# LİNEER CEBİR

1/2/2021, DERS 3

İlker Birbil

# TEMEL

$$A = \{x_1, x_2, \ldots, x_k\} \subseteq V$$

$span[A] \rightarrow$ A İÇİNDEKİ VEKTÖRLERİN TÜM DOĞRUSAL KOMBİNASYONLARI

$span[A] = V \rightarrow$ ÜRETEN KÜME

$\downarrow$

MİNİMAL SAYIDA VEKTÖR } A, TEMEL $\quad x_1, \ldots, x_k$ TEMEL VEKTÖRLER

$\downarrow$

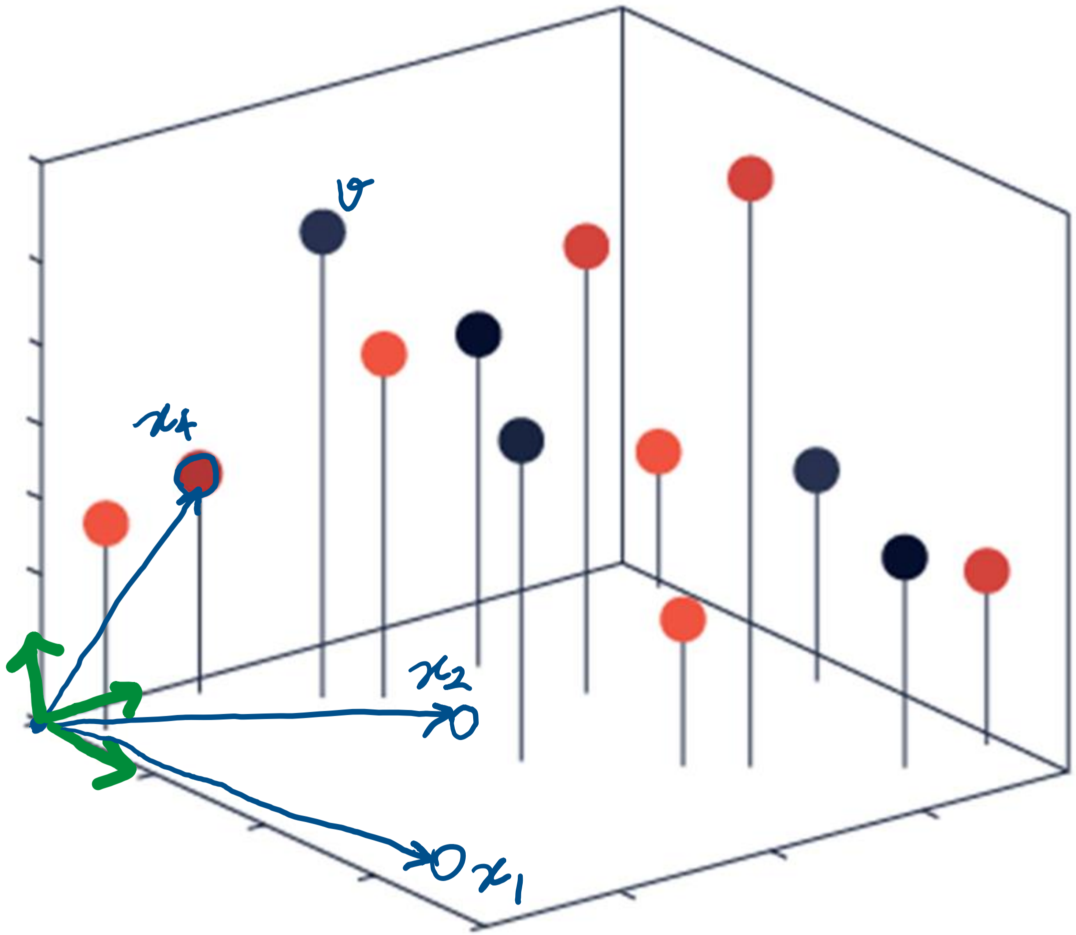
MAKSİMUM SAYIDA DOĞRUSAL BAĞIMSIZ VEKTÖR

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 0.5 \\ 0.8 \\ 0.4 \end{bmatrix}, \begin{bmatrix} 1.8 \\ 0.3 \\ 0.3 \end{bmatrix}, \begin{bmatrix} -2.2 \\ -1.3 \\ 3.5 \end{bmatrix} \right\}$$

# TEMEL

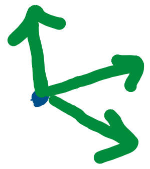



(kaynak)

$$B = \{x_1, x_2, x_4\}$$

$$\forall v \in \mathbb{R}^3, \exists \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}: \ v = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_4$$

$$\mathcal{B} = \left\{ \underset{1}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \underset{2}{\begin{bmatrix}0\\1\\0\end{bmatrix}}, \underset{3}{\begin{bmatrix}0\\0\\1\end{bmatrix}} \right\}$$



(SONLU UZAYLARDA)

$$\boxed{\underset{\text{BOYUT}}{\dim(V)} = \underbrace{|B|}_{\text{BAZ VEKTÖR SAYISI}}}$$

# TEMEL

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \\ -2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 3 \\ -4 \\ 3 \\ 5 \\ -3 \end{bmatrix}, \quad x_4 = \begin{bmatrix} -1 \\ 8 \\ -5 \\ -6 \\ 1 \end{bmatrix}$$

$$\sum_{i=1}^{4} \lambda_i x_i = 0 \qquad A\lambda = 0$$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -4 & 8 \\ -1 & 1 & 3 & -5 \\ -1 & 2 & 5 & -6 \\ -1 & -2 & -3 & 1 \end{bmatrix} \underset{1,2,3}{\rightsquigarrow \cdots \rightsquigarrow} \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B} = \{x_1, x_2, x_4\} \qquad \text{span}[\mathcal{B}] = U \subset \mathbb{R}^5$$

# RANK (KERTE)

$A \in \mathbb{R}^{m \times n}$ $\quad$ $rk(A)$: DOĞRUSAL BAĞIMSIZ VEKTÖR SAYISI

$$rk(A) = rk(A^T)$$

$$A = \begin{bmatrix} \Box & \Box & \cdots & \Box \end{bmatrix} \rightarrow \mathcal{A} = \{\Box, \Box, \ldots, \Box\} \rightarrow \operatorname{span}[\mathcal{A}] = \underbrace{U}_{\text{GÖRÜNTÜ ALTUZAYI}} \subseteq \mathbb{R}^m$$

$$\underset{\begin{bmatrix} \Box & \Box & \cdots & \Box \end{bmatrix}}{A}x = \underset{\blacksquare}{b} \ \checkmark \iff rk(A) = rk(\underbrace{A|b}_{\begin{bmatrix} \Box & \Box & \cdots & \Box & \blacksquare \end{bmatrix}})$$

$$A \in \mathbb{R}^{n \times n}, \quad A^{-1} \ \checkmark \iff rk(A) = n$$

# RANK (KERTE)

$W = \{x \in \mathbb{R}^n : Ax = 0\} \to \text{span}[W]$ : KERNEL (NULL) ALTUZAYI

$$\boxed{\dim(\text{span}[W]) = n - rk(A)}$$

$rk(A) = 3$

$$\begin{array}{rcrcrcrcrcr}
x_1 & - & 2x_2 & + & x_3 & - & x_4 & + & x_5 & = & 0 \\
 & & & & x_3 & - & x_4 & + & 3x_5 & = & -2 \\
 & & & & & & x_4 & - & 2x_5 & = & 1 \\
 & & & & & & & & 0 & = & 0
\end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$W = \{d_1, d_2\}$

$\text{span}[W] = \mathbb{R}^2$

$\dim(\text{span}[W]) = 2$

$$\left\{ x \in \mathbb{R}^5 : x = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \lambda_1 \underbrace{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{d_1} + \lambda_2 \underbrace{\begin{bmatrix} 2 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}}_{d_2}, \quad \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$

# DOĞRUSAL DÖNÜŞÜMLER

$$\forall x, y \in V \ \forall \lambda, \psi \in \mathbb{R} : \Phi(\lambda x + \psi y) = \lambda \Phi(x) + \psi \Phi(y)$$

$$V \longrightarrow W$$

$$\Phi : V \to W$$

(SONLU BOYUTLU UZAYLARDA)

$V$ VE $W$ BENZEŞ UZAYLAR $\iff \dim(V) = \dim(W)$

$\dim(V) = n \Rightarrow V$ VE $\mathbb{R}^n$ BENZEŞ UZAYLAR

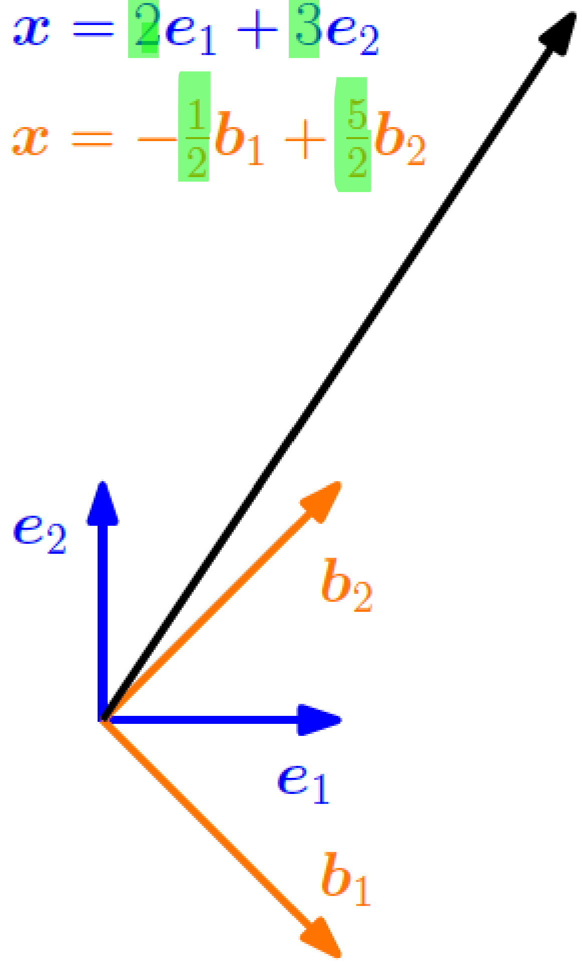
# DOĞRUSAL DÖNÜŞÜMLER

$\mathcal{B}=\{e_1,e_2\}$

$\mathcal{B}=\{b_1,b_2\}$

$$x = 2e_1 + 3e_2$$

$$x = -\frac{1}{2}b_1 + \frac{5}{2}b_2$$




FARKLI KOORDİNAT SİSTEMLERİ

DOĞRUSAL DÖNÜŞÜMLER

$\equiv$

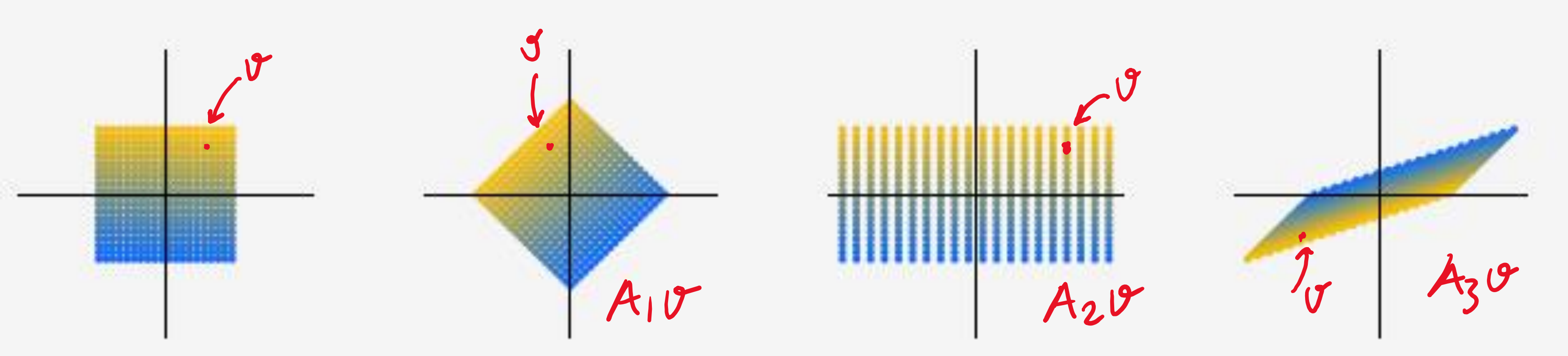
DÖNÜŞÜM MATRİSLERİ

# DOĞRUSAL DÖNÜŞÜMLER

DOĞRUSAL DÖNÜŞÜMLER

$\equiv$

DÖNÜŞÜM MATRİSLERİ




$$A_1 = \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix} \quad A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad A_3 = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$$

# AFİN ALTUZAYLAR

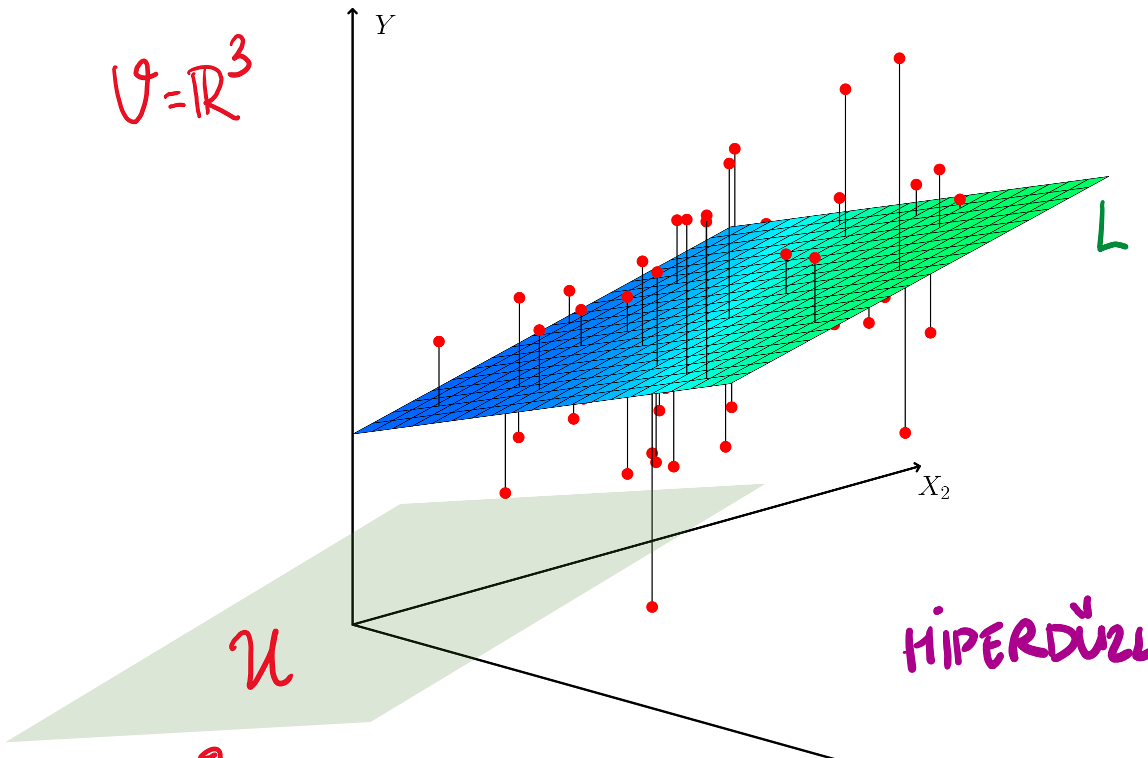
ALTUZAY

$$L = x_0 + U := \{x_0 + u : u \in U\} \subseteq V$$

$x_0 \in V$

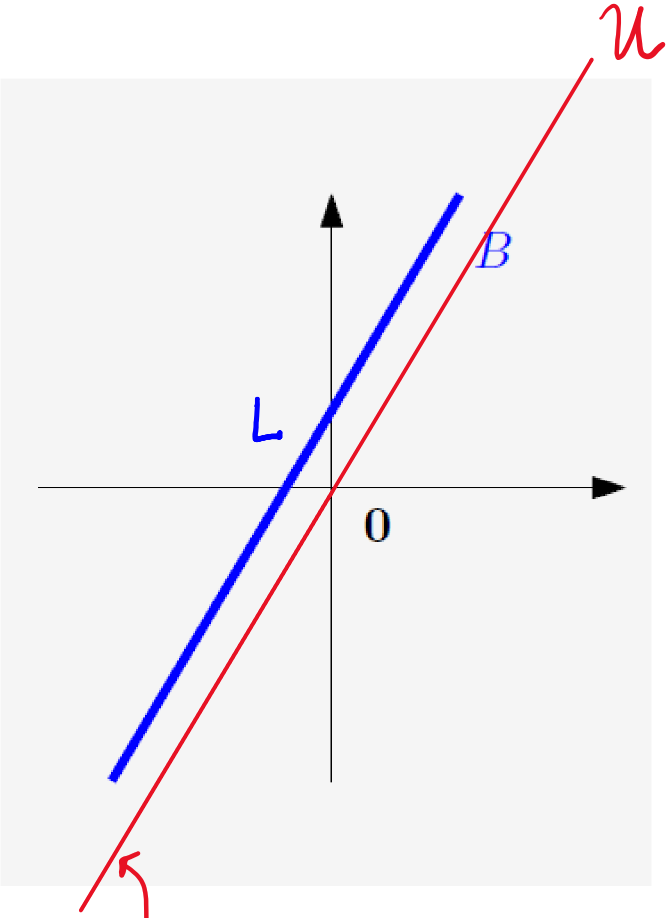
UZAY

$V = \mathbb{R}^3$




ALTUZAY

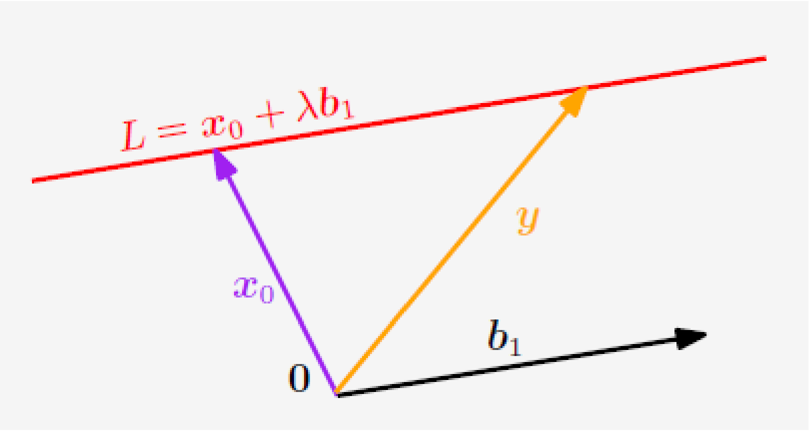
HİPERDÜZLEMLER

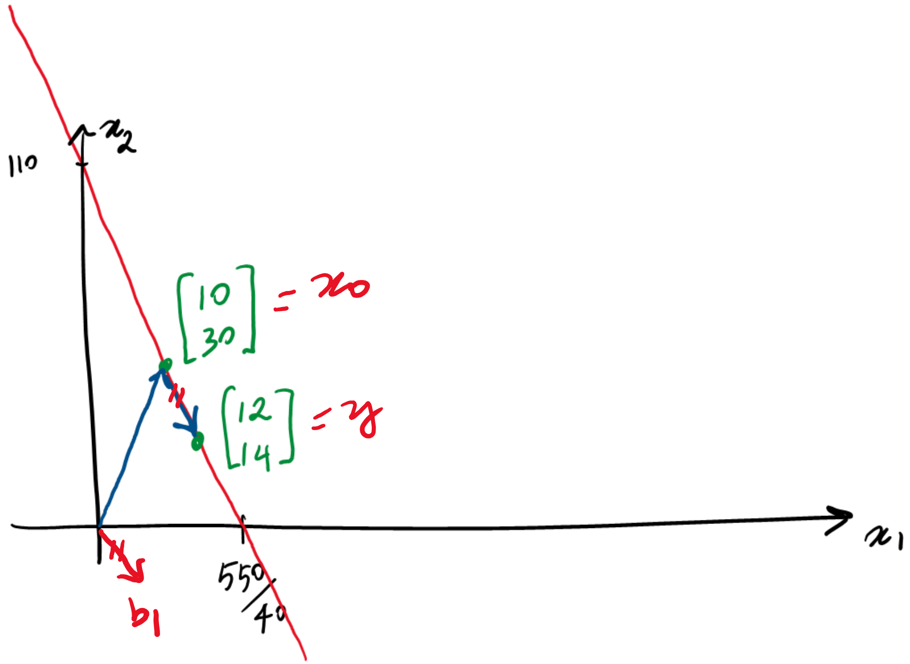



$V = \mathbb{R}^2$

ALTUZAY

# AFİN ALTUZAYLAR




$$(\mathbb{R}^n) \quad y = x_0 + \sum_{i=1}^{n-1} \lambda_i b_i, \quad \mathcal{B} = \{b_1, b_2, \dots, b_{n-1}\}$$

$$L = \{x \in \mathbb{R}^n : Ax = b\}, \quad A \in \mathbb{R}^{m \times n}$$

ÖDEV

2.5 Find the set $\mathcal{S}$ of all solutions in $\boldsymbol{x}$ of the following inhomogeneous linear

2.9 Which of the following sets are subspaces of $\mathbb{R}^3$?

2.13 Consider two subspaces $U_1$ and $U_2$, where $U_1$ is the solution space of the

2.15 Let $F = \{(x, y, z) \in \mathbb{R}^3 \mid x+y-z = 0\}$ and $G = \{(a-b, a+b, a-3b) \mid a, b \in \mathbb{R}\}$.

a. Show that $F$ and $G$ are subspaces of $\mathbb{R}^3$.
b. Calculate $F \cap G$ without resorting to any basis vector.
c. Find one basis for $F$ and one for $G$, calculate $F \cap G$ using the basis vectors previously found and check your result with the previous question.