Experiment 2: Measurement of g

Samiha Rahman
UID: 404792929
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TA: Nicholas Rombes III
Lab Partner: Rafi Hessami,
Ahmad Alshareef

WORKSHEET

I. Derivation

To derive equation 2.1, we start with the equation for velocity with constant acceleration

$$v_f = v_i + a(t_f - t_i)$$

The setup of the experiment allows us to find the average velocity for two different distances D and d using their respective time T_2 and T_1. We thus set the initial velocity and time to correspond with the data for between the photogates. Therefore, $v_i = \frac{d}{T_1}$ and since velocity changes linearly under constant acceleration, the time that the instantaneous velocity equals the average velocity is $t_i = \frac{T_1}{2}$. For the gap to the impact sensorportion , we let $v_f = \frac{D}{T_2}$. While the velocity is equal to v_f $\frac{T_2}{2}$ seconds after the lower photogate, since t is scaled to 0 from the highest photogate, we must set $t_i = T_1 + \frac{T_2}{2} = \frac{2T_1 + T_2}{2}$ to include the time between the photogates. Therefore, the velocity equation can be rewritten with these substitutions. We set $a = g$ and rearrange:

$$\frac{D}{T_2} = \frac{d}{T_1} + g\left(\frac{2T_1 + T_2}{2} - \frac{T_1}{2}\right)$$

$$\frac{D}{T_2} - \frac{d}{T_1} = g\left(\frac{T_1 + T_2}{2}\right)$$

$$g = \frac{2}{T_1 + T_2}\left(\frac{D}{T_2} - \frac{d}{T_1}\right)$$

II. Plots

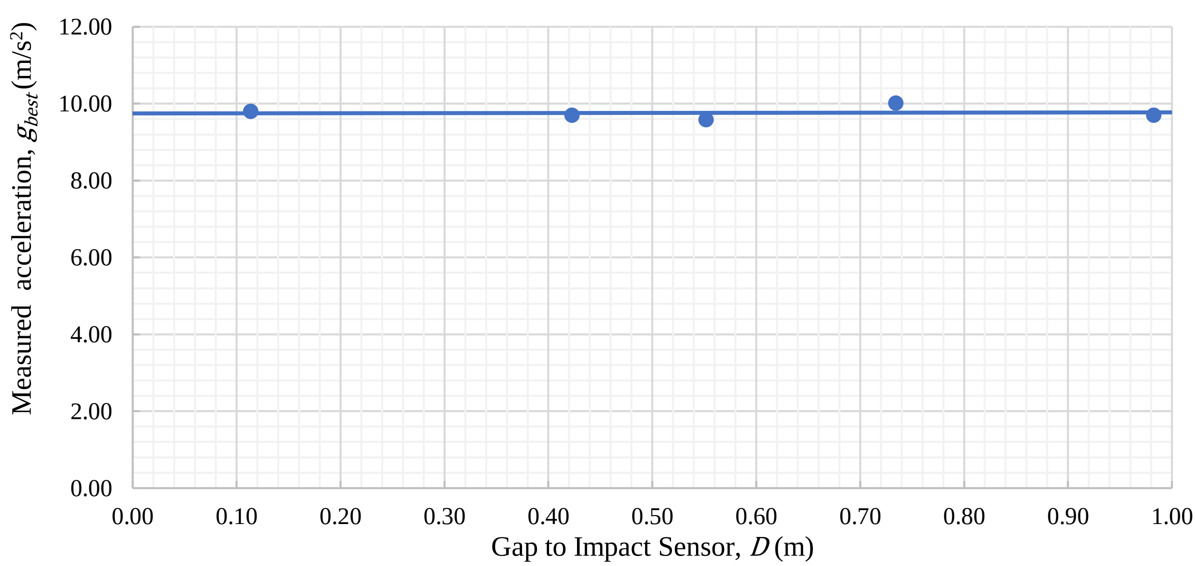

Figure 1. Relationship between g_{best} and Height of Ball in Freefall. The measured value of g for each height D was obtained through applying equation 2.1 to the best values T_1 and T_2. As per Excel's linear regression tool, the equation of the fitted line is $g(D) = (0.03 \pm 0.28)D\ s^{-2} + (9.74 \pm 0.18)\frac{m}{s^2}$.

We expect that g does not depend on D. The gravitation attraction between two objects depends on the masses and distance between the objects. However, since Earth's radius, $r \gg D$, the height above the surface of Earth, the added distance of D has no considerable effect on the acceleration. The acceleration due to gravity on Earth, which is approximately spherical, is therefore based only on the Earth's mass and radius and is independent of D.

This idea is supported by Figure 1. The line is near horizontal, implying a constant value of g regardless of D. The equation, $g(D) = (0.03 \pm 0.28)D\ s^{-2} + (9.74 \pm 0.18)\frac{m}{s^2}$ indicates that the linear dependence on D can be ruled out as the uncertainty of the slope overlaps with 0, which—based on the lab manual—means that the slope is consistent with 0. Therefore, all that should be taken from this figure is that $g = (9.74 \pm 0.18)\frac{m}{s^2}$

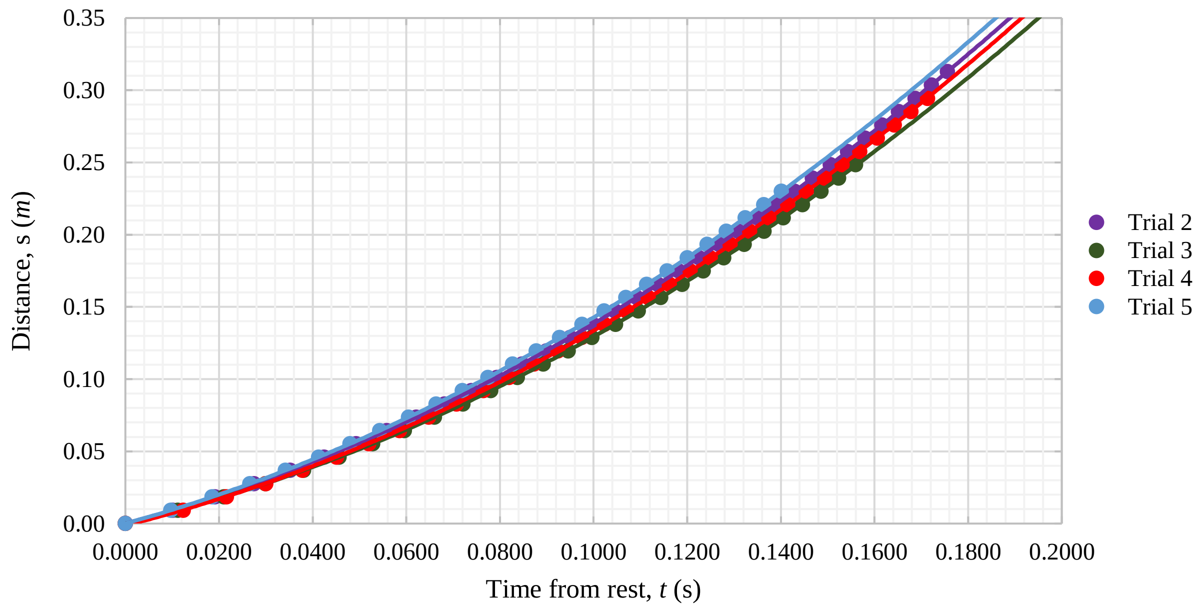

Figure 2. Distance vs. Time for Freefall Photogate Comb Trials 2-5. For visual clarity and comparison, the time has been scaled so that for each data set, $t = 0\ s$ and $s = 0\ m$ at the second block time (the first was discarded for unclear significance). This allows the graphs to be plotted in the same range. The uncertainties were found through applying regression on the upper and lower limits of $n\lambda$ for the distance column then using the overall range given to determine the uncertainty, which was considerably larger than the statistical uncertainty. The fitted parabolic curve for each dataset is the following: $s(t) = (5.27 \pm 0.29)t^2 \frac{m}{s^2}$ for Trial 2 (purple), $s(t) = (5.3 \pm 0.3)t^2 \frac{m}{s^2}$ for Trial 3 (green), $s(t) = (5.32 \pm 0.12)t^2 \frac{m}{s^2}$ for Trial 4 (red), $s(t) = (5.3 \pm 0.3)t^2 \frac{m}{s^2}$ for Trial 5 (blue). Differentiating each equation to get the measure of g yields $g = (10.5 \pm 0.6)\frac{m}{s^2}$ for Trial 2, $g = (10.6 \pm 0.6)\frac{m}{s^2}$ for Trial 3, $g = (10.64 \pm 0.24)\frac{m}{s^2}$ for Trial 4, and $g = (10.6 \pm 0.6)\frac{m}{s^2}$ for Trial 5.

Data Tables

Trial	Photogate spacing d (cm)	Gap to impact sensor D (cm)	Measured acceleration g (m/s^2)
1	8.25	11.34	9.8 ±0.3
2	8.25	42.25	9.67 ± 0.05
3	8.25	55.15	9.59 ± 0.09
4	8.25	73.41	10.02± 0.08
5	8.25	98.23	9.70 ±0.07

Table 1. Measured values of g for the multi-photogate method. Not included in the table is the ± 0.1cm uncertainty for d, which was obtained through subtraction of two height measurements, and the ± 0.05 cm uncertainty for d.

The uncertainty δg in the measured g value was obtained by calculating the systematic uncertainty. This uncertainty was calculated by finding the highest and lowest possible g values based on the upper and lower limits of $D$and d. The difference between the average of these values and one of the bounds yields the systematic uncertainty. The systematic contribution dominates the statistical contribution. The statistical uncertainty, which is obtained through formula STDEV.S()/SQRT(10) on Excel, is small enough to have little or no effect on the first nonzero digit of the δg, which is the only digit displayed. The sum of both contributions is still used for the δg for the purposes of rounding.

Conclusion

The measured acceleration in the ball drop method are relatively accurate compared to the photogate comb method. As can be seen in Table 1, the values come quite close to the expected value of $g = (9.7955 \pm 0.0003)\ m/s^2$. The bounds of the results for Trial 1 overlap this value. The highest possible values for the measurement Trial 2,3, and 5 are respectively $g = 9.72\ \frac{m}{s^2}$, $g = 9.68\ \frac{m}{s^2}$, and $g = 9.77\frac{m}{s^2}$, which all approach the accepted g. This is the same case for the lower limit value of $g = 9.94\frac{m}{s^2}$ for Trial 4. On the contrary, the photogate comb yielded less accurate measurements of g. All the measurements of g are greater than the expected value of g. The lower limits for Trials 2-5, $g = 9.9\ \frac{m}{s^2}$, $g = 10.0\ \frac{m}{s^2}$, $g = 10.4\ \frac{m}{s^2}$, and $g = 10.0\frac{m}{s^2}$ approach g, but not as close as the ball drop values. The photogate comb method is however more precise. As be seen in Figure 2, the graphs of each trial's regression curve overlapped each other. The differentiation of their equations to get the measurements of g were equivalent for Trial 3 and 5, but all trials are closely overlapped with each other, indicating the high precision of this method. On the other hand, as can be seen in Figure 1, the data points are close to the trend line but, not on the line itself.

The higher precision of the photogate comb method was expected as all the data points for a trial were done in one drop; this means that all factors affect the points uniformly, increasing the precision with each trial and thus the overall precision. For the ball drop method, each data point was obtained from 1 drop for a total of 10 drops per trial. While the factor mostly stay constant per drop, things such as the point of contact of the impact sensor and wind do not affect the points uniformly. This decreases precision within each trial and thus overall precision as expected.

The uncertainty of these values was found through the sum of the systematic and statistical uncertainties. Although some of the statistical uncertainties were small enough to not affect the first nonzero digit of δg , some of them still were enough to induce rounding to the next digit. It is better to include a higher uncertainty if possible as overestimating the accuracy or precision of results can lead to incorrect conclusions; therefore, the sum was used.

Extra Credit

The issue with the photogate comb is that it is difficult to keep in vertical. Since is it a bit long, a slight angle will ultimately tilt it away from the sensor. However, it is not long enough to understand whether the comb is perpendicular to the surface and if it will tilt away. To keep the comb vertical, we tried two methods. The first is putting the photogate comb through the ball ring used in the ball drop. This slightly adjusts the comb to keep it vertical till the last slot that goes through it; by this time, the comb is mostly through the sensor and remains vertical for the rest of the run. The second is dropping it through the ring by slipping it through two fingers. This theoretically will make it vertical to the desk as well as preventing rotation of the comb about the longer edge which may affect the block times. Both methods are performed with three trials each; the results are graphed and calculated for analysis of their accuracy and precision.

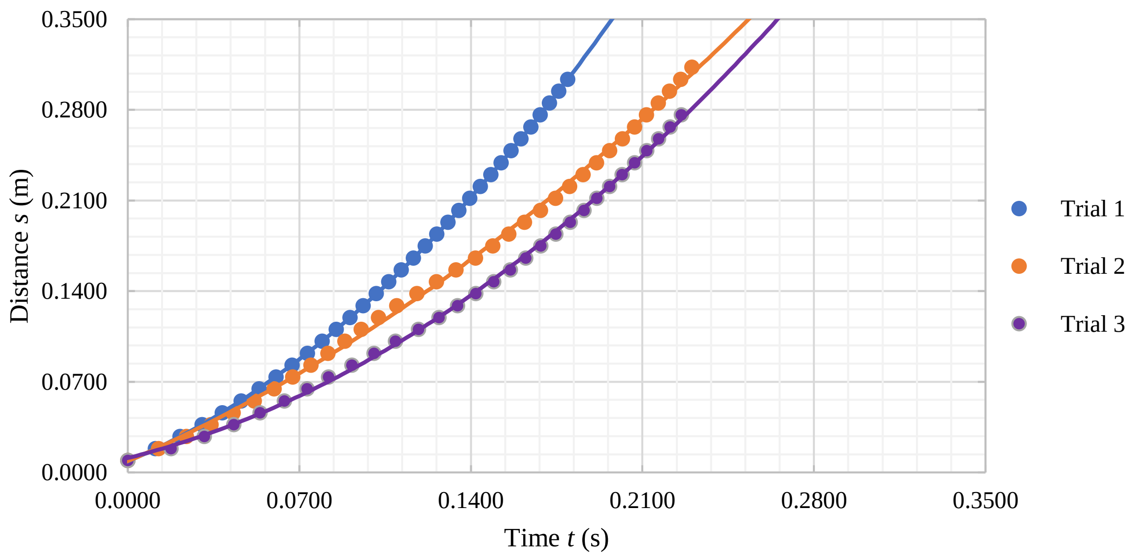

Figure 3. Distance vs. Time for Freefall Photogate Comb Through a Ring. The fitted parabolic curve for each dataset is the following: $s(t) = (4.72 \pm 0.24)t^2 \frac{m}{s^2} + 0.795t\frac{m}{s} + 0.009\ m$ for Trial 1 (blue), $s(t) = (1.99 \pm 0.12)t^2 \frac{m}{s^2} + 0.838t\frac{m}{s} + 0.008\ m$ for Trial 2 (orange), $s(t) = (3.04 \pm 0.10)t^2 \frac{m}{s^2} + 0.472t\frac{m}{s} + 0.01\ m$ for Trial 3 (purple). Differentiating each equation to get the measure of g yields $g = (9.44 \pm 0.48)\frac{m}{s^2}$ for Trial 1, $g = (3.97 \pm 0.24)\frac{m}{s^2}$ for Trial 2, and $g = (6.08 \pm 0.20)\frac{m}{s^2}$ for Trial 3. These values are less than what is expected for acceleration in freefall.

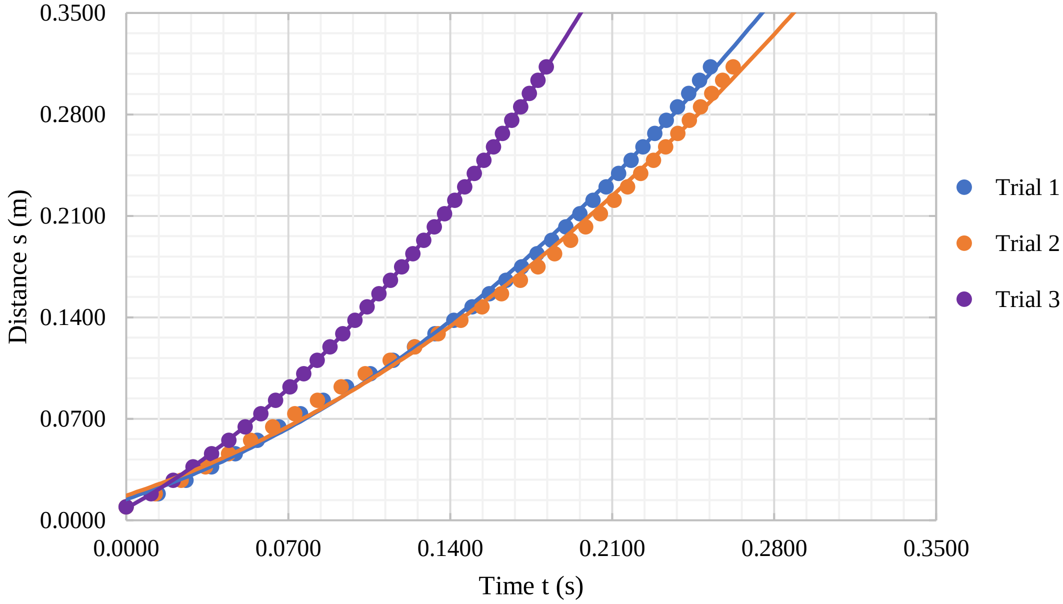

Figure 4. Distance vs. Time for Freefall Photogate Comb Through a Ring and Fingers. The fitted parabolic curve for each dataset is the following: $s(t) = (2.30 \pm 0.12)t^2 \frac{m}{s^2} + 0.571t\frac{m}{s} + 0.012\ m$ for Trial 1 (blue), $s(t) = (1.80 \pm 0.16)t^2 \frac{m}{s^2} + 0.608t\frac{m}{s} + 0.014\ m$ for Trial 2 (orange), $s(t) = (4.41 \pm 0.03)t^2 \frac{m}{s^2} + 0.871t\frac{m}{s} + 0.008\ m$ for Trial 3 (purple). Differentiating each equation to get the measure of g yields $g = (4.60 \pm 0.24)\frac{m}{s^2}$ for Trial 1, $g = (3.60 \pm 0.32)\frac{m}{s^2}$ for Trial 2, and $g = (8.83 \pm 0.06)\frac{m}{s^2}$ for Trial 3. These values are less than what is expected for acceleration in freefall. The data is also not as precise as that for just the ring (compare to Figure 3).

Figure 3 and 4 indicate that while both methods obtain more data points by successfully keeping the comb vertical, the resulting measurements of g are much less than the accepted value of $g = (9.7955 \pm 0.0003)\frac{m}{s^2}$. This was somewhat expected as there would be a factor of friction between the ring and photogate comb since they come in contact to maintain the vertical angle of the comb. However, the method of just using the ring is still more accurate and precise of the two methods. This is most likely due to the latter method also involving friction during the finger-comb contact. The fingers do not have a consistent value of friction imposed on the comb as it depends on the distance and exertion maintain by the owner of the fingers, something that is difficult to keep consistent. The lack of consistency and increase of friction thus eliminates the ring-and-finger method for keeping the comb vertical while producing valuable data. On the other hand, the ring, while also producing lower values due to friction, is promising in that each trial can still be done under the same conditions to maintain high precision. In addition, one of the measurements of g, $(9.44 \pm 0.48)\frac{m}{s^2}$, overlap with the accepted value of g. With lubrication or more frictionless material, the ring method may be a reasonable method to use for the photogate comb experiment in the future.

PRESENTATION MINI REPORT

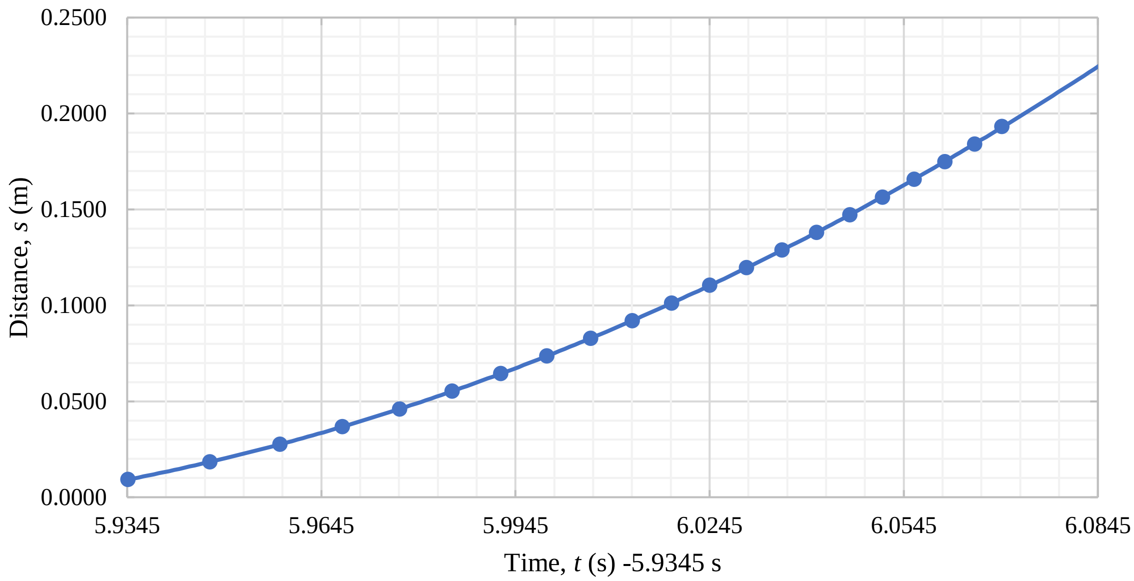

Figure 5. Measurement of g for Freefall Photogate Comb Trial 1. The blue dots represent each time the photogate comb blocked the photogate sensor. The gap between each slot edge of the comb is $\lambda = 0.0092\ m\ \pm 0.0005m$; the successive multiples of λ—n λ—are used to determine the distance covered by the comb at each block. The uncertainty was found through using the regression tool on the upper and lower limits of λ ($\lambda_{low} = 0.0087m$ and $\lambda_{high} = 0.0097m$) for the distance column and then using the overall range given to determine the systematic uncertainty. This value is considerably larger than just the statistical uncertainty given by the regression of the best distance column. The fitted parabolic curve is $s(t) = (5.27 \pm 0.29)t^2 \frac{m}{s^2} + (0.855)t\frac{m}{s}\ +\ 0.009\ m$. Differentiating the equation to get the measure of g gives $g = (10.5 \pm 0.6)\frac{m}{s^2}$.

Figure 5 models the freefall of the photogate comb in Trial 1. The photogate comb, which has uniformly spaced slots, was dropped above a photogate sensor fixed 42.80 ± 0.05 cm on a ring stand. It should be noted that while the photogate sensor was fixed at a constant measured height, the height is not of concern for this method—this is due to the fact concluded in the previous that height above Earth has no effect on the acceleration due to gravity. Because the comb blocks the sensor every time a slot edge passes through and the uniform spacing of $0.0092\ m\ \pm 0.0005m$ between these edges is known, a single run can yield as many as 34 data points (which is the number of slot edges on the comb used excluding the first). By reducing the need for repetition while still obtaining a fair set of data, the method has high precision within a single dataset. This idea is demonstrated from visual inspection of Figure 5 by the near-perfect fitting of the curve connecting the data. This high precision is not quite achieved with the previous ball drop method as each drop has the potential to be affected unequally by circumstances such as angle, consequent point of contact on the impact sensor, and wind—since the ball is lightweight. While the photogate comb is thus reliable in that it can provide highly precise data, the data itself is comparatively inaccurate. As explained earlier, the measure of the comb's acceleration due to gravity is obtained by differentiating the equation twice. This yields the equation $g = (10.5 \pm 0.6)\frac{m}{s^2}$, which already seems greater than the accepted value of $g = (9.7955 \pm 0.0003)\frac{m}{s^2}$. However, the uncertainty of the measurement, which doubled due to differentiation, still allows us to approach the accepted value through a low limit of $g_{low} = 9.9\ \frac{m}{s^2}$, which is only 1.0% higher in value. Therefore, for the purposes of measuring g, the photogate comb method is still a valid and reliable means of measurement.

Caption Word count: 134
Paragraph Word count: 336