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Hash functions and Message Authentication Code

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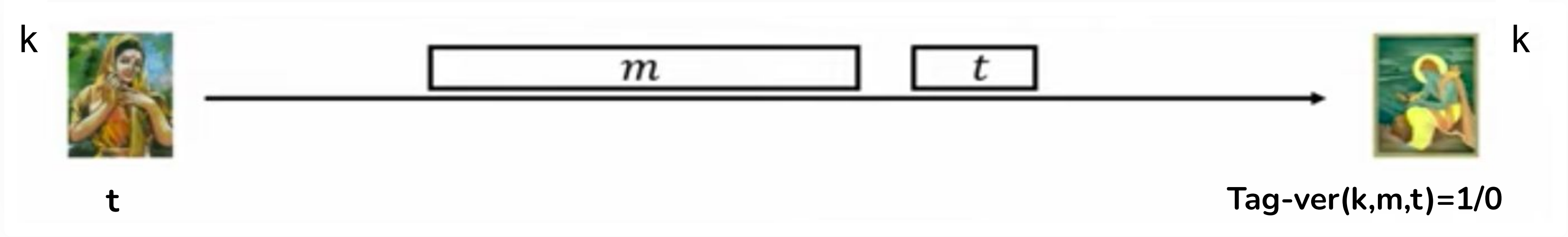
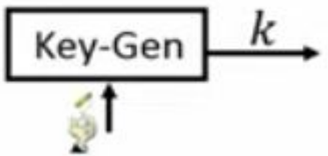
Message Integrity and Authenticity in Symmetric Key Setting



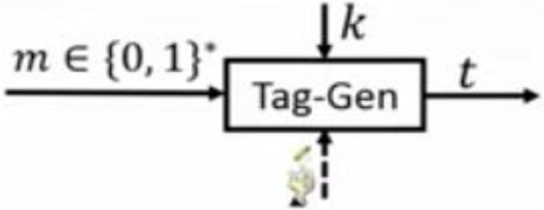
- **How can receiver verify whether the received message was indeed sent by a designated sender? —Message Authenticity**
- **How can receiver verify whether the received message was changed enroute?--- Message Integrity.**
- **Possible Solution: along with the message send a short verification tag.**
- **Message is accepted only if tag verification is successful.**

Message Authentication Code(MAC)

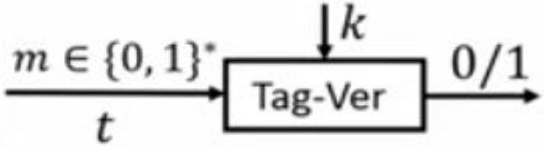
A Mac Π is a collection of three algorithms(key -gen, Tag- gen, Tag-ver).



- ❑ Key-Gen(1^n) $\to k \in$ K
- ❑ K: set of all output keys.
- ❑ Randomized Algorithm



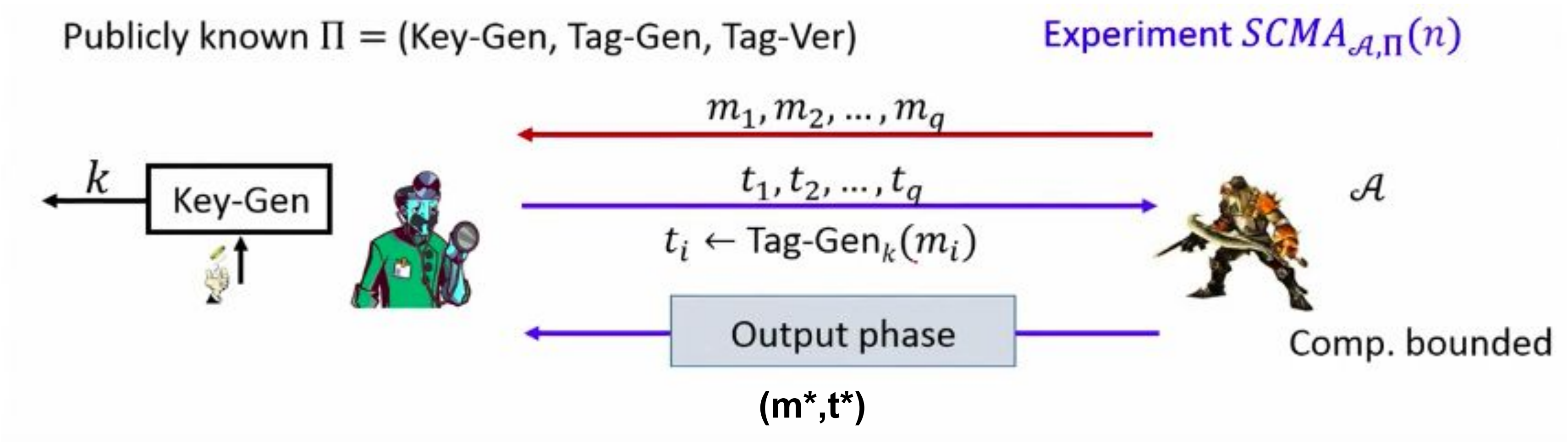
- ❑ Tag-Gen$_k$(m) -> t $\in$ T
- ❑ M: set of all possible messages.
- ❑ T: set of all possible output tags.
- ❑ Need not be randomized



- ❑ Tag-Verk(m,t) $\in$ {0,1}
- ❑ Deterministic Algorithm

❑ Correctness: for every $k \in$ K and m $\in$ M, the following should hold:

Tag-Ver$_k$(m, Tag-Gen$_k$(m)) = 1



A is said to win the experiment if:

$(m^*,t^*) \notin (m_1,t_1),(m_2,t_2)......(m_q,tq)$ and $\text{Tag-Ver}_k(m^*,t^*)=1$

Π is said to be (SCMA) strong chosen message Attack secure, if for every ppt A

$P[\text{A wins the experiment SCMA}_{A,\Pi}(n)] < negl(n)$

Secure MAC for Fixed-length messages from PRF.

- Let $F: \{0,1\}^n \times \{0,1\}^\ell \Rightarrow \{0,1\}^L$ be a secure PRF

$k \in_R \{0,1\}^n$

$x \in \{0,1\}^\ell$

$y = F(k,x) \in \{0,1\}^L$

- Using F, we construct a **deterministic MAC** with:
  - $\mathcal{K} = \{0,1\}^n$
  - $\mathcal{M} = \{0,1\}^\ell$
  - $\mathcal{T} = \{0,1\}^L$

Key-Gen $\xrightarrow{k \in_R \mathcal{K}}$

$m \in \mathcal{M} \rightarrow$ Tag-Gen (key k) $\xrightarrow{t \in \mathcal{T}}$

$t := F_k(m)$

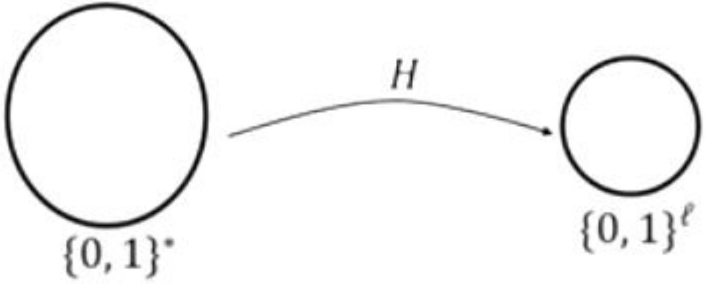
$(m,t) \rightarrow$ Tag-Ver (key k) $\rightarrow$ 1, iff $t = F_k(m)$

Cryptographic Hash functions

Tremendous application, both in symmetric key and public key world.

- **Primary Application :** Data Compression
- **Other applications: MAC, Key-derivation function, de-duplication, etc**

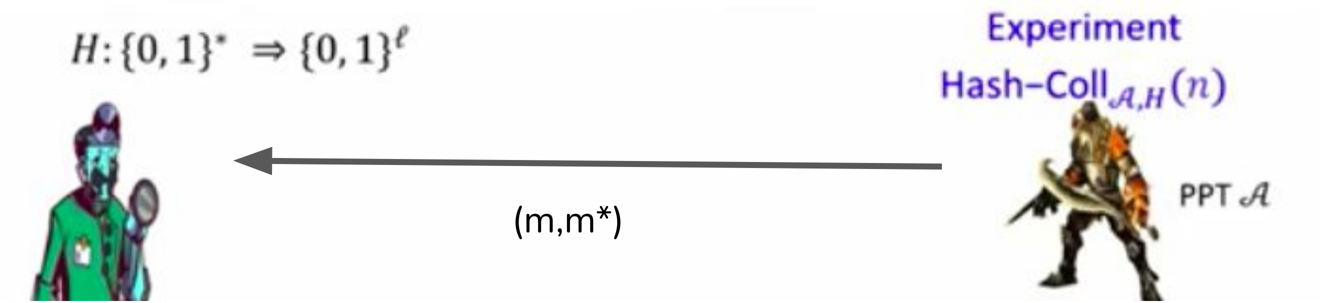
A Many to one function mapping arbitrary length bit strings to fixed length bit strings.



Main security property- Collision Resistant

- **Given a description of H, finding collisions for H must be computationally difficult.**

Collision Resistant Hash Function



- ❑ **H is a CRHF, if for every ppt A in Hash-CollA,H(n) their exists a negligible function negl(n):**
  P[A outputs m,m*: m≠m* and H(m) = H(m*)]<= negl(n)

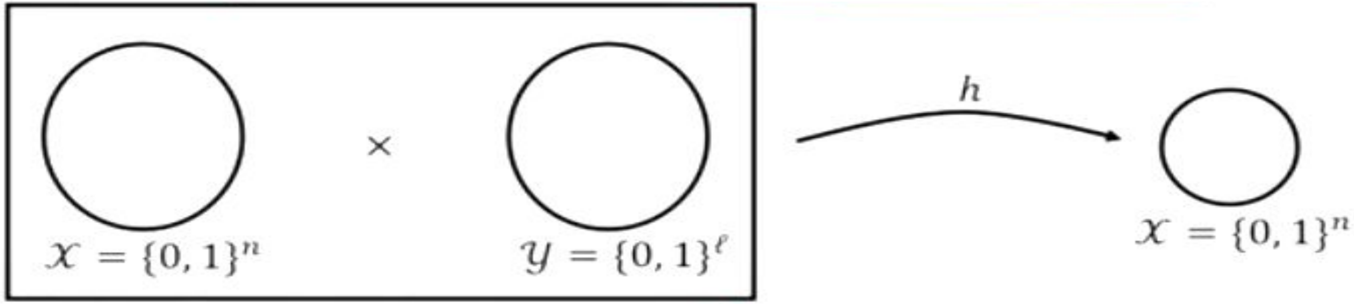
A technical issue with the above definition

- ❑ **Since |{0,1}*|>>{0,1}ℓ , collision must exist(pigeon-hole principle).**
- ❑ **There always exist a constant time A_{coll} hardcoded with a colliding pair(m,m*)**

Merkle Damgård Paradigm for design of CRHF

A well known two- stage approach for designing a CRHF (used in MD5, SHA-256).

- **Stage 1: Design a fixed length, collision resistant , compression function.**



h: $\{0,1\}^{n+\ell} \to \{0,1\}^n$

- **Stage 2: Design a CRHF H_{MD} for arbitrary length messages, using h as a black box.**

  *** Constructing a CRHF H_{MD} : $\{0,1\}^{\leq L}$ -> X, from h: X*Y-> X**

Constructing CRHF HMD :{0,1}<L ->X, from collision resistant h: X x Y->Y

$X=\{0,1\}^n$ $Y=\{0,1\}^\ell$

- For SHA256, n=256 and ℓ = 512

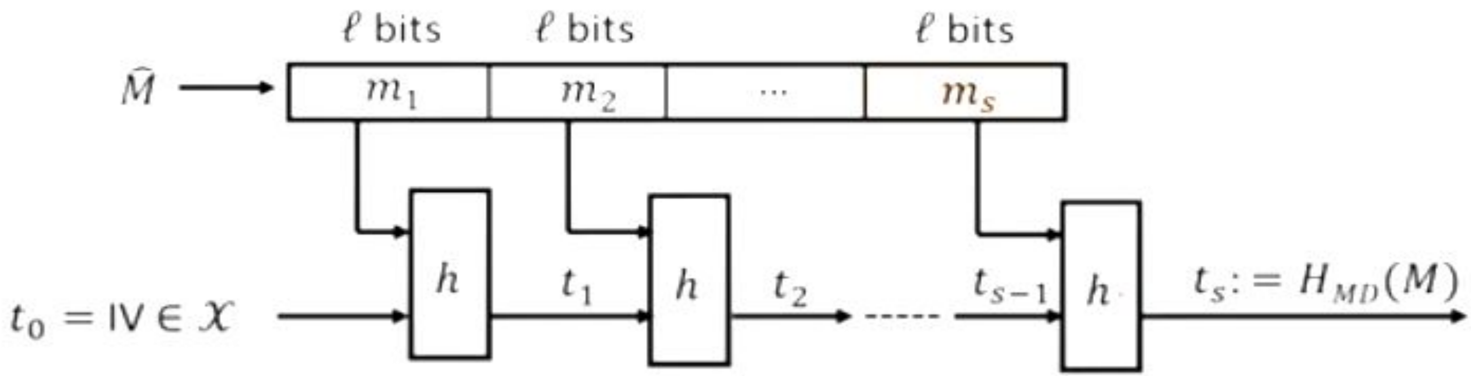
- Step 1: Encode the input $M \in \{0,1\}^{<L}$ for H_{MD}, to make encoded M as a multiple of ℓ bits.

$$M \in \{0,1\}^{\leq L} \xrightarrow{\text{Encode}} \hat{M} = M \,||PB \in \{\{0,1\}^\ell\}^{\leq \frac{L}{\ell}+1}$$

❖ $PB \stackrel{\text{def}}{=} 1000 \ldots 00||\langle s \rangle$, where $\langle s \rangle$ is a **fixed-length bit-string**, representing the **number of ℓ-bit blocks in M**

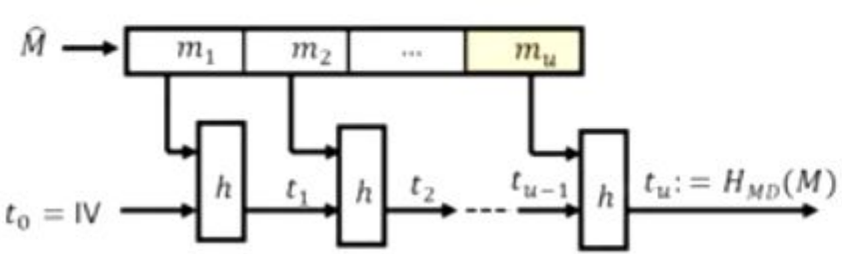
- Typically a 64-bit field -> $L \lesssim 2^{64}.\ell$ bits.
- If L is already a multiple of ℓ, then an additional dummy block added for PB.

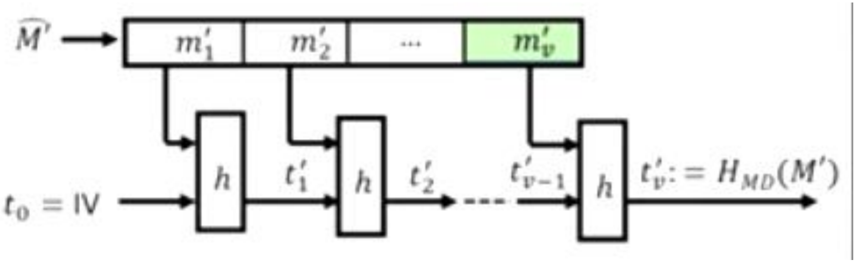
Step 2:Apply function H iteratively over the block of M and the previous outcome of h.



- IV : fixed, publicly known value, (say 0^n), some complicated string.
- Variable $t_0, t_1, \ldots, t_s \in X$ chaining variables.

- If h: X x Y-> X is a collision resistant function, then H_{MD}:{0,1}<L ->X is a CRHF
- Let there exist a ppt A_{MD} which output distinct M, M' ∈{0,1}<L such that P[H_{MD}(M)=H_{MD}(M')] = f(n), where f(n) is a non negligible function.

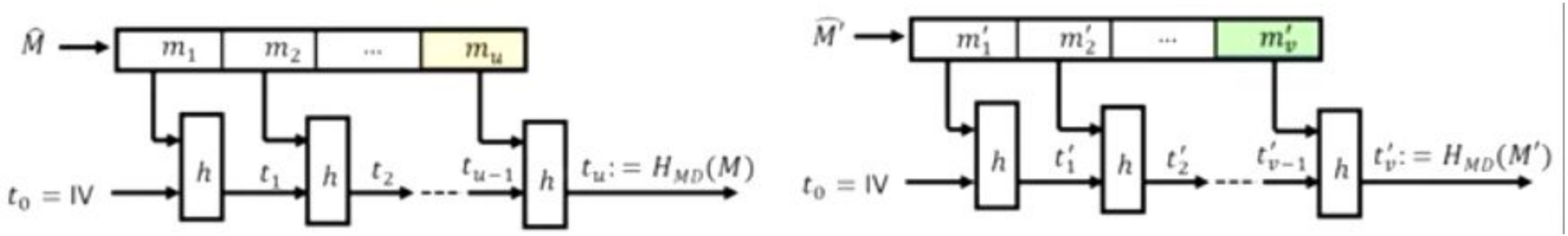




- ❑ Using A_{MD}, we construct a PPT A_h which outputs distinct pairs (t,m)(t*,m*)∈ X x Y:

P[h(t,m)=h(t*,m*)]= f(n)

- ❑ To find collision (t,m) (t*,m*) for h, A_h parses the hash chain H_{MD} and H_{MD}(M') from right to left.



- Since $H_{MD}(M) = H_{MD}(M') \Rightarrow h(t_{u-1}, m_u) = h(t'_{v-1}, m'_v)$

- If $(t_{u-1}, m_u) \neq (t'_{v-1}, m'_v)$, then the pair constitutes a collision for h

- **Else (tu-1, mu) = (t’v-1,m’v): M and M’ contains the same number of blocks–u=v.**
- **Consider the second-to- last invocation of h in the hash-chains.**

❑ $(t_{u-1}, m_u) = (t'_{u-1}, m'_u) : M \neq M'$, but **contains the same number of blocks**

Second-to-last invocation of h in the hash-chain $H_{MD}(M)$ and $H_{MD}(M')$

- ❖ If $(t_{u-2}, m_{u-1}) \neq (t'_{u-2}, m'_{u-1})$, then the pair constitutes a collision for h, as $t'_{u-1} = t_{u-1}$
- ❖ Else $(t_{u-2}, m_{u-1}) = (t'_{u-2}, m'_{u-1})$, with $m_u = m'_u$

➢ Consider the **third-to-last invocation** of h in the hash-chains

❑ The above process of scanning from right to left **will eventually find** an h-collision

➢ Else, we conclude that all the blocks of distinct M, M' are same --- a contradiction

Message Authentication using Hash functions

Mac for arbitrary long messages using a CRHF (Hash-and-Mac paradigm)

- Given an **arbitrary-length message**, compute its **fixed-length Mac-tag** in **two stages**:
  - **Step I: Hash** the arbitrary-length message to a **fixed-length string** using a CRHF
  - **Step II:** Compute the **Mac-tag on the message digest** (output of the CRHF)

Π_{MAC} = (Key-Gen, Tag-Gen, Tag-Ver) with key-space $\mathcal{K}$, message-space $\mathcal{M} = \{0,1\}^{\ell}$ and tag-space $\mathcal{T}$

+

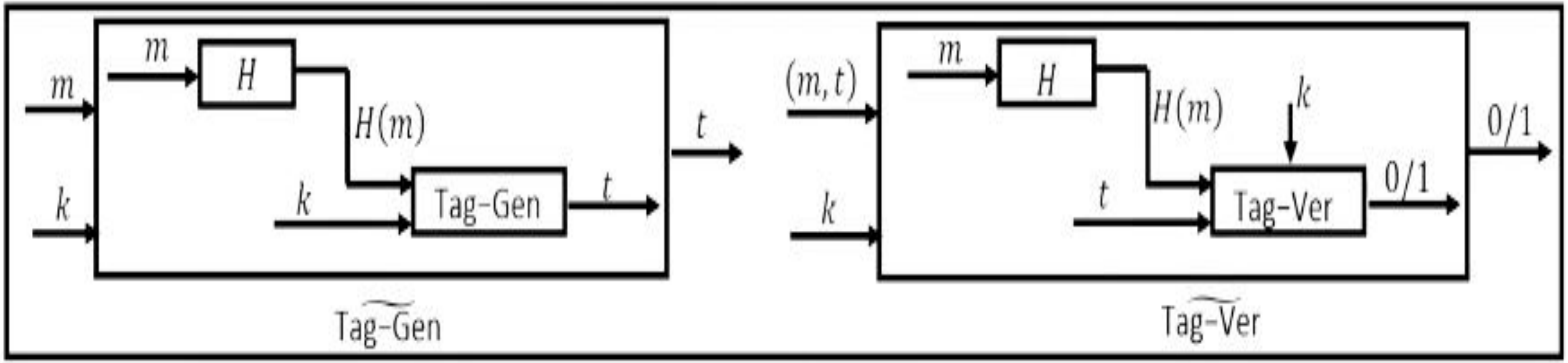
A CRHF $H: \{0,1\}^* \Rightarrow \{0,1\}^{\ell}$

⇒

$\widetilde{\Pi}_{MAC}$ = ($\widetilde{\text{Key-Gen}}$, $\widetilde{\text{Tag-Gen}}$, $\widetilde{\text{Tag-Ver}}$) with key-space $\mathcal{K}$, message-space $\mathcal{M} = \{0,1\}^*$ and tag-space $\mathcal{T}$

Π_{MAC} = (Key-Gen, Tag-Gen, Tag-Ver) with key-space $\mathcal{K}$, message-space $\mathcal{M} = \{0,1\}^{\ell}$ and tag-space $\mathcal{T}$

A CRHF $H: \{0,1\}^{*} \Rightarrow \{0,1\}^{\ell}$



What will I be studying in future?

- Birthday Attacks on cryptographic Hash functions.
- Many more applications of Hash functions
- Random oracle model and authentication Encryption.
- Security analysis of various Hash functions.
- MAC for arbitrary long messages.
- MAC for long messages using CRHF.
- And your suggestions are welcomed.

References

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THANK YOU