University of Dhaka
4^{th} Year B.S. (Honors) 2019-20
Department of Mathematics
Course No: MTH 450 (Math Lab IV)
Assignment 1 (ODEs & PDEs)

1	a) Solve the ODE $$y' = t^2 + y,\ \ 0 \le t \le 2,\ \ y(0) = 1,$$ using Euler's method for each of the values $h = 0.01, 0.1, 0.25, 0.5$. The exact solution of the above ODE is $$y_{exact} = 3e^t - t^2 - 2t - 2.$$ Plot the solutions obtained by Euler's method for different values of h along with the exact solution on the same diagram. b) Solve the above ODE using **ode23** and **ode45**. Also plot the solutions.
2.	Find the stability interval for Euler's method in solving the differential equation of the form: $y' = \lambda y$. Hence, perform the stability analysis of this method (graphically) for $\lambda = -2$.
3.	a) Solve the 2^{nd} order IVP $2y'' - 5y' + y = 0, y(3) = 6, y'(3) = -1$ by using **dsolve**. b) Write the above IVP as a system of first order linear differential equations. After converting the equation to a system of first order ordinary differential equations you get the following: (verify) $$x_1' = x_2, \qquad x_1(3) = 6$$ $$x_2' = -\frac{1}{2}x_1 + \frac{5}{2}x_2, \quad x_2(3) = -1$$ Now use ode45 to simulate the system. Also plot the solutions with different colors and markers and save the graph with .png extension.
4.	Determine the Laplace transform $F(s)$ of the function $f(t) = 3e^{-2t}\sin 5t + 4e^{-2t}\cos 5t$ and then find the inverse Laplace transform of the function $F(s)$.
5.	Solve the initial value problem (IVP) $y'' + 3y' + 2y = e^{-t},\ \ y(0) = 4,\ y'(0) = 5$, using Laplace transform. Plot the solution and interpret it geometrically.
6.	Solve the IVP $y'' + 3y' + 2y = f(t),\ \ y(0) = 2,\ y'(0) = 3$, where $f(t)$ is defined as: $$f(t) = \begin{cases} 1, & t < 3 \\ t - 2, & 3 < t < 6. \\ 2, & t > 6 \end{cases}$$ Interpret the solution geometrically.
7.	Solve the initial value problem $$y'' + 2y' + 10y = 1 + 5\,\delta(t-5), y(0) = 1, y'(0) = 2,$$ where $\delta(t)$ is Dirac Delta function. Interpret the solution geometrically.
8.	Solve the one-dimensional heat equation $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ \ u(0,t) = u(\pi,t) = 0, u(x,0) = 100$$ using PDE solver of MATLAB. Interpret your result.
9.	Solve the following nonlinear system of partial differential equations using PDE solver of MATLAB: $$u_{1_t} = u_{1_{xx}} + u_1(1 - u_1 - u_2)$$ $$u_{2_t} = u_{2_{xx}} + u_2(1 - u_1 - u_2)$$ $$u_{1_x}(t,0) = 0;\ \ u_1(t,1) = 1$$ $$u_2(t,0) = 0;\ \ u_{2_x}(t,1) = 0$$ $$u_1(0,x) = x^2;\ \ u_2(0,x) = x(x-2).$$ Also plot the solutions.