Leslie G. Valiant
Deborah Alves

Applied Mathematics 107

Homework 7

Due 5 pm, Thursday, April 9, 2015

Collaboration Policy: You may discuss the problems for this assignment with each other at a high level, but collaboration is limited to discussion of ideas only, and you should write up the solutions entirely on your own. Moreover, you should list the names of any collaborators on your solutions.

Problem 1

Show using a combinatorial interpretation that,

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

Problem 2

Prove the following. (Hints: Consider using combinatorial interpretation of what each side of an equation counts, induction, or the identity $(1+t)^n(1-t)^n = (1-t^2)^n$.)

(a) $\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}$

(b) $\sum_{i=0}^{k} \binom{n+i}{i} = \binom{n+k+1}{k}$

(c) $\sum_{i=0}^{n} (-1)^i \binom{n}{i}^2 = \begin{cases} 0 & \text{, if } n \text{ is odd} \\ (-1)^m \binom{2m}{m} & \text{, if } n = 2m \end{cases}$

Problem 3

In how many ways can $3r$ balls be selected from $2r$ identical red balls, $2r$ identical blue balls, and $2r$ identical white balls? (Selections are distinct if they do not have the same number of balls of each color.) Solve using generating functions.

Problem 4

Prove that the number of partitions of n into parts of distinct sizes is equal to the number of partitions of n into parts all of odd size, by calculating the generating functions of both sides.

Problem 5

Let δ_{nr} be 1 if $n = r$ and 0 otherwise. Using generating functions, prove that

$$\sum_{k=0}^{\infty} (-1)^{r+k} \binom{n}{k} \binom{k}{r} = \delta_{nr}$$