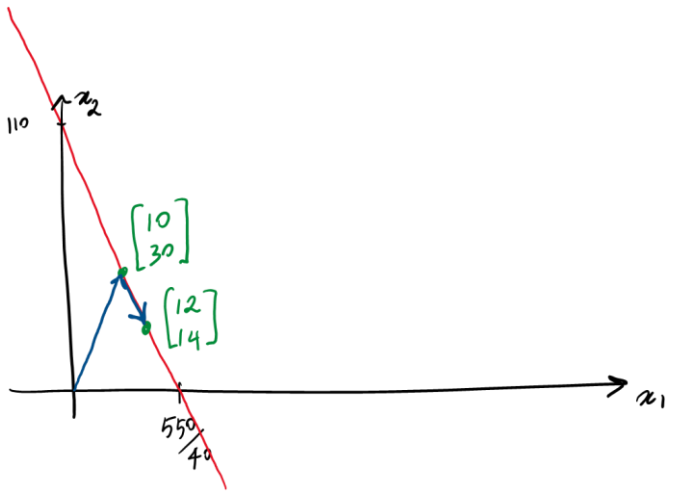
LİNEER CEBİR

1/2/2021, DERS 2

İlker Birbil

GENEL ÇÖZÜM

$$40x_1 + 5x_2 = 550$$



$$\underbrace{\begin{bmatrix}10\\30\end{bmatrix}}_{x_0} + \lambda \underbrace{\begin{bmatrix}2\\-16\end{bmatrix}}_{d} = \begin{bmatrix}10+2\lambda\\30-16\lambda\end{bmatrix}$$

GENEL ÇÖZÜM KÜMESİ

$$\{x \in \mathbb{R}^2 : x = x_0 + \lambda d,\ \lambda \in \mathbb{R}\}$$

$$A = \begin{bmatrix}40 & 5\end{bmatrix} \qquad b = 550$$

① $Ax_0 = b$

② $Ad = 0$

① VE ② ŞARTLARINI SAĞLAYAN HER x_0 VE d İÇİN DOĞRU

$$\underbrace{Ax_0}_{b} + \lambda \underbrace{Ad}_{0} = b$$

SATIR İŞLEMLERİ

$$Ax = b$$

① BİR SATIRI DİĞERİNE EKLEMEK

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\ &\vdots \\ \lambda(a_{m1}x_1 + \cdots + a_{mn}x_n) &= b_m\lambda \end{aligned}$$

② ③ $(\lambda \in \mathbb{R}\setminus\{0\})$

$$\begin{array}{rcrcrcrcr} -2x_1 & + & 4x_2 & - & 2x_3 & - & x_4 & + & 4x_5 & = & -3 \\ 4x_1 & - & 8x_2 & + & 3x_3 & - & 3x_4 & + & x_5 & = & 2 \\ x_1 & - & 2x_2 & + & x_3 & - & x_4 & + & x_5 & = & 0 \\ x_1 & - & 2x_2 & & & - & 3x_4 & + & 4x_5 & = & a \end{array}$$

①,②,③

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & x_4 & + & x_5 & = & 0 \\ & & & & x_3 & - & x_4 & + & 3x_5 & = & -2 \\ & & & & & & x_4 & - & 2x_5 & = & 1 \\ & & & & & & & & 0 & = & a+1 \end{array}$$

2 = 0 = -1 = 1 = -1

PİVOTLAR

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

GENEL ÇÖZÜM

$$x_0 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

GENEL ÇÖZÜM KÜMESİ

$$\left\{ x \in \mathbb{R}^5 : x = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \quad \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$

x_0 d_1 d_2

$$A(x_0 + \lambda_1 d_1 + \lambda d_2) = \underbrace{Ax_0}_{b} + \underbrace{\lambda_1 A d_1}_{0} + \underbrace{\lambda_2 A d_2}_{0} = b$$

MATRİS TERSİ

$$[A|I_n] \quad \rightsquigarrow \cdots \rightsquigarrow \quad [I_n|A^{-1}]$$

①, ②, ③

$A \in \mathbb{R}^{n \times n}$

A^{-1} ✓

$$Ax = b \iff x = A^{-1}b$$

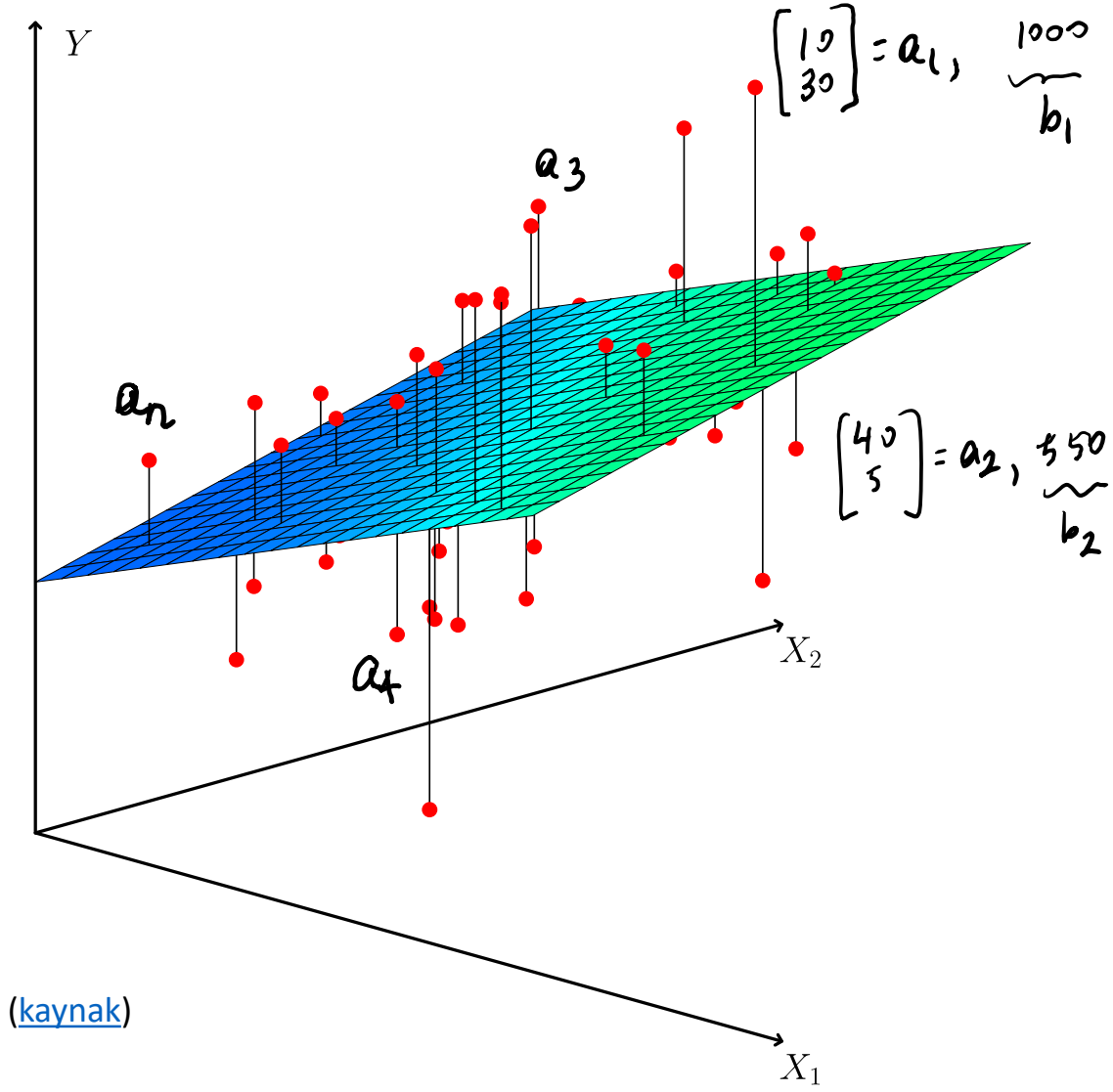
$A \in \mathbb{R}^{m \times n}$

$\text{rank}(A) = n$ ✓

$$Ax = b \iff A^\top Ax = A^\top b \iff x = (A^\top A)^{-1} A^\top b$$

MOORE - PENROSE TERSİ

DOĞRUSAL BAĞLANIM



(kaynak)

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 = b_n$$

$$Ax = b$$

$$x = (A^TA)^{-1}A^Ty$$

EN KÜÇÜK KARELER ÇÖZÜMÜ

★

VEKTÖR UZAYLARI

$$V = \mathbb{R}^n \qquad x, y \in V$$

$$x + y = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n) \in V$$

$$\lambda x = (\lambda x_1, \lambda x_2, \dots, \lambda x_n) \in V$$

$$V = \mathbb{R}^{m \times n} \qquad A, B \in V$$

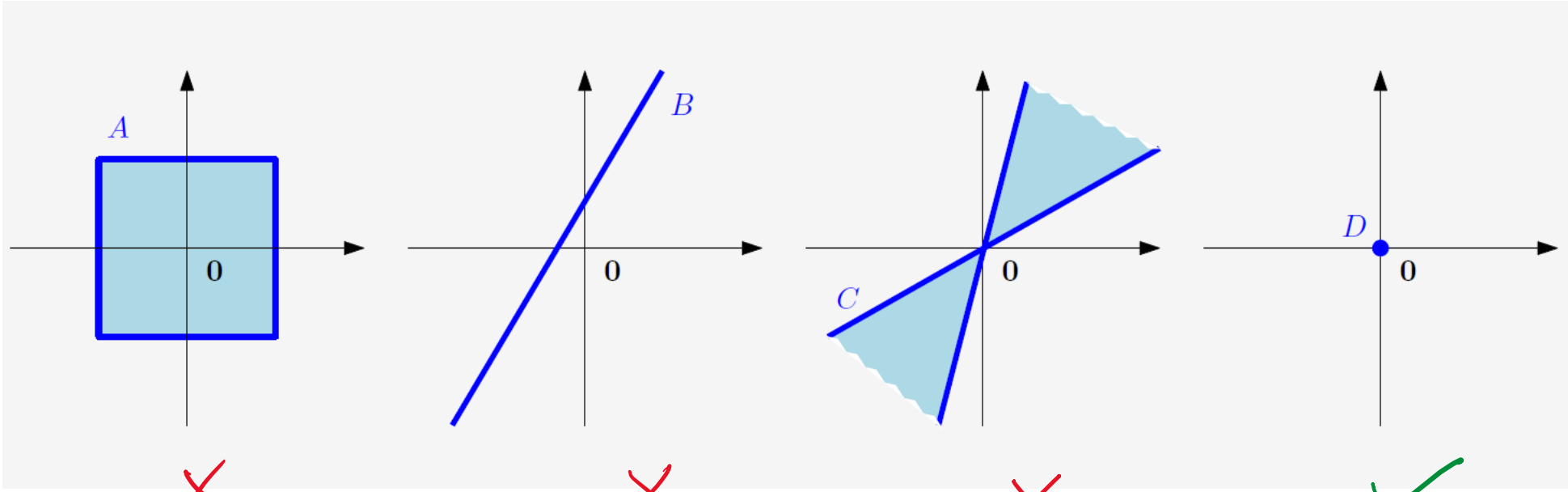
$$A + B = \begin{bmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & & \vdots \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{bmatrix} \in V \qquad \lambda A = \begin{bmatrix} \lambda a_{11} & \cdots & \lambda a_{1n} \\ \vdots & & \vdots \\ \lambda a_{m1} & \cdots & \lambda a_{mn} \end{bmatrix} \in V$$

VEKTÖR ALTUZAYLARI

$U \subseteq V$

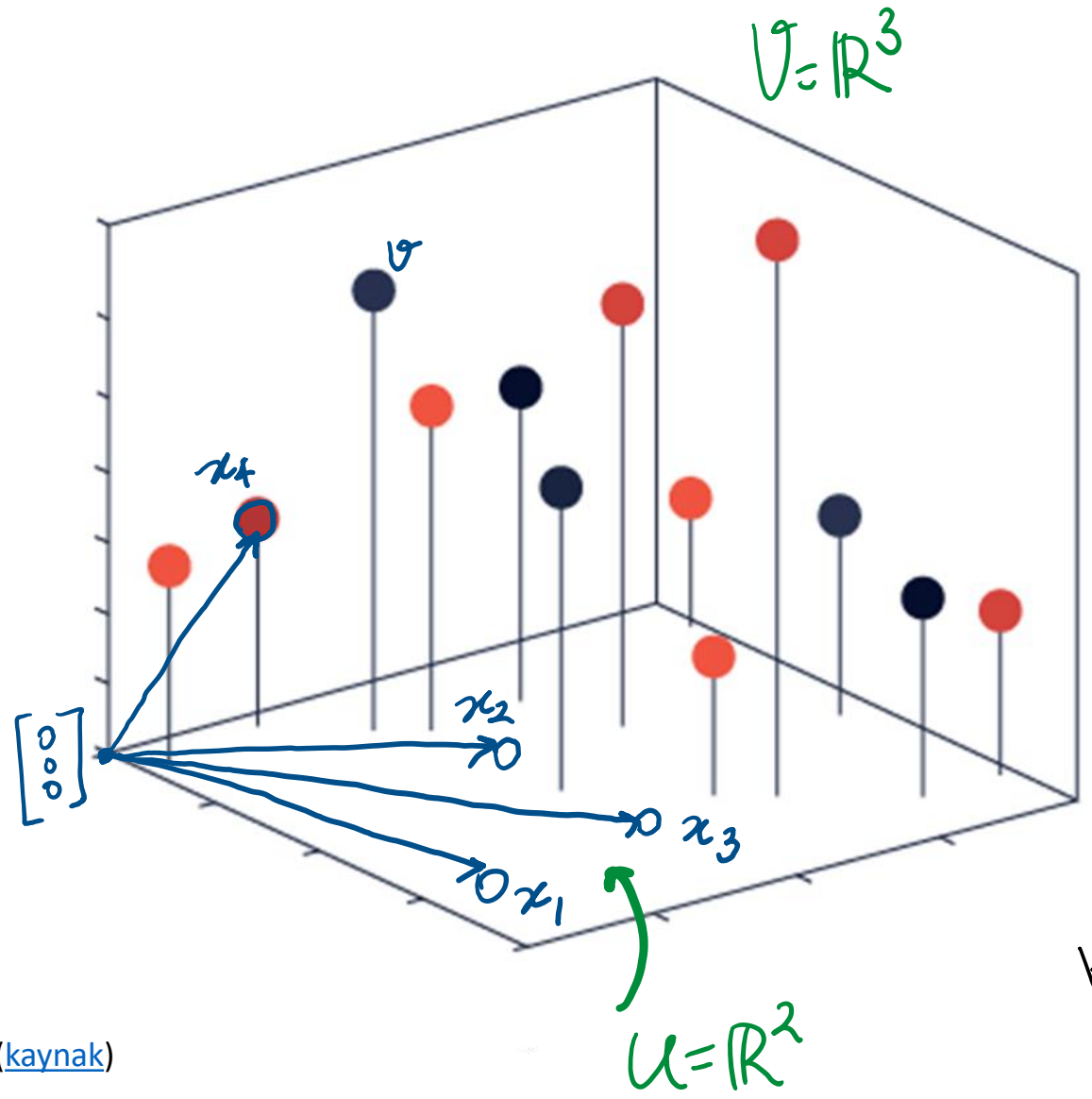
$\forall x,y \in U : x+y \in U$

$\forall x \in U : \lambda x \in U$



✗ ✗ ✗ ✓

DOĞRUSAL BAĞIMSIZLIK



(kaynak)

$x_1, x_2, \dots, x_k \in V$

$$v = \lambda_1 x_1 + \lambda_2 x_2 + \dots + \lambda_k x_k = \sum_{i=1}^{k} \lambda_i x_i \qquad \lambda_i \in \mathbb{R}$$

DOĞRUSAL KOMBİNASYON

$$\sum_{i=1}^{k} \lambda_i x_i = 0 \iff \lambda_1 = \lambda_2 = \dots = \lambda_k = 0$$

$\{x_1, \dots, x_k\}$ DOĞRUSAL BAĞIMSIZ VEKTÖRLER

$\{x_2, x_3\}$ ✓ $\{x_1, x_2, x_4\}$ ✓

$\{x_1, x_2, x_3\}$ ✗ $\{x_1, x_2, x_3, x_4\}$ ✗

$\forall v \in \mathbb{R}^3, \exists \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}: \; v = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_4$

$\forall v \in \mathbb{R}^2, \exists \lambda_1, \lambda_2 \in \mathbb{R}: \; v = \lambda_1 x_2 + \lambda_2 x_3$

DOĞRUSAL BAĞIMSIZLIK

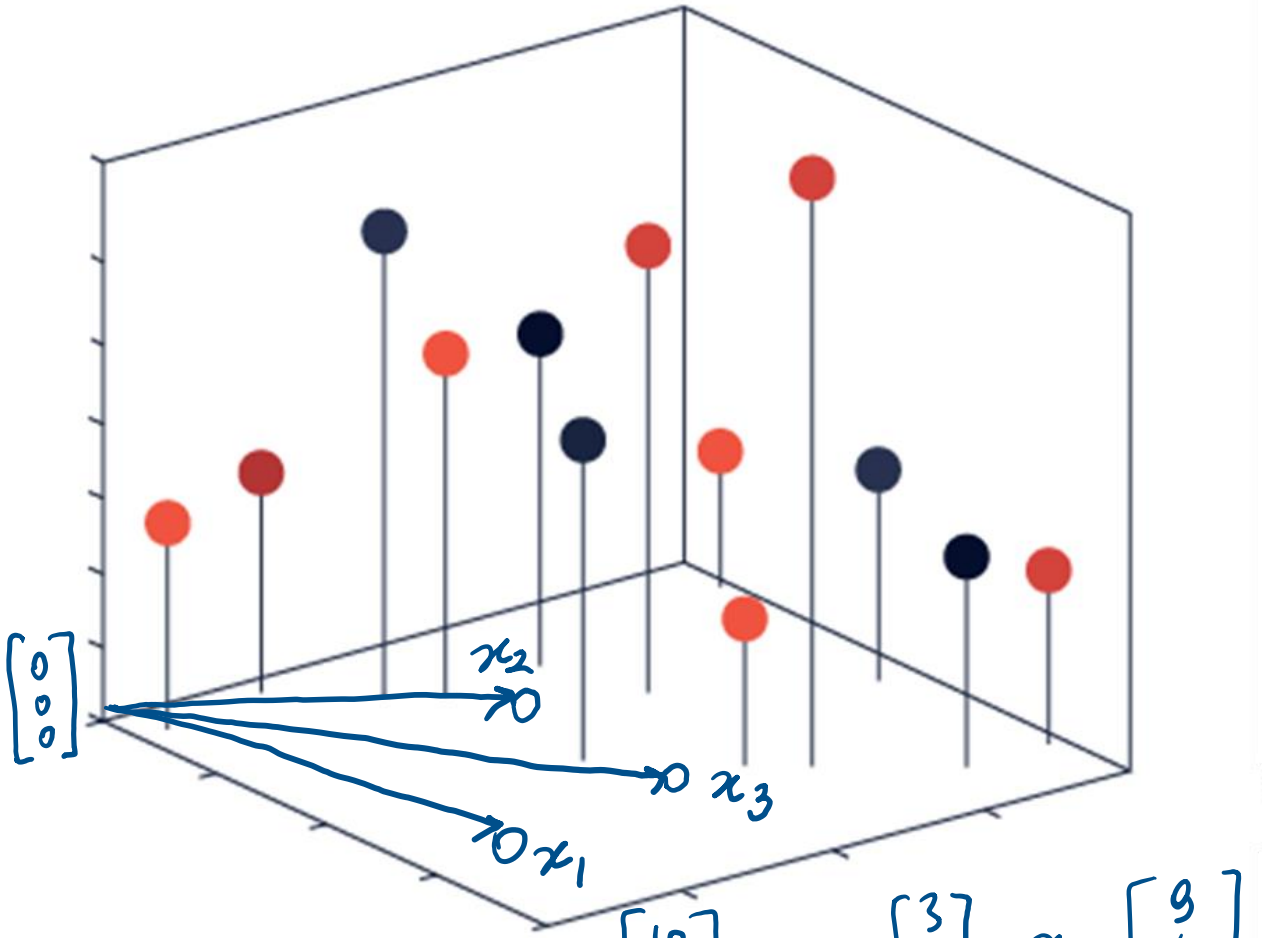
$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{R}^4$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -2 \\ -3 & 0 & 1 \\ 4 & 2 & 1 \end{bmatrix} \overset{①,②,③}{\rightsquigarrow \cdots \rightsquigarrow} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\{x_1, x_2, x_3\}$ DOĞRUSAL BAĞIMSIZ

$\forall v \in \mathbb{R}^4, \quad \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \overset{?}{=} v$

$v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$? HAYIR



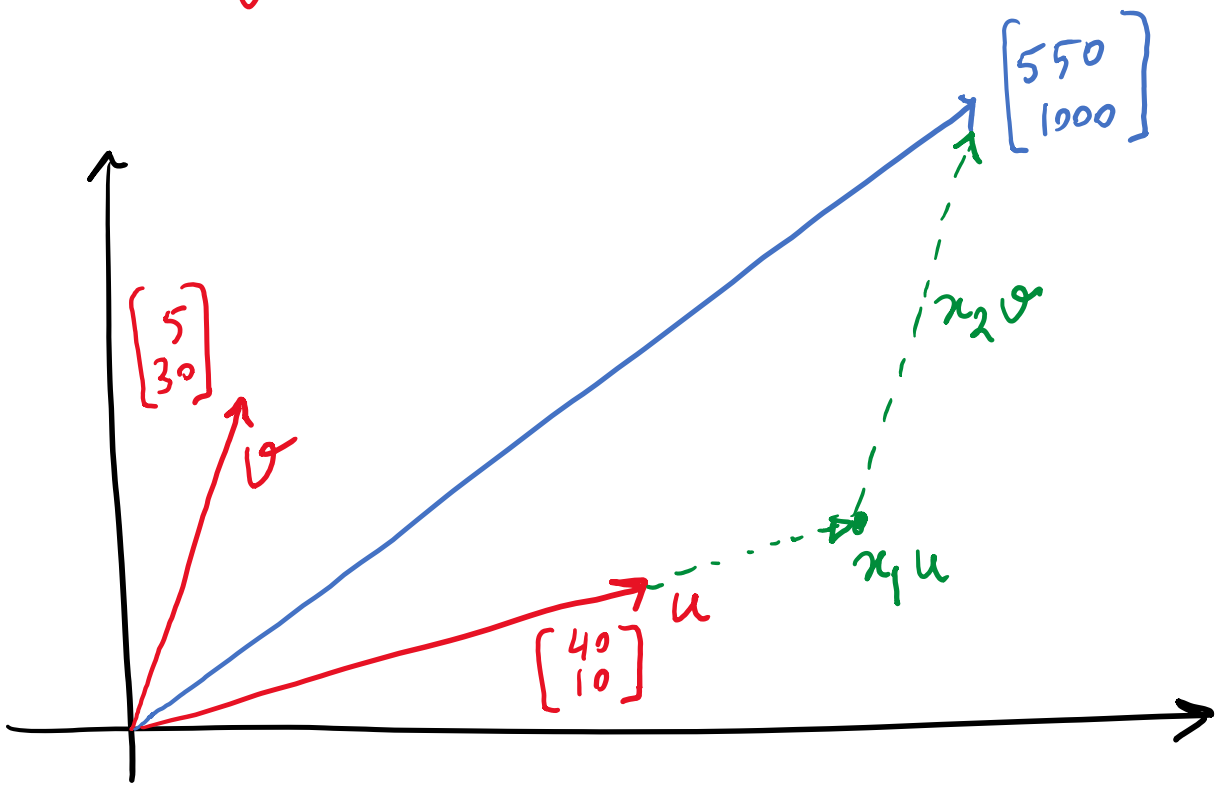
$x_1 = \begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$

(kaynak)

DOĞRUSAL BAĞIMSIZLIK

$$\begin{bmatrix} 40 \\ 10 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ 30 \end{bmatrix} x_2 = \begin{bmatrix} 550 \\ 1000 \end{bmatrix}$$

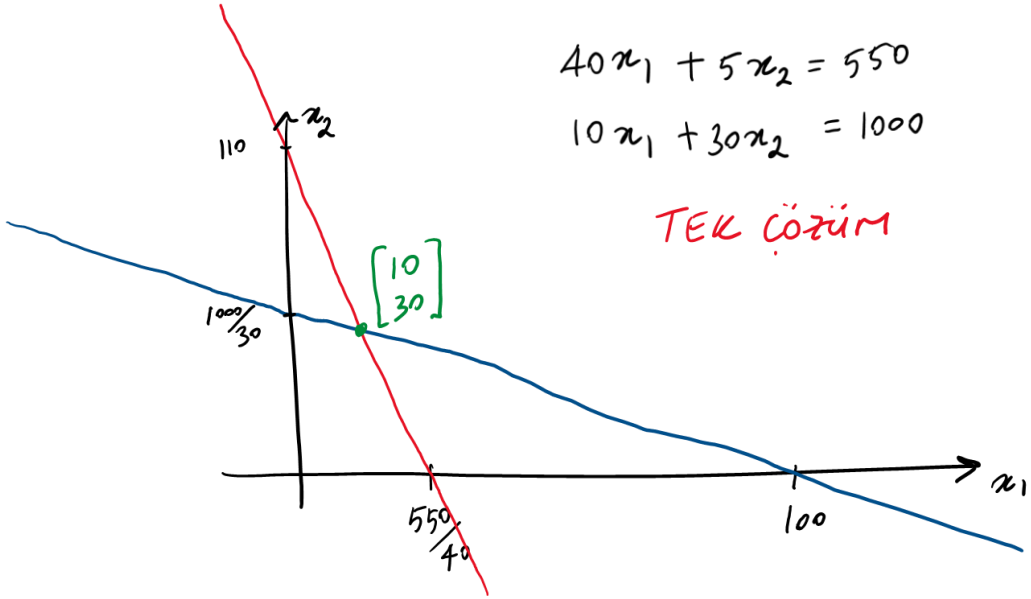
u v



$40x_1 + 5x_2 = 550$

$10x_1 + 30x_2 = 1000$

TEK ÇÖZÜM

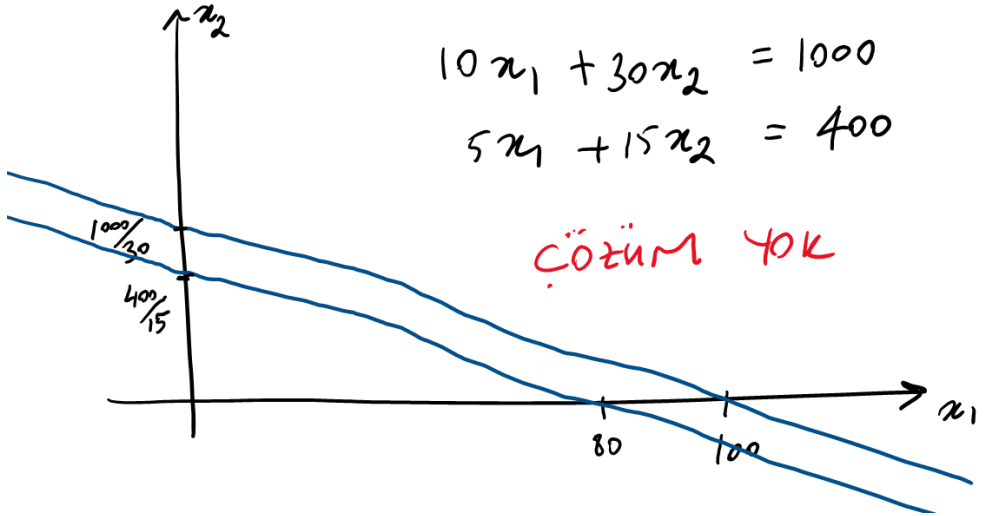


u, v DOĞRUSAL BAĞIMSIZ

DOĞRUSAL BAĞIMSIZLIK

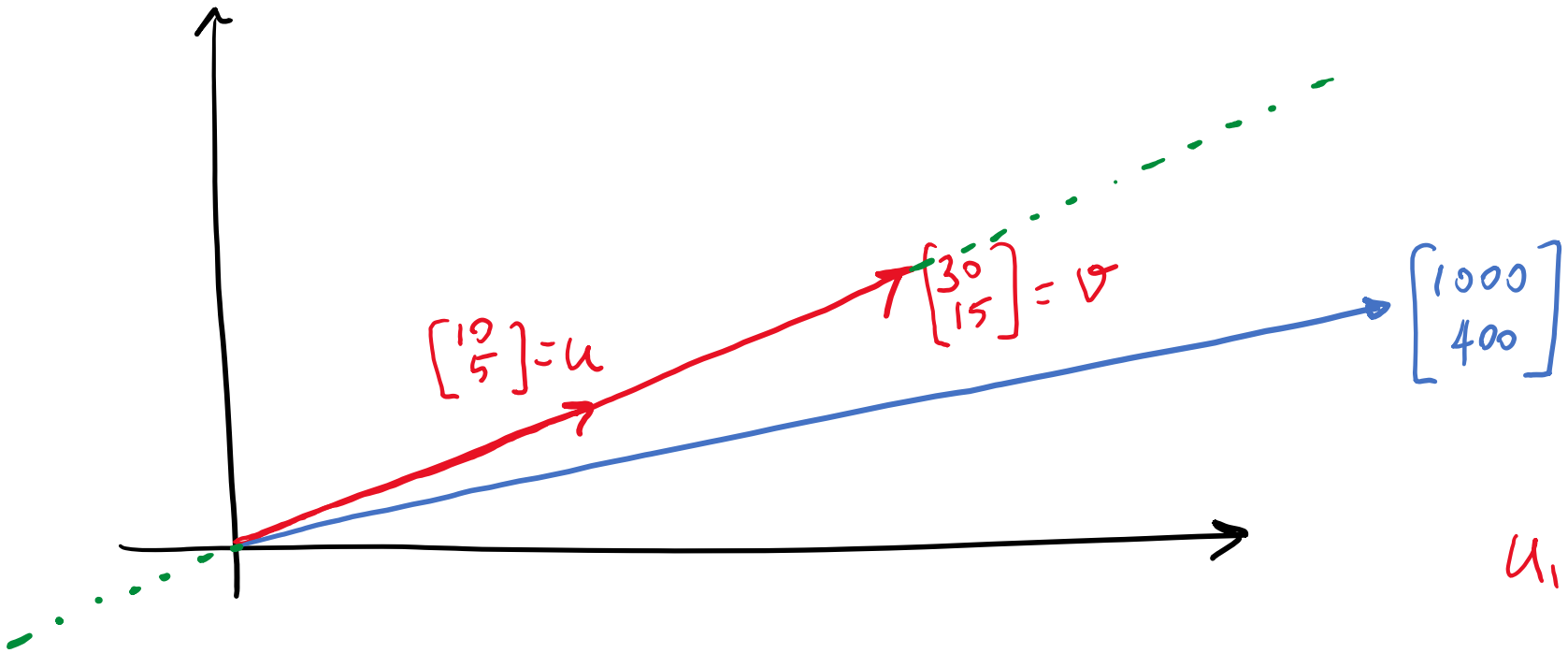
$$\begin{bmatrix}10\\5\end{bmatrix}x_1+\begin{bmatrix}30\\15\end{bmatrix}x_2=\begin{bmatrix}1000\\400\end{bmatrix}$$

u v



$$10x_1+30x_2=1000$$
$$5x_1+15x_2=400$$

ÇÖZÜM YOK



u, v DOĞRUSAL BAĞIMLI