Maths Class 9th

Chapter 8 – Quadrilaterals

Que 10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that

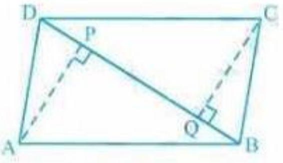


Fig. 8.21

(i) ΔAPB ≅ ΔCQD

(ii) AP = CQ

Solution :- (i) In ΔAPB and ΔCQD,

∠ABP = ∠CDQ …. (Alternate interior angles as AB||CD)

∠APB = ∠CQD …..(90° as AP and CQ are perpendiculars given)

AB = CD …. (ABCD is a parallelogram)

ΔAPB ≅ ΔCQD …. [AAS]

(ii) As ΔAPB ≅ ΔCQD.

AP = CQ [CPCT]

Que12. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.23). Show that

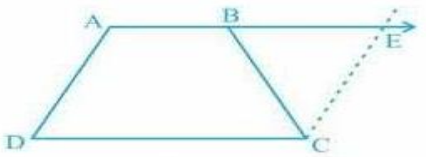


Fig. 8.23

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD

[**Hint:** Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Solution : (i) CE = AD (Opposite sides of a parallelogram)

AD = BC (Given)

, BC = CE

⇒∠CBE = ∠CEB also,

⇒∠A = ∠B

∠A+∠CBE = 180° (Angles on the same side of transversal and ∠CBE = ∠CEB)

∠B +∠CBE = 180° (As Linear pair)

(ii) ∠A+∠D = ∠B+∠C = 180° (Angles on the same side of transversal)

⇒∠A+∠D = ∠A+∠C (∠A = ∠B)

⇒∠D = ∠C

(iii) In ΔABC and ΔBAD,

AB = AB (Common)

∠DBA = ∠CBA

AD = BC (Given)

ΔABC ≅ ΔBAD [SAS congruency]

(iv) Diagonal AC = diagonal BD by CPCT as ΔABC ≅ ΔBAD.

Que 9- . In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.20).

Show that:

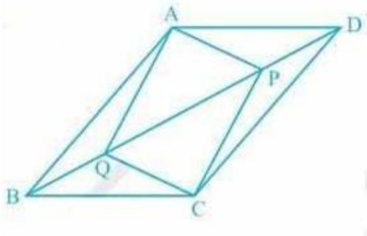


Fig. 8.20

(i) ΔAPD ≅ ΔCQB

(ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Solution :- **(i)** In ΔAPD and ΔCQB,

DP = BQ --------------------------------(Given)

∠ADP = ∠CBQ ---------(Alternate interior angles as AD||BC)

AD = BC -------------(Opposite sides of a parallelogram)

Thus, ΔAPD ≅ ΔCQB [SAS]

(ii) AP = CQ by CPCT as ΔAPD ≅ ΔCQB.

(iii) In ΔAQB and ΔCPD,

BQ = DP (Given)

∠ABQ = ∠CDP (Alternate interior angles)

AB = CD (Opposite sides of a parallelogram)

NCERT Solutions for Class 9 Maths Chapter 8 –

Quadrilaterals

Thus, ΔAQB ≅ ΔCPD [SAS congruency]

(iv) As ΔAQB ≅ ΔCPD

AQ = CP [CPCT]

(v) From the questions (ii) and (iv),

it is clear that APCQ has equal opposite sides and also has equal and opposite

angles. ,

APCQ is a parallelogram.

Que1. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Solution:

Let the common ratio between the angles be x.

We know that the sum of the interior angles of the quadrilateral = 360°

Now, $3x+5x+9x+13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$, Angles of the quadrilateral are:

$3x = 3\times12° = 36°$

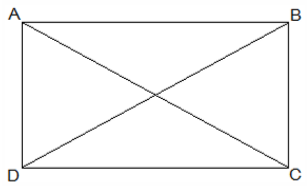
$5x = 5\times12° = 60°$

$9x = 9\times12° = 108°$

$13x = 13\times12° = 156°$

Que2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution: Given that, AC = BD



To show ABCD is a rectangle, we have to prove that one of its interior angles is a right-angle.

In ΔABC and ΔBAD,

AB = BA (Common)

BC = AD (Opposite sides of a parallelogram are equal)

AC = BD (Given)

Therefore, ΔABC ≅ ΔBAD [SSS]

∠A = ∠B [CPCT]

also,

∠A+∠B = 180° (Sum of the angles on the same side)

⇒ 2∠A = 180°

⇒ ∠A = 90° = ∠B .....Therefore, ABCD is a rectangle.

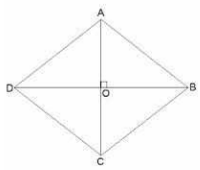
Hence Proved.

Que3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution: Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given that, OA = OC and OB = OD

and ∠AOB = ∠BOC = ∠OCD = ∠ODA = 90°



Maths Class 9th

Chapter 8 – Quadrilaterals

To show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus, we have to prove that ABCD is a parallelogram and AB = BC = CD = AD

In ΔAOB and ΔCOB,

OA = OC ---------------------(Given)

∠AOB = ∠COB --------------------(Opposite sides of a parallelogram are equal)

OB = OB ------------------(Common)

Therefore, ΔAOB ≅ ΔCOB -----------[SAS]

Thus, AB = BC -----------------[CPCT]

Similarly, we can prove,

BC = CD , CD = AD and AD = AB

AB = BC = CD = AD

Opposite sides of a quadrilateral are equal. Hence, it is a parallelogram.

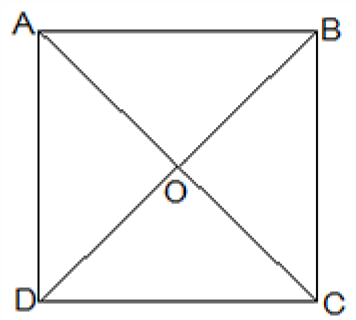
ABCD is rhombus as it is a parallelogram whose diagonals intersect at a right angle.

Hence Proved.

Que4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution: Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show that,



AC = BD and AO = OC and ∠AOB = 90°

Proof, In ΔABC and ΔBAD,

AB = BA …………….(Common)

∠ABC = ∠BAD = 90° ……. (ABCD is a square)

BC = AD …………………(Given)

ΔABC ≅ ΔBAD ………..[SAS]

Thus, AC = BD [CPCT] diagonals are equal.

Now, In ΔAOB and ΔCOD,

∠BAO = ∠DCO (Alternate interior angles, AB||CD)

∠AOB = ∠COD (Vertically opposite)

AB = CD (Given)

Thus, ΔAOB ≅ ΔCOD ………….[AAS]

AO = CO …………………..[CPCT].

Diagonal bisect each other. Now,

In ΔAOB and ΔCOB,

OB = OB …………………(Given)

AO = CO (diagonals are bisected)

AB = CB (Sides of the square)

ΔAOB ≅ ΔCOB [SSS]

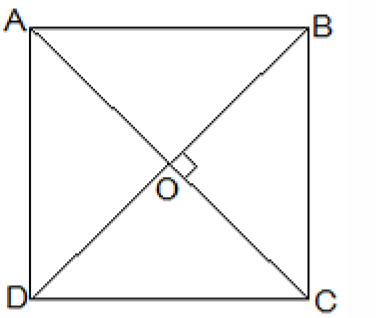
also, ∠AOB = ∠COB

∠AOB+∠COB = 180° (Linear pair)

Thus, ∠AOB = ∠COB = 90° Diagonals bisect each other at right angles

Que 5- Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution: Given that, ABCD be a quadrilateral and its diagonals AC and BD bisect each other at a right angle at O.



To prove that - The Quadrilateral ABCD is a square.

Proof, In ΔAOB and ΔCOD,

AO = CO …. …..(Diagonals bisect each other)

∠AOB = ∠COD …. ….(Vertically opposite)

OB = OD …. ….(Diagonals bisect each other)

ΔAOB ≅ ΔCOD [SAS]

Thus, AB = CD [CPCT] — (i)

also, ∠OAB = ∠OCD (Alternate interior angles, AB||CD)

⇒ AB || CD

Now, In ΔAOD and ΔCOD,

AO = CO ….. …… …..(Diagonals bisect each other)

∠AOD = ∠COD ….. …… ……(Vertically opposite)

OD = OD ….. …… ……(Common)

, ΔAOD ≅ ΔCOD ….. ….. ……[SAS]

Thus, AD = CD [CPCT] — (ii)

also, AD = BC and AD = CD

⇒ AD = BC = CD = AB — (ii)

also, ∠ADC = ∠BCD [CPCT]

and ∠ADC+∠BCD = 180° (co-interior angles)

⇒ 2∠ADC = 180°

⇒∠ADC = 90° — (iii)

One of the interior angles is a right angle.

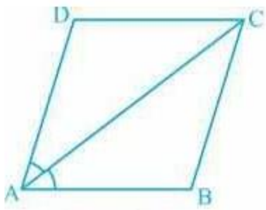
Thus, from (i), (ii) and (iii), given quadrilateral ABCD is a square.

Hence Proved.

Que 6. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.19). Show that

(i) it bisects ∠C also,

(ii) ABCD is a rhombus.



Solution: (i) In ΔADC and ΔCBA,

AD = CB (Opposite sides of a parallelogram)

DC = BA (Opposite sides of a parallelogram)

AC = CA (Common Side)

, ΔADC ≅ ΔCBA [SSS congruency]

Thus, ∠ACD = ∠CAB by CPCT

and ∠CAB = ∠CAD (Given)

⇒ ∠ACD = ∠BCA

Thus, AC bisects ∠C also.

(ii) ∠ACD = ∠CAD (Proved above)

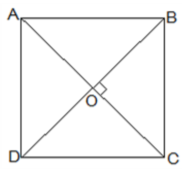
⇒ AD = CD (Opposite sides of equal angles of a triangle are equal)

Also, AB = BC = CD = DA (Opposite sides of a parallelogram)

Thus,

ABCD is a rhombus.

Que 7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.



Solution: Given that, ABCD is a rhombus.

AC and BD are its diagonals.

Proof, AD = CD (Sides of a rhombus)

∠DAC = ∠DCA (Angles opposite of equal sides of a triangle are equal.)

also, AB || CD

⇒∠DAC = ∠BCA (Alternate interior angles)

⇒∠DCA = ∠BCA

, AC bisects ∠C.

Similarly,

We can prove that diagonal AC bisects ∠A.

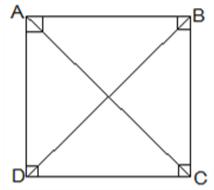
Following the same method,

We can prove that the diagonal BD bisects ∠B and ∠D.

Que 8. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:

(i) ABCD is a square

(ii) Diagonal BD bisects ∠B as well as ∠D.



Solution:

(i) ∠DAC = ∠DCA (AC bisects ∠A as well as ∠C)

⇒ AD = CD (Sides opposite to equal angles of a triangle are equal)

also, CD = AB (Opposite sides of a rectangle)

,AB = BC = CD = AD

Thus, ABCD is a square.

(ii) In ΔBCD, BC = CD

⇒ ∠CDB = ∠CBD (Angles opposite to equal sides are equal)

also, ∠CDB = ∠ABD (Alternate interior angles)

⇒ ∠CBD = ∠ABD

Thus, BD bisects ∠B

Now, ∠CBD = ∠ADB

⇒ ∠CDB = ∠ADB

Thus, BD bisects ∠B as well as ∠D.