Experiment 0:
Sensor Calibration and Linear Regression

Samiha Rahman
UID: 404792929
Date of Experiment: August 9, 2017
Lab Section: Monday/Wednesday,
11:30 AM
TA: Nicholas Rombes
Lab Partner: Rafi Hessami

Homework 0

1. Refer to cover page.
2. $a = \frac{\Delta v}{\Delta t} = (\Delta vbest \pm \delta\Delta v)/(\Delta tbest \pm \delta\Delta t)$

   Because a partial derivative and delta are redundant, we will refer to Δv as v anid Δt as t. Based on equation ii.14, the relation between δa and uncertainties δv and δt is (in m/s^2)

$$\delta a = \sqrt{\left(\frac{\partial a}{\partial v}\delta v\right)^2 + \left(\frac{\partial a}{\partial t}\delta t\right)^2}$$

   Where we assume the expression $v = vbest$, $t = tbest$. First, we solve for the following partial derivatives for our expressions:

$$\frac{\partial a}{\partial v} = \frac{\partial}{\partial v}\left(\frac{v}{t}\right) = \frac{1}{t}$$
$$\frac{\partial a}{\partial t} = \frac{\partial}{\partial t}\left(\frac{v}{t}\right) = -\frac{v}{t^2}$$

   We then substitute these solutions into the expression and simplify:

$$\delta a = \sqrt{\left(\frac{1}{t}\delta v\right)^2 + \left(-\frac{v}{t^2}\delta t\right)^2}\ \mathrm{m/s^2}$$

$$\delta a = \frac{1}{\sqrt{t^2}}\sqrt{(\delta v)^2 + \frac{v^2}{t^2}(\delta t)^2}\ \mathrm{m/s^2}$$
$$\delta a = \sqrt{\frac{v^2}{t^2}}\sqrt{\left(\frac{\delta v}{v}\right)^2 + \left(\frac{\delta t}{t}\right)^2}\ \mathrm{m/s^2}$$

   Since the uncertainty must be positive by convention, we use the absolute value of each measurement. We also use the assumptions $v = vbest$, $t = tbest$ to rewrite the expression with more clarity.

$$\delta a = \frac{|vbest|}{|tbest|}\sqrt{\left(\frac{\delta v}{|vbest|}\right)^2 + \left(\frac{\delta t}{|tbest|}\right)^2}\mathrm{m/s^2}$$

3. The maximum number of digits Capstone will display is 16 digits, with a maximum of 15 digits after the decimal point. If the sensor precision is exactly 4 digits, and data is taken with 10 displayed digits, the digits in 5 through 10 are somewhat nonsense. This is because due to the sensor precision, it is most likely the 4$^{\text{th}}$ digit at which there begins uncertainty. With the 4$^{\text{th}}$ digit in measurement already being uncertain, the digits following the 4$^{\text{th}}$ thus becomes meaningless and do not improve the uncertainty (only in rare cases may the 5$^{\text{th}}$ digit clarify/narrow the uncertainty). On the other hand, turning the display precision down enough to eliminate sensor fluctuation is not the best for taking good data because it might possibly ignore digits that are significant. Recording these significant digits are important for retaining both accuracy and precision as much as possible for further calculation and analysis.

4. Graph (with data table for reference of points)

Mass (g)	**Force (N)**	**Sensor Voltage (V)**
0.0	*0.000*	*0.120*
50.0	*0.490*	*0.048*
99.8	*0.978*	*-0.027*
199.8	*1.958*	*-0.180*
149.8	*1.468*	*-0.101*
349.6	*3.426*	*-0.411*
249.8	*2.448*	*-0253*
399.3	*3.913*	*-0.486*

Table 1: Raw Mass and Sensor Voltage Data; Calculated Force

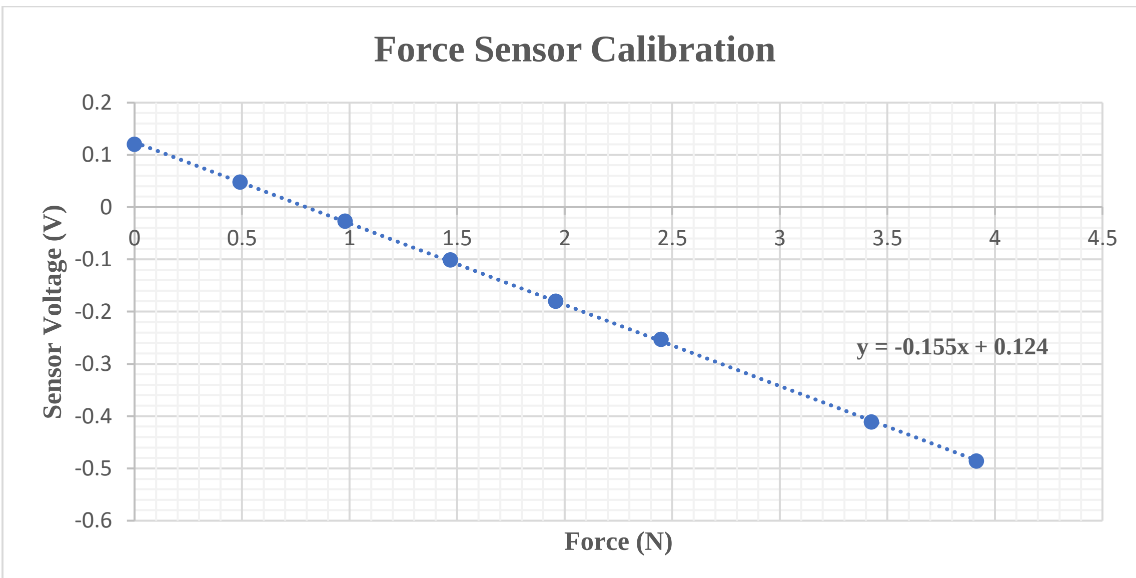

Figure 1. Linear Relationship between the Applied Tension to a Sensor and the Sensor's Output Voltage. *Using Microsoft Excel's chart-making and linear regression tools, the above graph was found with a line of best fit equation* $V = (-0.155 \pm 0.001)\ \frac{V}{N}F\ + (0.124 \pm 0.002)\ V$. *This linear relation is used to calibrate the sensor.*

5. The linear relation between the applied force and output voltage was found through Excel's linear regression tools. Since the sensor voltage fluctuated at the 3rd digit, all measurements were taken to 3 digits after the decimal point; such precision is thus maintained in the calculation of the force and the equation coefficients (with an exception for the standard error to include a nonzero digit). Therefore, the yielded linear equation is $V = (-0.155 \pm 0.001)\ \frac{V}{N}F\ + (0.124 \pm 0.002)\ V$. The negative force coefficient of $a = (-0.155 \pm 0.001)\ \frac{V}{N}$ make sense as the increasing magnitude of a downward force led to an increasing magnitude in negative voltage. However, the y-intercept of $b =$

$(0.124 \pm 0.002)\ V$ implies that the taring procedure is not effective. Technically, 0.000 N of force should output 0.000 V, which is not what happened. Therefore, we start with a positive initial voltage at 0 N that becomes negative with increasing force, demonstrating the ineffectiveness of the taring procedure.

6. To convert a measured sensor voltage V into force F, we start off with the yielded equation:

$$V = (-0.155 \pm 0.001)\ \frac{V}{N}F\ + (0.124 \pm 0.002)\ V$$

Which gives us components:

$$a = (-0.155 \pm 0.001)\ \frac{V}{N}$$
$$b = \ (0.124 \pm 0.002)\ V$$

To effectively use these components, it makes sense to transform the general F(V) formula to be V(F):

$$V = aF + b$$
$$F = cV + d = \frac{(V-b)}{a} = \frac{1}{a}V - \frac{b}{a}$$

Using the general form of the F(V) equation and manipulating the V(F) equation, we can solve for *c* and *d*:

$$c = \frac{1}{a} = \frac{1}{(-0.155 \pm 0.001)\ \frac{V}{N}}$$
$$c_{best} = \frac{1}{a_{best}} = \frac{1}{(-0.155)\ \frac{V}{N}} = \ -6.45\frac{N}{V}$$

$$d = \ -\frac{b}{a} = \frac{(0.124 \pm 0.002)\ V}{(-0.155 \pm 0.001)\ \frac{V}{N}}$$
$$d_{best} = \ -\frac{b_{best}}{a_{best}} = \ -\frac{0.124\ V}{(-0.155)\ \frac{V}{N}} = \ 0.800\ N$$

To find δc and δd, equation ii.23 is used, where for $f(x,y) = \frac{x}{y}$

$$\delta f = |f_{best}|\sqrt{\left(\frac{\delta x}{|\mathrm{x}best|}\right)^2 + \left(\frac{\delta y}{|ybest|}\right)^2}$$

So for c, where $c(x,y) = \frac{1}{a}$

$$\delta c = |-6.45|\sqrt{\left(\frac{0}{|\mathrm{x}best|}\right)^2 + \left(\frac{0.001}{|-0.155|}\right)^2} = 0.04\ \frac{N}{V}$$

And for d, where $d(x,y) = \frac{-b}{a}$

$$\delta d = |0.800|\sqrt{\left(\frac{0.002}{|0.124|}\right)^2 + \left(\frac{0.001}{|-0.155|}\right)^2} = 0.014\ N$$

Therefore, substituting in c and d into F = cV + d, we yield the final formula:

$$\mathrm{F} = (-6.45\ \pm 0.04)\frac{N}{V}\ V + (0.800\ \pm\ 0.014)N$$

7. It is possible for Frankie to get a B+ and for Avril to get a C+ despite both getting a numerical score of 84 because grades are scaled per section. Because of the department policy of grade quotas, a curve is often enforced in the course; to maintain fairness, the scales in each section are independent of each other. Since TA's may be easy or hard graders, it makes sense that a student's score be only analyzed relative to the rest of the students who are graded by the same TA. It is likely that Avril's TA was an easier grader, so the average grade in his section was higher than in Frankie's section. As a result, Avril fell a bit below average in the C+ range in his section whereas Frankie fell in the B+ range. Therefore, mean numerical scores do not have any direct correlation to a letter grade. Its correlation is determined by section so it is meaningless to compare numerical scores with a student in another section.