VEKTÖR ANALİZİ

2/2/2021, DERS 1

İlker Birbil

MODEL VE PARAMETRELER

GİRDİ y_i $\longrightarrow$ $\boxed{f\ ???}$ $\longrightarrow$ ÇIKTI $f(y_i)$

MODEL

$$P(y_i, x) \approx f(y)$$

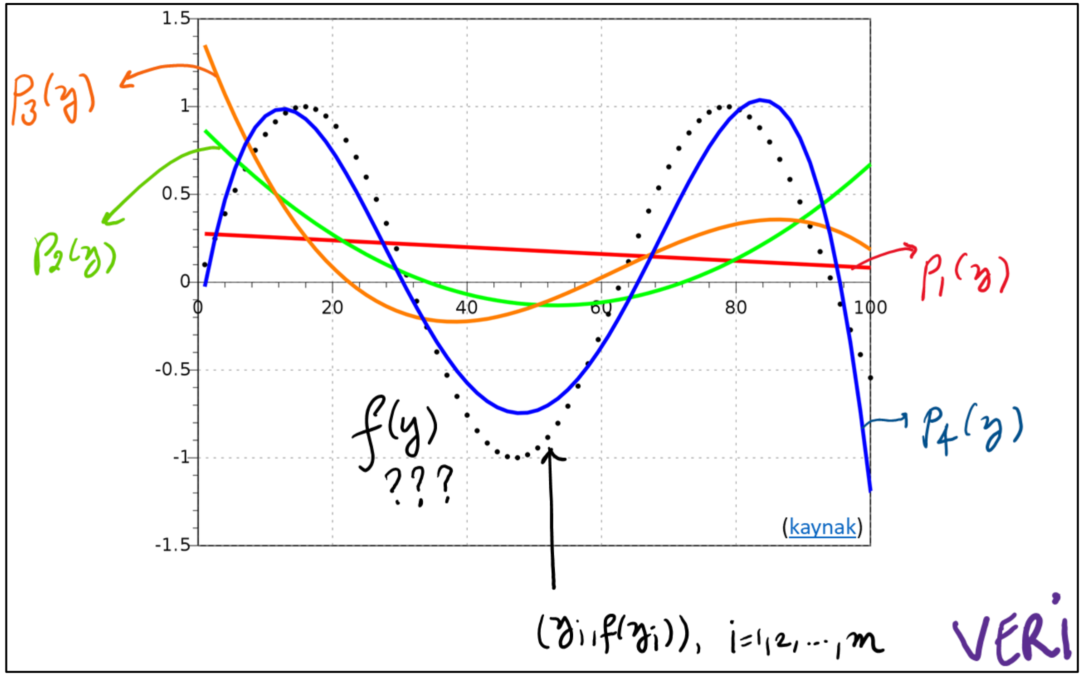
PARAMETRELER

$$\min\left\{ \sum_{i=1}^{m} \left(P(y_i, x) - f(y_i)\right)^2 : x \in \mathbb{R}^n \right\}$$

MODEL VE PARAMETRELER

GİRDİ y_i → $\boxed{f\ ???}$ → ÇIKTI $f(y_i)$

MODEL $\quad P(y, \underset{\substack{\uparrow \\ \text{PARAMETRELER}}}{x}) \approx f(y)$



$$\min\left\{ \sum_{i=1}^{m} \left(P(y_i, x) - f(y_i)\right)^2 : x \in \mathbb{R}^n \right\}$$

VEKTÖR ANALİZİ

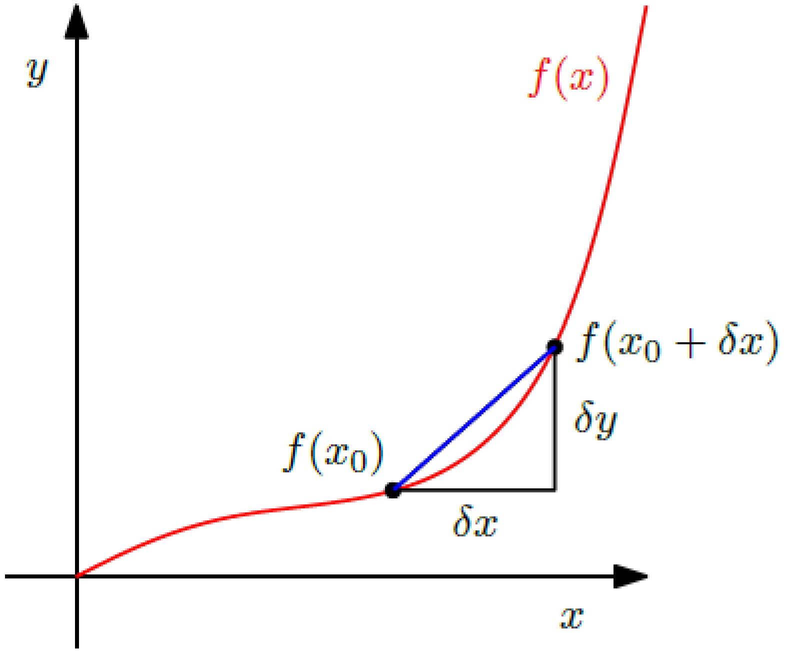
$x \in \mathbb{R}, \; f: \mathbb{R} \to \mathbb{R}$

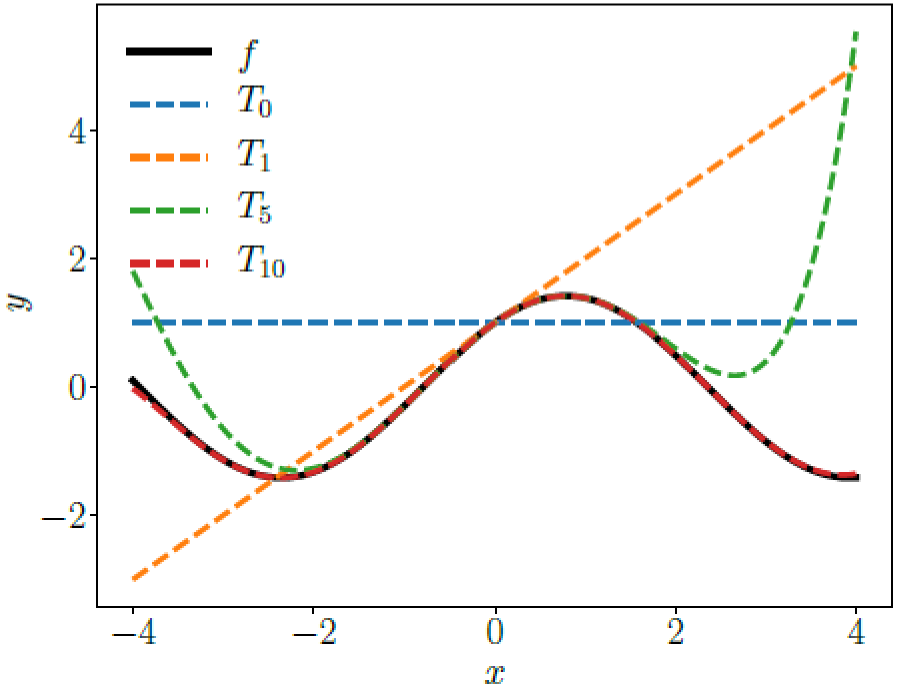
$$\frac{\mathrm{d}f}{\mathrm{d}x} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

TAYLOR SERİSİ

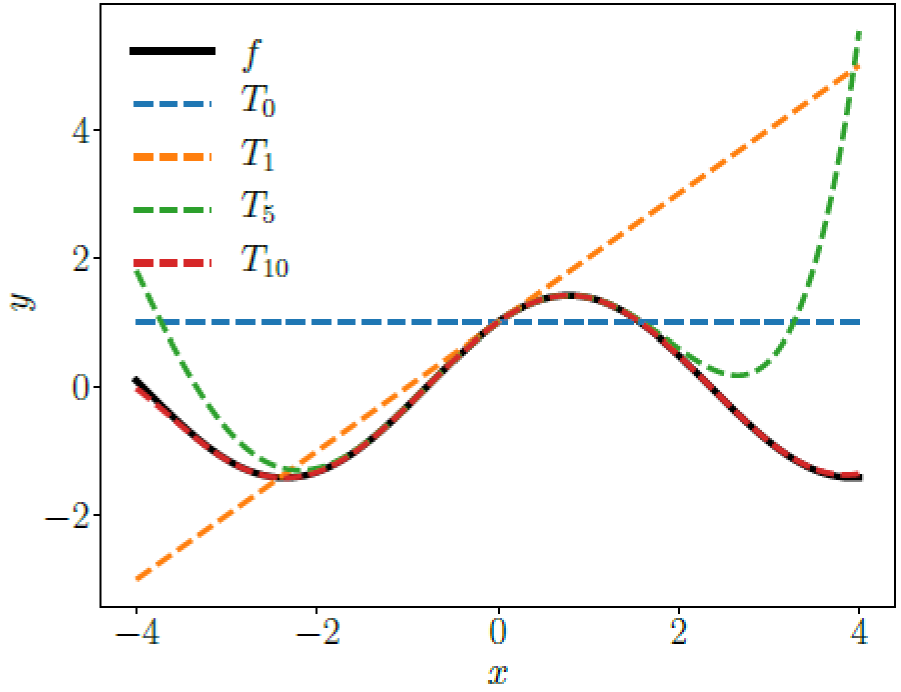
k. TÜREV

$$T_\infty(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$





$x \in \mathbb{R}, f: \mathbb{R} \to \mathbb{R}$



$$f(x) = \sin(x) + \cos(x)$$

$$\begin{aligned} T_\infty(x) &= \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k \\ &= 1 + x - \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 - \cdots \end{aligned}$$

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$(f(x) + g(x))' = f'(x) + g'(x)$$

$$(g(f(x)))' = (g \circ f)'(x) = g'(f(x))f'(x)$$

$x \in \mathbb{R}^n, f : \mathbb{R}^n \to \mathbb{R}$

$$\frac{\mathrm{d}f}{\mathrm{d}x} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{\partial f}{\partial x_1} = \lim_{h \to 0} \frac{f(x_1 + h, x_2, \dots, x_n) - f(\boldsymbol{x})}{h}$$

$$\vdots$$

$$\frac{\partial f}{\partial x_n} = \lim_{h \to 0} \frac{f(x_1, \dots, x_{n-1}, x_n + h) - f(\boldsymbol{x})}{h}$$

$$\nabla_{\boldsymbol{x}} f = \operatorname{grad} f = \frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{x}} = \begin{bmatrix} \frac{\partial f(\boldsymbol{x})}{\partial x_1} & \frac{\partial f(\boldsymbol{x})}{\partial x_2} & \cdots & \frac{\partial f(\boldsymbol{x})}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{1 \times n}$$

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$
$$(f(x) + g(x))' = f'(x) + g'(x)$$
$$\big(g(f(x))\big)' = (g \circ f)'(x) = g'(f(x))f'(x)$$

$$\frac{\partial}{\partial \boldsymbol{x}}(f(\boldsymbol{x})g(\boldsymbol{x})) = \frac{\partial f}{\partial \boldsymbol{x}} g(\boldsymbol{x}) + f(\boldsymbol{x}) \frac{\partial g}{\partial \boldsymbol{x}}$$
$$\frac{\partial}{\partial \boldsymbol{x}}(f(\boldsymbol{x}) + g(\boldsymbol{x})) = \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\partial g}{\partial \boldsymbol{x}}$$
$$\frac{\partial}{\partial \boldsymbol{x}}(g \circ f)(\boldsymbol{x}) = \frac{\partial}{\partial \boldsymbol{x}}(g(f(\boldsymbol{x}))) = \frac{\partial g}{\partial f} \frac{\partial f}{\partial \boldsymbol{x}}$$

ZİNCİR KURALI

$x \in \mathbb{R}$, $f: \mathbb{R} \to \mathbb{R}$

$$h(x) = (2x+1)^4 = g(f(x))$$
$$f(x) = 2x + 1$$
$$g(f) = f^4$$

$$h'(x) = g'(f)f'(x) = 4(2x+1)^3 \cdot 2 = 8(2x+1)^3$$

$x \in \mathbb{R}^n$, $f: \mathbb{R}^n \to \mathbb{R}$

$$f(x_1, x_2) = x_1^2 + 2x_2$$

$$x_1 = \sin t \qquad x_2 = \cos t$$

$$\begin{aligned} \frac{\mathrm{d}f}{\mathrm{d}t} &= \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t} \\ &= 2\sin t \frac{\partial \sin t}{\partial t} + 2\frac{\partial \cos t}{\partial t} \\ &= 2\sin t \cos t - 2\sin t = 2\sin t(\cos t - 1) \end{aligned}$$

ZİNCİR KURALI

$$f : \mathbb{R}^n \to \mathbb{R}, \quad g : \mathbb{R}^m \to \mathbb{R}^n \quad \left.\begin{matrix} g_1(y) \\ g_2(y) \\ \vdots \\ g_n(y) \end{matrix}\right\} \; y \in \mathbb{R}^m$$

$$\frac{\partial}{\partial y} f(g(y)) = \frac{\partial f}{\partial g} \frac{\partial g}{\partial y} = \underbrace{\begin{bmatrix} \frac{\partial f}{\partial g_1} & \cdots & \frac{\partial f}{\partial g_n} \end{bmatrix}}_{1 \times n} \underbrace{\begin{bmatrix} \frac{\partial g_1}{\partial y_1} & \cdots & \frac{\partial g_1}{\partial y_m} \\ \vdots & & \\ \frac{\partial g_n}{\partial y_1} & \cdots & \frac{\partial g_n}{\partial y_m} \end{bmatrix}}_{n \times m}$$

$$\frac{\partial}{\partial y} f(g(y)) \in \mathbb{R}^{1 \times m}$$

$x \in \mathbb{R}^n, f : \mathbb{R}^n \to \mathbb{R}^m$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{x}) \\ \vdots \\ f_m(\boldsymbol{x}) \end{bmatrix} \in \mathbb{R}^m \Bigg\} \; f_i : \mathbb{R}^n \to \mathbb{R}$$

JAKOBYEN

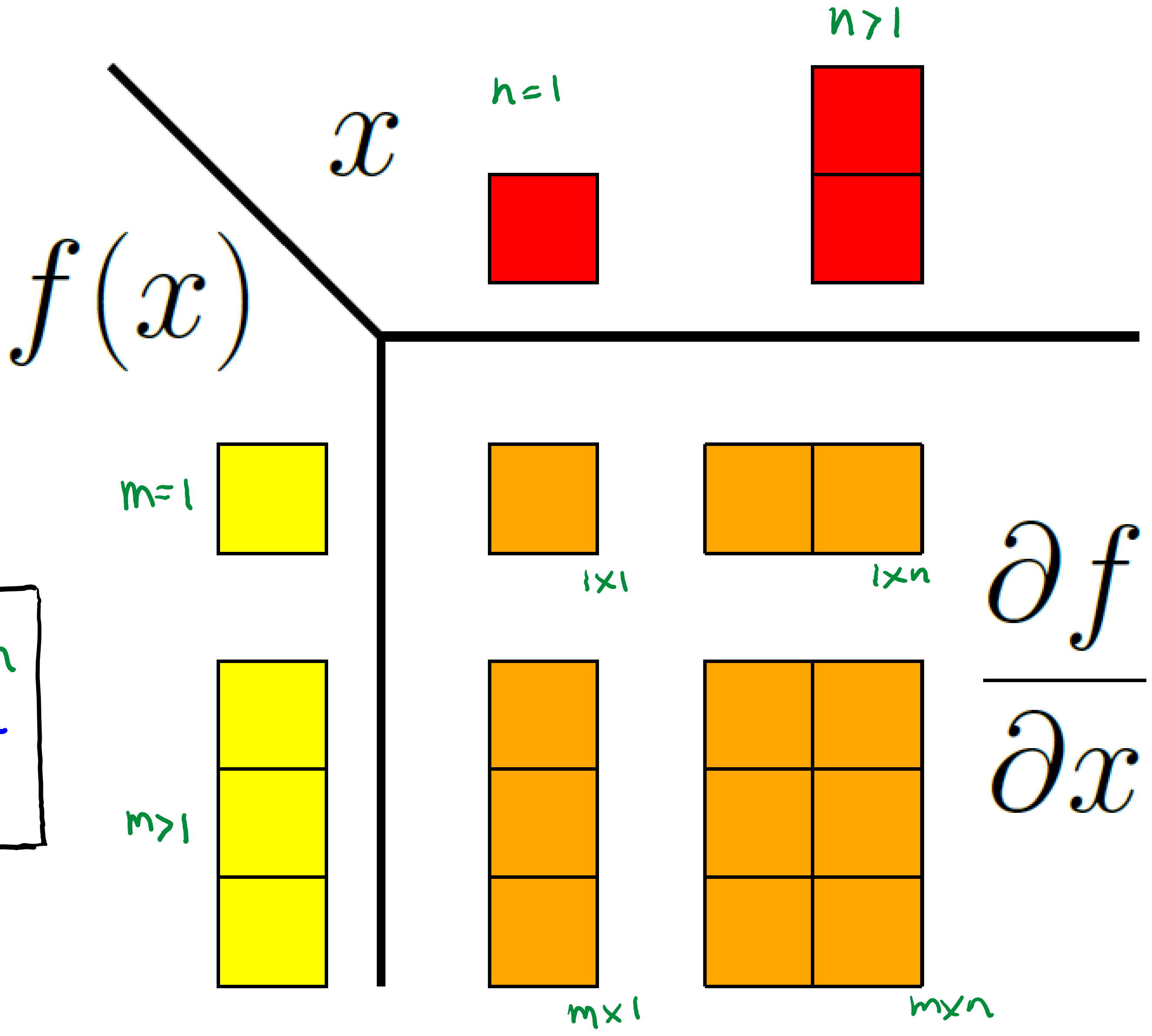
$$\boldsymbol{J} = \nabla_{\boldsymbol{x}} \boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{f}(\boldsymbol{x})}{\mathrm{d}\boldsymbol{x}} = \begin{bmatrix} \frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial x_1} & \cdots & \frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(\boldsymbol{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\boldsymbol{x})}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_m(\boldsymbol{x})}{\partial x_1} & \cdots & \frac{\partial f_m(\boldsymbol{x})}{\partial x_n} \end{bmatrix}_{m \times n}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x}\,, \qquad \boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^M, \quad \boldsymbol{A} \in \mathbb{R}^{M \times N}, \quad \boldsymbol{x} \in \mathbb{R}^N$$

$$f_i(\boldsymbol{x}) = \sum_{j=1}^{N} A_{ij} x_j \implies \frac{\partial f_i}{\partial x_j} = A_{ij}$$

$$\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial f_M}{\partial x_1} & \cdots & \frac{\partial f_M}{\partial x_N} \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & & \vdots \\ A_{M1} & \cdots & A_{MN} \end{bmatrix}$$

$x \in \mathbb{R}^n, f: \mathbb{R}^n \to \mathbb{R}^m$



$\boxed{f: \mathbb{R}^n \to \mathbb{R}^m}$