LİNEER CEBİR

1/2/2021, DERS 1

İlker Birbil

DOĞRUSAL MODELLER

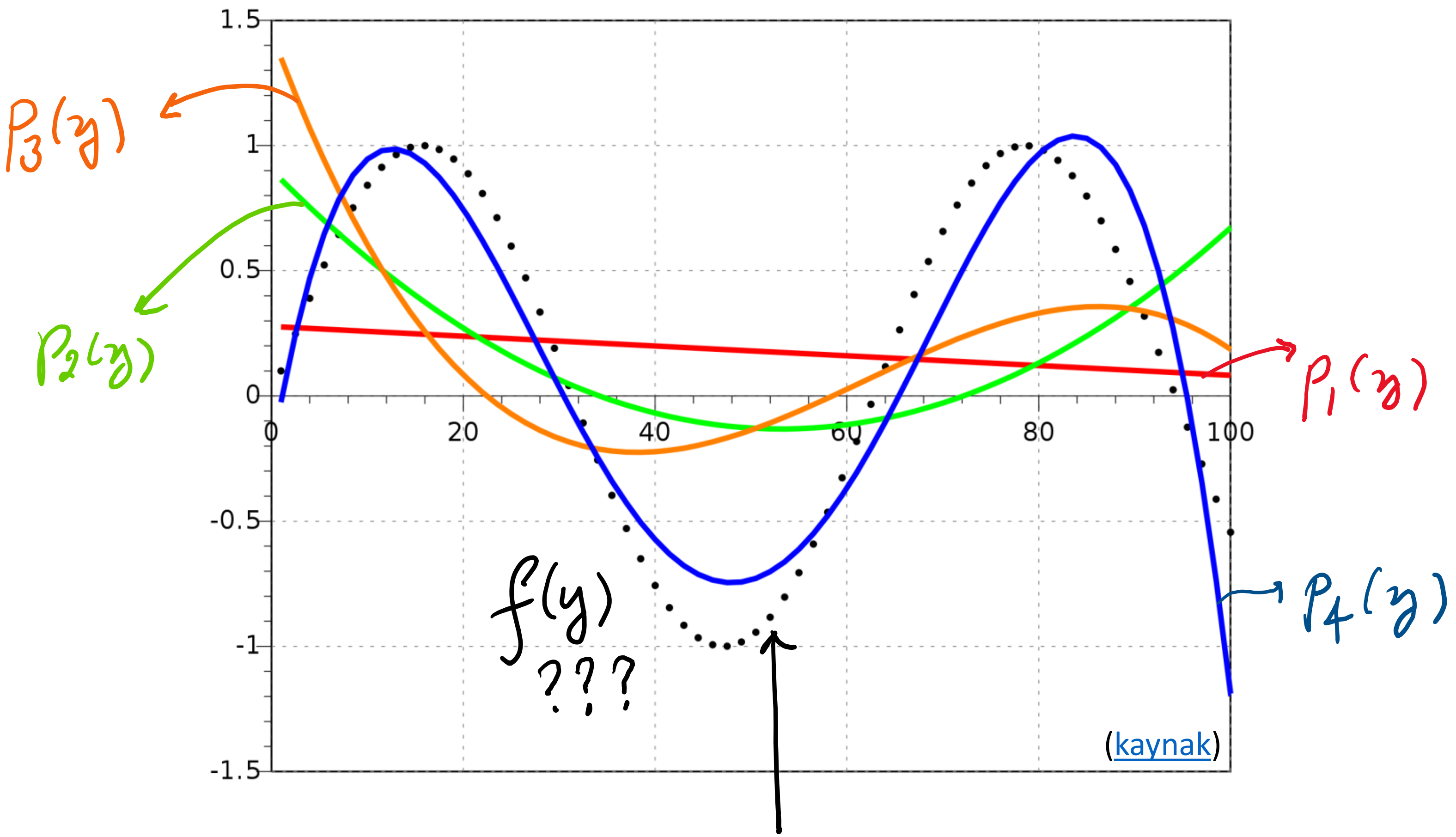
Kayıt No	Salon (m²)	Balkon (m²)	Fiyat (1000 TL)
1	40	5	550
2	10	30	1000
Yeni	20	15	?

$$\left.\begin{aligned} 40x_1 + 5x_2 &= 550 \\ 10x_1 + 30x_2 &= 1000 \end{aligned}\right\} \quad x_1, x_2 = ?$$

$$\downarrow$$

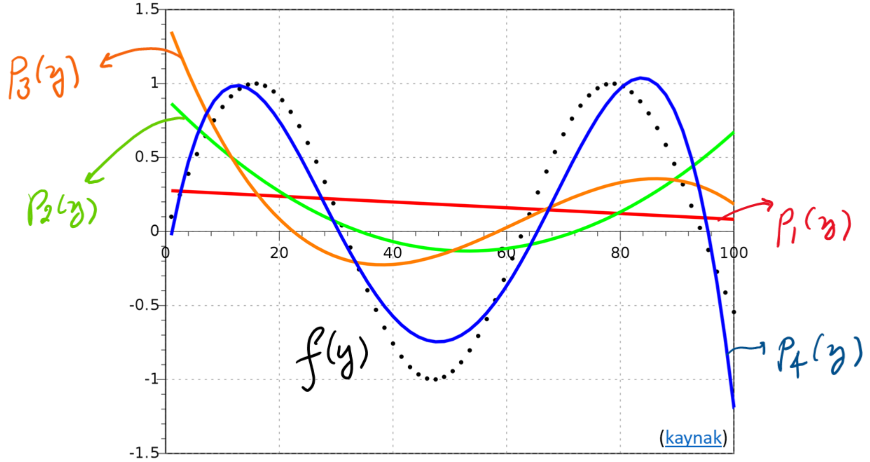
$$20x_1 + 15x_2$$

DOĞRUSAL MODELLER



$(y_i, f(y_i)), \; i=1,2,\ldots,m$ VERİ

DOĞRUSAL MODELLER



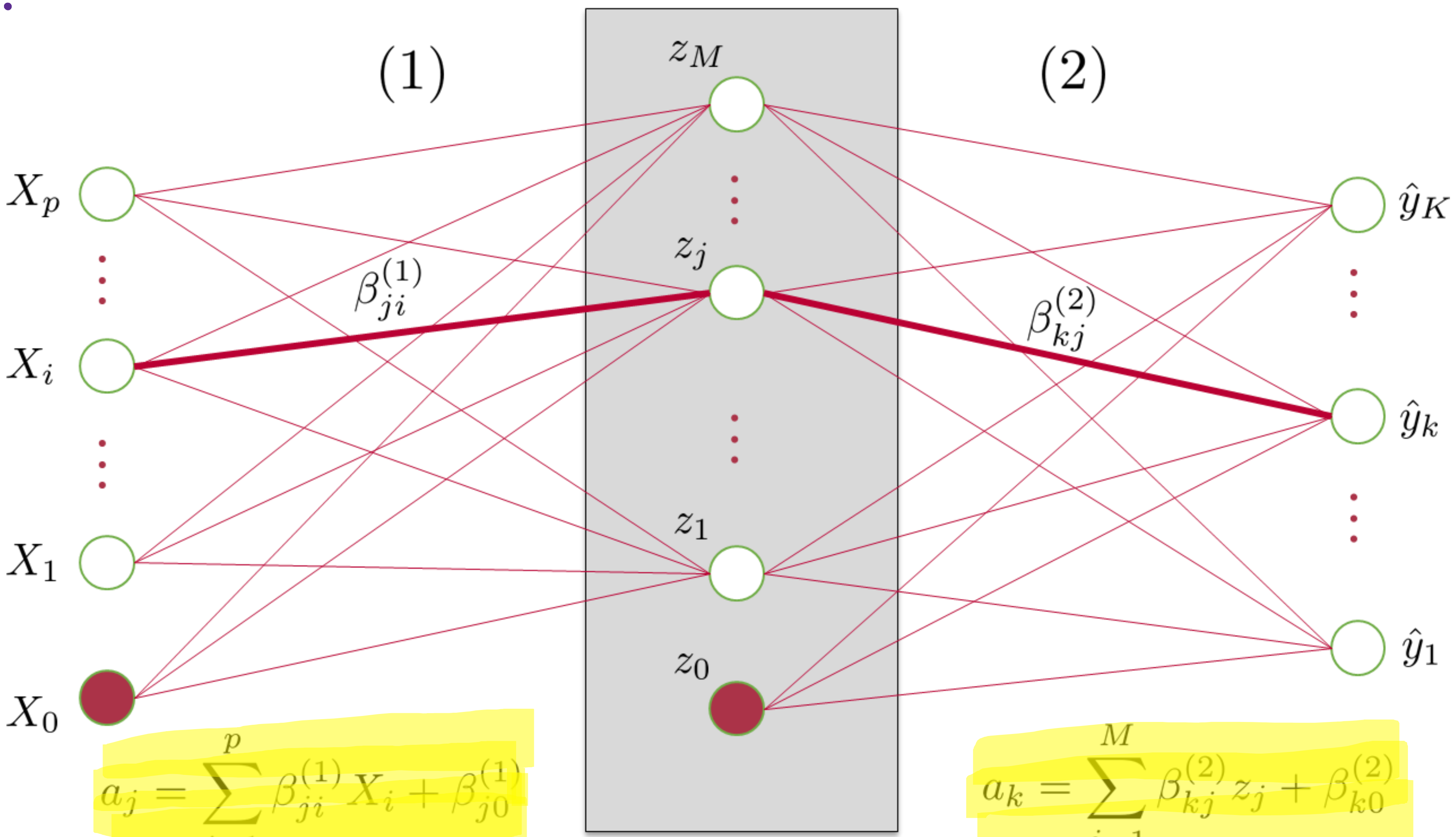
$$P_4(y) = y^4 x_1 + y^3 x_2 + y^2 x_3 + y x_4 + x_5 \approx f(y)$$

$$(y_i, f(y_i)), \quad i = 1, 2, \ldots, m$$

$$\left.\begin{array}{lcl} y = y_1 & & y_1^4 x_1 + y_1^3 x_2 + y_1^2 x_3 + y_1 x_4 + x_5 = f(y_1) \\ y = y_2 & & y_2^4 x_1 + y_2^3 x_2 + y_2^2 x_3 + y_2 x_4 + x_5 = f(y_2) \\ \vdots & & \vdots \qquad\qquad\qquad \vdots \\ y = y_m & & y_m^4 x_1 + y_m^3 x_2 + y_m^2 x_3 + y_m x_4 + x_5 = f(y_m) \end{array}\right\} \quad x_1, x_2, \ldots, x_5 \; ?$$

DOĞRUSAL MODELLER

???



$$a_j = \sum_{i=1}^{p} \beta_{ji}^{(1)} X_i + \beta_{j0}^{(1)}$$

$$z_j = h(a_j), \; j = 1, \ldots, M$$

$$a_k = \sum_{j=1}^{M} \beta_{kj}^{(2)} z_j + \beta_{k0}^{(2)}$$

$$\hat{y}_k = \sigma(a_k), \; k = 1, \ldots, K$$

VEKTÖRLER

$$40x_1 + 5x_2 = 550$$

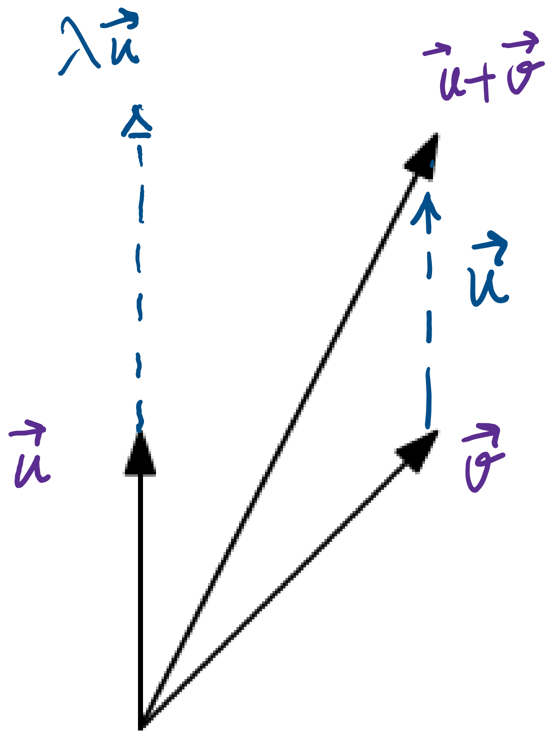
$$10x_1 + 30x_2 = 1000$$

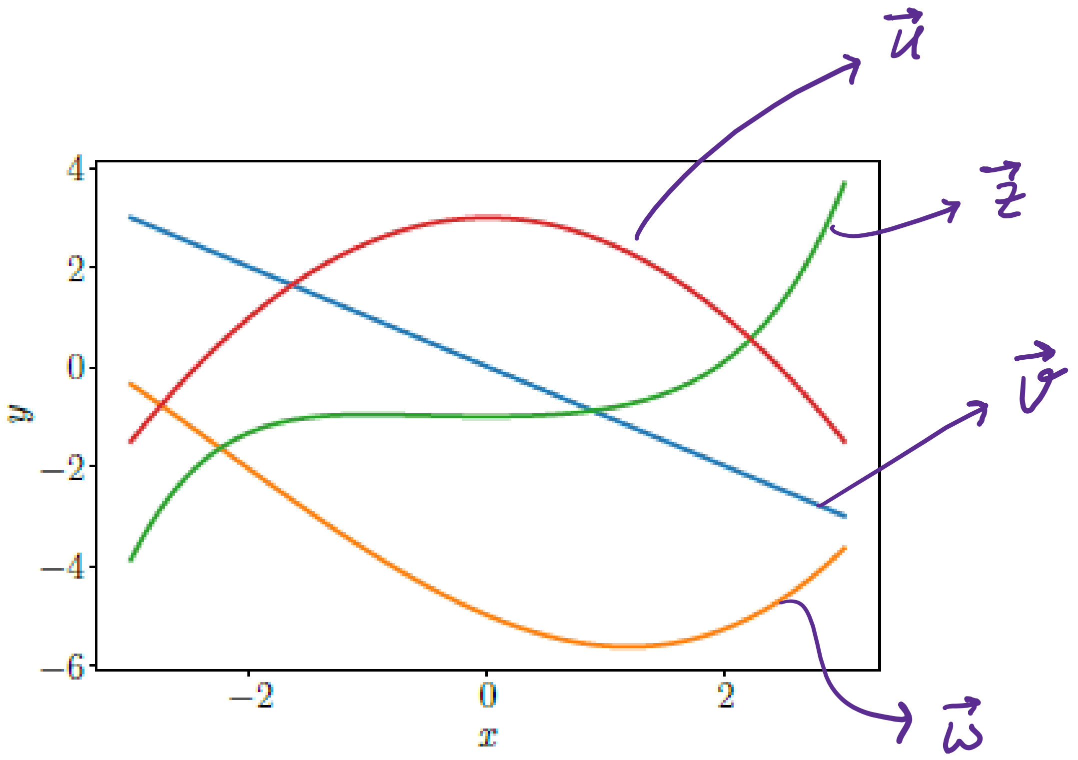
$$\underbrace{\begin{bmatrix} 40 \\ 10 \end{bmatrix}}_{\vec{u} \in \mathbb{R}^2} x_1 + \underbrace{\begin{bmatrix} 5 \\ 30 \end{bmatrix}}_{\vec{v} \in \mathbb{R}^2} x_2 = \begin{bmatrix} 550 \\ 1000 \end{bmatrix}$$

SÜTUN VEKTÖRLER

$$\vec{u}x_1 + \vec{v}x_2 = \begin{bmatrix} 40x_1 \\ 10x_1 \end{bmatrix} + \begin{bmatrix} 5x_2 \\ 30x_2 \end{bmatrix} = \begin{bmatrix} 40x_1 + 5x_2 \\ 10x_1 + 30x_2 \end{bmatrix}$$

VEKTÖRLER



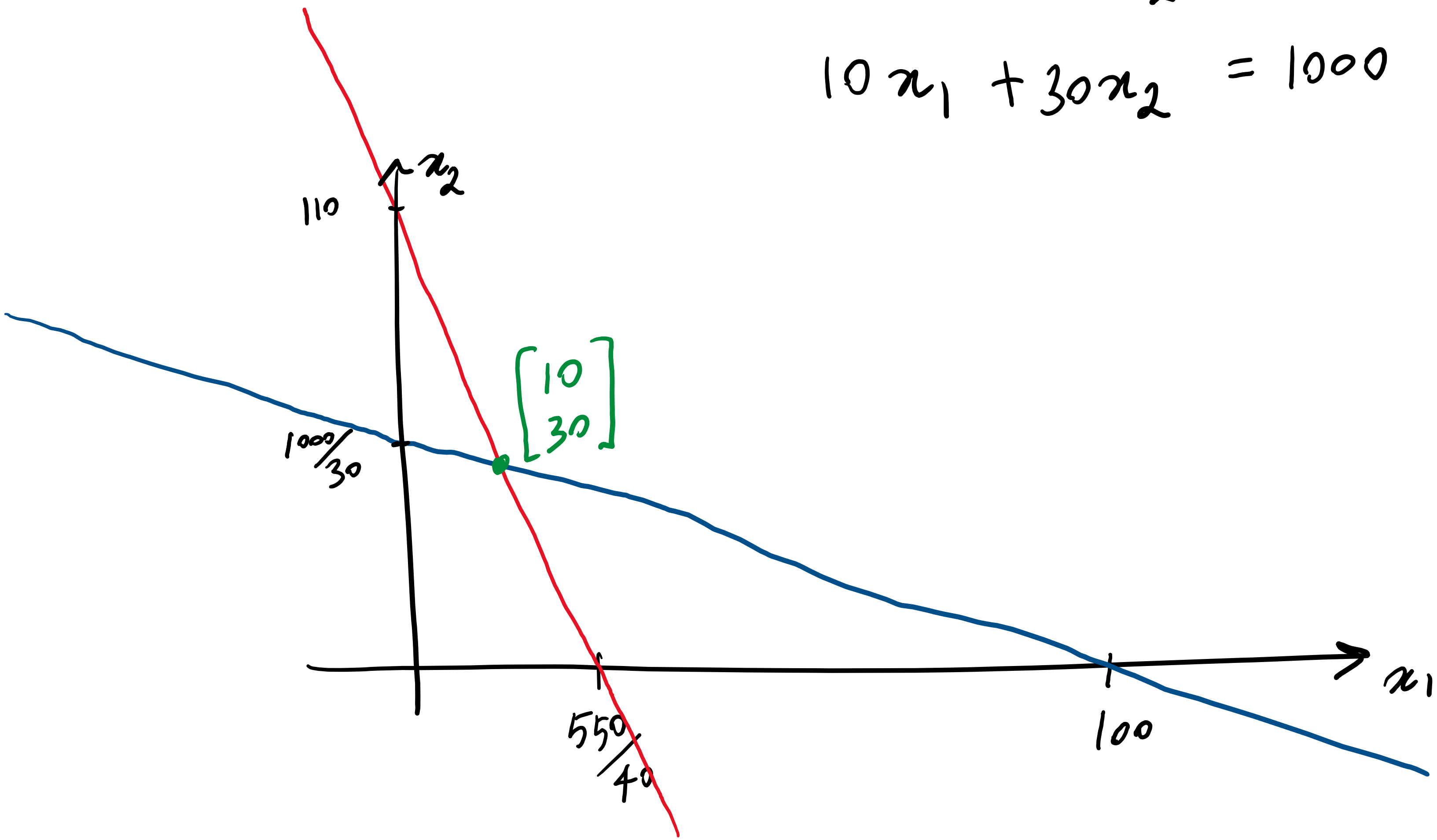


(!)

$$\left.\begin{aligned} \vec{u}+\vec{v} &= \vec{z} \\ \lambda\vec{u} &= \vec{w} \end{aligned}\right\} \quad \vec{z}, \vec{w} \text{ VEKTÖRLER}$$

DENKLEM SİSTEMLERİ

$$40x_1 + 5x_2 = 550$$
$$10x_1 + 30x_2 = 1000$$

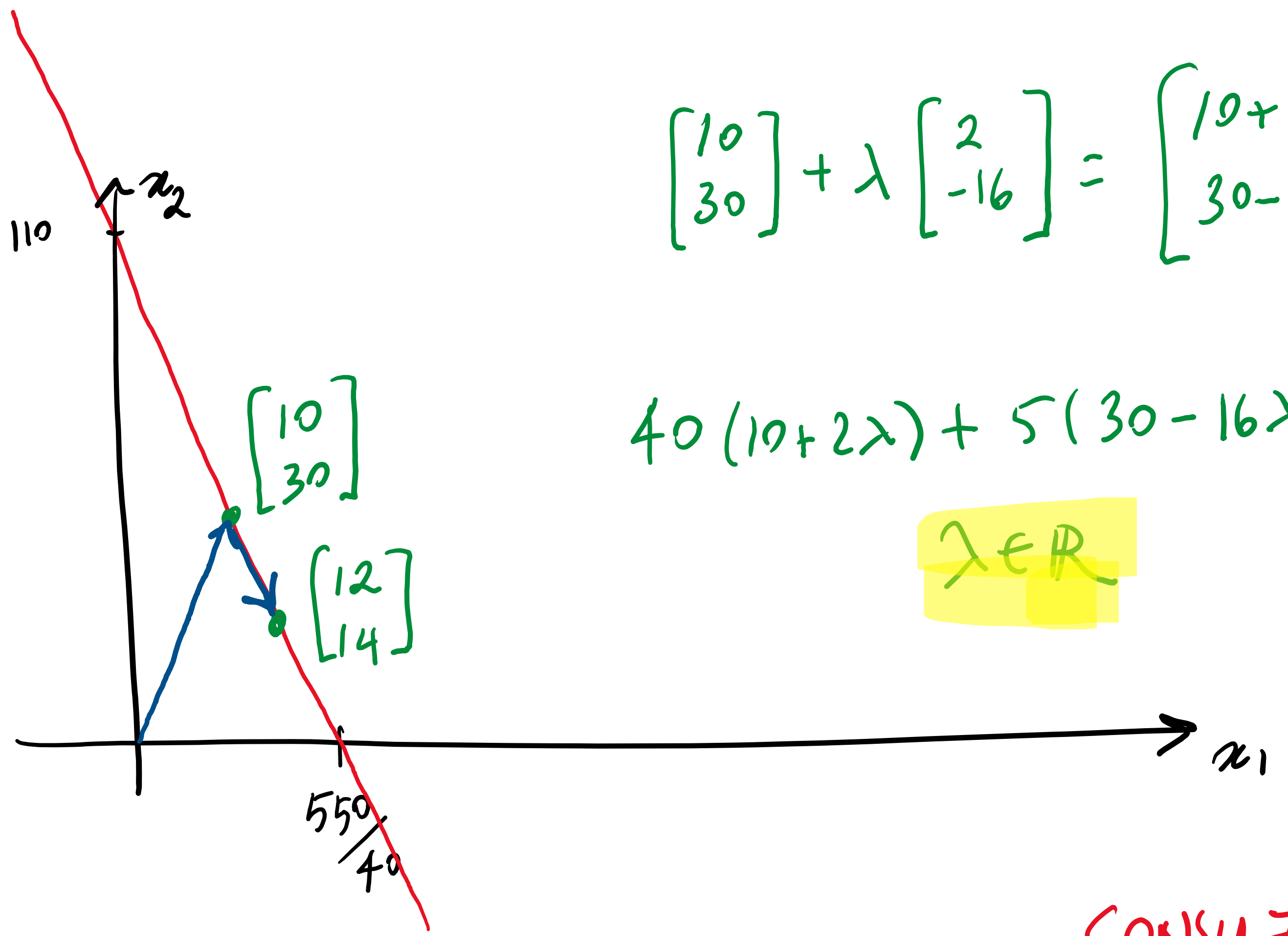


TEK ÇÖZÜM

DENKLEM SİSTEMLERİ

$$40x_1 + 5x_2 = 550$$

$$\begin{bmatrix} 10 \\ 30 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -16 \end{bmatrix} = \begin{bmatrix} 10+2\lambda \\ 30-16\lambda \end{bmatrix}$$



$$40(10+2\lambda) + 5(30-16\lambda) = 550$$

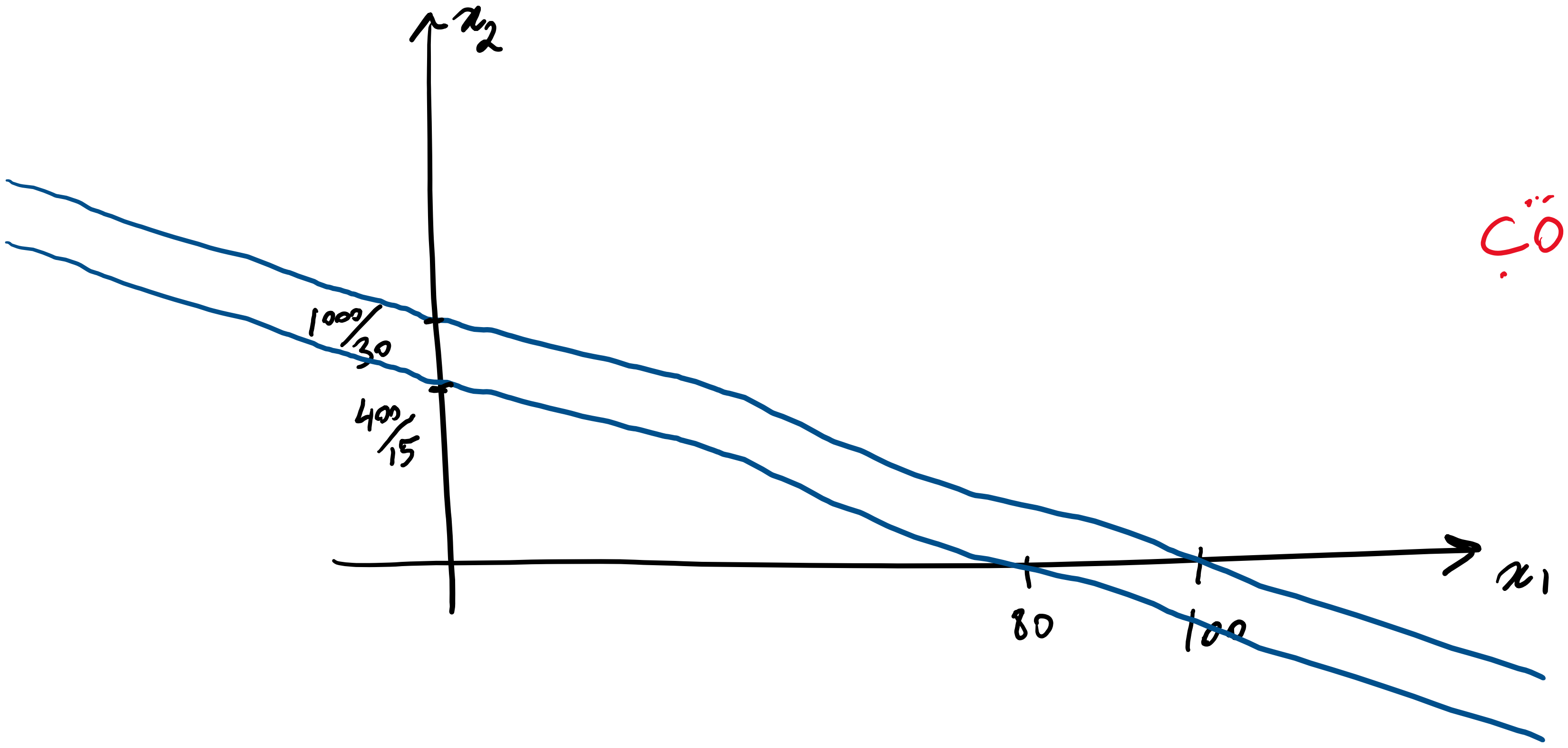
$\lambda \in \mathbb{R}$

$$\begin{bmatrix} 12 \\ 14 \end{bmatrix} - \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \begin{bmatrix} 2 \\ -16 \end{bmatrix}$$

SONSUZ ÇÖZÜM

DENKLEM SİSTEMLERİ

$$10x_1 + 30x_2 = 1000$$
$$5x_1 + 15x_2 = 400$$



ÇÖZÜM YOK

MATRISLER

$$\begin{bmatrix} 40 \\ 10 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ 30 \end{bmatrix} x_2 = \begin{bmatrix} 550 \\ 1000 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} x_1 + \begin{bmatrix} a_{12} \\ \vdots \\ a_{m2} \end{bmatrix} x_2 + \cdots + \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix} x_n = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}_{A \in R^{m \times n}} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$\underbrace{A\vec{x} = \vec{b}}_{Ax = b}$$

MATRISLER

$$\lambda A = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \vdots & \vdots & & \vdots \\ \lambda a_{m1} & \lambda a_{m2} & \cdots & \lambda a_{mn} \end{bmatrix}$$

$(\lambda \in \mathbb{R})$

$$A + B := \begin{bmatrix} a_{11} + b_{11} & \cdots & a_{1n} + b_{1n} \\ \vdots & & \vdots \\ a_{m1} + b_{m1} & \cdots & a_{mn} + b_{mn} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

$$\underbrace{A}_{n\times k}\underbrace{B}_{k\times m} = \underbrace{C}_{n\times m}$$

$$c_{ij} = \sum_{l=1}^{n} a_{il} b_{lj}, \qquad i = 1, \ldots, m, \quad j = 1, \ldots, k$$

c_{21}

$$AB = \begin{array}{c} \\ 2 \end{array}\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}}_{2\times 3} \underbrace{\begin{array}{c} \begin{array}{cc} 1 & \end{array} \\ \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \end{array}}_{3\times 2} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$AB \neq BA$

c_{23}

$$BA = \begin{array}{c} \\ 2 \\ \\ \end{array}\underbrace{\begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}}_{3\times 2} \underbrace{\begin{array}{c} \begin{array}{ccc} & & 3 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \end{array}}_{2\times 3} = \begin{bmatrix} 6 & 4 & 2 \\ -2 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

MATRİSLER

BİRİM MATRİS

$$I_n := \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{n \times n}$$

$$\forall A \in \mathbb{R}^{m \times n} : I_m A = A I_n = A$$

MATRİS TERSİ

$$A\underset{\substack{\downarrow \\ A^{-1}}}{B} = I_n = B\underset{\substack{\downarrow \\ B^{-1}}}{A}$$

$$AA^{-1} = I = A^{-1}A$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A+B)^{-1} \neq A^{-1} + B^{-1}$$

MATRİS DEVRİĞİ

$$A = \begin{bmatrix} \boxed{1} & \boxed{2} & \cdots & \boxed{n} \end{bmatrix}_{m \times n} \qquad A^T = \begin{bmatrix} \boxed{\quad 1 \quad} \\ \boxed{\quad 2 \quad} \\ \vdots \\ \boxed{\quad n \quad} \end{bmatrix}_{n \times m}$$

$$(A^\top)^\top = A$$

$$(A+B)^\top = A^\top + B^\top$$

$$(AB)^\top = B^\top A^\top$$

SİMETRİK MATRİS

$$A = A^\top$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 7 & 5 \\ 2 & 5 & 1 \end{bmatrix} = A^T$$

$$(A^{-1})^\top = (A^\top)^{-1} =: A^{-\top}$$