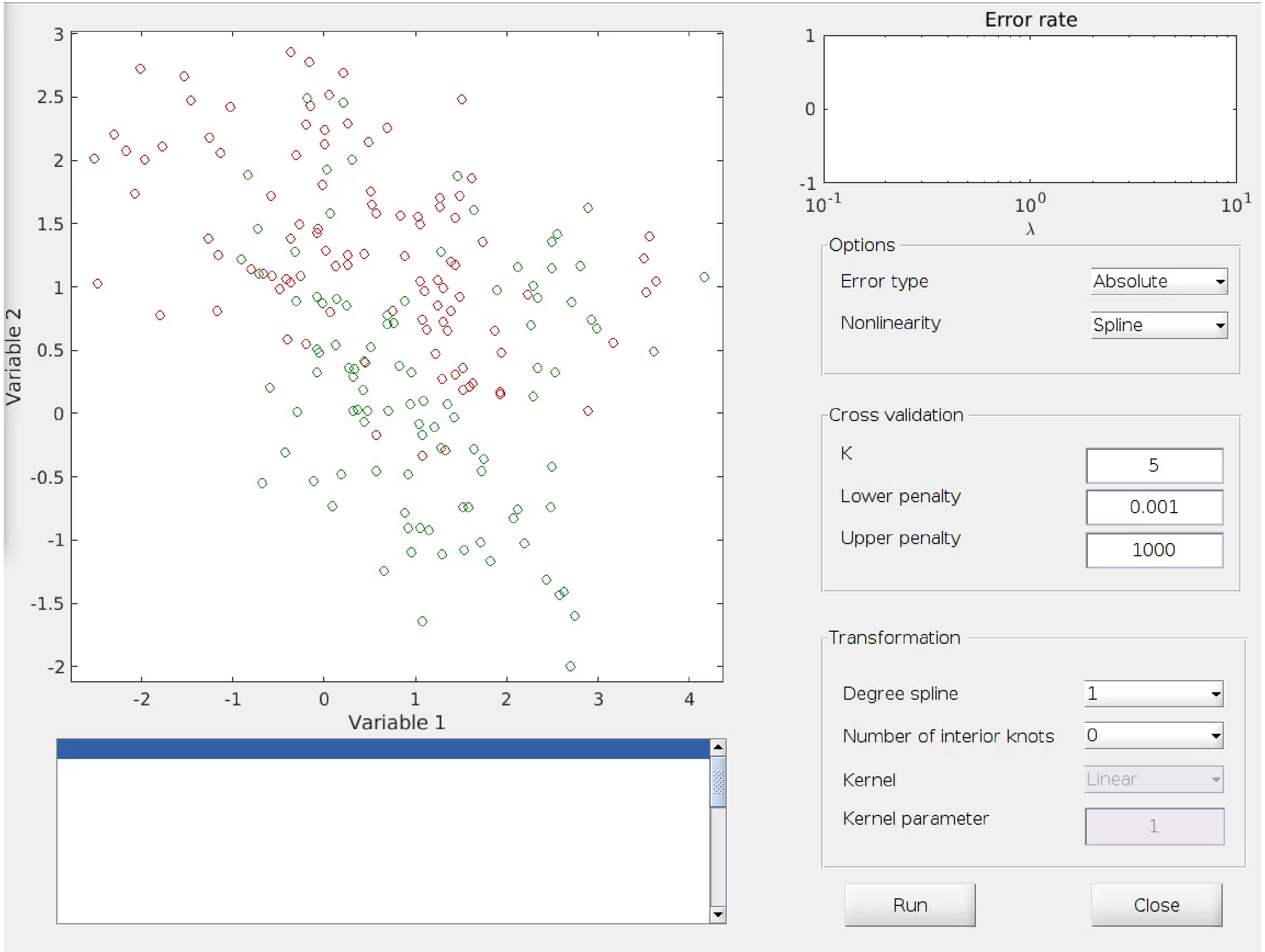
DESTEK VEKTÖR MAKİNELERİ

5/2/2021

İlker Birbil



Error rate
λ
Variable 1
Variable 2
Options
Error type
Absolute
Nonlinearity
Spline
Cross validation
K
5
Lower penalty
0.001
Upper penalty
1000
Transformation
Degree spline
1
Number of interior knots
0
Kernel
Linear
Kernel parameter
1
Run
Close

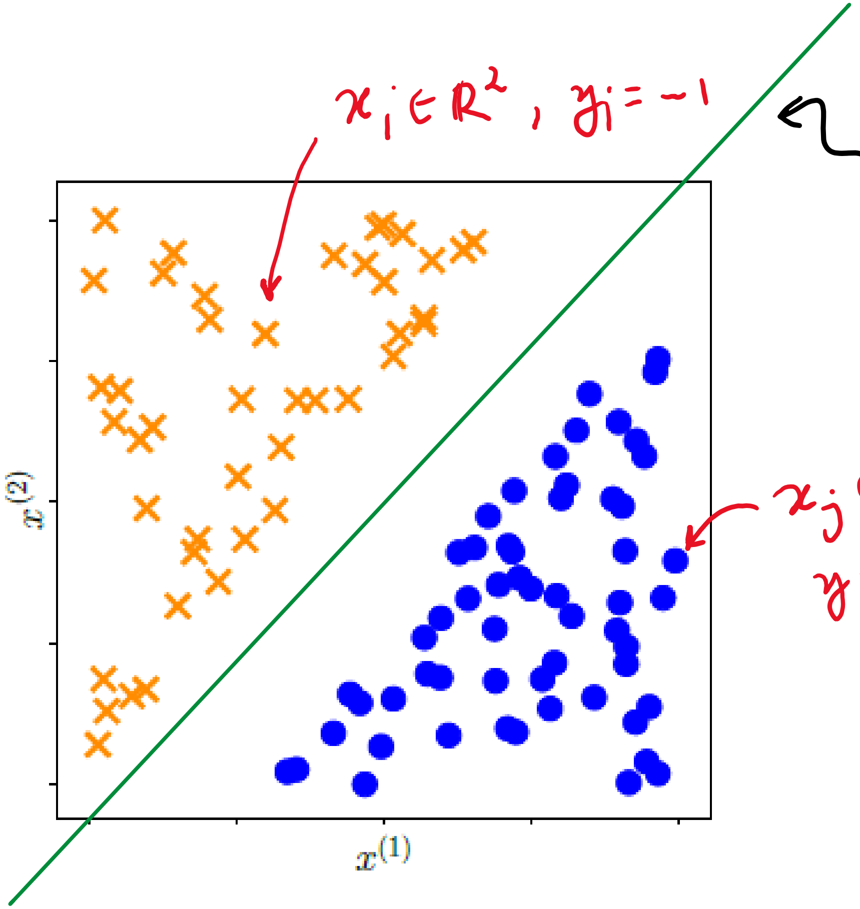
İKİLİ SINIFLANDIRMA

$$\{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_N, y_N)\}$$

$$\boldsymbol{x}_n \in \mathbb{R}^D$$

$$y_n \in \{+1, -1\}$$

$x_i \in \mathbb{R}^2$, $y_i = -1$

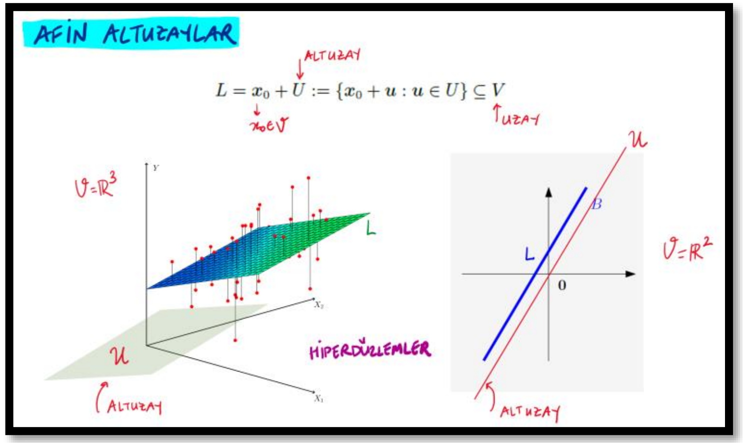


$\{x \in \mathbb{R}^D : w^T x + b = 0\}$

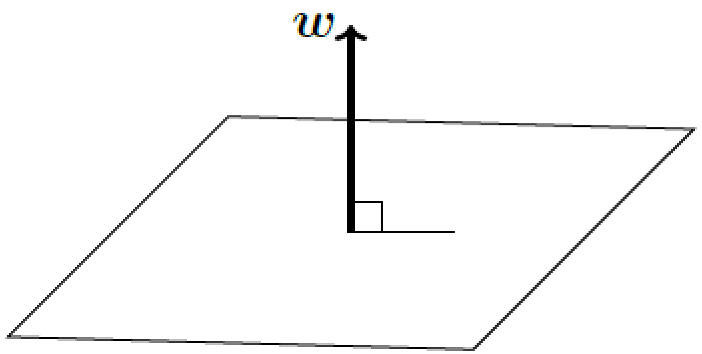
$w^T x = \langle w, x \rangle$

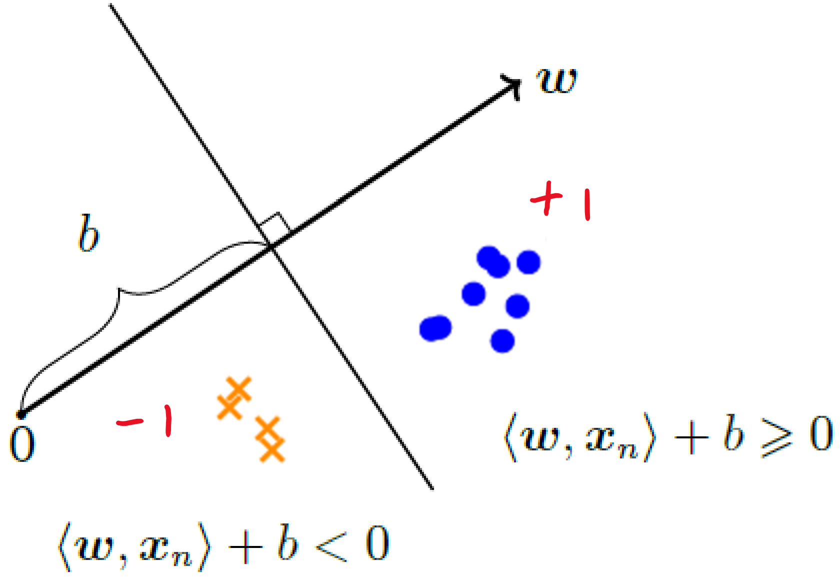
$x_j \in \mathbb{R}^2$

$y_j = +1$



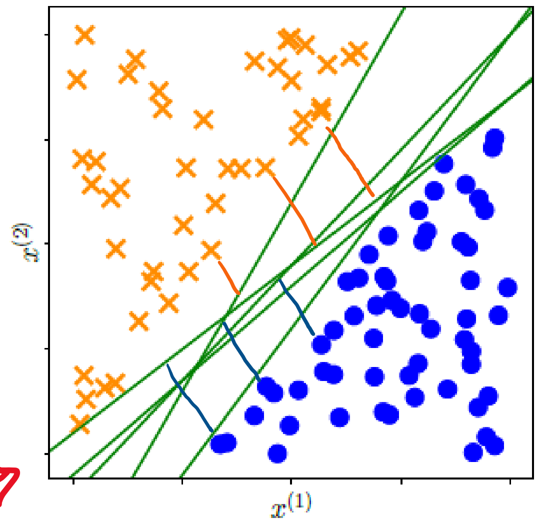
HİPERDÜZLEMLER



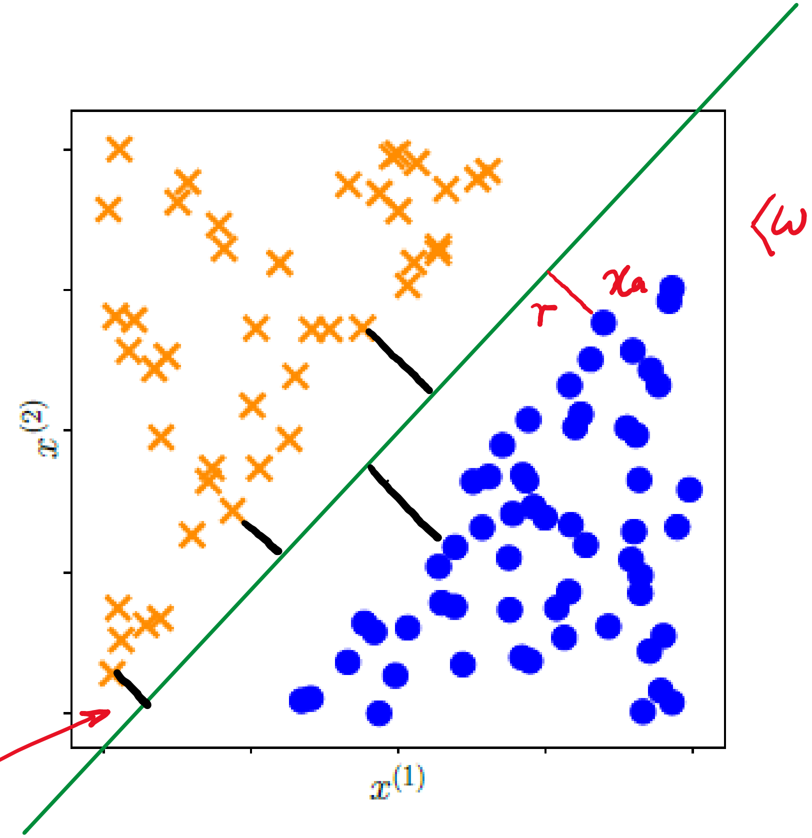


$$y_n(\langle w, x_n \rangle + b) \geqslant 0$$

?



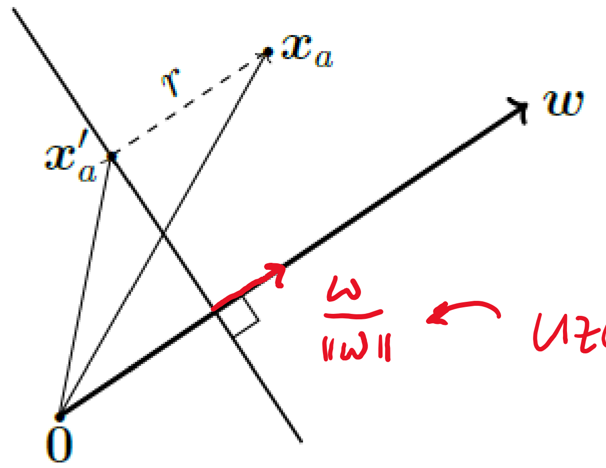
HİPERDÜZLEMLER



$\langle w, x_a \rangle + b > 0$

MARJİN

(MARGIN)

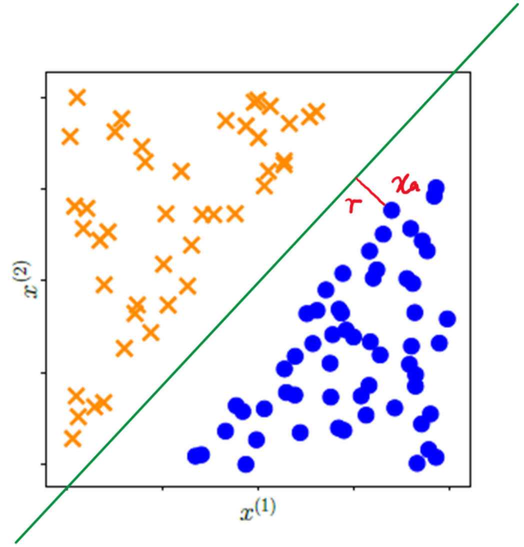


$\frac{w}{\|w\|}$ UZUNLUĞU = 1

$$x_a = x'_a + r\frac{w}{\|w\|}$$

$$y_n(\langle w, x_n \rangle + b) \geqslant r$$

OPTIMIZASYON



$$\text{enbüyükle } r$$
$$\text{öyle ki } \quad y_n(\langle w, x_n\rangle + b) \geq r,$$
$$\|w\| = 1,$$
$$r > 0.$$

$$\equiv$$

↑

TEOREM 12.1

$$\text{enküçükle } \tfrac{1}{2}\|w\|^2$$
$$\text{öyle ki } \quad y_n(\langle w, x_n\rangle + b) \geq 1.$$

BÜTÜN

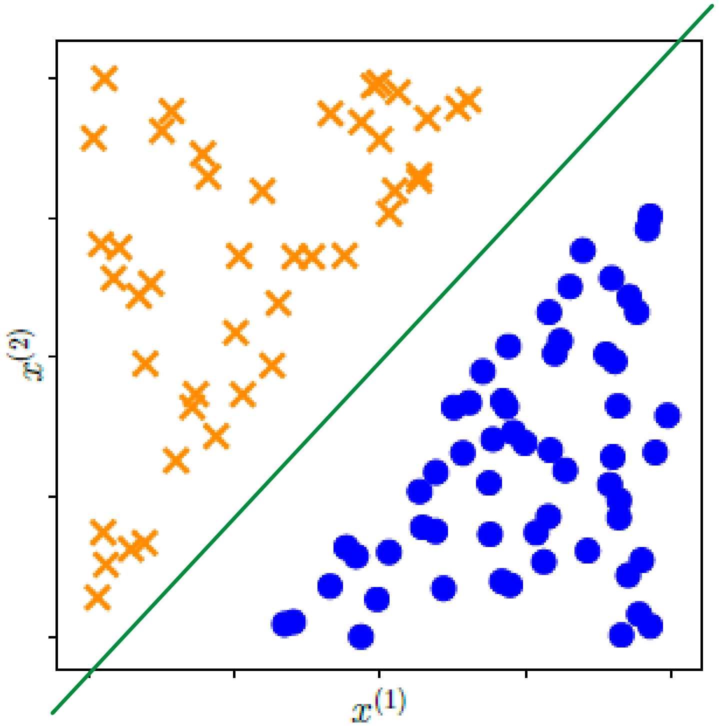
x_n'LER İÇİN

(TOPLAM N KISIT!)

KATI VE ESNEK MARJİN

enküçükle $\frac{1}{2}\|w\|^2$

öyle ki $y_n(\langle w, x_n\rangle + b) \geq 1$



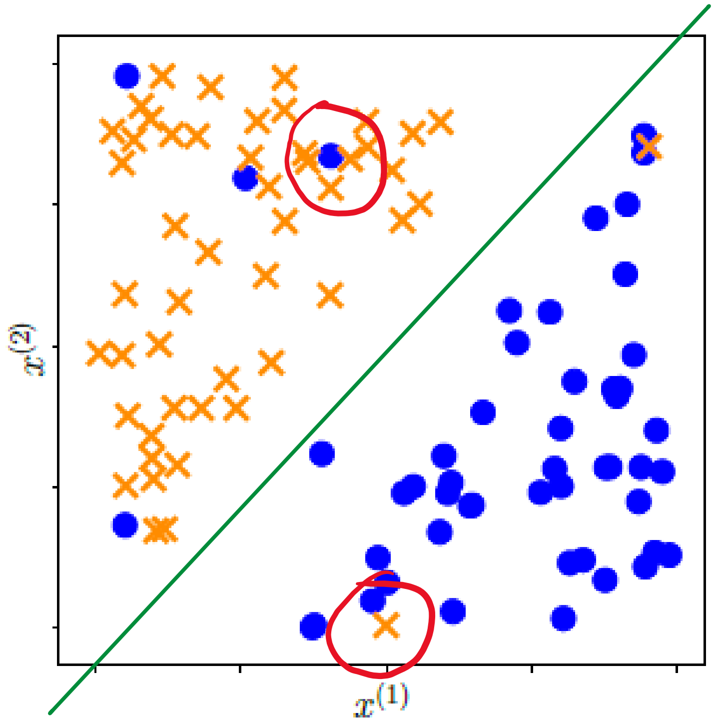
KATI (HARD) MARJİN

HİPERPARAMETRE

enküçükle $\frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n$

öyle ki $y_n(\langle w, x_n\rangle + b) \geq 1 - \xi_n$

$\xi_n \geq 0$



ESNEK (SOFT) MARJİN

DUAL MODEL

enküçükle $\frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n$

öyle ki $y_n(\langle w, x_n\rangle + b) \geq 1 - \xi_n$

$\xi_n \geq 0$

(TOPLAM N KISIT!)

$\alpha_n \geq 0$

$\gamma_n \geq 0$

$$\underbrace{\frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n - \sum_{n=1}^{N}\alpha_n\left(y_n(\langle w, x_n\rangle + b) - 1 + \xi_n\right) - \sum_{n=1}^{N}\gamma_n\xi_n}_{\mathcal{L}(w, b, \xi, \alpha, \gamma)}$$

$$\mathcal{L}(w, b, \xi, \alpha, \gamma) \rightarrow \underbrace{\frac{\partial \mathcal{L}}{\partial w}}_{=0\,?}, \underbrace{\frac{\partial \mathcal{L}}{\partial b}}_{=0\,?}, \underbrace{\frac{\partial \mathcal{L}}{\partial \xi}}_{=0\,?}$$

DUAL (EŞİZ) MODEL

$$\frac{\partial \mathfrak{L}}{\partial \boldsymbol{w}} = \boldsymbol{w}^\top - \sum_{n=1}^{N} \alpha_n y_n \boldsymbol{x}_n{}^\top = 0 \quad \Rightarrow \quad \boldsymbol{w} = \sum_{n=1}^{N} \alpha_n y_n \boldsymbol{x}_n$$

$$\frac{\partial \mathfrak{L}}{\partial b} = -\sum_{n=1}^{N} \alpha_n y_n = 0$$

$$\frac{\partial \mathfrak{L}}{\partial \xi_n} = C - \alpha_n - \gamma_n = 0$$

$$\mathcal{L}(\omega, b, \xi, \alpha, \gamma)$$

$$\tfrac{1}{2}\|\omega\|^2 + C\sum_{n=1}^{N} \xi_n - \sum_{n=1}^{N} \alpha_n\left(y_n\left(\langle \omega, x_n \rangle + b\right) - 1 + \xi_n\right) - \sum_{n=1}^{N} \gamma_n \xi_n$$

$$\mathfrak{D}(\xi, \alpha, \gamma) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle x_i, x_j \rangle + \sum_{i=1}^{N} \alpha_i + \sum_{i=1}^{N} \underbrace{(C - \alpha_i - \gamma_i)}_{=0} \xi_i$$

$$\alpha_n + \underbrace{\gamma_n}_{\geq 0} = C \quad \Rightarrow \quad \alpha_n \leq C$$

DUAL MODEL

$$-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle \boldsymbol{x}_i, \boldsymbol{x}_j \rangle + \sum_{i=1}^{N} \alpha_i$$

$$-\sum_{n=1}^{N} \alpha_n y_n = 0$$

$$\alpha_n \leq C$$

enküçükle $\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle x_i, x_j \rangle - \sum_{i=1}^{N} \alpha_i$

öyle ki $\sum_{i=1}^{N} y_i \alpha_i = 0$

$0 \leq \alpha_i \leq C$

DUAL DVM

PRIMAL-DUAL

PRIMAL

enküçükle $\frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n$

öyle ki $y_n(\langle w, x_n\rangle + b) \geq 1 - \xi_n$

$\xi_n \geq 0$

DUAL

enküçükle $\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle x_i, x_j\rangle - \sum_{i=1}^{N}\alpha_i$

öyle ki $\sum_{i=1}^{N} y_i \alpha_i = 0$

$0 \leq \alpha_i \leq C$

w^* ?

α^* ✓

α_n^*

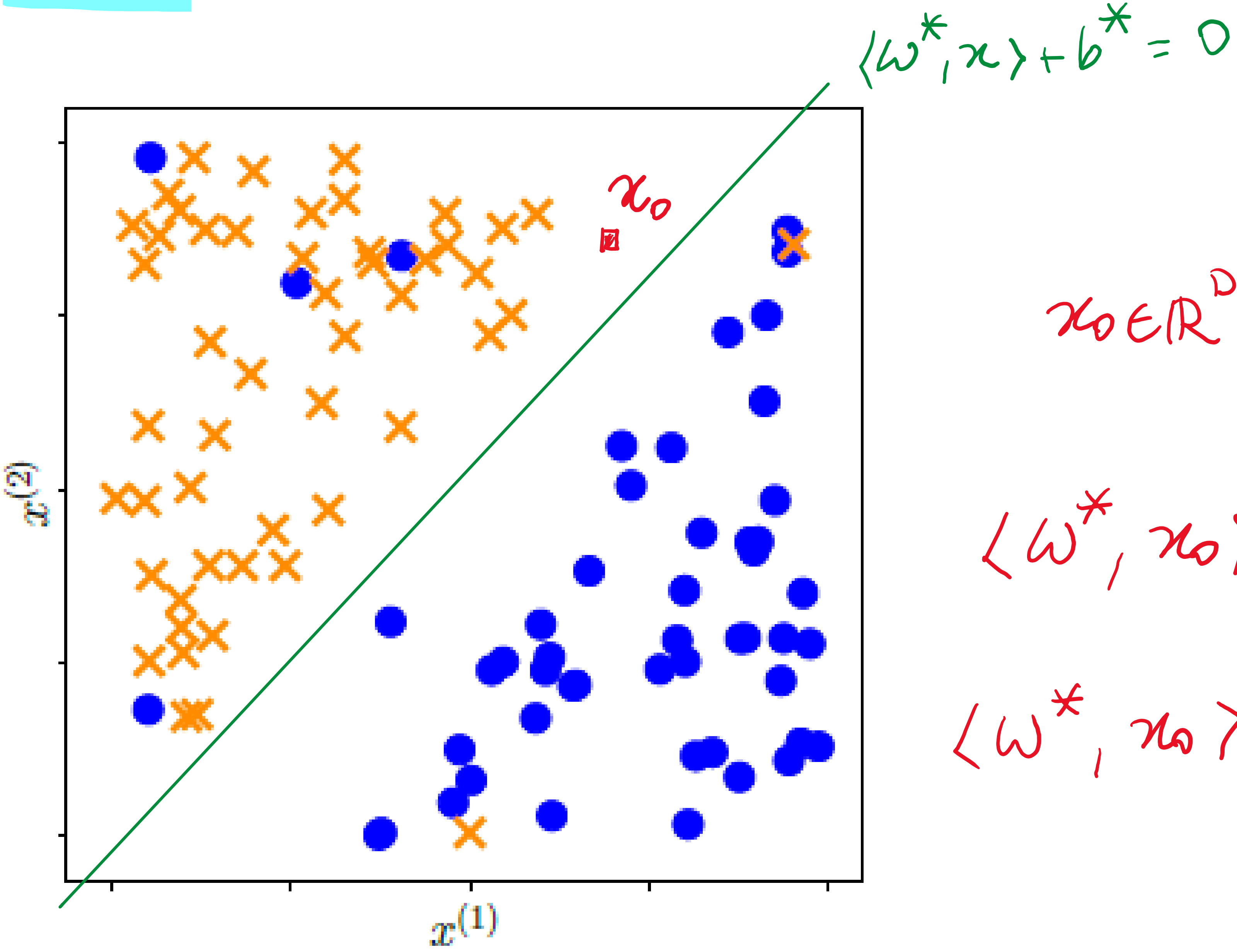
$$w = \sum_{n=1}^{N} \alpha_n y_n x_n$$

w^*

$$y_n(\langle w, x_n\rangle + b) \geq 0 \rightarrow b^*$$

...

TEST



$x_0 \in \mathbb{R}^D, \; y_0 = ?$

$\langle w^*, x_0\rangle + b^* > 0 \Rightarrow y_0 = 1$

$\langle w^*, x_0\rangle + b^* < 0 \Rightarrow y_0 = -1$

KERNEL

enküçükle $\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle x_i, x_j \rangle - \sum_{i=1}^{N} \alpha_i$

öyle ki $\sum_{i=1}^{N} y_i \alpha_i = 0$

$0 \leq \alpha_i \leq C$

$k(x_i, x_j)$

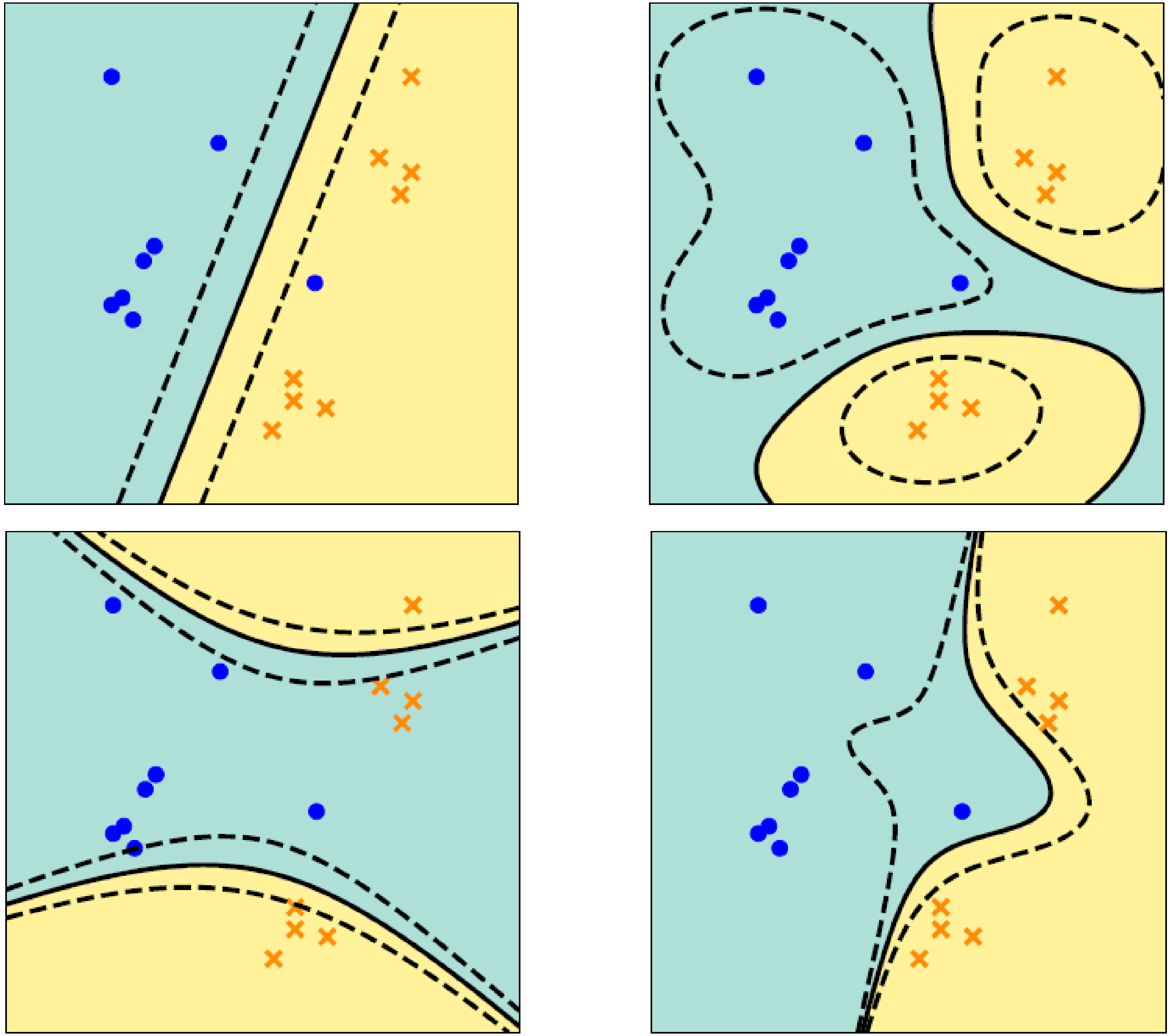
BENZERLİK FONKSİYONU

KERNEL : $k(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle_{\mathcal{H}}$

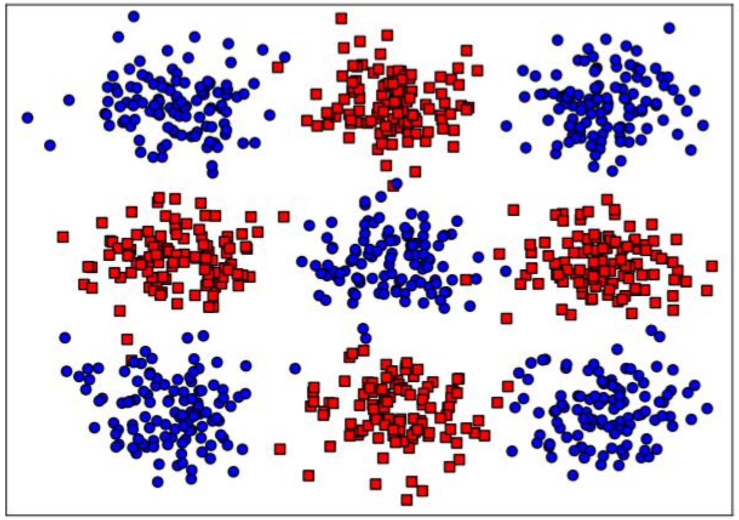
DÖNÜŞÜM

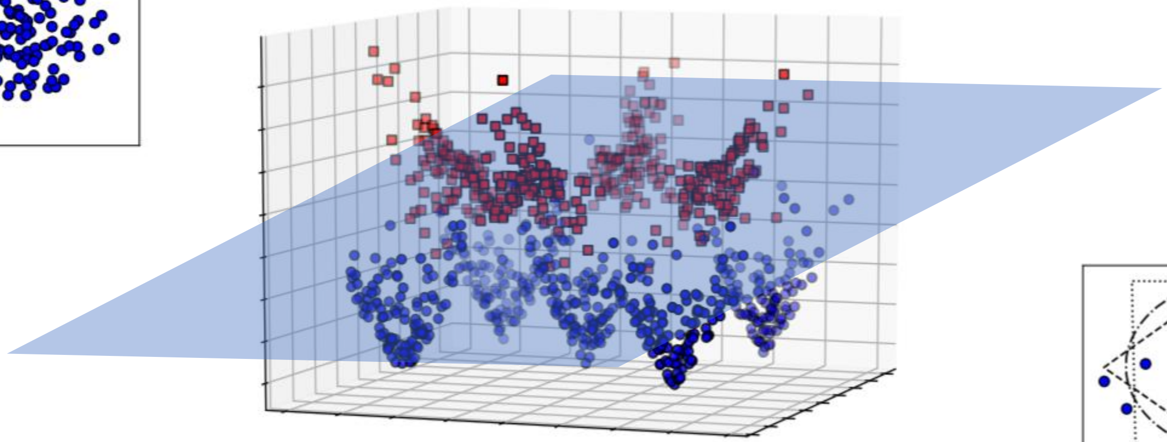
HILBERT UZAYI

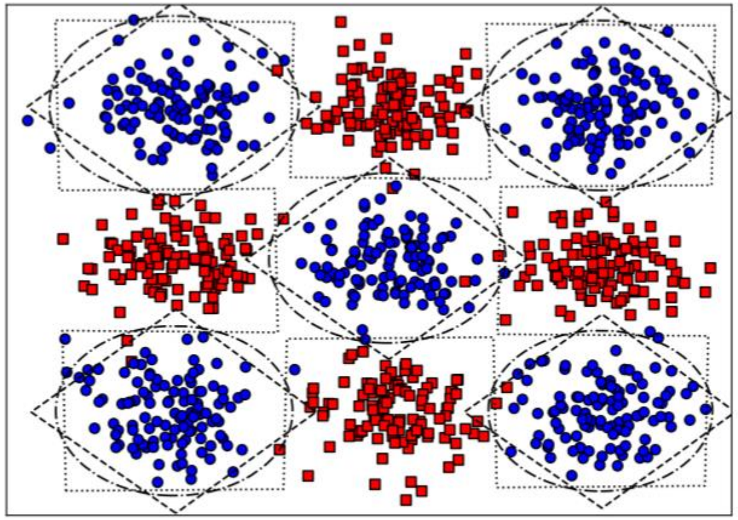
KERNEL



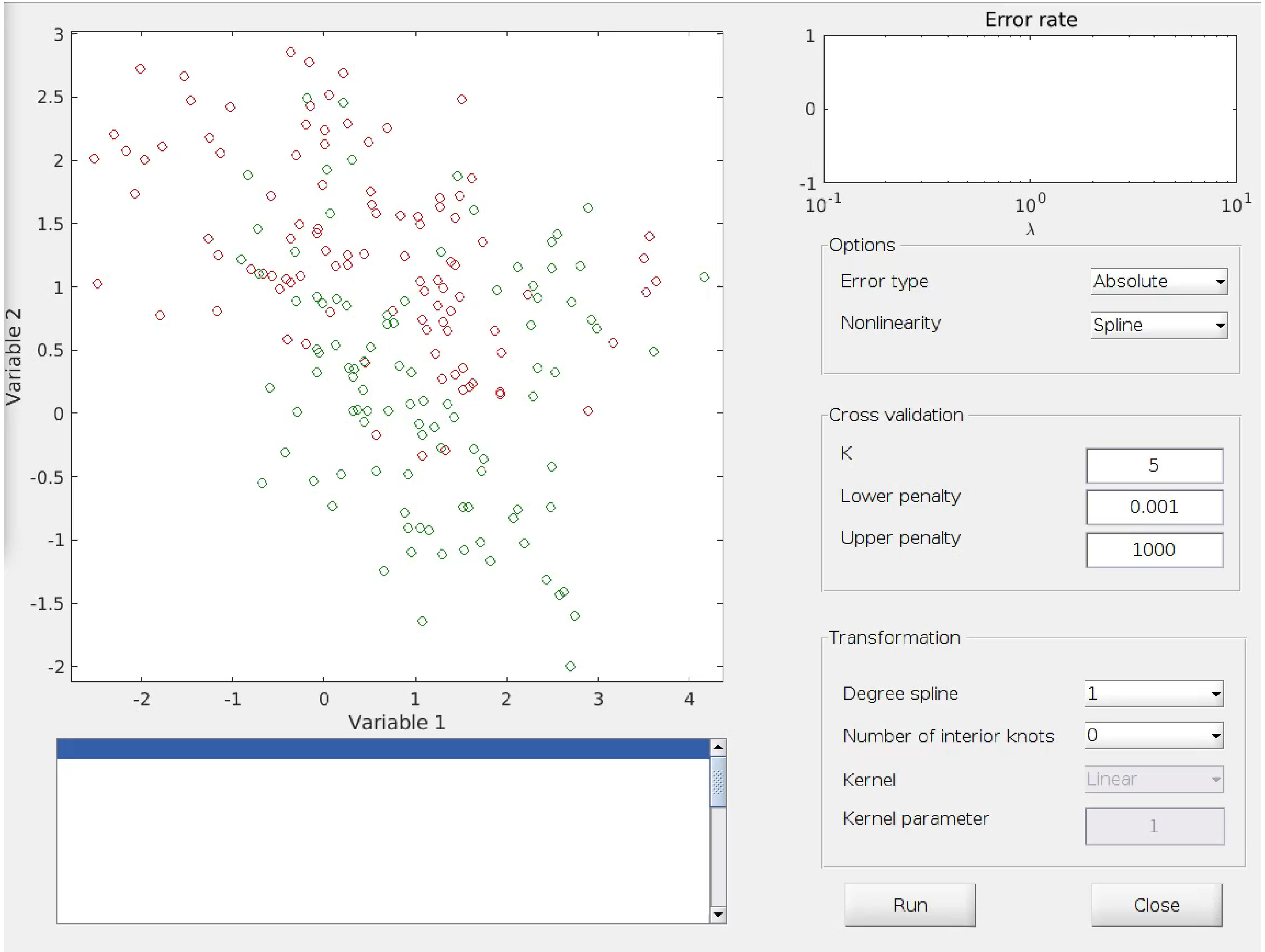
KERNEL







([kaynak](#))



Variable 2
Variable 1
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
3
4
Error rate
1
0
-1
10^{-1}
10^{0}
10^{1}
λ
Options
Error type
Absolute
Nonlinearity
Spline
Cross validation
K
5
Lower penalty
0.001
Upper penalty
1000
Transformation
Degree spline
1
Number of interior knots
0
Kernel
Linear
Kernel parameter
1
Run
Close

ÖDEV

12.3.2 Dual SVM: Convex Hull View