Persistent Homology of Cosmic web

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- N-body Simulations
- Topological Tools of Analysing Data
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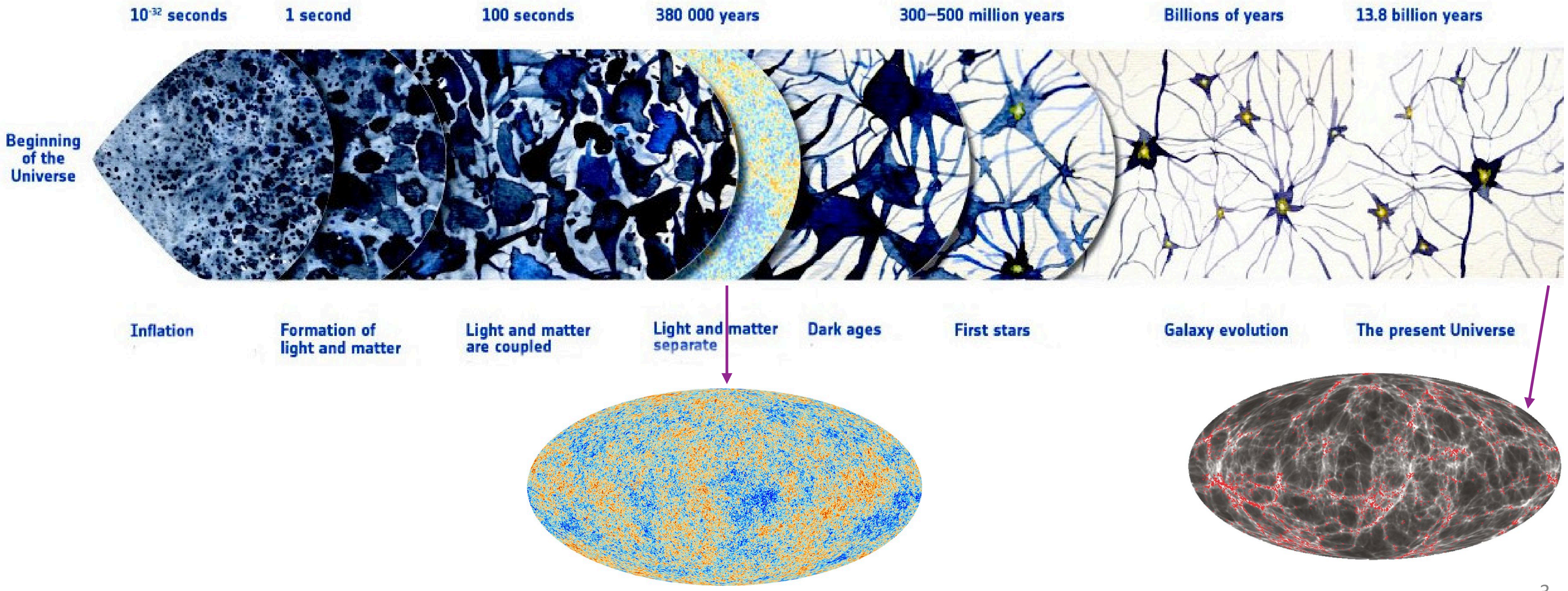
1-1-1 History of Cosmology



1-1-2 Theoretical Cosmology

$$a = (1 - z)^{-1}$$

- $$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$ *FLRW metric*
- $$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$$ *First Friedmann equation*
- $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$ *Second Friedmann equation*
- $$\dot{\rho} + 3H(\rho + p) = 0$$ *Energy conservation as a fluid*

1-1-3 Theoretical Cosmology

Fridmann equation $H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$

- $\rho(a) \propto \frac{1}{a(t)^{3(1+\omega)}}$
- $\Omega_{i0} = \frac{\rho_{i0}}{\rho_{crit}}$, $\rho_{crit} = 3H_0^2/8\pi G$

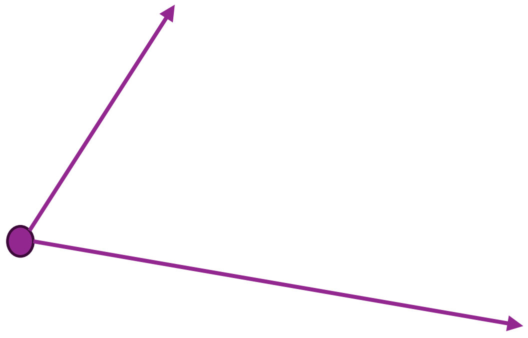
$$H(z) = H_0[\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0}(1+z)^4]^{1/2}$$

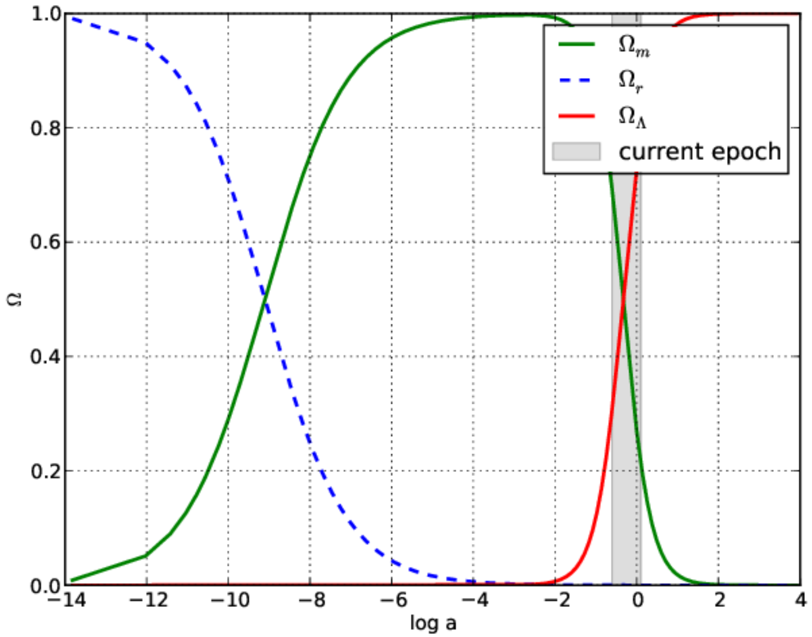
$\omega \equiv \frac{p}{\rho}$

- *Dark / Baryonic Matter* *0*
- *Radiation* *1/3*
- *Dark Energy* *-1*

1-1-4

Content	Equation of State	a	$\rho(density)$	$H(t)$
Matter	$\omega = 0$	$a \sim t^{2/3}$	$\rho \sim a^{-3}$	$H(t) = 2/3t$
Radiation	$\omega = 1/3$	$a \sim t^{1/2}$	$\rho \sim a^{-4}$	$H(t) = 1/2t$
Λ	$\omega = -1$	$a = \exp\{H(t - t_0)\}$	$\rho \sim const$	$H(t) = const$





1-2-1 Linear Structure Formation

$$P(k) = |\int \delta(x_1)\delta(x_2)e^{-ikx}dx|^2$$

$$\delta(x,t) \equiv \frac{\rho(x,t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

$$\begin{cases} \delta << 1 & \text{Linear Regime} \\ \delta \sim 1 & \text{Semi Linear Regime} \\ \delta >> 1 & \text{Totally Non-Linear Regime} \end{cases}$$

- $$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla} \cdot (1+\delta)\vec{v} = 0$$ Continuty Equation
- $$\frac{\partial \vec{v}}{\partial t} + \frac{1}{a}(\vec{v} \cdot \vec{\nabla})\vec{v} + \frac{\dot{a}}{a}\vec{v} = -\frac{1}{a}\vec{\nabla}\phi$$ Euler Equation
- $$\vec{\nabla}^2\phi = 4\pi G a^2 \rho_u \delta$$ Poisson equation

1-2-1 Linear Structure Formation

$$\delta(x,t) = D_+(t)\delta(x,t_0) + D_-(t)\delta(x,t_0)$$

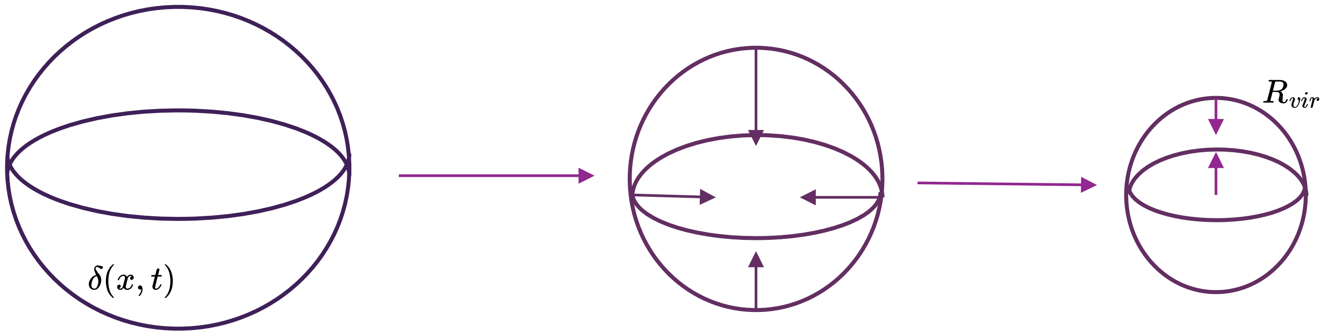
$$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} = (4\pi G\rho_u - \frac{c_s^2 k^2}{a^2})\delta$$

$$D(z) = \frac{5\Omega_m H_0^2}{2} H(z)\int_z^\infty \frac{1+z'}{H^3(z')}dz'$$

DE Dom	Matter Dom	Radiation Dom	δ
Const	a	Const	

1-2-1 Non-Linear Structure Formation

$$M = \frac{4\pi R_i^3 \bar{\rho}_i}{3}(1+\delta_i) \rightarrow \frac{d^2R}{dt^2} = -\frac{GM}{R^2} = -\frac{H_i^2 R_i^3}{2R^2}(1+\delta_i) \rightarrow \frac{1}{2}\dot{R}^2 - \frac{H_i^2 R_i^3}{2R}(1+\delta_i) = E \rightarrow E = -\frac{H_i^2 R_i^2}{2}\delta_i$$



$$R_t = \frac{(1+\delta_i)}{\delta_i} R_i \xrightarrow{\delta_i < 1} R_t \sim \frac{R_i}{\delta_i}$$

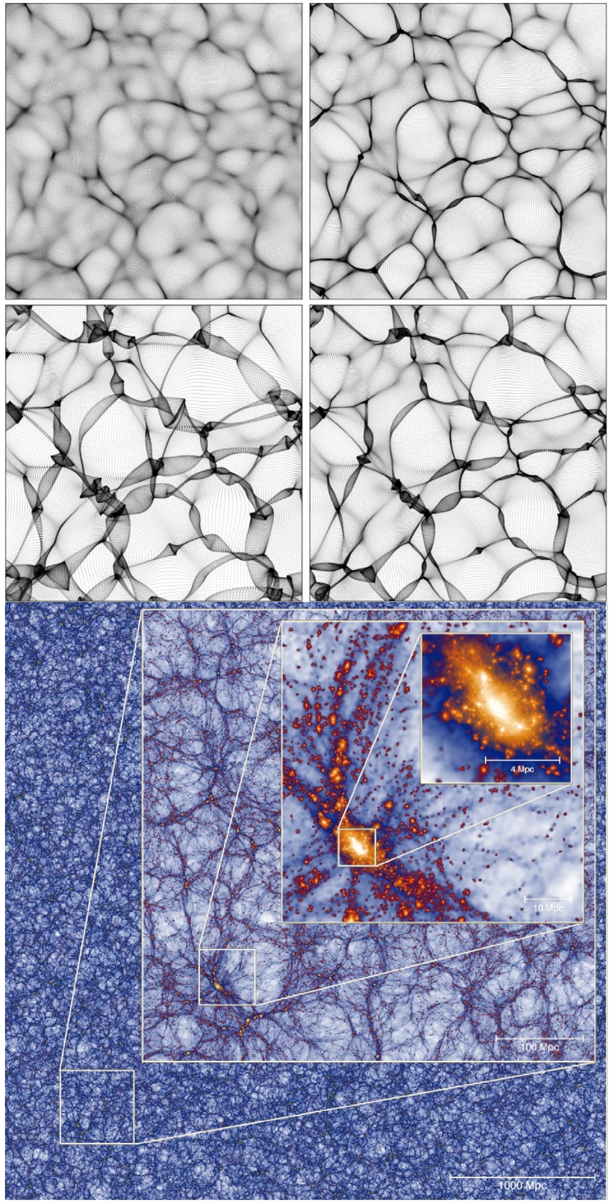
$$R_{vir} \equiv \frac{R_t}{2} \leftarrow 2K + U = 0, \quad \text{Where} \quad K = \frac{\dot{R}^2}{2} \quad \text{and} \quad U = -\frac{GM}{R}$$

1-2-1 Zel’dovich Approximation

$$\Psi(q) = \frac{2}{2H_0^2\Omega_{m,0}}\vec{\phi_0}(q) \longrightarrow \Psi_{ij} = \frac{2}{3H_0^2\Omega_{m,0}}\frac{\partial^2\phi_0(q)}{\partial q_i \partial q_j}$$

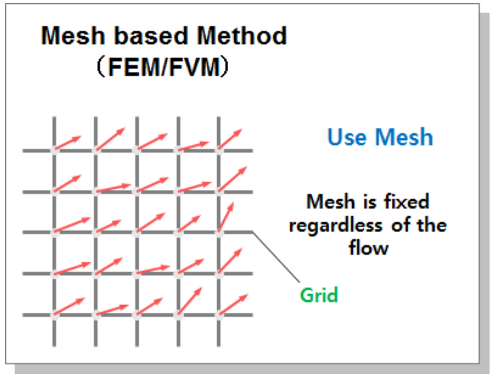
$$\rho(x,t) = \sum_q \frac{\rho_0}{|1 - D_+(t)\lambda_1(q)|.\,|1 - D_+(t)\lambda_2(q)|.\,|1 - D_+(t)\lambda_3(q)|}$$

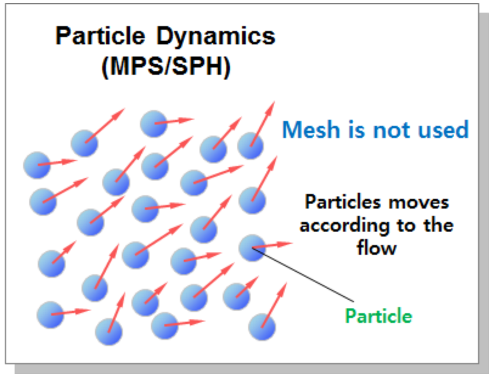
λ1	λ2	λ3	Components
-	-	-	Voids
+	-	-	walls
+	+	-	filaments
+	+	+	Clusters



Topological
Tool of
Analysing Data

3-1 N-Body Simulation





Name	Year Released	Simulation(s) Run	Hydro Method	Gravity Method
RAMSES	2002	HorizonAGN, NewHorizon	Adaptive Mesh Refinement (AMR)	Iterative Multigrid
GADGET-2/3	2005	EAGLE, MassiveBlack-II, Magneticum	Smoothed Particle Hydrodynamics (SPH)	TreePM
AREPO	2009	Illustris, IllustrisTNG, FABLE, MTNG	Moving Voronoi Mesh (MM)	TreePM
GIZMO	2014	Mufasa, Simba, FIREbox	Meshless Finite Mass, Volume (MFM/MFV)	TreePM
ChaNGa	2015	Romulus25	Smoothed Particle Hydrodynamics (SPH)	Tree
SWIFT	2018	FLAMINGO	Smoothed Particle Hydrodynamics (SPH), MFM/MFV	FMM-PM
GADGET-4	2020	CROCODILE	Smoothed Particle Hydrodynamics (SPH)	FMM-PM, TreePM

$$\frac{df(x(\vec{t}), u(\vec{t}), t)}{dt} = 0 \longrightarrow \frac{\partial f(\vec{x}, \vec{u}, t)}{\partial t} + \dot{r}\frac{\partial f(\vec{x}, \vec{u}, t)}{\partial r} + \dot{u}\frac{\partial f(\vec{x}, \vec{u}, t)}{\partial t} = 0$$

$$\rho \propto \int f d\vec{u}$$

3-2 Quijote Simulation

Quijote

Fiducial model (17,100 simulations)

$\Omega_m = 0.3175, \Omega_b = 0.049, h = 0.6711, n_s = 0.9624, \sigma_8 = 0.834,$
$w = -1, M_\nu = 0, \delta_b = 0, f_{NL} = 0, p_{NL} = 0, f(R) = 0$

Individual parameter variations (23,000 simulations)

ΛCDM	Dark energy	Massive neutrinos	Separate Universe	Primordial non-Gaussianities	Parity violation	Modified gravity
$\Omega_m, \Omega_b, h, n_s, \sigma_8$	w	M_ν	δ_b	$f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho}$	$p_{\rm NL}$	$f(R)$

Multiple parameter variations (42,816 simulations)

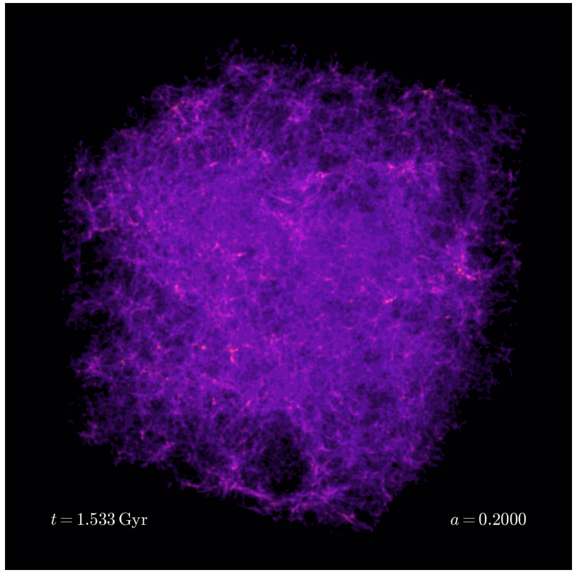
3 latin-hypercubes
$\Omega_m, \Omega_b, h, n_s, \sigma_8$
6,000 simulations

1 latin-hypercube
$\Omega_m, \Omega_b, h, n_s, \sigma_8, M_\nu, w$
2,000 simulations

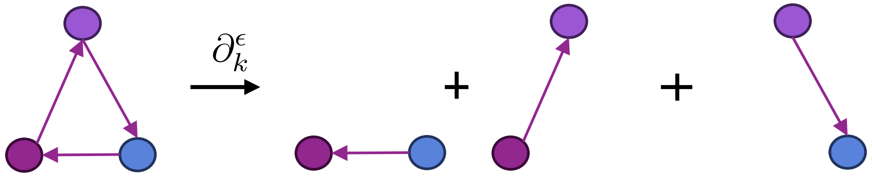
Big Sobol Sequence
$\Omega_m, \Omega_b, h, n_s, \sigma_8$
32,768 simulations

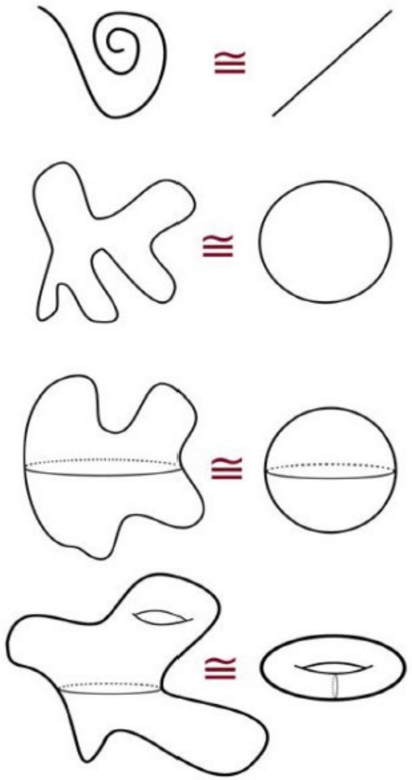
SB7
$\Omega_m, \Omega_b, h, n_s, \sigma_8, M_\nu, f_R$
2,048 simulations

3-3 Concept Simulation



4-1 Topological Tool of Analysing Data





k-th Homology Group:

$$H_k^\epsilon = \frac{\ker\ \partial_k^\epsilon}{\mathrm{im}\ \partial_{k+1}^\epsilon}$$

4-2 Homology Groups

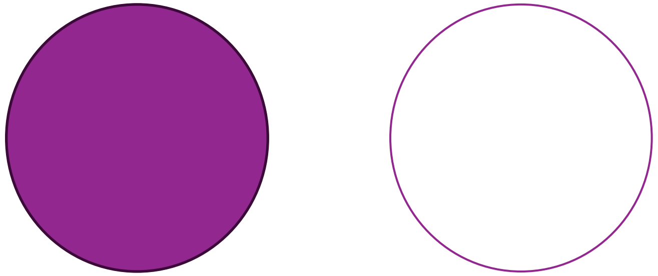
k-th Homology Group:

$$H_k^\epsilon = \frac{\ker\ \partial_k^\epsilon}{\text{im}\ \partial_{k+1}^\epsilon}$$

Boundaryless

Are [not] Boundaries

$$\beta_k^\epsilon = \dim\ H_k^\epsilon\ = \dim\ (\ker\ \Delta_k^\epsilon)$$



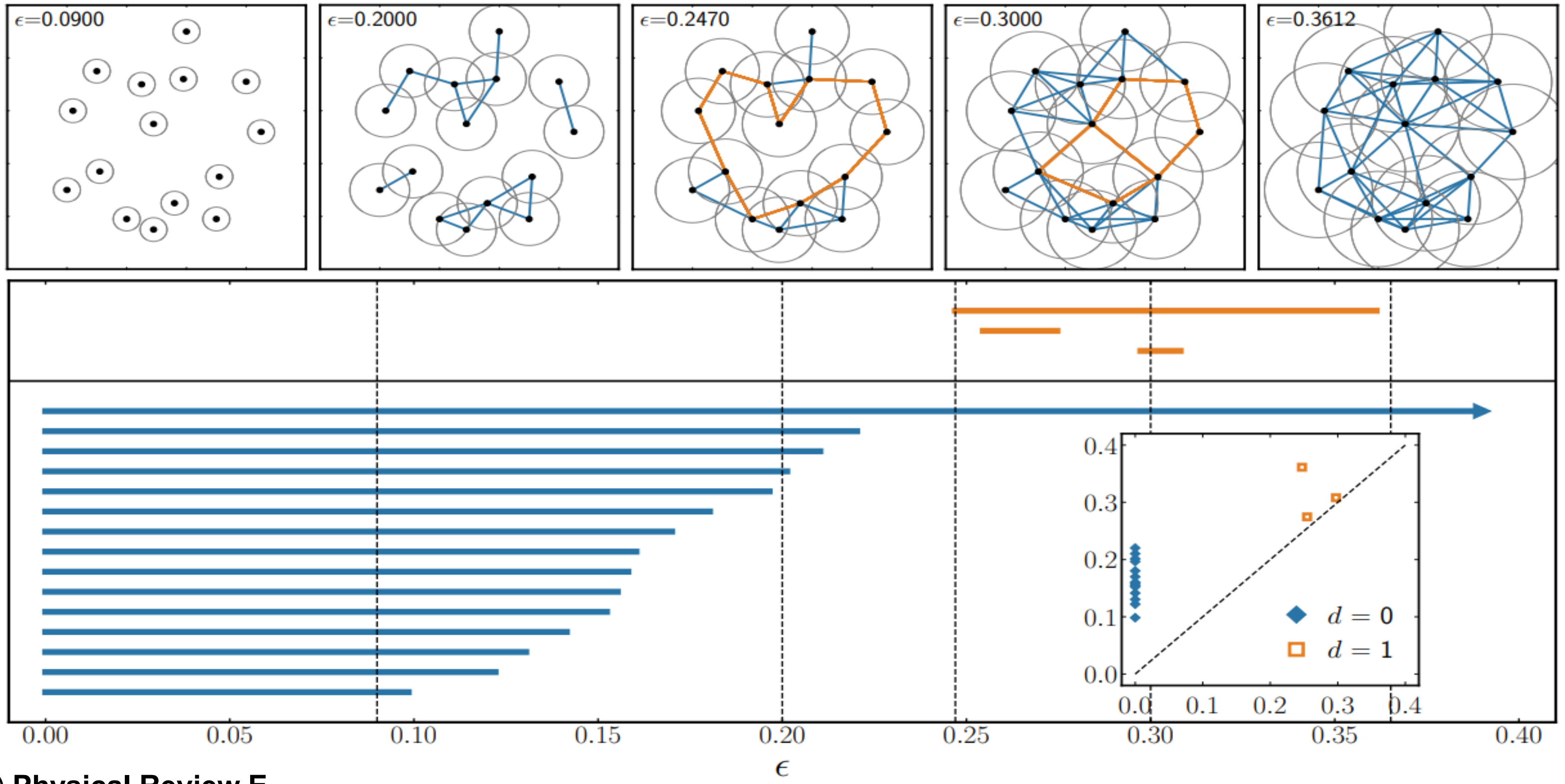
Topological space / Betti number						
β_0	1	1	1	1	1	1
β_1	0	0	1	0	2	4
β_2	0	0	0	1	1	1

Masoomy, H, et al. (2021) **Physical Review E**

4-3 Persistent Homology

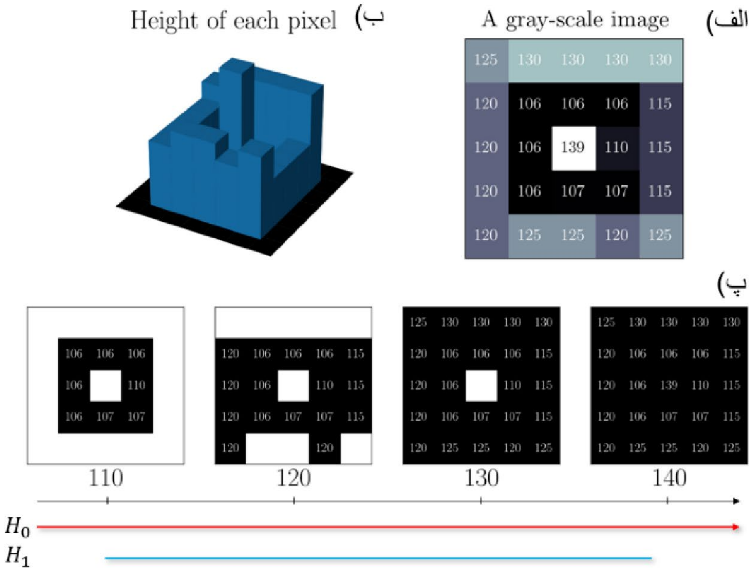
Here we look at the sequence of the topological structures:

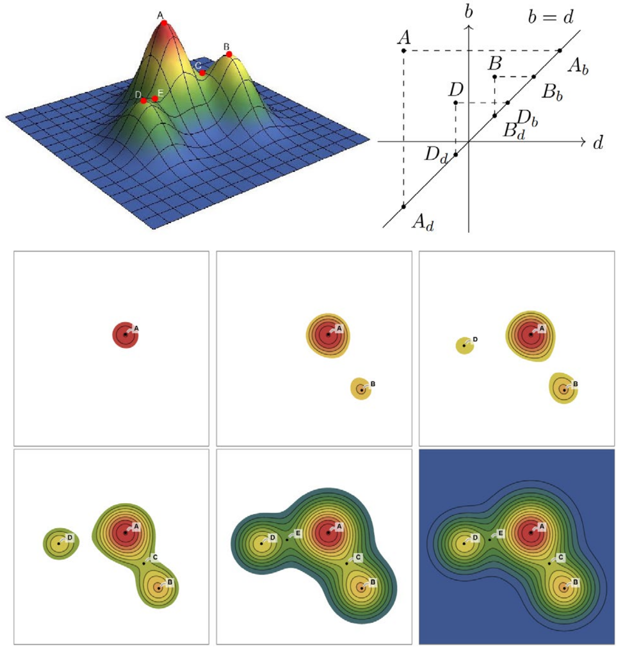
- *Persistent Diagrams*
- *Persistent Barcodes*



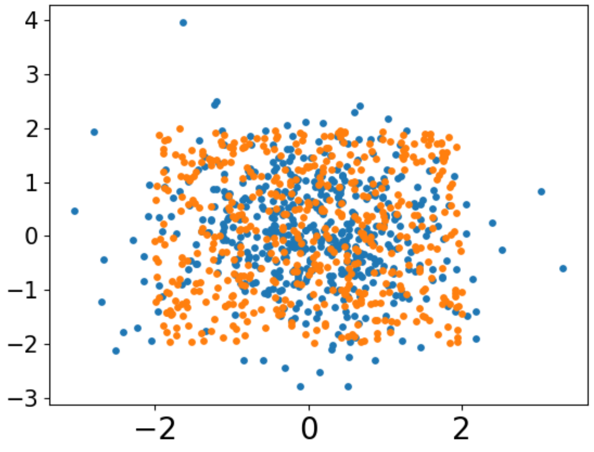
Masoomy, H, et al. (2021) **Physical Review E**
https://github.com/SinaGhafouri/Persistent-homology

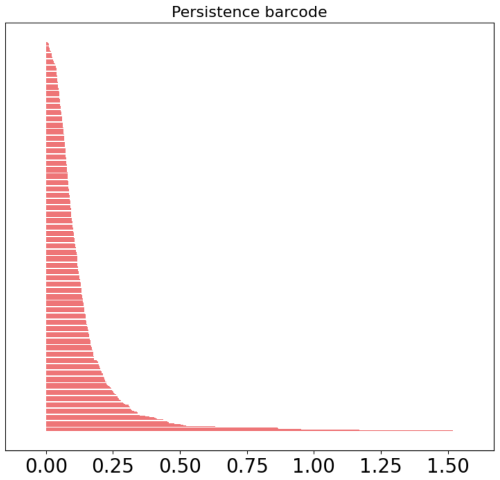
4-4 Cubical Persistent Homology

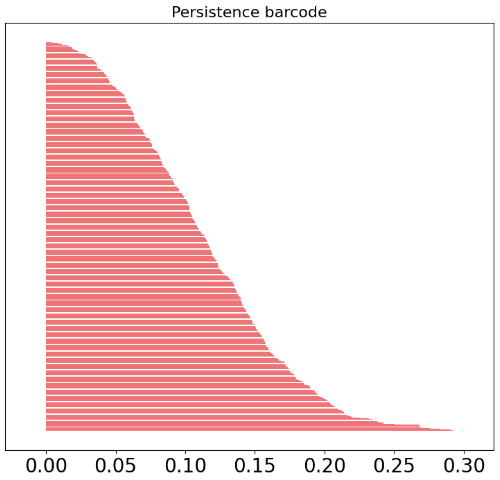


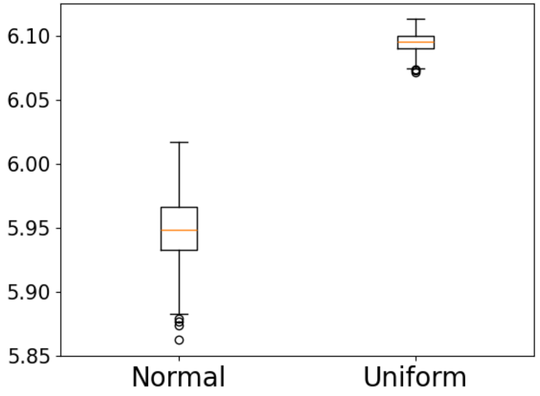


4-5 Persistent Entropy





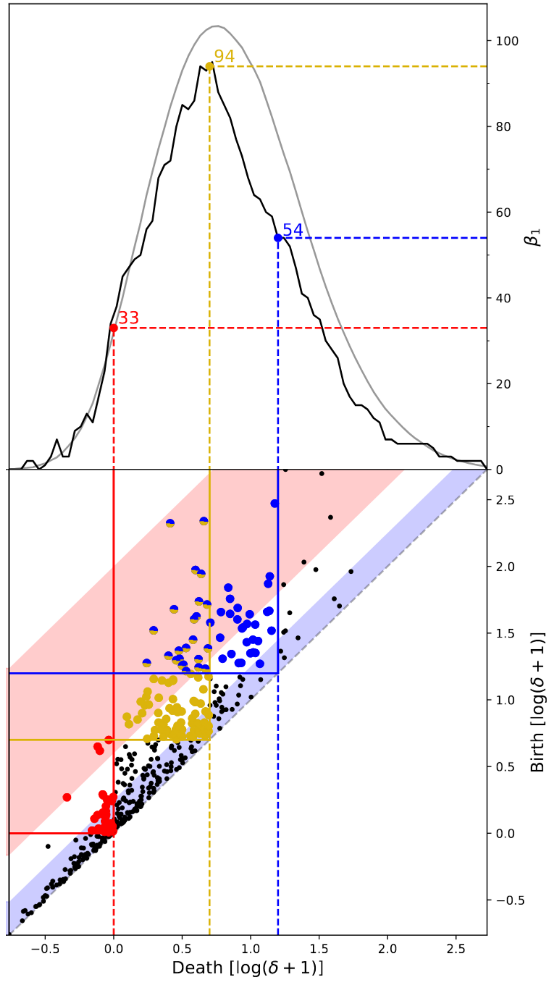




$$E_d = -\sum_{i=1}^{n_d} \frac{l_i^{(d)}}{\sum_{i=1}^{n_d} l_i^{(d)}} \ln \sum_{i=1}^{n_d} \frac{l_i^{(d)}}{\sum_{i=1}^{n_d} l_i^{(d)}}$$

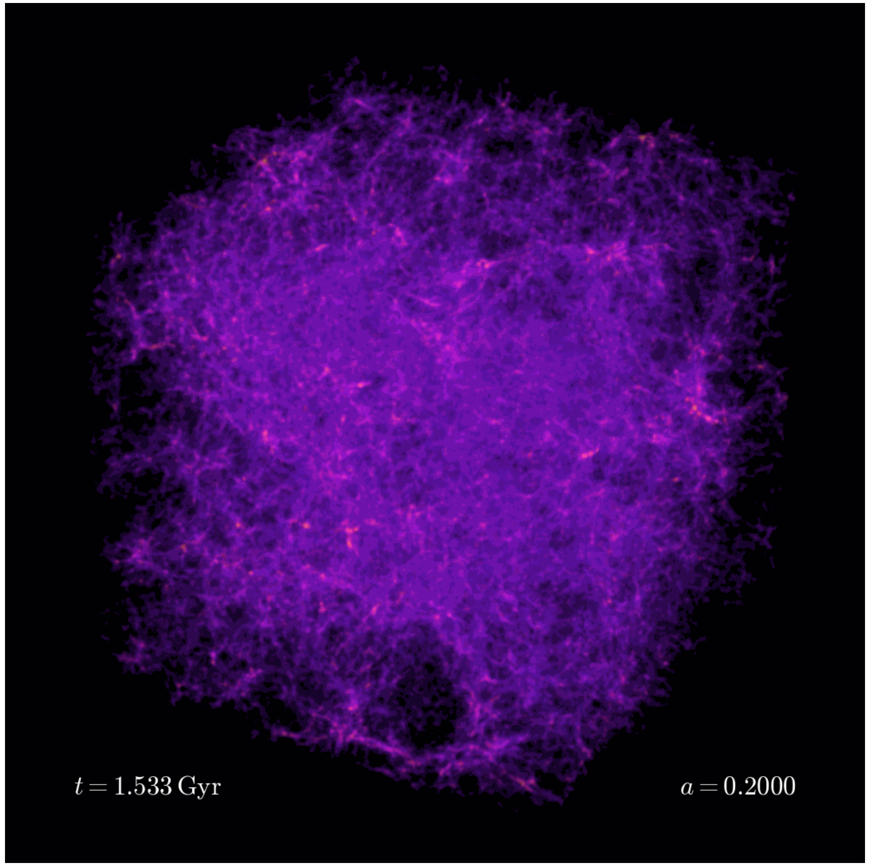
4-6 Features of Persistent Homology: Betti Curves

Topological space / Betti number	• (point)	/ (line)	○ (circle)	sphere	torus	double torus
β_0	1	1	1	1	1	1
β_1	0	0	1	0	2	4
β_2	0	0	0	1	1	1

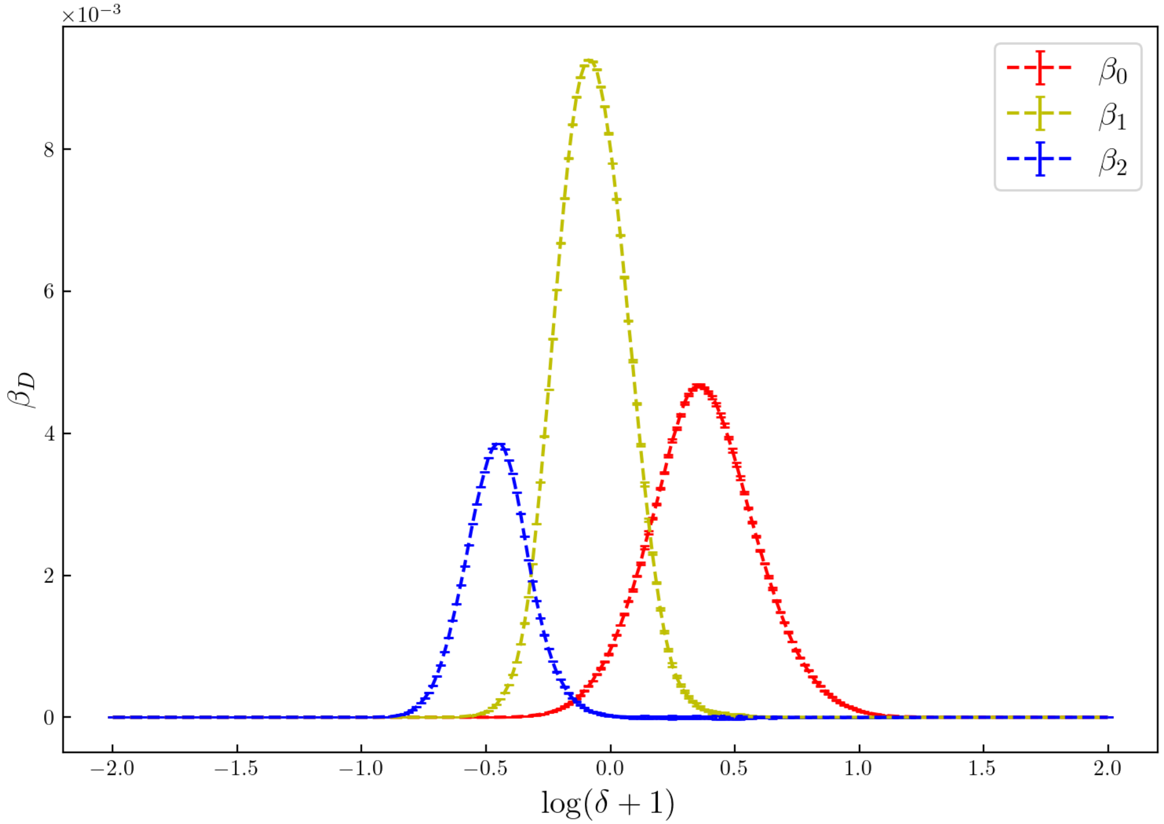


5 Pipeline of results

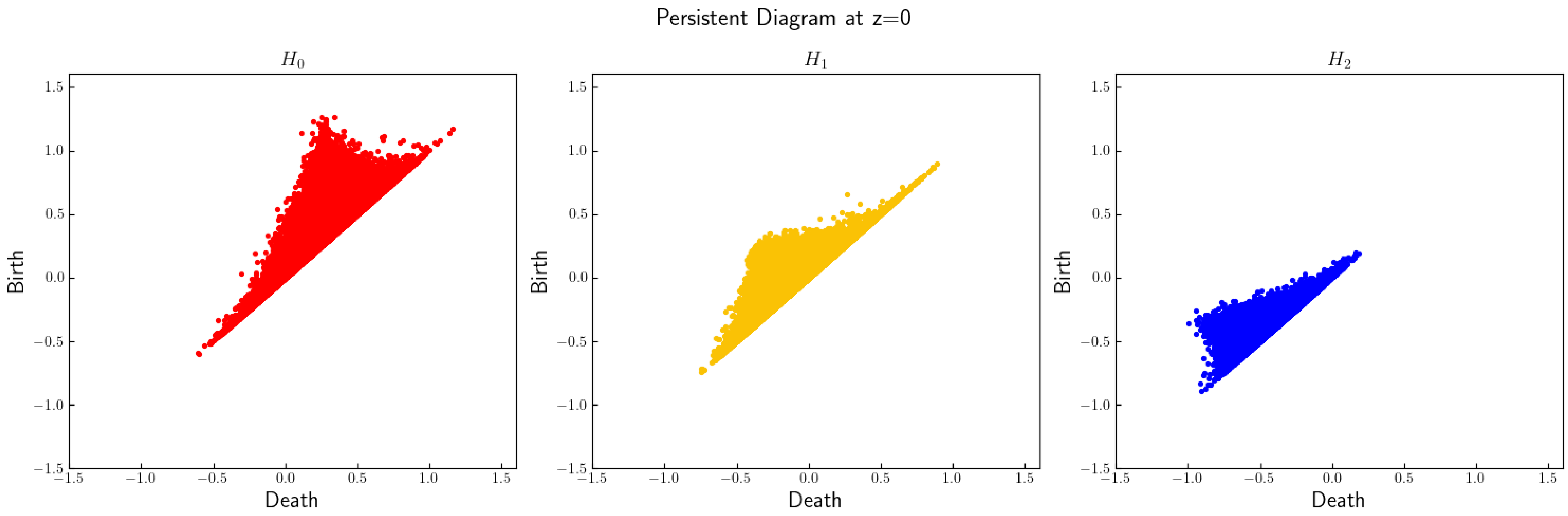
5-1-0 Date description



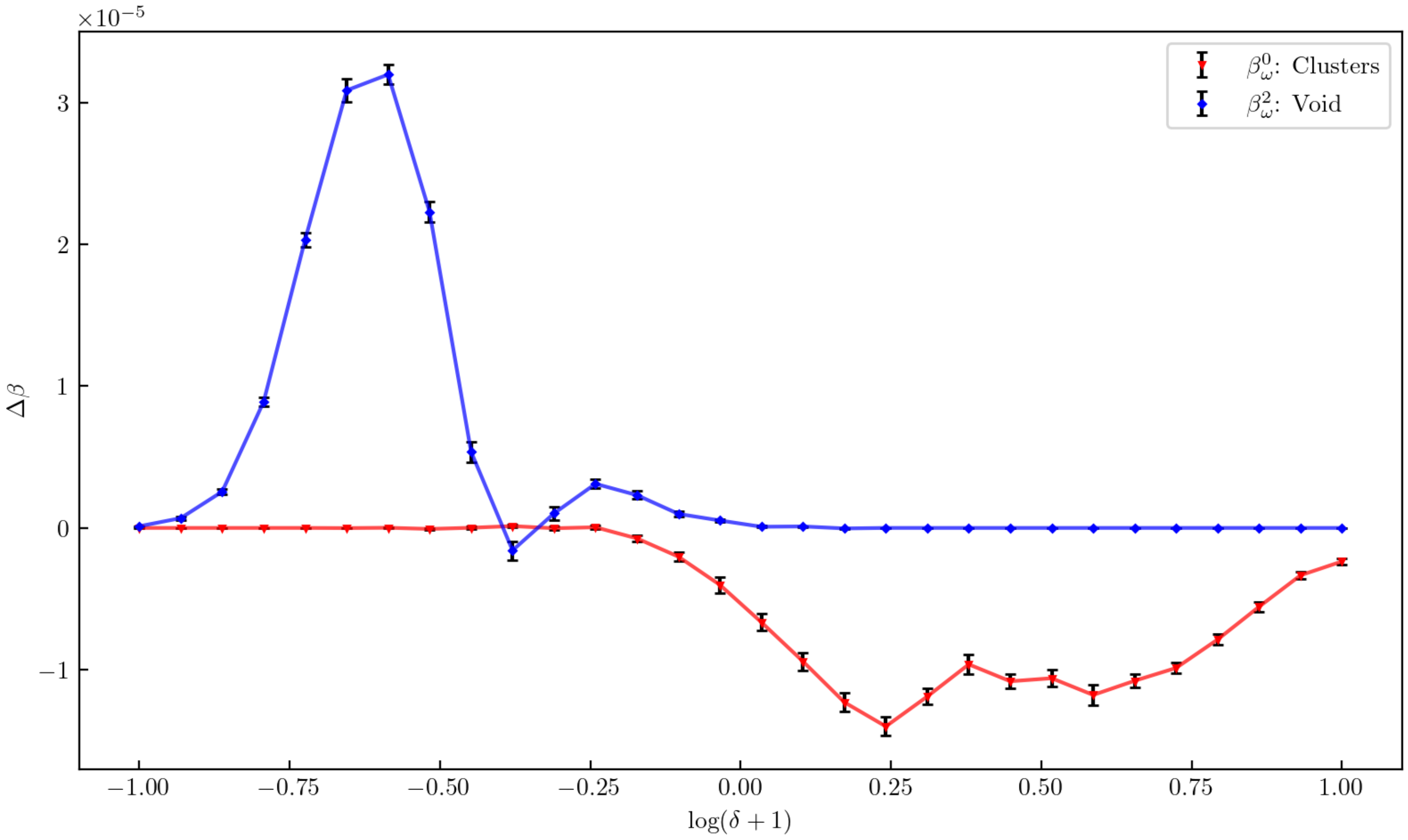
5-1-1 Betti curves at z=0



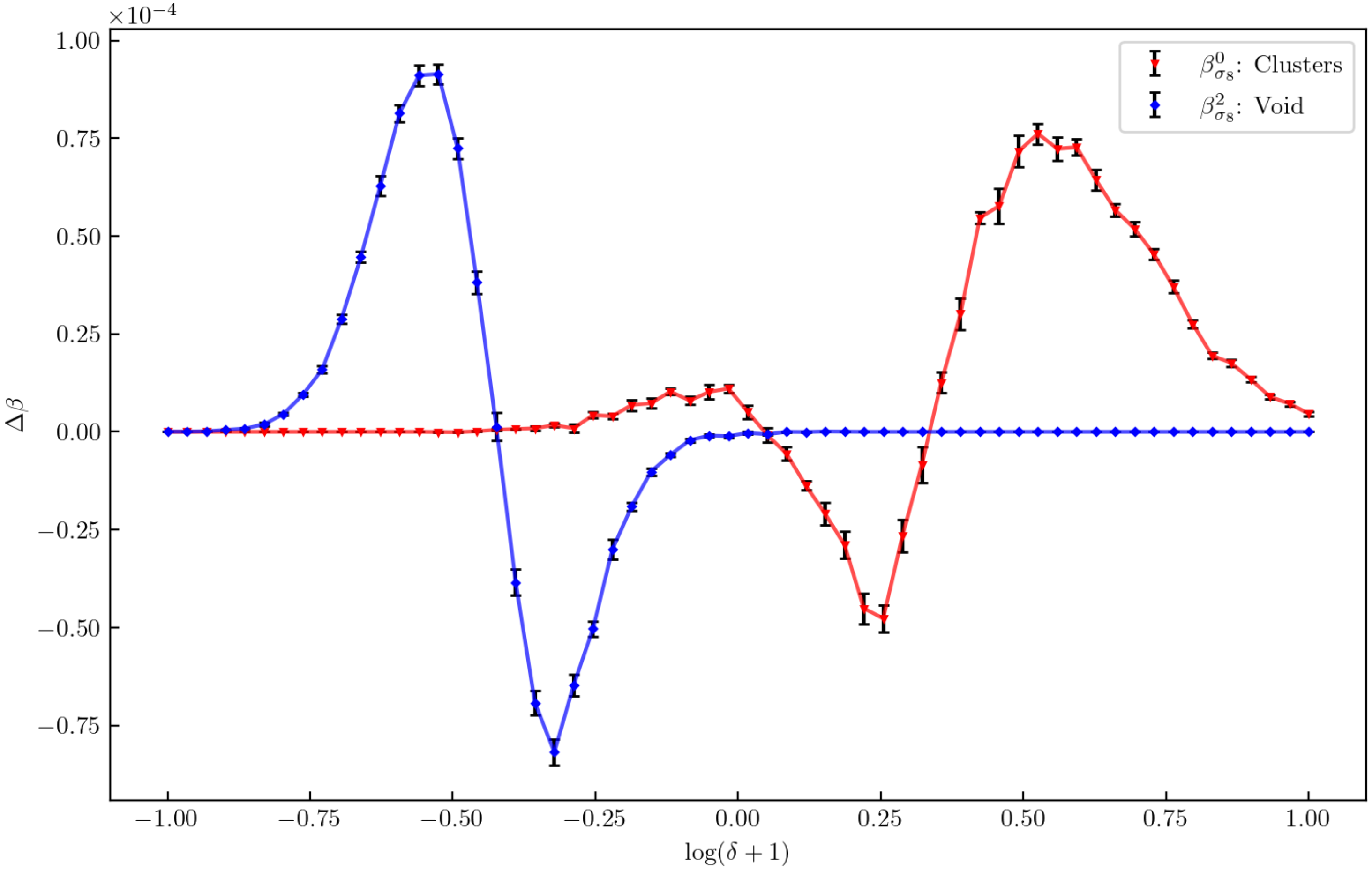
5-1-2 Persistent Diagrams at z=0



5-1-3

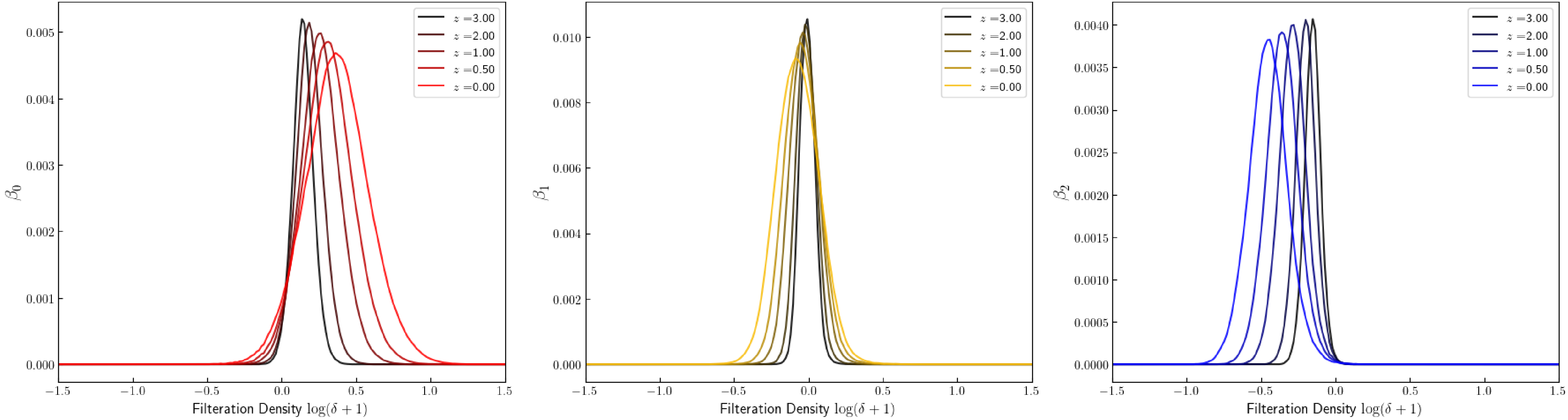

$\times 10^{-5}$
3
2
1
0
−1
$\Delta\beta$
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$\log(\delta+1)$
β^0_ω: Clusters
β^2_ω: Void

5-1-4

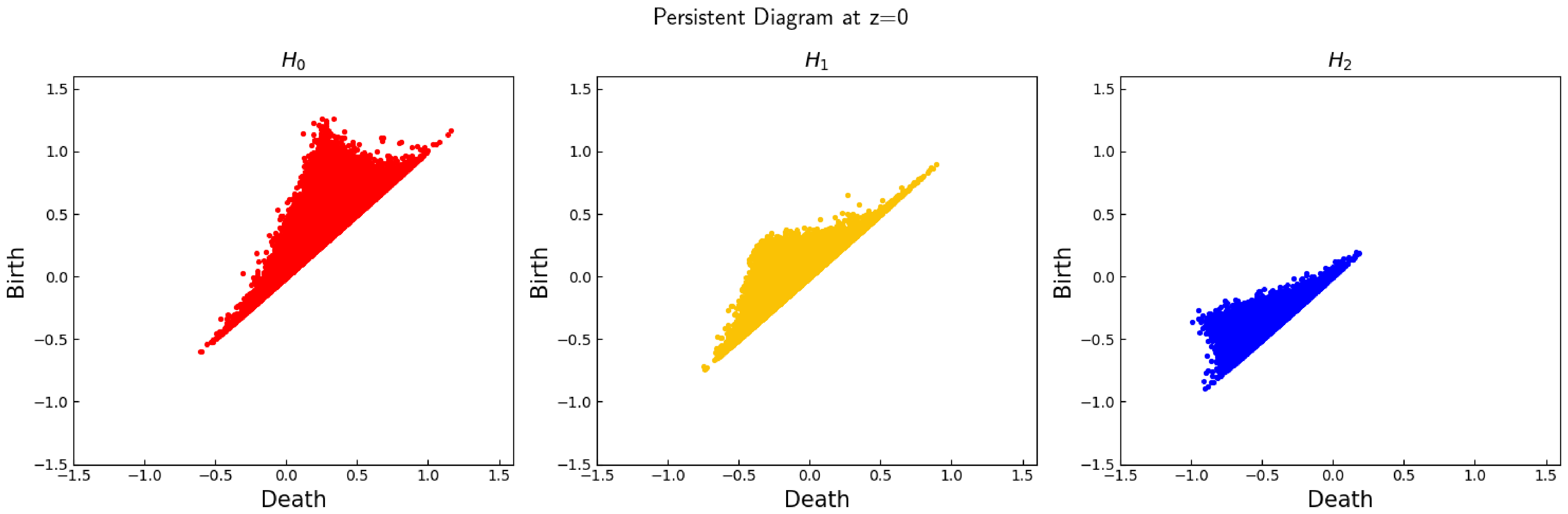

$\times 10^{-4}$
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
$\Delta\beta$
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$\log(\delta+1)$
$\beta^0_{\sigma_8}$: Clusters
$\beta^2_{\sigma_8}$: Void


Persistent
Homology of
Cosmic Web

5-2-1

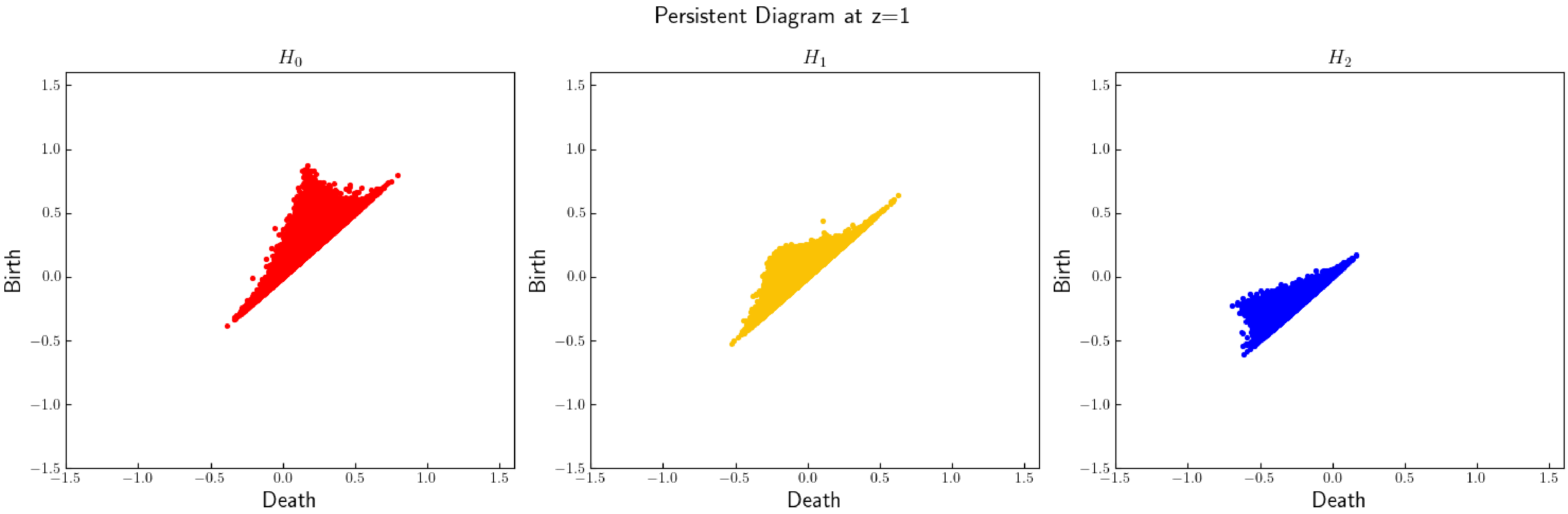

β_0
β_1
β_2
$z = 3.00$
$z = 2.00$
$z = 1.00$
$z = 0.50$
$z = 0.00$
Filteration Density $\log(\delta + 1)$

5-2-2


Persistent Diagram at z=0
H_0
H_1
H_2
Birth
Death
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5

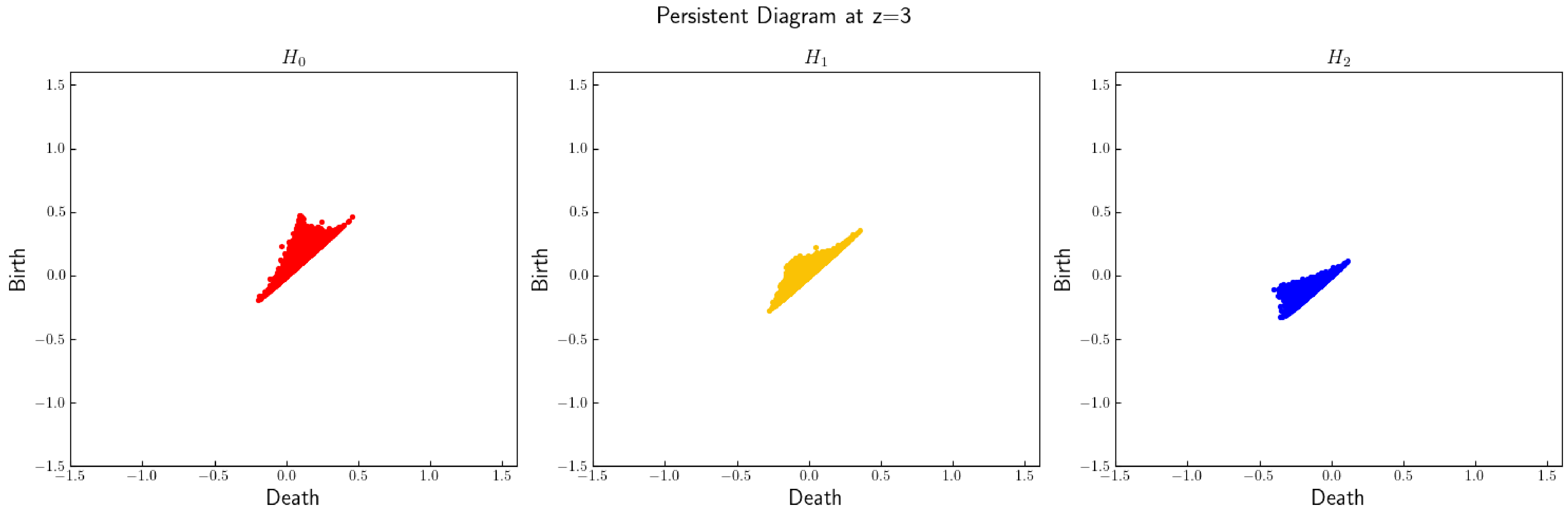
Persistent Homology of Cosmic Web

5-2-2



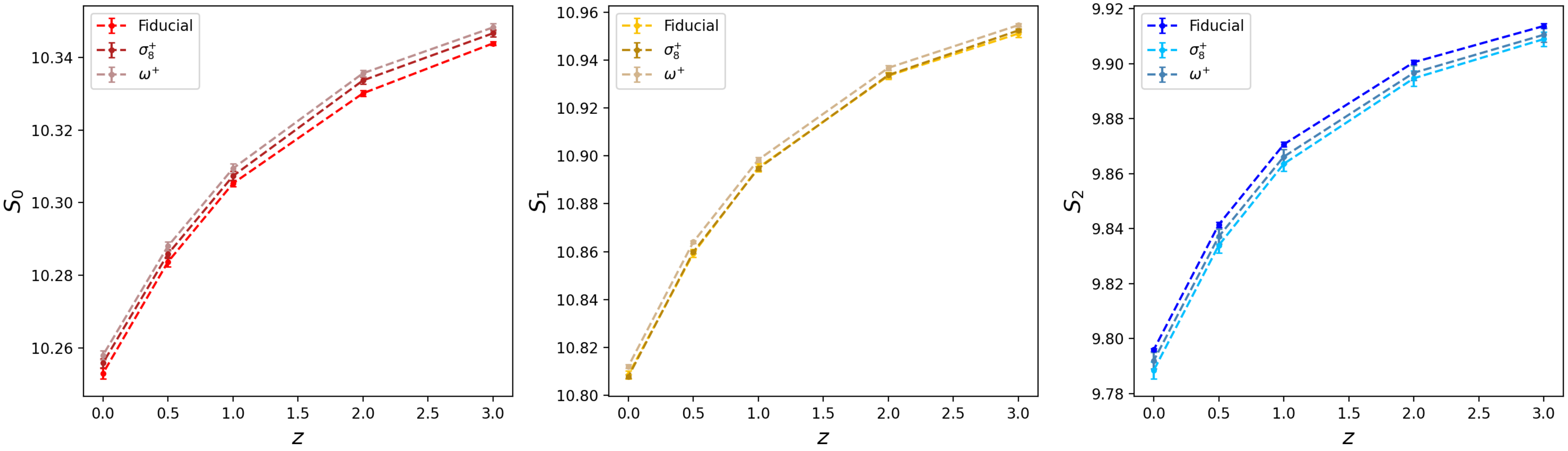
Persistent Homology of Cosmic Web

5-2-2




Persistent Homology of Cosmic Web

5-2-3


Fiducial
σ_8^+
ω^+
S_0
z
S_1
S_2

6 Conclusion

- *Using betti curves we saw that which features are* ***prominant*** *at different density thresholds .*
- *Using persistent diagram we saw the* ***most persistent features*** *at different thresholds.*
- *By betti curves and persistence diagrams at different redshifts we can see the hierarchical structure formation and results shows that β and β are highly affected by going to higher redshifts rather than β*
- *Persistent Entropy shows that evolution of* ***filaments are more sensitive*** *to changing redshifts rather than clusters and void.*
- *According to persistent entropy , voids are better for constraining* σ_8 *and clusters are better for constraining ω.*

7-3 Whats Next?

- *Using other changed parameters of Quijote Simulation and see the differences in betti curves and persistent diagrams*
- *After having these parameters we can use fisher forcast methods for constrain the parameters of our model.*
- *Using CONCEPT or Gadget4 simulation we can change the cosmological model like warm dark matter and then we can have a constrain on cosmological models*
- *Using CONCEPT or Gadget4 simulation we can switch to modified gravity and then we can have a constrain on gravity models*

Thank You for Your Attention!