

# **StarfishDB**: a Query Execution Engine for Relational Probabilistic Programming

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## What is Probabilistic Programming?

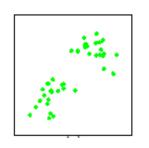
**Generative Story:** 
$$z \sim \mathsf{Categorical}(\phi)$$

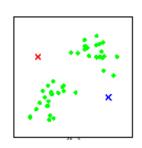
$$\mathbf{x} \sim \mathsf{Gaussian}(oldsymbol{\mu}_z, oldsymbol{\Sigma}_z)$$

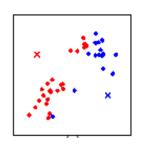
Data:

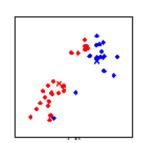
$$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$$
 where  $\mathbf{x}^{(i)} \in \mathbb{R}^M$ 

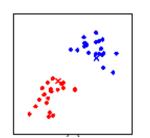
Goal: compute the posterior density of the generative process w.r.t. the data.

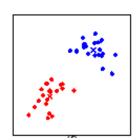






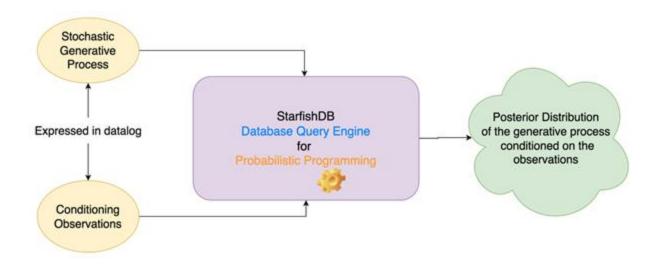






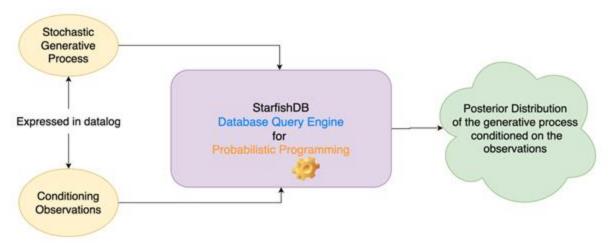
#### **Main Contributions**

 We introduce our own probabilistic programming language that is database-centric



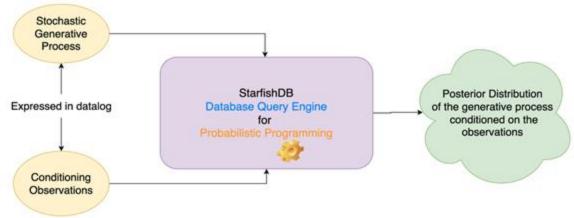
#### **Main Contributions**

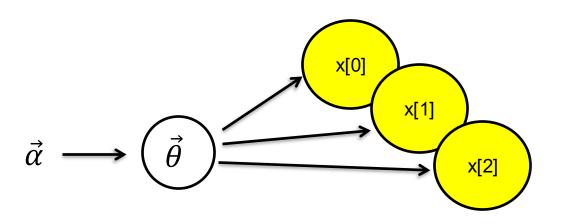
- We introduce our own probabilistic programming language that is database-centric
- We use probabilistic programming Datalog [1] to our framework



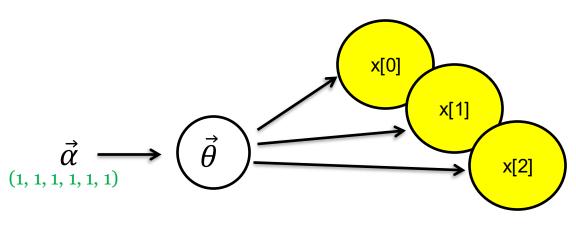
#### **Main Contributions**

- We introduce our own probabilistic programming language that is database-centric
- We use probabilistic programming Datalog [1] to our framework
- We leverage Just in time compilation to speed up inference



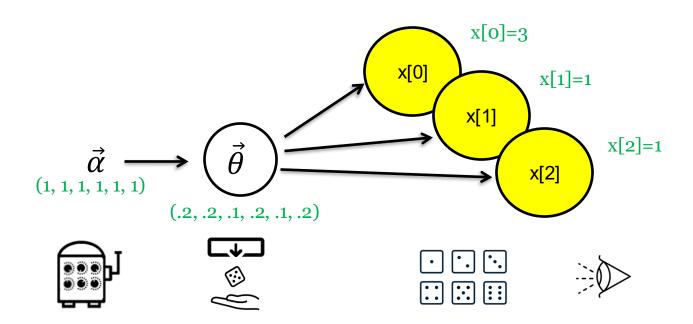


Pólya die  $\rightarrow$  a *Categorical* distribution (parametrized by  $\vec{\theta}$ ) with a *Dirichlet* prior (parametrized by  $\vec{\alpha}$ )

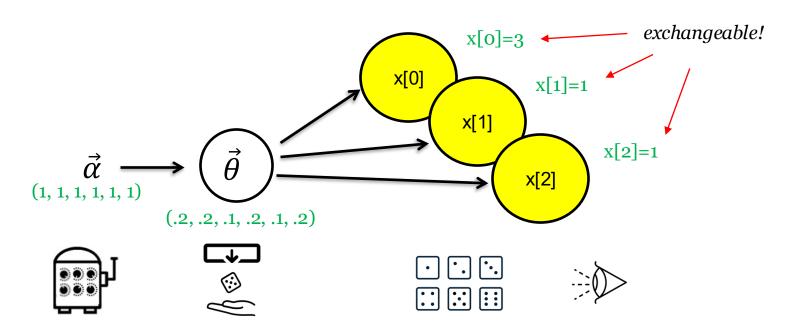




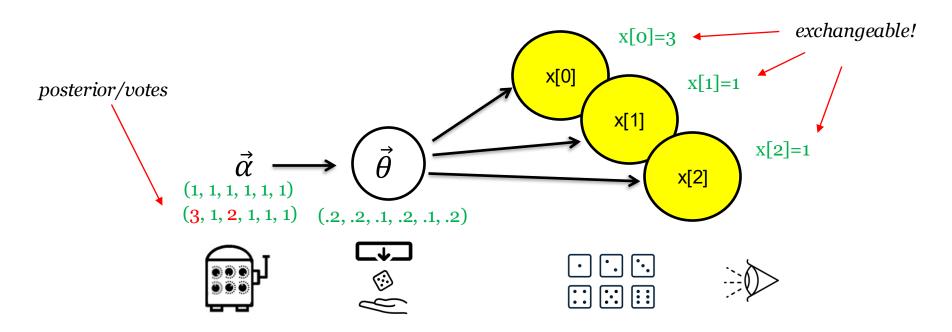
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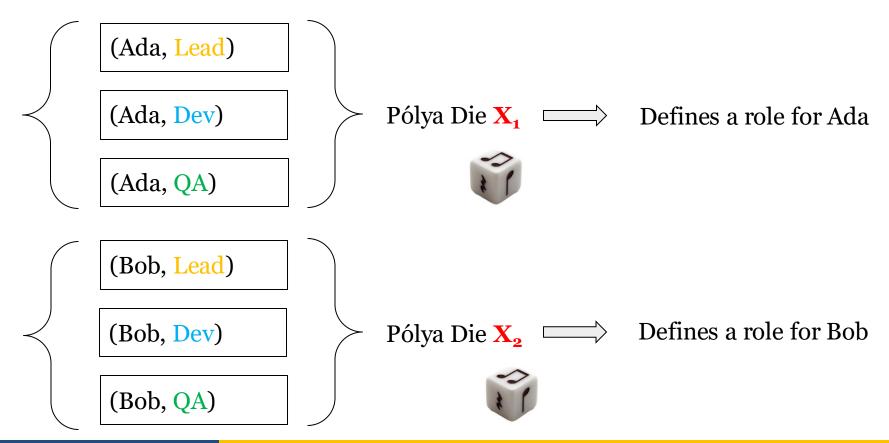


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## Toy Example: 2 dice, 2 constraints



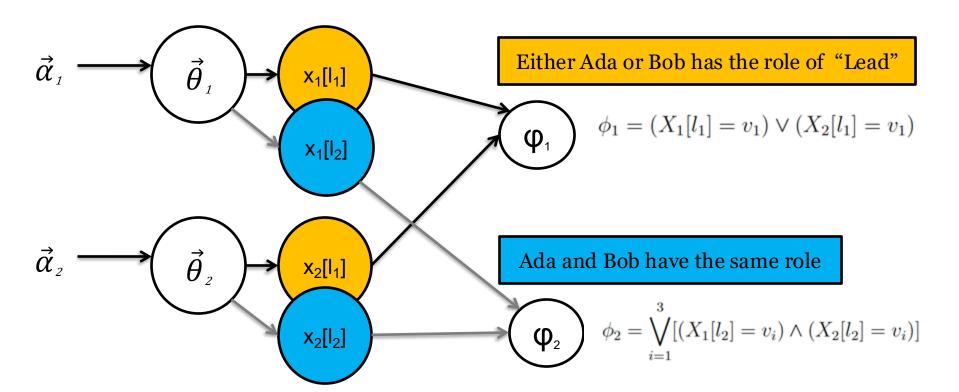
Example of a probabilistic relation:

EMP	ROLE		
	Lead (v <sub>1</sub> )		1 ( <b>a</b> <sub>1,1</sub> )
Ada (x <sub>1</sub> )	Dev (v <sub>2</sub> )	$\alpha_1$	1 (α <sub>1,2</sub> )
	QA (V <sub>3</sub> )		1 (\alpha_{1,3})
	Lead (v <sub>1</sub> )	$\alpha_2$	1 (\alpha_{2,1})
Bob (x <sub>2</sub> )	Dev (v <sub>2</sub> )		3 (α <sub>2,2</sub> )
	QA (v <sub>3</sub> )		4 (\alpha_{2,3})

There 9 possible worlds for Ada and Bob positions

EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Lead	Bob	Lead	Bob	Lead
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Dev	Bob	Dev	Bob	Dev
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	QA	Bob	QA	Bob	QA

## Toy Example: 2 dice, 2 constraints



EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Lead	Bob	Lead	Bob	Lead
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Dev	Bob	Dev	Bob	Dev
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	QA	Bob	QA	Bob	QA

c<sub>1</sub>: Either Ada or Bob has the of role "Lead"



$$\phi_1 = (X_1[l_1] = v_1) \lor (X_2[l_1] = v_1)$$

 $\tau_{1,1}$ : Ada is Lead, Bob is Lead

 $\tau_{1,2}$ : Ada is Dev, Bob is Lead

 $\tau_{1,3}$ : Ada is QA, Bob is Lead

 $\tau_{1.4}$ : Ada is Lead, Bob is Dev

 $\tau_{1.5}$ : Ada is Lead, Bob is QA

SAT 
$$(\Phi, \mathbb{X}) = {\mathbf{\tau}_{1,1}, \, \mathbf{\tau}_{1,2}, \, \mathbf{\tau}_{1,3}, \, \mathbf{\tau}_{1,4}, \, \mathbf{\tau}_{1,5}} \times {\mathbf{\tau}_{2,1}, \, \mathbf{\tau}_{2,2}, \, \mathbf{\tau}_{2,3}}$$

EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Lead	Bob	Lead	Bob	Lead
EMD	DO L F	EMB	DO 1 5		
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	Dev	Bob	Dev	Bob	Dev
EMP	ROLE	EMP	ROLE	EMP	ROLE
Ada	Lead	Ada	Dev	Ada	QA
Bob	QA	Bob	QA	Bob	QA

c<sub>2</sub>: Ada and Bob have the same role

$$\phi_2 = \bigvee_{i=1}^3 [(X_1[l_2] = v_i) \land (X_2[l_2] = v_i)]$$

 $\tau_{2,1}$ : Ada is Lead, Bob is Lead

 $\tau_{2,2}$ : Ada is Dev, Bob is Dev

 $\tau_{2,3}$ : Ada is QA, Bob is QA

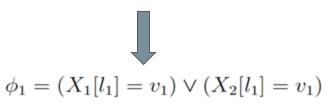
SAT 
$$(\Phi, \mathbb{X}) = {\mathbf{\tau}_{1,1}, \, \mathbf{\tau}_{1,2}, \, \mathbf{\tau}_{1,3}, \, \mathbf{\tau}_{1,4}, \, \mathbf{\tau}_{1,5}} \times {\mathbf{\tau}_{2,1}, \, \mathbf{\tau}_{2,2}, \, \mathbf{\tau}_{2,3}}$$

Constraints

Example of a probabilistic relation:

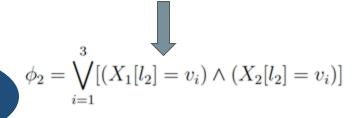
ROLE		
Lead (v <sub>1</sub> )	$\alpha_1$	1 (α <sub>1,1</sub> )
Dev (v <sub>2</sub> )		1 (α <sub>1,2</sub> )
QA (V <sub>3</sub> )		1 (\alpha_{1,3})
Lead (v <sub>1</sub> )	$\alpha_2$	1 (\alpha_{2,1})
Dev (v <sub>2</sub> )		3 (α <sub>2,2</sub> )
QA (V <sub>3</sub> )		4 (\alpha_{2,3})
	Lead (v <sub>1</sub> ) Dev (v <sub>2</sub> ) QA (v <sub>3</sub> ) Lead (v <sub>1</sub> ) Dev (v <sub>2</sub> )	Lead $(v_1)$ Dev $(v_2)$ QA $(v_3)$ Lead $(v_1)$ Dev $(v_2)$ $\alpha_2$

c<sub>1</sub>: Either Ada or Bob has the role of "Lead"





c<sub>2</sub>: Ada and Bob have the same role

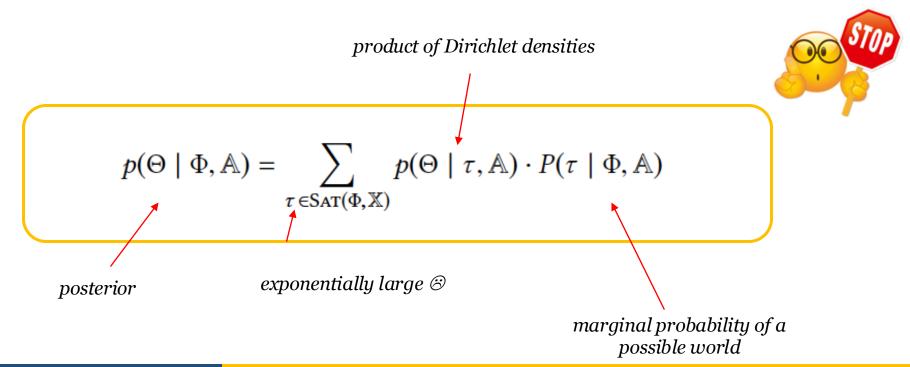


Generative process

Probabilistic program

#### Main Goal of a Probabilistic Program

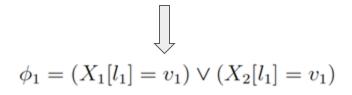
Compute the posterior distribution of generative process w.r.t constraints :



## Toy example: Gibbs Sampling

i	$S[\phi_1]$	$S[oldsymbol{\phi}_2]$
(init)	$(X_1[l_1] = v_1) \land (X_2[l_1] = v_3)$ Ada is lead in obs $l_1 \land$ Bob is QA in obs $l_1$	$(X_1 [l_2] = v_3) \wedge (X_2 [l_2] = v_3)$ Ada is QA in obs $l_2 \wedge$ Bob is QA in obs $l_2$

c<sub>1</sub>: Either Ada or Bob has the role of "Lead"



$$\phi_2 = \bigvee_{i=1}^{3} [(X_1[l_2] = v_i) \land (X_2[l_2] = v_i)]$$

#### Toy example: Gibbs Sampling

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0	$(X_1 [l_1] = v_2) \wedge (X_2 [l_1] = v_1)$ Ada is dev in obs $l_1 \wedge$ Bob is lead in obs $l_1$	$(X_1 [l_2] = v_1) \land (X_2 [l_2] = v_1)$ Ada is lead in obs $l_2 \land$ Bob is lead in obs $l_2$

c<sub>1</sub>: Either Ada or Bob has the role of "Lead"

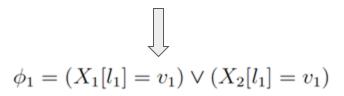
$$\oint \phi_1 = (X_1[l_1] = v_1) \lor (X_2[l_1] = v_1)$$

$$\phi_2 = \bigvee_{i=1}^{3} [(X_1[l_2] = v_i) \land (X_2[l_2] = v_i)]$$

## Toy example: Gibbs Sampling

i	$S[\phi_1]$	$S[\phi_2]$
<i>(</i> : •: )		(W. [1.] (W. [1.] o)
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1	$(X_1 [l_1] = v_3) \land (X_2 [l_1] = v1)$ Ada is QA in obs $l_1 \land$ Bob is lead in obs $l_1$	$(X_1 [l_2] = v_3) \wedge (X_2 [l_2] = v_3)$ Ada is QA in obs $l_2 \wedge$ Bob is QA in obs $l_2$

c<sub>1</sub>: Either Ada or Bob has the role of "Lead"



$$\phi_2 = \bigvee_{i=1}^{3} [(X_1[l_2] = v_i) \land (X_2[l_2] = v_i)]$$

i	$S[\phi_1]$	$S[\phi_2]$
(init)	$(X_1[l_1] = v_1) \land (X_2[l_1] = v_3)$ Ada is lead in obs $l_1 \land$ Bob is QA in obs $l_1$	$(X_1 [l_2] = v_3) \land (X_2 [l_2] = v_3)$ Ada is QA in obs $l_2 \land$ Bob is QA in obs $l_2$
0	$(X_1 [l_1] = v_2) \land (X_2 [l_1] = v_1)$ Ada is dev in obs $l_1 \land$ Bob is lead in obs $l_1$	$(X_1 [l_2] = v_1) \land (X_2 [l_2] = v_1)$ Ada is lead in obs $l_2 \land$ Bob is lead in obs $l_2$
1	$(X_1 [l_1] = v_3) \land (X_2 [l_1] = v1)$ Ada is QA in obs $l_1 \land$ Bob is lead in obs $l_1$	$(X_1 [l_2] = v_3) \wedge (X_2 [l_2] = v_3)$ Ada is QA in obs $l_2 \wedge \text{Bob}$ is QA in obs $l_2$
2	$(X_1 [l_1] = v_1) \wedge (X_2 [l_1] = v_1)$ Ada is lead in obs $l_1 \wedge$ Bob is lead in obs $l_1$	$(X_1 [l_2] = v_2) \wedge (X_2 [l_2] = v_2)$ Ada is dev in obs $l_2 \wedge Bob$ is dev in obs $l_2$
		•••

c<sub>1</sub>: Either Ada or Bob has the role of "Lead"

$$\oint \phi_1 = (X_1[l_1] = v_1) \lor (X_2[l_1] = v_1)$$

$$\phi_2 = \bigvee_{i=1}^{3} [(X_1[l_2] = v_i) \land (X_2[l_2] = v_i)]$$

weather(
$$\underline{C}, \underline{T}, w \in \{\text{sun}, \text{rain}\} \sim Cat[P]$$
)  $\leftarrow \text{city}(C, P), \text{ts}(T)$ .

```
city('Fargo', [.1, .9]), ts('noon')
```

```
weather('Fargo', 'noon', sun) with prob 0.1
weather('Fargo', 'noon', rain) with prob 0.9
```

weather(
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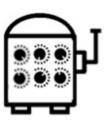
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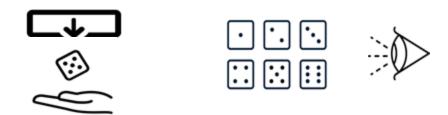
## **Probabilistic Programming Datalog**

(1, 1, 1, 1, 1, 1)

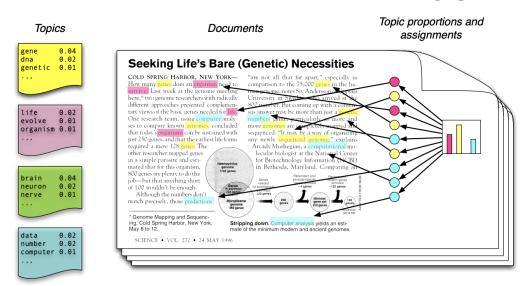


$$lp(\underline{VarId}, D, p \in \mathcal{S}_{|D|} \sim \mathcal{D}ir[\![H]\!]) \leftarrow dt(VarId, D, H)$$

## **Probabilistic Programming Datalog**



$$obs(\underline{\mathit{VarId}}, \mathit{ObsId}, v \in D \sim \mathit{Cat}[P]) \leftarrow \underbrace{\mathsf{lp}(\mathit{VarId}, D, P)}_{\mathsf{sample}(\mathit{VarId}, \mathit{ObsId})}$$



N documents  $\rightarrow N$  red dice K topics  $\rightarrow K$  blue dice



Red die: generate numbers between 1 and K

- Each topic is a distribution over words
- Each **document** is a mixture of corpus-wide topics
- Each word is drawn from one of those topics



Blue die: generate words from a fixed vocabulary

Source: Blei, ICML 2012 Tutorial

[3] Blei, David M., Andrew Y. Ng, and Michael I. Jordan. "Latent Dirichlet allocation

$$dt([red, D], ts, [1, 1, .., 1]) \leftarrow d(D, P, W)$$

Instantiate a red die for each document

 $dt([blue, T], ws, [1, 1, .., 1]) \leftarrow t(T).$ 



Instantiate a blue die for each topic

$$dt([red, D], ts, [1, 1, .., 1]) \leftarrow d(D, P, W)$$
. Instantiate a red die for each document  $dt([blue, T], ws, [1, 1, .., 1]) \leftarrow t(T)$ . Instantiate a blue die for each topic  $sample([red, D], P) \leftarrow d(D, P, W)$ .

Roll the red die for every document D and position P

 $dt([red, D], ts, [1, 1, ..., 1]) \leftarrow d(D, P, W)$ . Instantiate a red die for each document  $dt([blue, T], ws, [1, 1, ..., 1]) \leftarrow t(T)$ . Instantiate a blue die for each document  $sample([red, D], P) \leftarrow d(D, P, W)$ . Roll the red die for a given document D and position P  $sample([blue, T], [D, P]) \leftarrow d(D, P, W)$ , obs([red, D], P, T).



For every document D and position P, **roll the blue die** that corresponds to the topic sampled by rolling the red die

$$dt([red, D], ts, [1, 1, ..., 1]) \leftarrow d(D, P, W).$$

$$dt([blue, T], ws, [1, 1, ..., 1]) \leftarrow t(T).$$

$$sample([red, D], P) \leftarrow d(D, P, W).$$

$$sample([blue, T], [D, P]) \leftarrow d(D, P, W), obs([red, D], P, T).$$

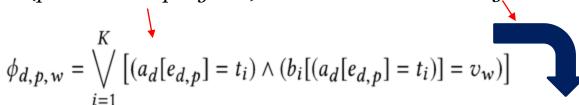
$$qa^*(D, P, W) \leftarrow d(D, P, W), obs([blue, T], [D, P], W).$$

Enforce the **condition** that the generated words must match the initial words that we observed in the corpus.

#### The Grounding Engine

Grounding

(probabilistic program)







$$\phi_{1,1,1} = [(a_1[e_{1,1}] = t_1) \land (b_1[a_1[e_{1,1}] = t_1] = v_1)] \lor [(a_1[e_{1,1}] = t_2) \land (b_2[a_1[e_{1,1}] = t_2] = v_1)] \lor \dots \lor [(a_1[e_{1,1}] = t_K) \land (b_K[a_1[e_{1,1}] = t_K] = v_1)]$$

$$\phi_{1,2,4} = [(a_1[e_{1,2}] = t_1) \land (b_1[a_1[e_{1,2}] = t_1] = v_4)] \lor [(a_1[e_{1,2}] = t_2) \land (b_2[a_1[e_{1,2}] = t_2] = v_4)] \lor \dots \lor [(a_1[e_{1,2}] = t_K) \land (b_k[a_1[e_{1,2}] = t_K] = v_4)]$$

$$\phi_{1,3,6} = [(a_1[e_{1,3}] = t_1) \land (b_1[a_1[e_{1,3}] = t_1] = v_6)] \lor [(a_1[e_{1,3}] = t_2) \land (b_2[a_1[e_{1,3}] = t_2] = v_6)] \lor \dots$$

$$\forall [(a_1[e_{1.3}] = t_K) \land (b_k[a_1[e_{1.3}] = t_K] = v_6)]$$

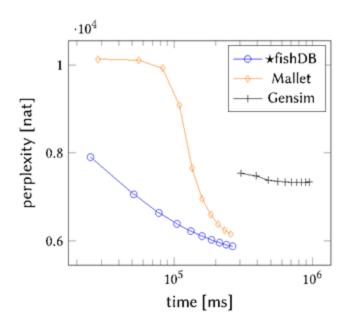
•••

... and many many others

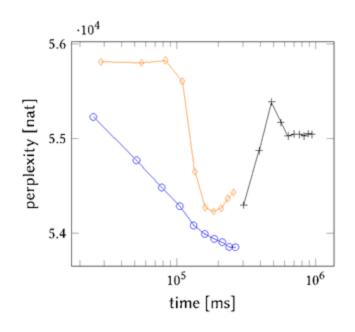
LLVM/ClangJIT<sup>[2]</sup>

[2] Finkel et al, Clangjit: Enhancing C++ with just-in-time compilation

#### **Experimental Evaluation: LDA**

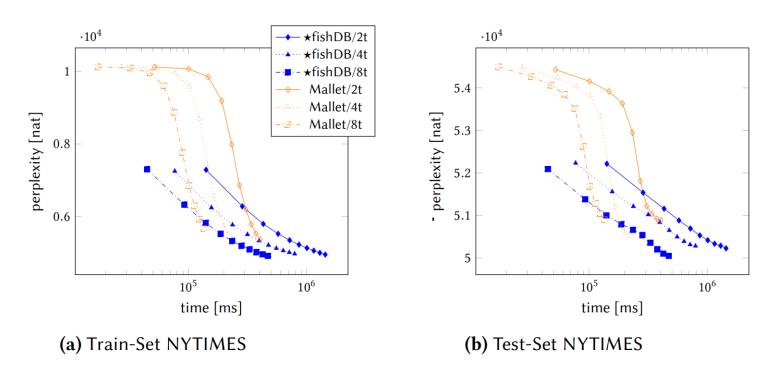


(a) Train-set NYTIMES



(b) Test-set NYTIMES

#### **Experimental Evaluation: LDA**



LDA Multithreaded 100 topics benchmarking on NYTIMES using 2, 4 and 8 threads

#### **Future work**

#### a) Query-Driven Variational Inference

Blei, David M., Alp Kucukelbir, and Jon D. McAuliffe. "Variational inference: A review for statisticians." *Journal of the American statistical Association* 112.518 (2017): 859-877.

#### b) Fairness through rel. constraints

Salimi, Babak, Luke Rodriguez, Bill Howe, and Dan Suciu.

"Interventional fairness: Causal database repair for algorithmic fairness" In *Proceedings of the 2019 International Conference on Management of Data*, pp. 793-810. 2019.

Alireza Pirhadi, Mohammad Hossein Moslemi, Alexander Cloninger, Mostafa Milani, and Babak Salimi. 2024. OTClean: Data Cleaning for Conditional Independence Violations using Optimal Transport. Proc. ACM Manag. Data 2, 3, Article 160 (June 2024)

#### c) Non-parametric Bayesian models

Grohe, Martin, and Peter Lindner. "Infinite probabilistic databases." Logical Methods in Computer Science 18 (2022).

## Thank You ^^

Any Questions?



#### References

[1] Bárány, Vince, Balder Ten Cate, Benny Kimelfeld, Dan Olteanu, and Zografoula Vagena. "Declarative probabilistic programming with datalog." ACM Transactions on Database Systems (TODS) 42, no. 4 (2017): 1-35

[2] Finkel, Hal, David Poliakoff, Jean-Sylvain Camier, and David F. Richards. "Clangjit: Enhancing C++ with just-in-time compilation." In 2019 IEEE/ACM International Workshop on Performance, Portability and Productivity in HPC (P3HPC), pp. 82-95. IEEE, 2019.

[3] Blei, David M., Andrew Y. Ng, and Michael I. Jordan. "Latent Dirichlet allocation." Journal of machine Learning research 3, no. Jan (2003): 993-1022.