{\alpha_1}{\alpha_1+2\alpha_2}[i - i_d^m]$, we can back out the upper support of the distribution by $\bar{i}_d = i[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$. Differentiate the upper bound of the support of distribution G with respect to inflation γ. We obtain $\frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} = \frac{1}{\beta}[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$ and it satisfies that $0 < \frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} < \frac{1}{\beta}$.

Thus we have established that the upper bound of the support of the distribution shifts to the right, and it becomes wider at a rate less than $1/\beta$ as inflation γ goes up. $\square$

F.3 Proof of Proposition 5: Deposit-rates spread and inflation

Deposit-rates spread. Following Drechsler et al. (2017) and Choi and Rocheteau (2023a), we define the average interest rate spread on deposits as the difference between the central bank policy interest rate and the average of deposit interest rates across banks:

$$s^d(\gamma) = i_f(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma), \tag{F.2}$$

where the distribution G is characterized in Lemma 15.

Proof. We first consider the average posted deposit rates spread and make a few observations before we show how it changes with respect to the change in inflation. Recall that all deposit interest rate i_d in the support of the distribution G must be smaller than the policy interest rate $i_f(\gamma)$ in a noisy deposit search equilibrium, given $\alpha_1^d \in (0,1)$. This implies that $i_f(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$, since all banks earn a positive expected profit on deposit operation in equilibrium by marking down the deposit rate that they post. Then it establishes that the deposit spread is positive. That is, $i_f(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$ implies that $s(\gamma) > 0$, for a given $\gamma > \beta$ and $\alpha_1^d \in (0,1)$.

Next, we consider how the average posted deposit rates spread $s^d(\gamma)$ moves with respect to the change in inflation. Let the function $\tilde{i}^d(\gamma)$ to denote the average posted deposit rates, *i.e.*, $\tilde{i}^d(\gamma) := \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$.

Differentiate Equation (F.2) with respect to γ, we obtain

$$s^d_\gamma(\gamma) = i_{f,\gamma}(\gamma) - \tilde{i}^d_\gamma(\gamma) \equiv i_{f,\gamma}(\gamma) - \left[- \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma) \mathrm{d}i_d \right]. \tag{F.3}$$

We show that the average deposit rate is increasing with respect to inflation from the result in Lemma 6, *i.e.*, $\tilde{i}^d_\gamma(\gamma) > 0$ since $G_\gamma(\cdot) < 0$. We also show that the growth rate of the support of the distribution G is less than $1/\beta$ in Lemma 6. It follows that the integral function $\tilde{i}^d_\gamma(\gamma)$ must be also less than $1/\beta$. Hence, we have $\frac{1}{\beta} > \tilde{i}^d_\gamma(\gamma) > 0$.

Next, recall that the growth rate of the policy interest rate is given by $i_{f,\gamma}(\gamma) = 1/\beta$. Combining this result with the inequality above, then $i_\gamma(\gamma) > \tilde{i}^d_\gamma(\gamma)$ implies that $s^d_\gamma(\gamma) = i_{f,\gamma}(\gamma) - \tilde{i}^d_\gamma(\gamma) > 0$. This establishes that the average posted deposit-rates spread is increasing with inflation. Moreover, it follows that the growth rate of the average posted deposit-rates spread is also bounded such that $\frac{1}{\beta} > s^d_\gamma(\gamma) > 0$ since $i_\gamma(\gamma) > i_\gamma(\gamma) - \tilde{i}^d_\gamma(\gamma)$ holds.

From the result in Lemma 6, we have established that the average transacted deposit rate cannot grow faster than the posted rate, which is also slower than the policy rate grows. It follows that the average transacted deposit rate is also increasing in γ and bounded. $\square$

G Statistical Calibration of the Model

G.1 Baseline calibration

Our approach is to match the empirical money demand and average loan spread in the macro data, where we measure the latter in an SME by

$$\bar{\mu}(\tau) = \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \left[\frac{i}{i_f(\tau)} - 1\right] \mathrm{d}F(i, \mathbf{z}), \tag{5.1}$$

where $\gamma = 1 + \tau$.[29]

For identification, the bank's loan contact probabilities (α_0, α_1) directly affect the loan rate distribution, $F(\cdot, \mathbf{z})$, and thus banks' average loan-rate spread over the policy rate $i_f = (\gamma - \beta)/\beta$. Likewise, the bank's deposit contact probabilities affect the deposit rate distribution G, and, therefore, the interest spread on deposits. The CM utility function, U, is assumed to be logarithmic. With quasi-linear preferences, real CM consumption is then given by $x^{\star} = (U^{'})^{-1}(A)$, where the scaling parameter, A, determines the relative importance of CM and DM consumption. The DM utility function is given by (2.2). The DM production function is linear and has no parameter to be estimated. The parameters (A, σ) are identified through the model-implied aggregate real money demand relationship with i_f.

We set the model period to a year and calibrated it to annual data. There are nine parameters: $(\tau, \beta, \hat{\delta}, \sigma, A, n, \alpha_0, \alpha_1, \alpha_1^d)$. We assumed in the baseline model that there are no redistributive policies $(\tau_b = \tau_s = 0)$, and $\tau_1^e + \tau_2^e = 0$. The baseline policy is just the long-run inflation target, γ, with $\tau = (\gamma - 1) = \tau_2$.

External calibration. Some parameters can be determined directly by observable statistics. We use the Fisher relation to determine the money growth rate, τ, and discount factor, β. The share of inactive buyers (depositors) $\tilde{n} \equiv 1 - n$ is set to match the average share of household depositors with commercial banks per thousand adults in the United States.[30]

Internal calibration. We set α_1^d to match with the average deposit spread of 1.29% estimated byWang et al. (2022), who also use the same data as Drechsler et al. (2017). We then jointly choose the pairs (σ, A), and (α_0, α_1) to match, respectively, the aggregate relationships between nominal interest and money demand, and between nominal interest and average gross loan spread. These empirical relations are estimated by auxiliary fitted-spline functions. Intuitively, each pair of these parameters is identified by the shift (or position) and the overall shape of the respective spline approximations of the empirical relations.

Our parameter values and targets are summarized in Table 1. Figure 6 provides the respective scatter-plots of the two empirical relationships (*blue circles*) just mentioned, the empirical spline models (*dashed-red lines*, "Fitted Model"), and our calibrated model's predictions (*solid-green lines*, "Model") for these relations.

In Figure 6, the model's fit to the average loan spread (the solid green line in Panel b) is not perfect, especially at low nominal interest rates. This is due to a tension between matching both real money demand

[29] We use $\left(\frac{\text{bank prime loan rate}}{\text{federal funds rate}} - 1\right)$ as a proxy for the average loan spread over the opportunity cost of holding money. As a robustness check, we also consider $\left(\frac{\text{finance rate on personal loans}}{\text{federal funds rate}} - 1\right)$. The two measures are qualitatively similar. The data for the finance rate on personal loans at commercial banks can be found in the FRED Series (TERMCBPER24NS). We use the data on the bank prime loan rate as it is a longer time series. Alternatively, we could use the three-month T-bill rate to be consistent with the empirical money demand in Lucas and Nicolini (2015). Since the time series for the three-month T-bill rate and fed funds rates behave similarly, this would not alter the general shape of our average loan spread.

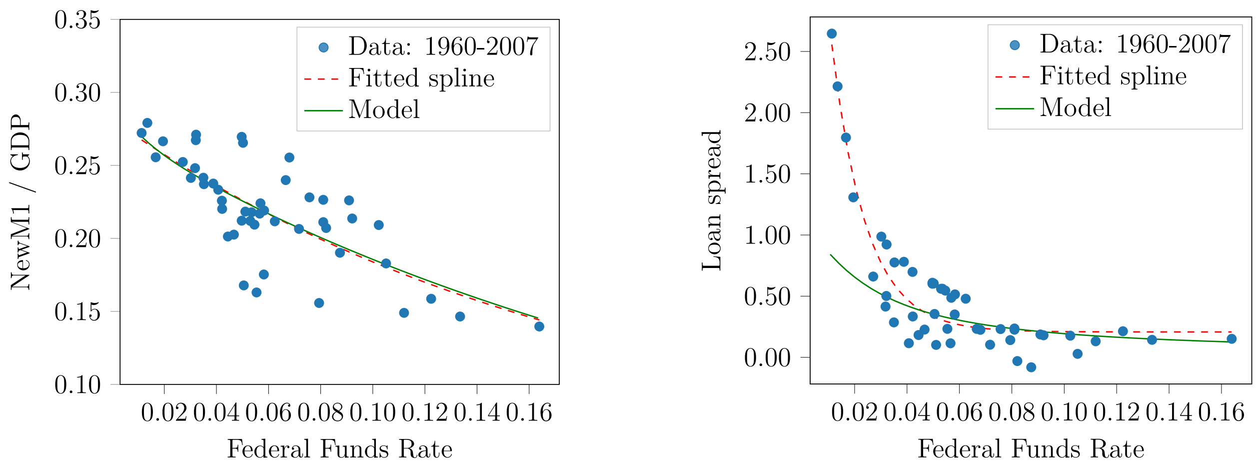
[30] Source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators".

Table 1: Calibration and targets.

Parameter	Value	Empirical Targets	Description
$1+\tau$	$(1+0.042)$	Inflation rate[a]	Inflation rate
$1+i_{\mathrm{f}}$	$(1+0.061)$	Effective federal funds rate[a]	Nominal interest rate
β	0.982	-	Discount factor, $(1+\tau)/(1+i_{\mathrm{f}})$
σ	0.1923	Aux reg. (i_{f},M/PY)[c]	CRRA (DM q)
A	0.6484	Aux reg. (i_{f},M/PY)[c]	CM preference scale
$\tilde{n}$	0.35	household depositors[d]	Proportion of inactive DM buyers
α_0, α_1	$0.0662, 0.1085$	Aux reg. (i_{f}, loan spread)[e]	Prob. $k = 0, 1$ lending bank contacts
α_1^d	0.1411	Average (deposit spread)	Prob. $k = 1$ depository contacts

[a] Annual nominal interest and inflation rates.
[b] National average percent of consumers with new brankruptcies.
[c] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i_{f}) and Lucas and Nicolini (2015) New-M1-to-GDP ratio (M/PY).
[d] Household depositors with commercial banks per 1000 adults for the United States.
[e] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i_{f}) and average loan spread, (bank loan prime rate - i_{f})/i_{f}.

Figure 6: Aggregate money demand and average loan spread—model and data.



(a) Calibration and money demand data

(b) Calibration and average loan spread data

and the average loan spread. In the model, a lower nominal policy interest leads to a lower cost of holding money, and thus, higher real money demand, reducing the average loan rate by Lemma 1. Since the policy rate $i_f(\tau)$ is fixed by the inflation rate, τ, the average loan spread has to be lower. Nonetheless, we view the fit under the benchmark calibration to be reasonable.

H Ination, pass-through and loan/deposit dispersion: Empirics

H.1 Loan rate Data

Branch-level interest rate data. *RateWatch* provides monthly interest rate data at the branch level for several types of consumer lending products. Our baseline analysis focuses on unsecured consumer loans within a particular class. By focusing on posted loan rates (rather than the rates on specific loans) we minimize the effects of both observed and unobserved heterogeneity across borrowers and loans. Also, this measure is the most consistent with our theoretical model's setting where there is equilibrium rate dispersion for a single type of consumer loan product. Specifically, we choose the most commonly used product for personal loans: Personal Unsecured Loans for Tier 1 borrowers.[31] Our primary sample includes 496,942 branch-month observations from January 2003 to December 2017, involving 11,855 branches. To calculate each branch's loan spread over the federal funds rate, we collect daily effective federal funds data from the Federal Reserve H15 report.

Bank and county controls. We obtain commercial banks' information from their call reports. Specifically, we collect information on each commercial bank's reliance on deposit financing, leverage ratio, credit risk, and bank size.

The Federal Deposit Insurance Corporation (FDIC) provides branch-level deposit holdings information, for all FDIC-insured institutions. This can be found in the Summary of Deposits (SOD) dataset. We use this data set to approximate each branch's local market competition and the impact of its commercial-bank-branch network. To control for potential local-market competition effects, we calculate each branch's deposit share in its county, the Herfindahl-Hirschman Index (HHI) in each county's deposit holdings, and the number of branches in the county. To measure one branch's parent commercial bank's branch network, we calculate one branch's deposit share in its parent bank, the Herfindahl-Hirschman Index (HHI) of the commercial bank's deposit holdings, and the number of branch counts in the commercial bank.

We also control for county-level socioeconomic information. This includes median income, the poverty rate, population, and the average house price, all obtained from census data. We also have county-level unemployment and number of business establishments from the Bureau of Labor Statistics, county-level real GDP, and GDP growth from the Bureau of Economic Analysis to control for local economic activity.

H.2 Loan rate spreads

We use two measures of loan rate spreads: (1) the raw spread of lending rates over the federal funds rate; and (2) an orthogonalized spread using a set of control variables.

The raw spread. We calculate each branch's loan spread relative to the federal fund rate (FF). Specifically, the branch-level raw spread is calculated as

$$Spread_{b,i,c,s,t} = \frac{(1 + Rate_{b,i,c,s,t}) - (1 + FF_t)}{1 + FF_t}. \tag{H.1}$$

In this definition, b stands for a bank branch, i for the parent bank to which the branch belongs, c for the county in which the branch is located, s for the state and t for the date that *RateWatch* reports the branch rate information.

Residual or orthogonalized spreads. Differences in branch-level loan pricing could simultaneously be explained by local socioeconomic factors, deposit market competition, bank-branch networks, characteristics of banks, and other fixed effects. These factors could determine locally different demands for loans and costs of bank funds. These confounding features, however, will not be captured in our simpler model structure. In our model, the distribution of loan rate spreads will result from the single feature of noisy consumer search in equilibrium. To maintain consistency with our model, it is useful to focus on an empirical measure of the residual spread accounting for as many of these factors as possible.

[31] As a robustness check, we also use mortgage rates as the alternative variable to calculate loan spreads. Specifically, we choose the 30-year Fixed Mortgage rate with an origination size of \$175,000. Our key results still hold when we use mortgage rates. Our results continue to hold if we use rates on personal loans with different borrower qualities (*i.e.*, different borrower "tier" definition).

We thus orthogonalize the branch-level spread with respect to these potential factors to obtain a measure of a residual loan spread. We use this OLS regression to obtain the residual $\epsilon_{b,i,c,s,t}$:

$$Spread_{b,i,c,s,t} = a_0 + a_1 X_{b,i,c,s,t} + a_2 X_{i,t} + a_3 X_{c,s,t} + \epsilon_{b,i,c,s,t}. \tag{H.2}$$

Here, $X_{b,i,c,s,t}$ represents branch-specific control variables including local deposit market competition and bank branch networks, $X_{i,t}$ represents commercial bank control variables and $X_{c,s,t}$ represents county-level socio-economic control variables. We then re-scale $\epsilon_{b,i,c,s,t}$ to match the mean and standard deviation of raw spreads in our full sample and use it as our alternative specification for the loan rate spread. A detailed summary can be found in our Online Appendix J.1 (Table 5).

H.3 The dispersion and mean of the loan spread

We estimate OLS regressions of the dispersion of spreads ($Dispersion_t$) on their monthly average, ($\overline{Spread}_t$):

$$Dispersion_t = b_0 + b_1 \overline{Spread}_t + \epsilon_t. \tag{H.3}$$

In the (H.3), b_1 is the coefficient of interest and standard errors are clustered by month. The average spread, $\overline{Spread}_t$, refers to either the raw or orthogonalized spread. We consider two measures of dispersion: the monthly standard deviation (SD_t) coefficient of variation (CV_t) of the spreads.

H.4 Results: loan rate dispersion

We illustrate first our main empirical findings graphically using simple scatter plots. In Figure 7, we have the correlations between our two measures of loan spread dispersion and the average spread.

Consider now the relationship between the standard deviation and mean of spreads in the top two panels of Figure 7. The left panel depicts the relationship using the raw spreads, while the right uses the orthogonalized spread, *i.e.*, our residual measure after controlling for various local, market, and social confounding factors. The standard deviations of both measures of loan spreads are positively correlated with their averages. In particular, the correlation is 0.752.

The bottom two panels of Figure 7, show that the coefficients of variation of spreads are *negatively* correlated with their averages. This holds for both raw (*left panel*) and orthogonalized (*right panel*) measures. The correlation for the case of the raw loan spread is -0.857.

Next, we report regression results for our two measures of loan spreads from estimating Equation (H.3). The results are summarized in Table 2. From Columns (1) and (2) of the table, we can see positive significant relationships between standard deviations and averages for the respective raw and orthogonalized loan spread measures. For the raw spread, the coefficient in Column (1) indicates that a one-percentage-point increase in the average spread is associated with a 0.146-percentage-point increase in its standard deviation. Alternatively, for the orthogonalized spread, Column (2) indicates that a one-percentage-point increase in the average is associated with a 0.192-percentage-point increase in the standard deviation.

We report state-level results in Online Appendix J. We consider the dispersion of spreads at the state level using the standard deviation of branch spreads from state s in month t. At the national level, the standard deviations of spreads are positively related to their average levels in the state-month panel data, after controlling for state and time-fixed effects. There is a corresponding, but noisier result for the coefficient of variation at the national level.

Figure 7: Spread dispersion and averages at the national level (January 2003 to December 2017). Dispersion measures: SD (*standard deviation*). Data source: *RateWatch*, "Personal Unsecured Loans (Tier 1)." Least squares regression lines with 95% (bootstrapped) confidence bands (shaded patches) superimposed.

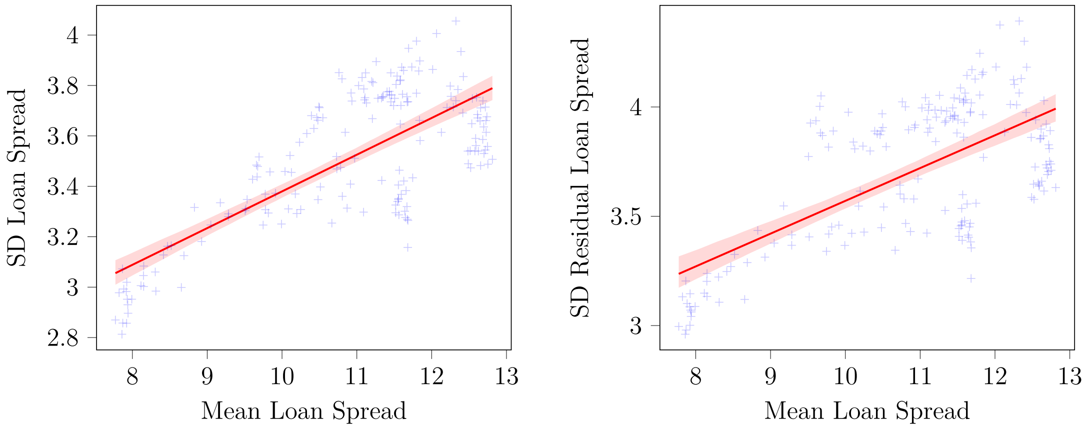


Table 2: Regression of spread dispersion on averages (national data).

	Raw loan spread	Orthogonalized spread
	(1)	(2)
	SD_t	SD_t
$\overline{Spread}_t$	0.146***	0.192***
	(0.004)	(0.010)
Constant	1.924***	1.621***
	(0.039)	(0.111)
N	180	180
adj. R^2	0.554	0.333

Note: Standard errors in parentheses. $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

H.5 Deposit rate data

Branch-level interest rate data. We obtain weekly interest-rate information on an identical deposit product at each branch from *RateWatch*. Specifically, we use rates for one of the most commonly used time deposit products in the United States, the twelve-month certificate of deposits (CD).[32] This strategy of focusing on posted rates for a class of identical deposit products allows us to rule out any observable (and unobservable) pricing heterogeneity across depositors and deposit products.

Our primary sample includes 1,428,900 branch-weekly observations from 12,381 branches, between January 2001 and December 2007.[33] Our sample covers 49 states and the District of Columbia. We drop Hawaii due to insufficient branch-level observations to calculate state-level dispersion. To calculate each branchs deposit spread against the federal funds rate, we collect daily effective federal funds data from the U.S. Federal Reserve H15 report.

[32]We focus on fixed-term time deposits to be consistent with our theoretical model. In the model, households use time deposits to save idle money balances in contrast to demand deposits, which help to smooth out consumption expenditures. While we do not report this in the paper, we have also conducted the empirical analysis using other deposit products and have obtained the same results.

[33]We choose not to include observations beyond 2008 to avoid the near-zero-lower-bound interest rate environment similar to Choi and Rocheteau (2023a).

The deposit spread. We follow Drechsler et al. (2017) and define the deposit spread as the difference between federal funds rate (FF_t) and branch-level deposit rate ($Rate_{b,s,t}$).[34] Specifically, we calculate each bank branch's deposit spread as

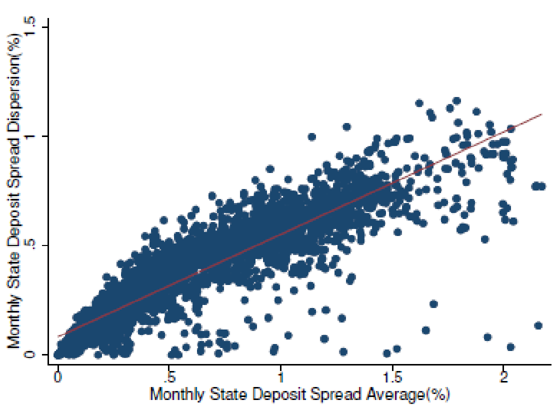
$$Spread_{b,s,t} = FF_t - Rate_{b,s,t}, \tag{H.4}$$

where b denotes the bank branch, s the state, and, t the date for which *RateWatch* reports. We then calculate the mean ($\overline{Spread}_{s,t}$) and the standard deviation ($Dispersion_{s,t}$) of branch-level deposit spreads within a particular state s and a period t.

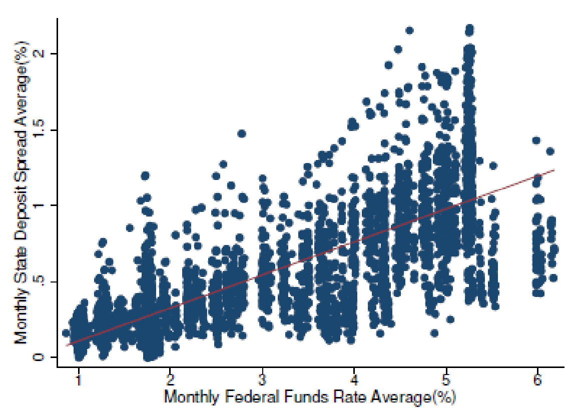
Figure 8 depicts the data visually and summarizes our results. Specifically, Panel 8a shows a positive relationship between the monthly standard deviation and the average of deposit spreads at the state level. Panel 8b shows a positive relationship between the average deposit spreads and the federal funds rate. This latter finding is consistent with the findings of both Drechsler et al. (2017) and Choi and Rocheteau (2023b).

Figure 8: Dispersion (standard deviation) and average of deposit spreads

(a) Dispersion vs. mean



(b) Mean vs. Federal Funds Rate



H.6 Results: deposit rate dispersion

To test formally the significance of the relationship observed in Figure 8a, we estimate the following regression equation by OLS:

$$Dispersion_{s,t} = b_0 + b_1\overline{Spread}_{s,t} + b_2 Z_s + b_3 Z_t + \epsilon_{s,t}, \tag{H.5}$$

where Z_s and Z_t are state and time-fixed effects and standard errors are clustered by state.

Table 3 summarizes the regression results for Equation H.5. All columns show a positive and statistically significant relationship (b_1) between our measure of the dispersion of the deposit spread and its mean. Column (4) suggests that an increase of 10 basis points in the average deposit spread is associated with an increase of 3.4 basis points in the standard deviation of the spread after controlling for state-fixed effects and time-fixed effects.

[34] We also use an alternative specification of the deposit spread: $Spread_{b,s,t} = \frac{FF_t - Rate_{b,s,t}}{FF_t}$ following Wang (2022) and find consistent results.

Table 3: State-Month Regression Results: Dependent Variable: $Dispersion_{s,t}$

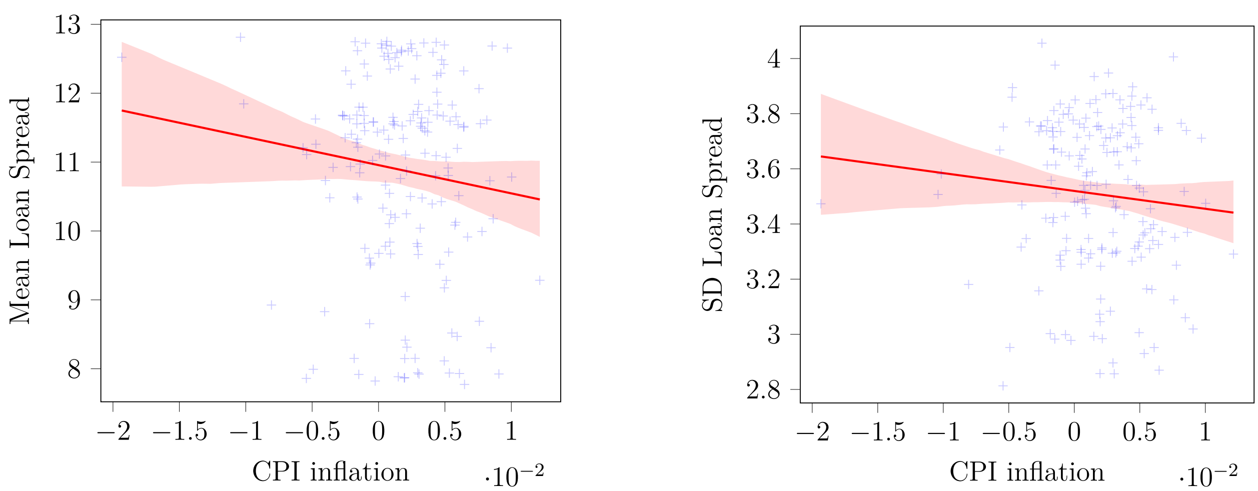
	(1)	(2)	(3)	(4)
$\overline{Spread}_{s,t}$	0.482***	0.360***	0.492***	0.335***
	(0.017)	(0.040)	(0.015)	(0.038)
Constant	0.080***	0.148***	0.075***	0.162***
	(0.006)	(0.021)	(0.008)	(0.021)
Month FEs	No	Yes	No	Yes
State FEs	No	No	Yes	Yes
Observations	4155	4155	4155	4155
Adjusted R^2	0.856	0.894	0.885	0.922

These findings complement those of Drechsler et al. (2017) documenting a positive relationship between monetary policy and the average deposit spread. We find a similar relationship and provide additional evidence on it, demonstrating a similar positive relationship between the average spread and its dispersion. These findings, summarized in Figure 8 and Table 3, are consistent with our theoretical model and complement those of Choi and Rocheteau (2023a).

I Inflation, the average, and dispersion of the loan spread: Data

Figure 9 depicts the correlations of monthly CPI inflation and average loan-rate spreads, with the dispersion measure in *RateWatch* data—standard deviation (SD)—for January 2003 to December 2017. These two panels provide U.S. data counterparts to those for the model in Figure 4.

Figure 9: Correlations between inflation, the average loan spread, and the dispersion of spreads.



(a) The average loan spread (%)

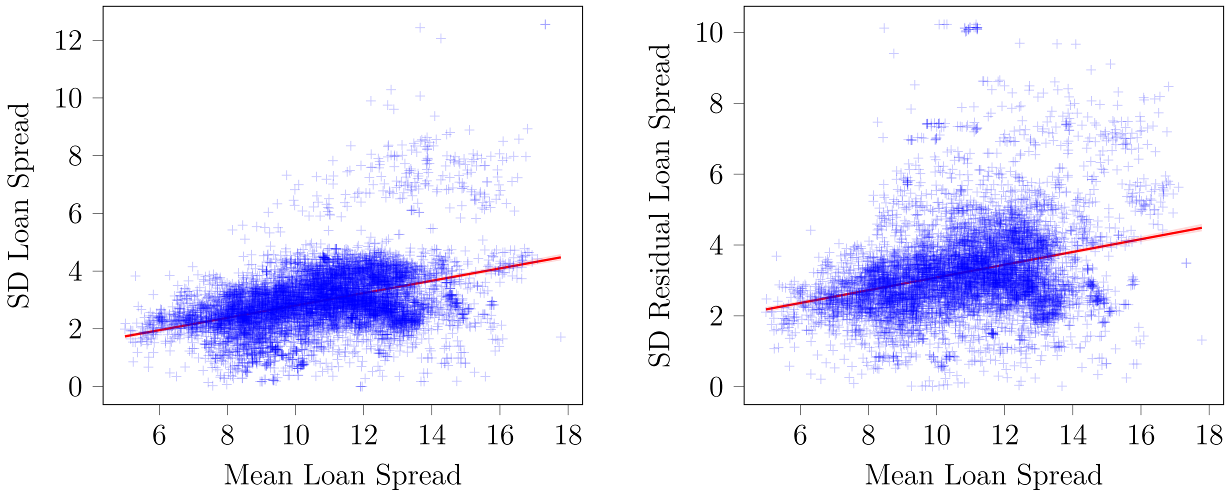
(b) The dispersion of loan spreads (SD)

J Empirical analysis of loan spreads at the state level

In this section, we calculate the standard deviations and means of loan spreads. There are 8,464 usable observations of the variables at the state and month level. This allows us to construct a panel dataset.

In Figure 10, we can see that the spreads' standard deviation and average are positively correlated at the state-month level.

Figure 10: State-month-level relationship between the dispersion and average of the loan spread. Dispersion measures: SD: *standard deviation*. Data source: *RateWatch*, "Personal Unsecured Loans (Tier 1)."



We estimate by OLS the relationship between the standard deviation and the average of the loan spread:

$$Dispersion_{s,t} = b_0 + b_1\overline{Spread}_{s,t} + b_2 Z_s + b_3 Z_t + \epsilon_{s,t} \tag{J.1}$$

The index s stands for a particular state and t stands for the month of observation. We cluster standard errors by state and month.

Table 4 reports the regression results. Columns (1) to (3) use the raw spreads and Columns (4) to (6) the orthogonalized ones. (Table 5 in Section J.1 for the controls used to define the orthogonalized spreads.) All columns show positive and statistically significant relationships between the standard deviations and averages. The magnitude of the coefficient is also economically significant. From column (6), the coefficient indicates that a one-percentage-point increase in orthogonalized spread average is associated with a 0.286-percentage-point increase in the standard deviation after controlling for state and time fixed effects.

Table 4: OLS regressions: Averages and Std. Deviations of State Level Loan Spreads, Jan. 2003 to Dec. 2017.

	Spread dispersion: $Dispersion_{s,t}$					
	Raw spread			Orthogonalized spread		
	(1)	(2)	(3)	(4)	(5)	(6)
	State FE	Time FE	Both FE	State FE	Time FE	Both FE
$\overline{Spread}_{s,t}$	0.179***	0.290***	0.353***	0.220***	0.304***	0.286***
	(0.030)	(0.094)	(0.077)	(0.055)	(0.079)	(0.084)
State fixed effects	X		X	X		X
Time fixed effects		X	X		X	X
N	8237	8237	8237	7463	7463	7463
adj. R^2	0.618	0.178	0.646	0.538	0.203	0.577

Note: Standard errors in parentheses. $^{*}\ p < 0.1$; $^{**}\ p < 0.05$; $^{***}\ p < 0.01$

J.1 Control variables list

In Table 5, we describe the controls used in constructing our orthogonalized loan spreads.

Table 5: Control variables to obtain the orthogonalized spread.

(a) Panel A: County variables

Variable	Data source	Frequency	Details
Real GDP	BEA	Annual	Annual county real GDP
GDP growth	BEA	Annual	Real GDP growth
Establishments	BLS	Annual	Number of establishments within county
Unemployment	BLS	Annual	County unemployment rate
House price	U.S. Census	Annual	Average housing pricing in the county
Median income	U.S. Census	Annual	Median Household Income
Population	U.S. Census	Annual	ln(Total population)
Poverty	U.S. Census	Annual	Proportion of county population under poverty line

(b) Panel B: Local competition

Variable	Data source	Frequency	Details
Within county share	SOD	Annual	Total branch deposits / Total county deposits
County deposit HHI	SOD	Annual	HHI of county's deposit holdings
County branch count	SOD	Annual	Number of branch counts in the county

(c) Panel C: Bank branch network

Variable	Data source	Frequency	Details
Within bank share	SOD	Annual	Total branch deposits / Total bank deposits
Bank deposit HHI	SOD	Annual	HHI of bank's deposit holdings across its branches
Bank branch count	SOD	Annual	Number of branch counts in the commercial bank

(d) Panel D: Commercial bank controls

Variable	Data source	Frequency	Details
Deposit reliance	Call reports	Quarter	Total deposits / Total liabilities
Leverage	Call reports	Quarter	Total equity / Total assets
Credit risk	Call reports	Quarter	Allowance for Loan and Lease Losses/Total Loans
Bank size	Call reports	Quarter	ln(Total assets)

J.2 Different household loan products

In this section, we show that the main evidence, based on a particular high-quality-consumer loan product in Section 6, is robust to alternative loan-product definitions. Here, we redo the analysis using other household loan products, namely personal unsecured loans, credit cards, fixed-rate mortages, variable-rate mortgages, new vehicle auto loans, and used vehicle auto loans.

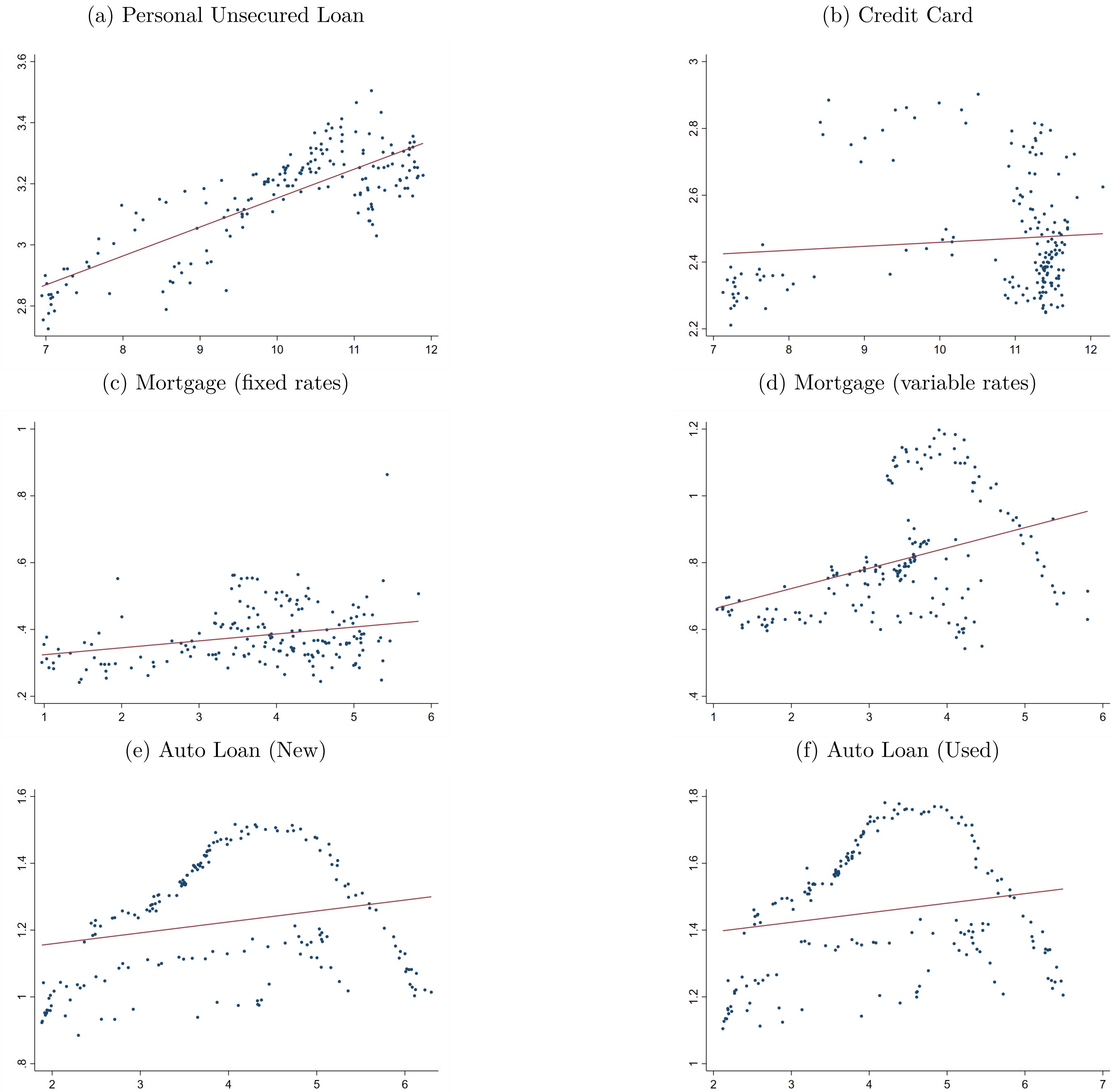
Table 6 provides details of each loan product. Figure 11 corroborates the raw spread results in Figure 7, where the standard deviation is the measure of dispersion.

Table 6: Different household loan products information.

Product type	Observations	Descriptions
Personal Unsecured Loan	718,748	Personal unsecured loan with tier 1 borrowers
Credit Card	182,118	Credit card with Visa
Mortgage (fixed rates)	331,558	30-Year fixed rate mortgage ($175k loan amount)
Mortgage (variable rates)	194,740	5-Year adjustable-rate mortgage ($175k loan amount)
Auto Loan (New)	878,797	Auto loan for new vehicles (60 mths term)
Auto Loan (Used)	804,871	Auto loan for (<24 mths) used vehicles (36 mths term)

Consistent with our main finding, there is a positive relationship between the standard deviation and the average loan spread for all six different household loan products. While these figures are graphical summaries, more formal regression results confirming the same relationships are also available upon request.

Figure 11: Loan spread dispersion measures (Y-axis) and average (X-axis) at the national level (January 2003 to December 2017). Dispersion measures: SD (*standard deviation*). Both variables are shown in percentage points. Data source: *RateWatch*.



K Aggregate demand shocks in the baseline model

We provide the details of the stochastic version of the baseline model used in Section 7 of the paper here. In particular, we characterize the SME with aggregate demand shocks and we set up the Ramsey optimal policy problem for aggregate demand stabilization. The optimal policy exercise here is in the same spirit as Berentsen and Waller (2011). In contrast to the perfectly-competitive banking environment of Berentsen et al. (2007) and Berentsen and Waller (2011), our model now has non-trivial consequences for the design of optimal monetary policy in response to aggregate demand shocks.

K.1 Aggregate demand shocks

The model admits two simple types of aggregate demand shocks. Let n, the fraction of active DM buyers now fluctuate randomly. An increase in this fraction raises the demand for DM goods and increases the number of potential borrowers. Let ϵ be a multiplicative shock to the utility of DM consumption for active buyers. An increase in ϵ raises demand for both goods and loans by each active DM buyer. Let $n < 1$ lie in $[\underline{n}, \overline{n}]$, and, $\epsilon > 0$ in $[\underline{\epsilon}, \overline{\epsilon}]$. Define $\omega = (n, \epsilon) \in \Omega$ denote the aggregate state vector, and $\psi(\Omega)$ be its probability density.

K.2 Monetary policy

The central bank commits to an overall long-run inflation target τ (or equivalently, a price path) and engages in state-contingent liquidity management which varies prices and loan interest rates in the DM. The sequence of central bank actions is as follows. First, a uniform monetary injection, τM, is made to all buyers at the beginning of the period (before ω is realized). Second, contingent on ω, the central bank makes a non-negative lump-sum transfer to buyers in the DM, $\tau_1(\omega)$.[35] We assume the central bank can tax only in the CM, hence the restriction that $\tau_1(\omega) \geq 0$. Next, DM interactions among buyers, banks, and sellers take place as described above followed similarly by CM interactions. Lastly, we assume that any state-contingent injection of liquidity made in the DM is undone in the CM: *i.e.*, $\tau_2(\omega) = -\tau_1(\omega)$.[36]

Given the assumption that the DM state-contingent policy will be undone in the subsequent CM, the total change to the aggregate money stock is deterministic and given by

$$M_{+1} - M = (\gamma - 1)M = \tau M, \tag{K.1}$$

where $\gamma = 1 + \tau$ is the growth in the money supply. As such, we consider only a stationary monetary equilibrium where end-of-period real money balances are both time and state invariant, *i.e.* $\phi M = \phi_{+1} M_{+1} = z$, for all $\omega \in \Omega$. In a stationary monetary equilibrium, money supply growth is

$$\frac{\phi}{\phi_{+1}} = \frac{M_{+1}}{M} = \frac{p_{+1}}{p} = \gamma = 1 + \tau. \tag{K.2}$$

Thus, the central bank follows a price-level targeting policy via a given trajectory for the money stock, as in Berentsen and Waller (2011).

K.3 Characterization of SME with shocks

The market structure of the model is the same as in baseline except that ϵ and n are random variables now.[37] The shock process and monetary policy are the same as described in the main text. We now highlight the new features of this version of the model.

[35] The central bank could treat the active and inactive buyers differently and could make transfers to sellers. We ignore this channel because such policies are redundant. Moreover, it is without loss of generality that we let $\tau_s(\omega) = 0$.

[36] As described in Berentsen and Waller (2011), this policy can be thought of as a repo agreement where the central bank sells money in the DM and promises to buy it back in the CM.

[37] If we treat ϵ and n as parameters and set $\epsilon = 1$, then we are back to the deterministic baseline case.

Ex-post households with at least one lending bank contact. In events with probability measure α_1 and α_2, and for all $\epsilon \in \omega \in \Omega$, the buyer's optimal demand for DM consumption and loan is respectively characterized by

$$q_b^{1,\star}(z,\mathbf{z},\omega) = \begin{cases} \epsilon^{\frac{1}{\sigma}}\left[\rho(1+i)\right]^{-\frac{1}{\sigma}} & \text{if } 0<\rho\leq\tilde{\rho}_i \text{ and } 0\leq i\leq\hat{i} \\ \frac{z+\tau_b Z}{\rho} & \text{if } \tilde{\rho}_i<\rho<\hat{\rho} \text{ and } i>\hat{i} \\ \epsilon^{\frac{1}{\sigma}}\rho^{-\frac{1}{\sigma}} & \text{if } \rho\geq\hat{\rho} \text{ and } i>\hat{i} \end{cases}, \tag{K.3}$$

and,

$$\xi^{\star}(z,\mathbf{z},\omega) = \begin{cases} \epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z+\tau_b Z) & \text{if } 0<\rho\leq\tilde{\rho}_i \text{ and } 0\leq i\leq\hat{i} \\ 0 & \text{if } \tilde{\rho}_i<\rho<\hat{\rho} \text{ and } i>\hat{i} \\ 0 & \text{if } \rho\geq\hat{\rho} \text{ and } i>\hat{i} \end{cases}, \tag{K.4}$$

where $\hat{\rho} := \hat{\rho}(z,\mathbf{z},\omega) = \epsilon^{-\left(\frac{1}{\sigma-1}\right)}(z+\tau_b Z)^{\frac{\sigma}{\sigma-1}}$, $\tilde{\rho}_i := \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, and, $\hat{i} = \epsilon(z+\tau_b Z)^{-\sigma}\rho^{\sigma-1} - 1 > 0$.

Ex-post households with zero lending bank contact. The buyer's optimal demand for DM consumption (for events with probability measure α_0) is

$$q_b^{0,\star}(z,\mathbf{z},\omega) = \begin{cases} \frac{z+\tau_b Z}{\rho} & \text{if } \rho\leq\hat{\rho} \\ \epsilon^{\frac{1}{\sigma}}\rho^{-\frac{1}{\sigma}} & \text{if } \rho\geq\hat{\rho} \end{cases}, \tag{K.5}$$

where $\hat{\rho} := \hat{\rho}(z,\mathbf{z},\omega) = \epsilon^{-\left(\frac{1}{\sigma-1}\right)}(z+\tau_b Z)^{\frac{\sigma}{\sigma-1}}$.

Firms. The firm's optimal production plan satisfies $c_q(q_s) = p\phi$.

Hypothetical monopolist lending bank. We can derive the closed-form loan-price posting distribution similar to the baseline, except that the distribution is both state and policy-dependent now. Given a realization of shock ω, this bank's "monopoly" profit function is $\Pi^m(i) = n\alpha_1 R(i)$. To pin down a monopoly loan price, differentiate the bank's "monopoly" profit function wrt. i, the (stationary variable version) FOC is

$$-\underbrace{z+\tau_b Z}_{f(i)} + \underbrace{\frac{1}{\sigma}\epsilon^{\frac{1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}\left[(\sigma-1)+\frac{1+i_d}{1+i}\right]}_{g(i)} = 0, \tag{K.6}$$

which needs to hold for each realization of state $\omega \in \Omega$.

Observe that in Condition (K.6), for a given individual state z, aggregate state Z, trend inflation rate τ, state ω, and $\omega \mapsto \tau_b(\omega)$, $f(i)$ is a constant w.r.t. i, and $g(i)$ is decreasing in i. Thus, as in the earlier, baseline model, there exists a unique monopoly-profit-maximizing price i^m that satisfies the above FOC for each realization of state $\omega \in \Omega$.

Once we pin down this $i^m(\mathbf{z},\omega)$ in an SME, then we use the equal profit condition combining with the upper support of the distribution $\bar{i}(\omega) := \min\{i^m(\mathbf{z},\omega), \hat{i}(\mathbf{z},\omega)\}$ to derive the lower support of the distribution $\underline{i}(\mathbf{z},\omega)$, which together pin down the closed-form loan-price posting distribution for each realization of state

$\omega \in \Omega$.

Real money demand. Similar to the baseline case, we differentiate the DM value function with respect to m, update one period and substitute that into the CM first-order condition. Convert the result using stationary variables and combine that with the *ex-post* optimal goods demand functions in Equations (K.3) and (K.5) in DM, and we get the Euler equation for real money demand as

$$\begin{aligned}
\frac{\gamma-\beta}{\beta} &= \theta\left(z,\mathbf{z},\omega\right)-1 \\
&+\int_{\omega\in\Omega} n\mathbb{I}_{\{\rho<\hat{\rho}\}}\alpha_0\left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma}-1\right]\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]i\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right] \\
&\times\left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma}-1\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega,
\end{aligned} \tag{K.7}$$

and,

$$\begin{aligned}
\theta\left(z,\mathbf{z},\omega\right)-1 :=& \int_{\omega\in\Omega}\left(1-n\right)\left(1+i_d\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\alpha_0\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\bar{i}}^{i^{\mathrm{m}}}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\bar{i}}^{i^{\mathrm{m}}}\mathbb{I}_{\{\hat{\rho}\leq\rho\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&-1.
\end{aligned}$$

Note that the integral limits $(\bar{i}, i^{\mathrm{m}}, \underline{i})$ and cut-off prices $(\tilde{\rho}_i, \hat{\rho})$ are also functions of $(z, \mathbf{z}, \omega)$. The LHS of Condition (K.7) captures the marginal cost of accumulating an extra unit of real money balance at the end of each CM, and the RHS captures the expected marginal utility value of that extra unit of money balance (evaluated at the beginning of next DM before the shock is realized and before buyer types, matching and trading occur).

Loan price-posting distribution. We restrict to the case $\alpha_1 \in (0, 1)$ for the stochastic version here. The distribution of loan (interest-rate) price posts is given by:

$$F\left(i,z,\mathbf{z},\omega\right)=1-\frac{\alpha_1}{2\alpha_2}\left[\frac{R\left(\bar{i}(z,\mathbf{z},\omega)\right)}{R\left(i(z,\mathbf{z},\omega)\right)}-1\right], \tag{K.8}$$

and, $\text{supp}\,(F(\cdot,z,\mathbf{z},\omega)) = \left[\underline{i}(z,\mathbf{z},\omega),\bar{i}(z,\mathbf{z},\omega)\right]$, and, given $\bar{i}(z,\mathbf{z},\omega) = \min\{i^m(z,\mathbf{z},\omega),\hat{i}(z,\mathbf{z},\omega)\}$, $\underline{i}(z,\mathbf{z},\omega)$ solves: $R\left(\underline{i}(z,\mathbf{z},\omega)\right) = \frac{\alpha_1}{\alpha_1+2\alpha_2}R\left(\bar{i}(z,\mathbf{z},\omega)\right)$, where the (real) bank profit per customer served is $R\left(i,z,\mathbf{z},\omega\right) = \left[\epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}\left(1+i\right)^{-\frac{1}{\sigma}} - \left(z+\tau_b Z\right)\right]\left(i-i^d\right)$.

Observe that in Equations (K.3) and (K.4), all the cutoff functions (in terms of the relative price of DM goods or lending interest rate) now depend on the optimal policy function, $\omega \mapsto \tau_b(\omega)$ function, and also on the $\omega := (\epsilon, n)$ states of the economy.

Similarly, the support of the posted loan interest rate distribution in Equation (K.8) now also depends on a given $\omega \mapsto \tau_b(\omega)$ function, and also on $\omega := (\epsilon, n)$. This can be seen from the optimal monopoly rate that solves the Condition (K.6), from households' reservation interest rate $\hat{i}(z,\mathbf{z},\omega)$, and from the associated lowest possible loan rate of the distribution $\underline{i}(z,\mathbf{z},\omega)$.

The key difference between ϵ shocks and n shocks is that the former induces one extra moving part—a direct effect of policy outcomes $\tau_b(\omega)$ on the support of $F(\cdot,z,\mathbf{z},\omega)$. The latter shock implies one less moving part.

K.4 The central bank

We compare an *active* central bank conducting an optimal policy of the type described above to two alternatives. First, to a *passive* central bank that undertakes no policy actions in response to shocks (*i.e.* $\tau_1(\omega) = \tau_2(\omega) = 0$ for all $\omega \in \Omega$). Second, to an active central bank under the assumption that $\alpha_2 = 1$ and with the restriction that the deposit rate remains constant removed. This later case replicates the policy experiment conducted in Berentsen and Waller (2011).

An active central bank commits to an *ex-ante* optimal policy that maximizes social welfare in a stationary (Markov) monetary equilibrium:[38]

$$\begin{aligned}
\max_{\{q_b^0(\cdot,\omega),\ q_b(\cdot,\omega),\tau_1(\omega)\}_{\omega\in\Omega}} \quad & U\left(x\right) - x - c(q_s(\mathbf{z},\omega)) \\
& + \int_{\omega\in\Omega} n\alpha_0\epsilon u\left[q_b^0\left(\mathbf{z},\omega\right)\right]\psi\left(\omega\right)\mathrm{d}\omega \\
& + \int_{\omega\in\Omega} n\int_{\underline{i}(\mathbf{z},\omega)}^{\bar{i}(\mathbf{z},\omega)}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i,\mathbf{z},\omega\right)\right)\right] \\
& \times \epsilon u\left[q_b\left(i,\mathbf{z},\omega\right)\right]\mathrm{d}F\left(i,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega
\end{aligned} \tag{K.9}$$

subject to the constraint on policy: $\frac{\gamma-\beta}{\beta} = \tau + \tau_1(\omega) + \tau_2(\omega)$, and, $\tau_2(\omega) = -\tau_1(\omega)$. The optimal policy prescribes a set of state-contingent liquidity injections, $\tau_1(\omega)$.[39]

The objective of the active central bank is similar to Berentsen and Waller (2011). New insights arise from the equilibrium varying dispersion of loan spreads since $F(i;\omega)$ is now both a state and policy-dependent object. We explain what the new insights are in Section L below.

L Optimal stabilization policy

This appendix expands on the summary from Section 7 of in the main paper.

[38] The equilibrium definition in Section 3.6 is expanded straightforwardly to account for variation in the state, ω, and policy, $\tau_1(\omega)$.

[39] We write these as functions of an SME state-policy vector augmented by ω—*i.e.*, $(\mathbf{z},\omega)$.

L.1 The optimal policy: An example

For illustration, we consider a policy exercise using only shocks to the number of active DM buyers, n, and holding ϵ fixed at one. We do this for simplicity as optimal policy in response to both types of shock is qualitatively similar. The key difference between shocks to marginal utility (ϵ) and to the measure of active buyers (n) is that in the absence of a policy response the former shifts the distribution of loan rates whereas the latter leaves it unchanged. As such the effect of the optimal policy on the average loan-rate spread is simpler in the case of shocks to n. We assume that n is distributed uniformly on $\{n_1, \ldots n_4\}$ where $n_i < n_{i+1}, i = 1, ..., 3$.

Table 7 depicts the result of our optimal policy exercise. As noted above we compare three economies, the first in the table being our Benchmark calibrated economy with imperfectly competitive lending and four aggregate demand states as described above. We compare this economy under the optimal policy with an active central bank to two alternatives. The first is the same economy with the central bank remaining passive in response to shocks and the second is the case considered by Berentsen and Waller (2011). In this case lending is perfectly competitive, the lending rate equals the deposit rate and varies in response to both shocks and the policy response to them. In all cases we fix the long-run inflation target at $\tau > \beta - 1$, and set it to 0.042, reflecting the average of 4.2% annual inflation throughout the sample period of our calibration. Thus, banks' marginal cost of funds is also fixed at $i_d = \gamma/\beta - 1$.[40]

The first line for each case in the table reports the optimal DM transfer of additional liquidity for each state. When the central bank is passive, this is of course zero in all states. Before describing the optimal active policy in the benchmark economy, it is useful to review the optimal policy in the economy of Berentsen and Waller (2011). In the absence of an active policy in that case, as the measure of active buyers increases deposits decline and loan demand increases, putting upward pressure on the loan rate. This, however, is sub-optimal, as the return on deposits rises precisely when relatively few inactive buyers hold them, limiting their insurance function. As such, the optimal policy counteracts this—increasing liquidity when the supply of deposits would otherwise be low and lowering (raising) the loan (and deposit) rate when aggregate demand is high (low).

In our benchmark economy, in the absence of active policy fluctuations in aggregate demand do not affect either the return on deposits (because it is determined in the external interbank market) or on the distribution of loan rates (because fluctuations in n alone do not affect the upper bound of the loan rate distribution). Optimal policy in this case hinges on the effect of banks' market power on consumption per active buyer in the DM.

As aggregate demand increases, the aggregate welfare cost of a given loan-rate spread increases as lenders extract surplus from a larger share of the population. The central bank can counteract this to an extent by injecting liquidity in the DM, inducing banks to reduce their loan market spreads in hopes of making more and larger loans. A higher DM liquidity injection ($\tau_b(n)$) thus lowers both the average loan spread and its dispersion directly by reducing the maximum (*i.e.* monopoly) loan rate. There is, however, a counteracting force that can dominate when the fraction of active buyers becomes sufficiently large.[41] This can be seen in the non-monotonicity of the optimal policy. Liquidity injections *lower* all buyers' real money balance, inducing increased dispersion of the loan spread. The net welfare consequence of a given liquidity injection, and thus the optimal state-contingent policy depends on the relative magnitude of these two opposing forces,

[40] If $\tau = \beta - 1$, *i.e.* the *Friedman Rule*, then holding money is costless and there is no need for either banking or stabilization policy of any kind.

[41] These can be deduced from Equations (K.6), (K.7) and (K.8) in Appendix K.

Table 7: **Optimal policy in response to aggregate demand shocks**

	States			
Measure of active buyers n:	$n_1 = 0.7$	$n_2 = 0.75$	$n_3 = 0.8$	$n_4 = 0.85$
I. Benchmark Economy with Active Central Bank:				
Amount of transfer $z\tau_1$	0.0002	0.0052	0.0498	0.0447
DM consumption q_b	0.4586	0.4919	0.5298	0.5623
Average loan interest rate i	0.0788	0.0785	0.0764	0.0767
Average loan interest spread μ	0.2897	0.2852	0.2509	0.2546
II. Benchmark Economy with Passive Central Bank:				
Amount of transfer $z\tau_1$	0	0	0	0
DM consumption q_b	0.4600	0.4928	0.5256	0.5585
Average loan interest rate i	0.0781	0.0781	0.0781	0.0781
Average loan interest spread μ	0.2783	0.2783	0.2783	0.2783
III. Perfect Competition with Active Central Bank:				
Amount of transfer $z\tau_1$	0.3570	0.4118	0.4392	0.4666
DM consumption q_b	0.4726	0.5475	0.6060	0.6672
Loan interest rate i	0.0388	0.0307	0.0253	0.0202
Loan interest spread μ	0	0	0	0
Welfare gains: Consumption Units				
Active vs. Passive Policy in the Benchmark (I vs. II):	0.0338%			
Perfect vs. Imperfect Competition with Active Policy (III vs. I):	0.6981%			

Note: $\tau = 0.042$. Each row depicts the state-contingent variable in level. The *ex-ante* welfare gain is the percentage deviation from the benchmark with a passive central bank, measured as compensating variation in consumption units.

the first of which raises welfare when aggregate demand increases and the second of which mitigates these gains.

In our example exercise here, as long as the demand shock is not too big the injection is increasing in n. The central bank thus increases the transfer as aggregate demand increases, lowering the average loan spread in the higher demand states at the expense of allowing it to rise when demand is lower. Only in the highest demand state (with $n = .85$) is the latter effect sufficient to blunt this trend. The result is a non-monotonicity in the DM liquidity injection. It increases with n to a point and then declines. The decline is, however, rather small. It remains the case that in the highest-demand state, the average loan spread is lower (and DM consumption per active buyer higher) than it would be under the passive policy.

Overall, the optimal policy raises DM consumption and lowers the average loan spread when aggregate demand is high and does the opposite when it is low. The policy thus raises welfare, although the gains are small relative to the overall losses from imperfect competition in lending. The latter can be seen by comparing the imperfectly competitive benchmark with the economy of Berentsen and Waller (2011) under their respective optimal policies.